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Abstract
This paper characterizes the role of capital and labor mobility in the shifting of capital taxes in Harberger-type 2×2 general-equilibrium model with partially-mobile factors. We first provide and examine an intuitive decomposition of the incidence of a selective capital tax into a "specificity effect" and a "mobility effect". Then we extend Harberger's incidence theorems to a partially-mobile factors context. The relevance of factor mobility differentials is examined. We also show that a surprising number of elements of Harberger's theorems carry over in the partially-mobile-factors model.
1. INTRODUCTION: TAXATION AND FACTOR MOBILITY

It has long been recognized that the flexibility and the nature of the responses of an economy to policy shocks depend crucially on the degree of mobility of its primary factors of production. In general equilibrium models used to study the comparative static effects of exogenous perturbations, the assumptions concerning which factors are intersectorally mobile and which are sector specific are often critical to the results.

The distributional impact of a tariff imposed by a small open economy offers a well-known example. Under perfect factor mobility, if the protected sector is relatively labor-intensive, labor in both the protected and the non-protected sectors should favor the tariff, whereas capital-owners should oppose it (Stolper and Samuelson, 1941). Now suppose that labor is immobile. In this setting, the tariff rises the real wage in the protected sector and lowers it in the rest of the economy, irrespective of factor intensities in production (Jones, 1971). Imperfect mobility implies a conflict of interests between labor in the two sectors.

Abandoning the assumption of perfect mobility can also dramatically affect the distributional incidence implications of tax policy. The case of a selective capital income tax (SCIT) is revealing. Just over thirty years ago, Harberger (1962) analyzed the incidence of a SCIT in a simple 2x2 general-equilibrium model with perfect factor mobility. The Harberger model has since become the standard tool of incidence analysis. In this model, the introduction of a SCIT in one sector drives a wedge between the returns to capital in the two industries. The taxed sector will tend to substitute labor for relatively expensive capital, thus making capital owners worse-off. At the same time, however, as production of the taxed industry falls, the wage-rental ratio rises (falls) if the taxed industry is capital-(labor) intensive. The overall effect of SCIT upon the wage-rental ratio is "a priori" ambiguous.

Harberger's analysis has been extended in many directions. The assumption of perfect factor mobility has been relaxed in a number of papers (for example, McLure, 1971; Ratti and Shome, 1977; Bhatia, 1989). These extensions posit the existence of a fixed (or a quasi-
fixed) factor. In this context, if capital is immobile a SCIT will be borne by the owners of the taxed capital, regardless factor substitution and factor intensities. On the other hand, when capital is perfectly mobile and labor is sector-specific, the rental to capital falls (rises) if the elasticity of technical substitution is greater (smaller) than the elasticity of demand for the good produced by the taxed industry. Surprisingly, this sharp contrast in conclusions has not been followed by any attempt to explain systematically the relationship between factor mobility and tax shifting.

The need to reconcile these very different results is important from a policy perspective. Imperfect mobility is a "fact of life". Further, if one adheres to the notion that mobility increases over time\(^2\), tax policy involves time horizons in which factors cannot be assumed to be perfectly mobile or completely immobile. Rather, a more sensible choice would be a time frame so short that factor supplies do not change, but long enough to allow market clearing, for any degree of factor mobility. The objective of this paper is to characterize the role of mobility in the shifting process in a Harberger-type model with partially-mobile factors (section 2). In order to do this, we first provide and examine an intuitive decomposition of the incidence of the SCIT into a "specificity effect" and a "mobility effect" (sections 3 and 4). The second part of the paper derives results about tax incidence. Our purpose is to assess the generality of Harberger's incidence theorems when factors are partially mobile (sections 5 and 6). The last section summarizes the main conclusions (section 7).

2. THE MODEL\(^3\)

Consider a closed economy that produces two final commodities, \(X_1\) and \(X_2\), using homogeneous capital, \(K\), and labor, \(L\). Technologies are CRS. Under competition, the behavior of producers is completely described by the equality of price and average cost:

\[ p = c_1(r, \tau, w) \]  

(1)
\[ 1 = c_2(r_2, w_2) \]  

where \( p \) is the relative price of \( X_1 \) in terms of \( X_2 \) (numéraire), \( c_i \) (\( i=1,2 \)) is the unit-cost function with the standard properties, \( r_i \) and \( w_i \) are the net rewards of capital and labor, and \( \tau_{ki} = 1 + t_{ki} \), where \( t_{ki} \) is a selective "ad valorem" tax on capital used in sector 1.

Capital and labor are imperfectly mobile. In our context of homogeneous factors, immobility can arise from two main sources\(^4\). First, factor movements may be inhibited by physical barriers, government restrictions or union pressures. Second, factors may be "preferentially specific", a situation in which a factor prefers being employed in a particular sector rather than being hired in the other industry (see Lancaster, 1958, Manning and Sgro, 1975, or Casas, 1984). Reluctancy to shift among occupations gives rise to a premium paid out to the factor employed in the least preferred sector. Suppose, for instance, that preferences are additive in goods and factors, i.e. \( U = U_0(X_1, X_2) - U_F(L_1, L_2, K_1, K_2) \). Further, assume that \( U_F \) is a CES function of the form:

\[ \phi[(1-\rho)^{-\eta_1}L_1^{1+1/\eta_1} + \rho^{1/\eta_1}L_2^{1+1/\eta_1}]^\alpha + \phi[(1-\xi)^{-\eta_2}K_1^{1+1/\eta_2} + \xi^{1/\eta_2}K_2^{1+1/\eta_2}]^\alpha \]

\[ 0 < \rho < 1 \, , \, 0 < \xi < 1 \]

where \( \eta_j \) (\( j : K, L \)), \( \rho \) and \( \xi \) are constants. Maximization of \( U \) subject to \( pX_1 + X_2 = w_1L_1 + w_2L_2 + r_1K_1 + r_2K_2 \) with respect to \( L_i \) and \( K_i \) yields the following first order conditions:

\[ \frac{L_1}{L_2} = \frac{\rho}{1-\rho} \left( \frac{w_1}{w_2} \right) ^\alpha, \quad 0 \leq \eta_L < \infty \]  

\[ \frac{K_1}{K_2} = \frac{\xi}{1-\xi} \left( \frac{r_1}{r_2} \right) ^\alpha, \quad 0 \leq \eta_K < \infty \]

where \( \eta_j (j : K, L) \) is the relative supply elasticity of the \( j \)-th factor to sector 1 with respect to
the net earnings ratio. The degree of factor mobility is given by \( \eta \): for an arbitrary change in the rental ratio, the size of the reallocation of factor \( j \) across industries increases with the magnitude of \( \eta \). Mussa (1982) and Grossman (1983) derive mobility conditions analogous to (3)-(4) in models of heterogeneous factors of production.

Full employment of factors is ensured by perfect flexibility of factor returns:

\[
L_1(r_1,r_{K1},w_1,X_1) + L_2(r_2,w_2,X_2) = L
\]

\[
K_1(r_1,r_{K1},w_1,X_1) + K_2(r_2,w_2,X_2) = K
\]

where the terms in the left-hand side are factor demands. Total factor supplies are fixed. Equations (5)-(6) - which state that the total supply of a factor equals the sum of the quantities of that factor demanded by both sectors - do not imply that there is a single market for factors. Although firms demand factors of production which are homogeneous, imperfect mobility may lead to an equilibrium featured by \( w_1 \neq w_2 \) and \( r_1 \neq r_2 \), i.e. the equilibrium rental rates may differ across sectors. The markets for \( L_1 \) and \( L_2 \) (\( K_1 \) and \( K_2 \)) are not completely separated except when labor (capital) is immobile.

Preferences over goods are represented by a single homothetic utility function. Aggregate demand for \( X_1 \) is:

\[
X_1 = X_1(p, I)
\]

where \( I = pX_1 + X_2 \). This definition of income is valid when the SCIT is "small" and revenues are returned back to consumers in a lump-sum fashion. In equilibrium, Walras' Law allows to ignore the demand function for \( X_2 \).

In order to analyze the incidence of taxation, the model can be solved for the change in factor prices using the convenient properties of the by now standard Jones' algebra (see Jones, 1965, and Atkinson and Stiglitz, 1980). This approach allows to express the rate of change of a variable as a function of the tax and a set of parameters that characterize the behavior of the economy: the elasticities of technical factor substitution, \( \sigma_i \); the elasticities
of factor mobility, \( \eta_i \); the compensated elasticity of demand for \( X_i \), \(-\varepsilon\); the shares in the total supply of factor \( j \) of the amount of this factor employed in sector \( i \), \( \lambda_{ij} \); and the shares of the \( j \)-th factor in the value of the \( i \)-th product, \( \theta_{ji} \). A circumflex over a variable indicates a proportional rate of change: \( \hat{z} = d\log z \).

Differentiating totally (1)-(7) and noting the above definitions of parameters, the incidence of a SCIT upon \( r_j \) can be expressed as:

\[
\hat{r}_j = \left| \Sigma \right|^{-1} \left[ \eta_L \lambda_{1i} \theta_{L2} \Omega - \eta_L \Sigma_B - \eta_R \lambda_{2i} \theta_{L1} \sigma_1 \varepsilon - \sigma_1 \sigma_2 \varepsilon \right] \hat{r}_{ki} \tag{8}
\]

with

\[
\left| \Sigma \right| = \eta_L \lambda_{1i} \Sigma_A + \eta_L \Sigma_B + \eta_R \Sigma_C + \sigma_1 \sigma_2 \varepsilon
\]

\[
\Omega = \Delta \theta_{ki} \varepsilon - \delta_1 \sigma_1
\]

\[
\Sigma_A = \varepsilon \Theta \Lambda + \delta_1 \sigma_1 + \delta_2 \sigma_2
\]

\[
\Sigma_B = \lambda_{1i} \theta_{L2} \sigma_1 \varepsilon + \lambda_{L2} \sigma_2 (\theta_{L1} \sigma_1 + \theta_{ki} \varepsilon)
\]

\[
\Sigma_C = \lambda_{2i} \theta_{L2} \sigma_1 \varepsilon + \lambda_{L2} \sigma_2 (\theta_{ki} \sigma_1 + \theta_{L1} \varepsilon)
\]

where the short-hand expressions used above are: \( \Lambda = \lambda_{L1} \lambda_{ki} - \lambda_{ki} \lambda_{L2} \), \( \Theta = \theta_{L1} \theta_{L2} = \theta_{ki} \), \( \delta_1 = \lambda_{ki} \lambda_{L1} \theta_{ki} + \lambda_{ki} \lambda_{L2} \theta_{L1} \), and \( \delta_2 = \lambda_{ki} \lambda_{L2} \). Factor \( j \) is perfectly mobile (completely immobile) as \( \eta \to \infty \) (\( \eta = 0 \)). Sector \( 1 \) is said to be relatively labor-intensive in the physical (the value) sense if \( \Lambda (\Theta) > 0 \). At an initial equilibrium, \( \Lambda \Theta > 0 \) ensures stability (see Neary, 1978).

In order to understand the results in the paper, it is crucial to note that we are not assuming that the existence of an intersectoral difference in, say, wage rates will induce a transfer of labor. This postulate superimposes a dynamic argument on a static analytical framework. Further, it implicitly identifies the long-run equilibrium with a situation of perfect factor mobility. Rather, we assume that whatever the initial wage rates in the two sectors, a change in the ratio \( w_1/w_2 \) is required to induce a movement of labor across industries. This movement will come to an end, with the size of the reallocation determined by the change in \( w_1/w_2 \). The new equilibrium will be stable even though it may involve
different wage rates in the two sectors.

3. THE SPECIFICITY AND THE MOBILITY EFFECTS OF A SCIT

Here we aim at an expression that separates the incidence of the tax upon impact from the general equilibrium effects that take place once factors are allowed to move in response to the tax. To do this, we can reexpress (8) as:

\[ \hat{r}_1 = \hat{r}_1^S + \hat{r}_1^M \]  

(9)

where the **specificity effect** (superscript \( S \)) is the tax induced response of \( r_1 \) if capital were immobile, and the **mobility effect** (superscript \( M \)) represents the portion of the tax that capital in sector 1 succeeds in passing on to other factors of production through mobility.

Trivially, with \( \eta_k=0 \), \( \hat{r}_1^S = -\hat{\tau}_{KL} \), i.e. \( r_1 \) falls by the amount of the tax. Subtracting from (8), the mobility effect can be written as:

\[ \hat{r}_1^M = \Sigma |^{-1} \left[ \eta_L \eta_K \Pi_A + \eta_K \Pi_B \right] \hat{\tau}_{KL} \]  

(10)

where \( \Pi_A = \theta_{KL} \theta_{LZ} \theta_+ + \theta_{KL} \delta_L \sigma_L + \delta_2 \sigma_j \) and \( \Pi_B = \sigma_2 \lambda_{KL} (\theta_{KL} \sigma_L + \theta_{LZ} \epsilon) \). A necessary condition for any shifting to take place is that capital be mobile. On the other hand, equation (10) indicates that the degree of labor mobility largely determines the proportion of the tax that is shifted. The link between mobility and shifting that the former decomposition establishes is best characterized in two main results:

**Proposition 1.** Sufficient conditions for capital in sector 1 to bear less than the full burden of the tax, i.e. \( r_1^M > 0 \), are: (i) \( \Delta \geq 0 \), for all \( \eta_k > 0 \), \( \eta_L > 0 \), and (ii) \( \eta_L = 0 \), for all \( \eta_k > 0 \).

Suppose that \( \epsilon = 0 \) initially. Then, as labor is substituted for capital at a fixed level of output of \( X_1 \), the net return to capital will start to rise. If we now allow \( \epsilon \neq 0 \), an additional
factor intensity differential effect will further encourage shifting when $\Lambda \neq 0$ by creating an economy-wide excess demand for the factor intensively used in the untaxed sector as industry 1 cuts down production. Result (ii), first proved by McLure (1971) for $\eta_{L} \to \infty$, emerges as a special case when the factor intensity differential does not play any role. Provided that $X_2$ is not produced by means of a Leontief-type technology, capital in sector 1 always gains from mobility when labor is sector-specific.

Is it possible that capital in sector 1 actually loses from mobility when trying to escape the tax? Under the assumption of perfect capital mobility, capital may end up bearing more than the full amount of the tax (Harberger, 1962). Equation (10) indicates that this cannot occur when any factor is immobile. This implies that although mobility is necessary for any shifting to be possible, it is not sufficient to improve the position of capital owners.

**Proposition 2.** Necessary conditions for capital in sector 1 to bear more than the full burden of the tax, i.e., $\pi_1^M < 0$, are that both factors be mobile and that sector 1 be relatively capital-intensive. These, together with either: (i) $\epsilon \to \infty$ and $\sigma_2 = 0$, or (ii) $\epsilon \to \infty$ and $\eta_{L} \to \infty$, or (iii) $\sigma_1 \to 0$ and $\sigma_2 \to 0$, suffice to ensure a negative mobility effect.

Propositions 1 and 2 suggest that analyses of tax incidence based upon the specific-factors model ($\eta_L = 0$) could yield misleading results when capital and/or labor are in fact partially mobile. According to this intuition, which is confirmed by the remaining results in the paper, if the degree of factor mobility is positive but "small", both the size and the sign of the mobility effect could be substantially different from those associated to the immobility benchmark.

4. TAX SHIFTING UNDER INCREASED MOBILITY

The available literature on tax incidence has neglected the analysis of the effects of changes in the degree of mobility upon tax shifting. This is not surprising, since the existing
models can be derived as special cases of (1)-(7) when \( \eta_j \) is either 0 or \( \infty \). From equation (10), we can obtain:

\[
\frac{\partial \tau^M}{\partial \eta_K} = |\Sigma|^{-2} \left[ \Pi_A (\eta_K \Sigma_B + \eta_L \sigma_1, \sigma_2) + \Pi_B (\eta_L \Sigma_B + \sigma_1, \sigma_2) \right] \hat{\tau}_{KI} \tag{11}
\]

\[
\frac{\partial \tau^M}{\partial \eta_L} = |\Sigma|^{-2} \left[ \Pi_A (\eta_K \Sigma_C + \eta_K \sigma_1, \sigma_2) - \Pi_B (\eta_K \Sigma_B + \eta_K \Sigma_B) \right] \hat{\tau}_{KI} \tag{12}
\]

Expressions (11) and (12) indicate how the incidence pattern of a SCIT is modified as a result of an exogenous change in mobility conditions. Suppose that these can be modified by government policy. Since \( \Pi_B \) is non-negative, the qualitative effect of changes in factor mobility upon tax shifting depends on the sign of \( \Pi_A \). Capital in sector 1 will favor policies intended to increase capital mobility when \( \Pi_A > 0 \) (industry 1 relatively labor-intensive or even "moderately" capital-intensive) and oppose those intended to increase the mobility of labor when \( \Pi_A < 0 \) (industry 1 "highly" capital-intensive). From Proposition 2 we know that the mobility effect may be harmful for capital in sector 1 only if this industry is relatively capital-intensive and there are no immobile factors. When a negative factor intensity differential effect dominates, owners of capital in sector 1 will favor policies to reduce its impact. Restrictions on labor mobility will always do the job. The case for an increase in capital mobility is just symmetric.

It is worth noting that when factor substitution elasticities in both industries are zero, tax shifting becomes independent of factor mobility considerations. Capital in sector 1 gains (loses) relative to the immobility benchmark as \( \Theta > (\leq) 0 \). This Harberger-type result -which is in the flavor of result 7 in Bhatia (1989)- generalizes to situations featured by any degree of factor mobility, provided that no factor is completely "tied" to its sector of employment (i.e. \( \eta_j > 0 \)).
5. TAX INCIDENCE AND AGGREGATE FACTOR SHARES (I): PROPORTIONAL BURDEN PROPOSITIONS

We now turn to the issue of the incidence of taxation upon the functional distribution of income. How the burden of the SCIT will be shared between capital and labor as a whole when both factors are partially mobile? In the Harberger world, this question is analyzed in a very simple fashion. Under the assumption of perfect factor mobility, if labor is chosen as the *numéraire*, we just need to focus on the values taken by $\rho$. Clearly, $\rho=0$ implies that capital and labor bear the tax in proportion to their initial shares in national income. On the other hand, capital as a whole will bear the entire burden of the tax if $\rho=-\lambda_{kl}\hat{\pi}_{kl}$: the return to capital falls by the amount of the revenue from the tax on capital in sector $1$.

Taking these two situations as benchmark cases, Harberger (1962, 227-230) presented and proved ten celebrated theorems, each highlighting how demand, substitution and factor intensities determine the final incidence of the corporate tax. Despite of its well-known shortcomings -essentially those of the standard $2\times2$ Heckscher-Ohlin-Samuelson model of international trade theory-, the Harberger model remains as the basic workhorse for applications in the theory of tax incidence. How partial mobility modifies Harberger results?

The notion of aggregate burden borne by a factor can be made explicit by stating the following definition:

**Definition 1.** Capital and labor bear the tax in proportion to their initial shares in national income if

$$\Psi = \frac{d}{dt_{kl}} \left( \frac{r_{1}K_{1}+r_{2}K_{2}}{w_{1}L_{1}+w_{2}K_{2}} \right) = 0$$

(13)

Using the standard procedure of normalizing all the initial prices to unity, equation (13) can expanded as:
In order to solve equation (14) we need expressions for the rates of change in $r_2$, $w_1$, $w_2$ and $p$. These are readily obtained as (see Appendix):

$$
\frac{1}{d_{KL}}(K/L)(\lambda_{KL} r_2 + \lambda_{KL} r_2 - \lambda_{KL} w_1 - \lambda_{KL} w_2) = 0. \tag{14}
$$

It is now a straightforward matter to derive and explicit expression for the distributional coefficient $\Psi$:

$$
\Psi = (K/L) | \Sigma |^{-1} \left[ \eta_L \eta_K \Omega - \eta_L \lambda_{KL} \Sigma_B - \eta_K \lambda_{KL} \sigma_1 \right] \tag{19}
$$

where $\Psi = \lambda_{KL} \lambda_{KL} \sigma_2 (\sigma_1 - \epsilon) + \lambda_{KL} (\lambda_{KL} + \lambda_{KL}) \sigma_1 \epsilon$. Capital as a whole will bear more (less) of the tax, in proportion to its initial share in national income, as $\Psi < (>) 0$.

We can now set out a generalized version of Harberger’s first theorem by using equation (19) to establish

**Proposition 3:** Necessary conditions for labor to bear a greater burden of the tax, in proportion to its initial share in national income, than capital, are: (i) $\Lambda > 0$, or (ii) $\epsilon > \sigma_1$, for all $\eta_i > 0$ ($i$: $L$, $K$).

When both factors are partially mobile, it is no longer sufficient that the taxed sector be relatively capital-intensive for capital to bear a higher burden. Indeed, if labor is sufficiently tied to its sector of employment, capital might improve its position *vis-a-vis* labor.
despite of the fact that the taxed industry be relatively capital-intensive. A high degree of labor specificity tends to minimize the impact of the factor intensity differential (beneficial to labor in both industries), and makes it possible that labor in sector 1 loses so much that labor's share in national income falls. This can occur only if the elasticity of demand for \( X_1 \) is large enough relative to the elasticity of factor substitution in the taxed sector. This reasoning establishes that condition \((i)\) is no longer necessary -as in Harberger's first theorem- if \((ii)\) holds, and viceversa. As a limiting case of the above result, equation (19) implies the following proposition, which we state without further comments:

**Proposition 4:** When labor is sector-specific, labor as a whole will bear a smaller burden of the tax, in proportion to its initial share in national income, than capital, if \( \sigma_1 \geq \epsilon \), for all \( \eta_K > 0 \). On the other hand, only if \( \epsilon > \sigma_1 \) can labor bear the greater burden.

Harberger's theorems 2 and 3 carry over unmodified in the partially-mobile-factors model. According to these, if the elasticity of factor substitution in sector 1 is as great of greater than the elasticity of demand for the taxed product -the elasticity of substitution in demand between \( X_1 \) and \( X_2 \) in theorem 3-, capital must bear a greater burden of the tax. Using our model, we just have to show that \( \Omega < 0 \) and \( \psi > 0 \) when \( \sigma_1 \geq \epsilon^2 \). Let \( \sigma_1 = \epsilon = \kappa \). Then it can be shown that \( \Omega \) collapses to \( -\lambda_{11} \lambda_{22} < 0 \). Trivially, \( \sigma_1 \geq \epsilon \) implies \( \psi > 0 \). Interestingly enough, this result holds for any positive degree of factor mobility, thus providing a nice generalization of the perfect mobility result.

Theorems 4 and 5, however, do not carry over in the general case. The latter is only applicable to the perfect mobility case, since it refers to "the" post-tax rate of return to capital. Theorem 4 establishes that the higher is the elasticity of factor substitution in the untaxed sector, the greater will be the tendency for labor and capital to bear the tax in proportion to their respective initial income shares. As \( \sigma_2 \) goes to \( \infty \) \( \lim \psi = 0 \) only under special circumstances. One such case is Harberger's perfect mobility scenario. When factors are imperfectly mobile, the proportional burden result will hold if both \( \sigma_i \) and \( \epsilon \) are close to
zero, or when \( \sigma_1 = 0 \) and both the intensity differential (\( \Lambda \)) and the factor mobility differential (\( \eta_K - \eta_L \)) tend to zero.

The last of Harberger's propositions stated in proportional-burden terms refers to the fixed coefficients case (theorem 8): when \( \sigma_1 = \sigma_2 = 0 \), labor will bear more of the tax than in proportion to its initial income share when the taxed industry is relatively labor-intensive. From equation (19) it can be seen that this proposition carries over unmodified:

\[
\Psi = (K/L)(\theta_{KL}/\Theta) > (<) 0 \quad \text{as} \quad \Theta > (<) 0, \quad \text{for all } \eta_L > 0, \eta_K > 0 \quad (20)
\]

An interesting advantage of explicitly introducing imperfect factor mobility in the Harberger framework is that it allows to analyze the role of factor mobility differentials upon the distributional incidence of a SCIT. This is perhaps best illustrated by the following

**Proposition 5:** The elasticity of factor substitution in the taxed sector is zero, the mobility differential (\( \eta_K - \eta_L \)) and the factor intensity differential (\( \Lambda \)) jointly determine which factor will bear a greater burden of the tax, in proportion to its initial share in national income. In particular, for all \( \eta_L > 0, \eta_K > 0 \): (i) if \( \eta_K = \eta_L \), \( \Psi \geq (<) 0 \) as \( \Lambda \geq (<) 0 \), (ii) if \( \Lambda = 0 \), \( \Psi \geq (<) 0 \) as \( \eta_K \geq (<) \eta_L \), and (iii) if \( \eta_K > (<) \eta_L \) and \( \Lambda \geq (<) 0 \), \( \Psi \geq (<) 0 \).

There is nothing comparable to proposition 5 in the literature on tax incidence. In order to clarify the intuition behind these results, notice that if \( \sigma_1 = 0 \) we have:

\[
\Psi = \theta_{KL}(K/L) \cdot [\eta_L \eta_K \Lambda + \sigma_2 (\eta_K \lambda_L \lambda_{KL} - \eta_L \lambda_L \lambda_{KL})]^{-1} \quad (21)
\]

When both factors are equally mobile, all that matters is the factor intensity differential effect, which influences the returns to homogeneous factors in the same direction, regardless the particular industry that employs them. The Harberger model is a particular case in which result (i) applies. On the other hand, result (ii) states that when both industries use identical factor proportions, the distributional outcome hinges upon the sign of the factor mobility differential: capital owners gain relative to labor if capital is more mobile across industries.
than labor, and vice versa. Once we allow for differences in factor intensities, we can be sure of the distributional outcome only if both the factor intensity differential and the factor mobility differential influence the wage-rental ratio in the same direction. Thus, when sector 1 is labor-intensive and capital is more mobile than labor, capital bears the tax less than in proportion to its income share, and vice versa. Hence result (iii). Incidentally, notice that since factor substitution does not operate in the taxed industry, proposition 5 provides a generalization of Mieszkowski's (1967) "output effect" of a SCIT.

An interesting extension of the above, which also holds a fortiori from proposition 4, is now stated without proof in the form of

**Proposition 6:** If labor is sector specific and the taxed sector employs a fixed-proportions technology, the tax will burden labor more than in proportion to its initial income share, for all $\sigma_k > 0$.

### 6. TAX INCIDENCE AND AGGREGATE FACTOR SHARES (II):

**FULL BURDEN PROPOSITIONS**

In section 3 we derived conditions under which the owners of capital in sector 1 bear the full burden of the tax with the purpose of providing an intuitive interpretation of the shifting mechanism. Consider now the role that partial mobility plays in the determination of the circumstances under which capital as a whole bears the full burden. For analytical purposes, it will be useful to state the following

**Definition 2:** Capital will bear the full burden of the tax if the income accruing to capital gross of the tax in ratio to total labor income remains unchanged, i.e.

$$\Phi = \frac{d}{dr_{KI}} \left( \frac{\tau_{KI}r_1K_1 + r_2K_2}{w_1L_1 + w_2L_2} \right) = 0 \quad (22)$$

If we notice that $\Phi = \Psi + (K/L)\lambda_{KI}$, some manipulation of expression (19) yields:
\[
\Phi = (K/L) \mid \Sigma \mid ^{-1} \left[ \eta_L \eta_K (\phi_2 \sigma_1 + \phi_3 \sigma_2) - \eta_K (\phi_2 \sigma_1 \epsilon + \phi_3 \sigma_2 - \phi_4 \sigma_2 \epsilon) \right]
\]  
(23)

where \( \phi_1 = \lambda (\lambda_{K1} \theta_{K2} + \lambda_{K2} \theta_{K1}) \), \( \phi_2 = \lambda (\lambda_{L1} \theta_{L2} + \lambda_{L2} \theta_{L1}) \), \( \phi_3 = \lambda \lambda_{K1} \theta_{K1} \) and \( \phi_4 = \lambda \lambda_{K2} \theta_{K2} \). Notice that \( \phi_2 \) and \( \phi_4 \) are positive, whereas \( \phi_1 \) and \( \phi_3 \) have the sign of \( \Lambda \).

Unlike in the proportional-burden analysis above, all of Harberger’s full-burden theorems carry over in the partially-mobile factors model. Theorem 6 establishes that when the capital-labor ratio is the same in both industries, capital will bear the full burden of the tax if \( \sigma_1 = \sigma_2 \) and more (less) than the full burden when \( \sigma_1 > (<) \sigma_2 \). With partial factor mobility, when \( \Lambda = 0 \), the coefficient of \( \eta_L \eta_K \) collapses to \( \lambda_{K1} \lambda_{K2} (\sigma_2 - \sigma_1) \). In addition, when factor proportions are equal the term in \( \eta_K \) reduces to \( \lambda_{K1} \lambda_{K2} (\sigma_2 - \sigma_1) \), \( j : K, L \). Hence the generalized proposition:

\[
\Phi \geq (<) 0 \text{ as } \sigma_2 \geq (<) \sigma_1, \text{ for all } \eta_L > 0, \eta_K > 0 .
\]  
(24)

Theorem 7 in Harberger states that if the elasticity of demand for the good produced by the taxed industry is zero and \( \sigma_1 = \sigma_2 \), capital will bear the full burden when the initial factor proportions are the same in both sectors, and more (less) that 100 per cent of the burden of the tax as \( \Lambda > (<) 0 \). Let \( \sigma_1 = \sigma_2 = \xi \). Then equation (23) reduces to:

\[
\Phi = -(K/L) \mid \Sigma \mid ^{-1} \left[ \eta_L \eta_K \lambda_{K2} \theta_{K1} \xi + \eta_K \lambda_{K2} \theta_{K1} \xi^2 \right]
\]  
(25)

which proves that if \( \sigma_1 = \sigma_2 \) and \( \epsilon = 0 \),

\[
\Phi \geq (<) 0 \text{ as } \Lambda \leq (>0, \text{ for all } \eta_L > 0, \eta_K > 0 .
\]  
(26)

Much econometric evidence has found that the elasticities of substitution in production are close to 1. Further, empirical research often assumes that preferences over goods are homothetic, which implies that the elasticity of substitution in consumption between \( X_1 \) and \( X_2 \), \( \epsilon^* (\epsilon^* = -d \log (X/Y)/d \log p \mid _{u}) \), equals 1. Thus, it is not surprising that Harberger’s theorem 9 is regarded as one of the most compelling results: capital will bear the full burden of the
tax when \( \sigma_1 = \sigma_2 = \epsilon^* = 1 \). Theorem 10 simply generalizes the former for any positive values of \( \sigma_1, \sigma_2 \) and \( \epsilon^* \), as long as \( \sigma_1 = \sigma_2 = \epsilon^* \). Interestingly, we can prove that the former conditions imply that \( \Phi = 0 \) for all \( \eta_k > 0, \eta_k > 0 \).

First note that \( \epsilon = (X_2/Z)\epsilon^* \), where \( X_2/Z \) is the initial share of good \( X_2 \) in national income. Consider now the sign of the coefficient in \( \eta_k \eta \eta_k \) in equation (23):

\[
\text{sgn}(\phi_1 \epsilon - \lambda_{X_2} \delta_1 \sigma_1 + \lambda_{X_2} \delta_2 \sigma_2) = \text{sgn}(\phi_1 X_2 - \lambda_{X_2} \delta_1 Z + \lambda_{X_2} \delta_2 Z),
\]

where normalization of all initial prices to unity allow to write \( Z = X_1 + X_2 \), and \( \gamma = \sigma_1 = \sigma_2 = \epsilon^* \). Notice now that

\[
\phi_1 X_2 = K \frac{K_2}{K} \left( 1 + \frac{X_1}{X_2} \right) \Lambda = \frac{K_1 K_2}{KK_1} \Lambda Z
\]

and

\[
(-\lambda_{X_2} \delta_1 + \lambda_{X_1} \delta_2) Z = \left[ \frac{K_1 K_2 (L_2 K_1 - L_1 K_2)}{K_2 L X_1} \right] Z = -\frac{K_1 K_2}{KK_1} \Lambda Z
\]

Expressions (28) and (29) establish that the sign of (27) is zero. Thus, if theorem 10 is to carry over in the partially-mobile-factors case, the sign of the coefficient \( \eta_k \) must be zero. In effect,

\[
\text{sgn}(\phi_2 \sigma_1 \epsilon + \phi_3 \sigma_1 \sigma_2 - \phi_4 \sigma_2 \epsilon) = \text{sgn}^2(\phi_2 X_2 + \phi_3 Z - \phi_4 X_2) = 0,
\]

which completes the proof of the generalized theorem.

It may be useful to complete the analysis of full-burden results by stressing the interaction between technical substitution, factor intensities and mobility. Some inspection of equation (23) leads to the following

**Proposition 7:** For all \( \eta_k > 0 \): (i) when \( \sigma_1(\sigma_2) = 0, \) capital will bear less (more) than the full burden of the tax if the taxed industry is relatively labor-(capital-)intensive, for all \( \eta_k > 0, \) and (ii) when \( \eta_k = 0, \) if \( \sigma_1(\sigma_2) = 0 \) capital will always bear less (more) than the full
burden of the tax.

The results in proposition 7 reaffirm the basic intuition of previous findings and complement propositions 2, 5 and 6: capital mobility is a necessary condition for any shifting to be possible, but it does not suffice to ensure that the mobility effect will improve the position of capital owners. In the absence of factor substitution possibilities in the untaxed sector or when the taxed industry is capital-intensive, the reduction in net capital earnings could well exceed the amount of the tax.

7. CONCLUSIONS

The thirty years following the appearance of Harberger's model have been the most fruitful ones in both the theoretical and the empirical development of tax incidence analysis. Despite its obvious limitations, this model remains as the simplest, yet rigorous analytical tool for examining incidence issues in general equilibrium. In the Harberger's model, the assumption of perfect factor mobility plays a crucial role in the shifting process: factor taxes are shifted as factors move across industries in response to earnings differentials. Subsequent extensions have introduced the assumption of factor specificity, which has caused dramatic changes in results about incidence. Surprisingly, this sharp contrast in conclusions has not been followed by attempts to explain systematically the relationship between factor mobility and tax shifting.

The primary methodological objective of this paper has been to clarify and characterize the role of factor mobility in the shifting process in a Harberger-type model with partially-mobile factors. Three features of the analysis are noteworthy. First, the relationship between the shifting possibilities of a selective factor tax and the mobility conditions of the economy has been explicitly examined with the help of the intuitive notion of the "mobility effect". Use of this concept has enabled us to establish propositions that explain the degree of success of the taxed factor in passing on the tax burden to other factors of production in terms of the
relative factor intensities and the degrees of mobility of capital and labor. Second, the analysis has shown that when capital and labor are partially mobile, the incidence of a SCIT hinges upon the factor mobility differential. The sign of Mieszkowski’s "output effect" depends only upon the difference in factor intensities -as in the perfect mobility scenario- when capital and labor are equally mobile. However, if the two industries have the same capital-labor ratio, capital owners gain (relative to the "proportional burden" benchmark) if capital is more mobile across industries than labor, and vice versa.

Third, sections 5 and 6 have demonstrated that when incidence analysis focuses upon aggregate factor shares, the assumption of perfect factor mobility is unnecessarily restrictive. Although Harberger’s fundamental proposition (theorem 1) has to be modified when mobility is imperfect -the fact that the taxed industry be relatively labor-intensive is no longer necessary for labor to bear the tax more than in proportion to labor’s share in national income-, most of Harberger theorems remain valid for any positive degrees of mobility of capital and labor. Furthermore, the previous results show how stringent is incidence analysis based upon the specific-factors approach. When capital is immobile, capital owners bear the full burden of the tax. However, if the degree of capital mobility is positive but arbitrarily "small", capital’s share in the burden could be even smaller than its share in national income.

These findings are relevant not just in terms of their generality but also because they help to illuminate the nature of the relationship between tax shifting and factor mobility. In most instances, despite of the existence of rental differentials, the incidence of the tax upon aggregate factor shares when factors are partially mobile is qualitatively the same as in the perfect mobility case. This implies that the basic role of mobility is to determine the distribution of the burden between homogeneous factors employed in different sectors. To sum up, as regards to the robustness of its main implications in a partially-mobile-factors context, the Harberger model can still be praised as an attractive and convenient simplification for tax incidence analysis.
Notes

1 The analysis extends trivially to selective labor income taxes (e.g. payroll taxes), selective employment incentives (e.g. exemptions in social security contributions) and selective capital subsidies (e.g. industrial promotion schemes).

2 Section 2 briefly deals with this issue.

3 The basic structure of the model follows the standard general equilibrium model of international trade theory as formulated by Jones (1965). For applications to tax incidence in general equilibrium, see Atkinson and Stiglitz (1980).

4 In heterogeneous factors models, immobility can also stem from "aptitudinal specificity", a situation in which a factor is more efficient in producing a good than another. Even if, say, capital is instantaneously transferable between sectors, capital would be imperfectly mobile if different capital units contribute differently to the stock of "efficiency capital" (see Mussa, 1982, or Grossman, 1983).

5 Algebraic treatment of general equilibrium models is a straightforward but tedious exercise even in the simple $2 \times 2$ model with perfect mobility. In our case, algebra is greatly simplified if we note that (1) perfect competition and CRS implies:

$$\dot{X}_i = \theta_L \dot{L}_i + \theta_K \dot{K}_i .$$

and that (2) the definition of the elasticity of technical substitution allows to write:

$$\ddot{K}_i - \dot{K}_i = \sigma \phi_i \dot{X}_1$$

Once eliminated the rates of change in $X_i$, $K_i$ and $L_i$, the model can be reduced to a three-equation system in the percentage changes in $w_1/r_1$, $w_2/r_2$ and $p$. The solution of the system may be combined with the zero-profit conditions to yield formal expressions for the tax-induced changes in factor prices (for details, see Appendix).

6 This popular Marshallian assumption is controversial. Differences in preferences, in labor skills and in capital efficiency can be a feature of long-run equilibrium (for example, see Herber and Kemp, 1971, and Grossman and Shapiro, 1982). Further, physical or legal restrictions that inhibit mobility can be long-lasting.

7 The elasticity of substitution in demand between $X_1$ and $X_2$ is always greater (in absolute value) than $\epsilon$. For a simple proof, differentiate the consumer budget constraint and use the zero-degree homogeneity of compensated demands. Thus, Harberger's theorem 3 is a simple corollary of theorem 2.

8 This equivalence follows from the proof suggested in footnote 7.

9 The second term in brackets in equation (30) can expanded as:
\[
\frac{K}{K} \left\{ \frac{L,K}{LX_2} + \frac{L,K}{KX_2} \right\} + \frac{K,K}{KL_1} AZ - \frac{K}{KL_1} \left\{ \frac{L,K}{LX_1} + \frac{L,K}{KX_1} \right\} X_2
\]

Noting that \(X_j = Z - X_j\), this expression reduces to:

\[
\frac{K,K}{K^2LX_1} \left( L_{LZ}X_LX_L - X_LX_L + L_{L}Z + KL_2KX_2 \right).
\]

From the facts that \(Z = K + L\) and \(X_j = L_2 + K_2\), and using the definition of \(L\), the term in brackets becomes \(Z(L_Z - X_L) + K_L - L_2 = Z(L_Z - X_L) + K_L = Z(L_Z - L_2) = 0\).

**References**


Jones, R.W. (1971), "A three-factor model in theory, trade, and history", in J.N. Bhagwati et al. (eds.): *Trade, balance..."
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**Appendix**

Derivation of equations (8), (15), (16), (17) and (18)

We just need to reduce equations (1)-(7) to a three equation system in \((\phi_1 - \delta_1), (\phi_2 - \delta_2)\) and \(\beta\). First, notice that the zero-profit conditions (1)-(2) imply:

\[
\dot{p}_1 = \beta + \theta_{x1}(\psi_1 - \delta_1) - \theta_{x1}^* x_1
\]

\[
\dot{p}_2 = \theta_x(\psi_2 - \delta_2)
\]

\[
\dot{p}_1 = \beta - \theta_{x1}(\psi_1 - \delta_1) - \theta_{x1}^* x_1
\]

\[
\dot{p}_2 = -\theta_x(\psi_2 - \delta_2).
\]

The full-employment equations (5)-(6) can be rewritten as:

20
\( \lambda_{L}f_{1} - \lambda_{L}f_{2} = 0 \quad (A.5) \)

\( \lambda_{X}K_{1} = \lambda_{X}K_{2} = 0 \quad (A.6) \)

Subtract (A.5) from (A.6) to get:

\( \lambda_{X}K_{1} + \lambda_{X}(K_{2} - L_{2}) + \lambda_{X}f_{2} = 0 \quad (A.7) \)

From (A.5), \( L_{2} = -\langle \lambda_{X}/\lambda_{X} \rangle L_{1} \). Substituting in equation (A.7) for the mobility conditions (3)-(4) and using the definition of \( \sigma_{2} \) and equations (A.1)-(A.4), expression (A.8) becomes the first equation of the system:

\[
-\langle \lambda_{X}\beta_{X}^{L}\eta_{X} + \lambda_{X}\beta_{X}\lambda_{X}\eta_{X} \rangle (\hat{w}_{1} - \hat{w}_{2}) + (\sigma_{2} + \lambda_{X}\beta_{X}\lambda_{X}\eta_{X}) (\hat{w}_{2} - f_{2}) + (\lambda_{X}\eta_{X} - \lambda_{X}\eta_{X}) \theta = \theta_{X}(\lambda_{X}\eta_{X} - \lambda_{X}\eta_{X}) \hat{y}_{12} \quad (A.8)
\]

The second equation is obtained as follows. The mobility condition (3) can be restated as \( L_{2} = \hat{L}_{1} - \eta_{X}(\hat{w}_{1} - \hat{w}_{2}) \). On the other hand, perfect competition and CRS imply \( \hat{X}_{1} = \theta_{X}f_{1} + \theta_{X}K_{1} = \hat{L}_{1} + \theta_{X}(\hat{K}_{2} - L_{2}) \). Thus, if we use this result and the definition of \( \sigma_{1} \), (A.5) becomes:

\[
\hat{X}_{1} - \theta_{X} \sigma_{1} (\hat{w}_{1} - \hat{w}_{2}) - \lambda_{X} \eta_{X} (\hat{w}_{1} - \hat{w}_{2}) = 0 \quad (A.9)
\]

Using the demand condition (7) and substituting for \( (\hat{w}_{1} - \hat{w}_{2}) \) from (A.1)-(A.2), we have:

\[
-\langle \theta_{X}(\sigma_{1} + \lambda_{X}\lambda_{X}^{2}) \rangle (\hat{w}_{1} - \hat{w}_{2}) + \theta_{X}\lambda_{X}\eta_{X}(\hat{w}_{2} - f_{2}) + (\epsilon + \lambda_{X}\lambda_{X}^{2}) \theta = -\theta_{X}(\sigma_{1} + \lambda_{X}\lambda_{X}^{2}) \hat{y}_{12} \quad (A.10)
\]

Analogous substitutions allow to rewrite equation (A.6) as:

\[
[\theta_{X}(\sigma_{1} + \lambda_{X}\lambda_{X}^{2})] (\hat{w}_{1} - \hat{w}_{2}) - \theta_{X}\lambda_{X}\lambda_{X}^{2}(\hat{w}_{2} - f_{2}) - (\epsilon + \lambda_{X}\lambda_{X}^{2}) \theta = (\theta_{X}^{L} - \theta_{X}^{L}) \lambda_{X}\lambda_{X}^{2} \hat{y}_{12} \quad (A.11)
\]

In order to derive the incidence expression (8), we must solve:

\[
\Sigma \begin{bmatrix} \hat{w}_{1} - \hat{w}_{1} \\ \hat{w}_{2} - \hat{w}_{2} \\ \theta \end{bmatrix} = \begin{bmatrix} \theta_{X} \langle \lambda_{X}\eta_{X} - \lambda_{X}\eta_{X} \rangle \\ -\theta_{X}(\sigma_{1} + \lambda_{X}\lambda_{X}^{2}) \\ \theta_{L} \theta_{L} - \theta_{X} \lambda_{X}\lambda_{X}^{2} \hat{y}_{12} \end{bmatrix} \quad (A.12)
\]

where \( \Sigma \) is the coefficient matrix of the system (A.8), (A.10)-(A.11). Solutions for \( (\hat{w}_{1} - \hat{w}_{1}) \), \( (\hat{w}_{2} - \hat{w}_{2}) \) and \( \theta \) can be combined with (A.1)-(A.4) to yield equations (8) and (15)-(18) in the text.