The underlying structure in Atanassov’s IFS

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Abstract

In this paper we shall stress the key relevance of addressing a specific structure for the valuation space in Atanassov’s IFS, which may help to establish certain differences with other related models. In particular, we point out that each one of the valuation states proposed by Atanassov plays a different role, so they should be distinguished from their particular properties with respect to the other valuation states. In the same way, we realize that those three valuation states are not interchangeable and that each state should in principle allow a clear definition, as in any standard classification problem, always subject to potential direct estimation. Considering for example a transition (directed) graph in order to connect states provides in addition a new framework for useful more general models, taking simultaneously into account several logics when needed and avoiding the still binary heritage in Atanassov’s model.

Key words: Atanassov’s Intuitionistic Fuzzy Sets, Interval Valued Fuzzy Sets, Type-2 Fuzzy Sets, L-Fuzzy sets.

1 Introduction.

Since the introduction of Atanassov’s intuitionistic fuzzy sets [4] (see also [5, 6, 7, 10, 28]), quite a number of papers have been published developing his model both from a theoretical and a practical point of view (see [11] for an overview of this an other formal generalizations of fuzzy sets [32] and some of their applications). Nevertheless, the serious scientific criticism received about the term intuitionistic [29] and even about the originality of the model itself
might many researchers not pay attention on the key issues Atanassov 
was addressing with his proposal (see also [8, 9]). Based upon a paper of
the authors recently published [20] in this paper we pretend to stress that Atanassov 
model, if viewed as a classification system in the sense of [1, 2], is offering
a contextual framework for fuzzy sets which is extremely close to linguistic
modelling needs. As it will be shown below, the most basic unit for knowledge
requires three valuation states, as Atanassov is postulating.

Let us briefly remind here that, given a fuzzy property, Atanassov model [4]
is defined as a particular type-2 fuzzy set (see [14, 16, 17])

\[ \mu : X \rightarrow [0, 1]^3 \]

where \( X \) represents the family of objects under consideration in such a way that
the degrees of membership, non-membership and indeterminacy are associated
to each object \( x \in X \), summing 1 these three values, for all \( x \in X \). In this way
the Zadeh’s fuzzy sets are extended by imposing a simultaneous view into these
three fuzzy sets.

In fact, nothing should be said about a fuzzy set without taking into account
the whole family of fuzzy sets directly related to our fuzzy set of interest. For
example, nothing should be said about tallness without taking into account shortness. But Atanassov wrote in his model that the standard alternative
to a fuzzy set was its negation, keeping in this way an unconsciously binary
framework, which leaded as a consequence to a third state (indeterminacy) difficult to explain.

Moreover, Atanassov did not realize that his model was not simply made
out of three states, but from three states that require certain structure behind.
This is being clear from the names he uses for his three classes. His indetermi-
nacy is clearly different in nature to membership and non-membership, but this
difference in nature is not being reflected anywhere in his model, apart from
the different name they receive. Things are not different because they have a
different name, but the other way round: things get different names because
they are different.

In this paper we offer a model being consistent to Atanassov’s intuition. The
introducing of a particular structure (than can become more complex either in
the number and relation between valuation states or in the logic allowing their
manipulation) will avoid the isomorphism with interval valued fuzzy sets, since
each model will require different structures (see [20]).

2 About Atanassov’s negation

Our main argument is to view Atanassov’s model as a classification model. As
pointed out in [21], this decision making aid framework is in general closer to
a fuzzy approach, rather than pursuing the crisp decision some models have as
output (see also [22]). But in a classification framework it is a standard require-
ment in practice that classes under consideration allow a direct intuition. In
this sense, conceiving a fuzzy set together its negation as the basic classification
system should not be considered as appropriate. In fact, it looks quite unnatural
to ask somebody to what extent John is not-tall.

The most terrible heritage of binary logic is perhaps the constant temptation
to use simple models in order to escape complexity instead of approaching such
a complexity. For example, we can view the question is John tall? as a classification problem between two possible answers, yes and no. This is because we realize that any question requires the possibility of more than one possible answer, at least something and its opposite. But the opposite of a concept within a classification framework is not its negation, but its dual or antonym concept (see, e.g., [23, 24]). And it has experimentally shown that people confuse duality and negation only within a very particular contexts where concepts are truly crisp [25] (note that from a classification point of view, “no” is the dual concept of “yes”, not its negation).

If we accept that the basic piece in our approach should be a classification unit, this unit should be at least formed by a concept and its antonym (see also [30]). But of course we should open the possibility to more complex non-binary classification units. For example, and coming back to the question is John tall?, most people will ask for additional intermediate possible answers apart from the extremes yes and no.

Many standard classification units are based upon a finite family of linearly ordered valuation states.

3 About Atanassov’s indeterminacy

As already pointed out, choosing a concept together with its negation as the basic classification unit produces as a consequence a difficulty in explaining the role of determinacy in Atanassov’s model. From a logical point of view it is difficult to justify that something is left outside a concept and its negation. But once negation is changed into antonym, of course there is room (and necessity) for a third state in order to capture what is not captured between a concept and its dual or antonym. For example, ignorance should appear in a quite general context, since ignorance use to be the previous status to knowledge process. The introduction of this extra ignorance valuation state, outside the standard classification valuation possibilities, seems to us a relevant originality in Atanassov’s proposal. In fact, it is quite surprising, even in a crisp framework, that when there is no information some mathematical models still force to choose between truth and falsehood, or define a probability distribution between both states, when the only clear thing is that we know nothing (i.e., the degree of membership to ignorance is 1).

Atanassov’s two-states plus ignorance classification unit is obviously related to Ruspini’s partition [26], but it must be clearly stated that we can not assume that our classification should be a fuzzy partition in the sense of [26], which can perhaps be associated to ideal classification units in some particular context (its existence is neither assured in general, see [2]).

If a classification unit is based upon a finite family of linearly ordered valuation states, adding an ignorance state, the new classification unit will be consistent to Atanassov’s main arguments.

4 About the underlying structure

As already pointed out, states can not be properly understood without a structure justifying names and expected properties of each name. Notice that, for
example, classical 5-states valuation unit (given by none, Poor, Average, Good and Very good) is not just a bunch of states, but they define a very particular structure: those 5 valuation states define a linear order.

When we consider only two valuation states, the question of the existence of a particular structure explaining roles of each state seems no relevant at all. But as pointed out in [20], the underlying structure of Interval Values Fuzzy Sets [27, 31] and the underlying structure of Atanassov’s IFS are not the same, as they both are different from Lukasiewicz’s model (true, possible, false) which has also three valuation states. As already pointed out in [20], these three models have three valuation states, but depending on the model each middle state has a different relation with the remaining two states.

Valuation spaces require the definition of their particular structure, most probably a directed graph, which will hopefully show the transition learning procedure.

5 About the manipulation of states and objects

Once a directed graph had being fixed, we may wonder about combinations of states, so a logic must be defined between states, but of course unconnected classes can not be combined. The same applies to objects, which may require a different logic (see [18, 19]).

6 The model

Let us then assume

1.- A well-defined (crisp) finite set of objects, $\mathcal{X}$.

2.- A finite valuation space $\mathcal{C}$ of valuation states, fuzzy in nature but well defined from a representation point of view (perhaps a finite family of linguistic labels representing a concept).

Following [20], we stress the relevance of certain type-2 fuzzy sets [14, 16, 17] given by a mapping

$$\mu : \mathcal{X} \to [0, 1]^\mathcal{C}$$

once they are provided with a particular structure (including the ignorance state) in such a way that

A1 $\mathcal{X}$ is a well-defined non-empty, but finite, set of objects such that

A1.1 There exists a crisp directed graph $(\mathcal{X}, \mathcal{P})$ showing physical immediacy between two distinct objects $x, y \in \mathcal{X}$ ($p_{xy} = 1$ in case there is immediacy between $x, y \in \mathcal{X}$ and $p_{xy} = 0$ otherwise).

A1.2 There exists a logic on $\mathcal{X}$ allowing a consistent evaluation of questions about objects.

A2 $\mathcal{C}$ is a finite valuation space, with at least three elements, such that

A2.1 There is a crisp directed graph $(\mathcal{C}, \mathcal{R})$, in such a way that $r_{ij} = 1$ in case there is immediacy between $i, j \in \mathcal{C}$ and $r_{ij} = 0$ otherwise.
A2.2 There exists a logic in \( C \), allowing a consistent evaluation of questions about valuation states.

A3 There exists an ignorance state \( I \in C \) such that

\[ A3.1 \] For every \( i \in C \) there exist a path connecting \( i \) from \( I \).

\[ A3.2 \] \( \mu_x(I) = 1, \forall x \in X \), and \( \mu_x(i) = 0, \forall x \in X, \forall i \neq I \), when there is no available information (complete ignorance).

Additional conditions can be introduced. For example, we may impose the following condition that may be assumed by default, when there is no information about the structure of objects:

\[ B1 \] \( X \) is a well-defined non-empty, but finite, set of objects such that its associated crisp directed graph \((X, P)\) verifies that \( p_{xy} = 0, \forall x \neq y \), so reference to the structure of objects can be avoided (no immediacy between two objects).

Analogously, we can also impose the following condition that may be assumed by default, when there is no information about the structure of the valuation space:

\[ B2 \] \( C \) is a finite valuation space, with at least three elements, such that its associated crisp directed graph \((C, R)\) verifies that for all \( r_{ij} = 1, \forall i \neq j \), so reference to the structure of states can be partially avoided (immediacy between every two valuation states).

So we can conclude with the following definition as a model that in our opinion fits main Atanassov’s requirements, but allowing an explanation of the dispute on Atanassov’s original model, once the underlying classification structure is fixed (see [20] for details).

**Definition 6.1** Let \( \mu_A : X \rightarrow [0, 1]^C \) be a function where \( X \) represents a finite family of objects and \( C \) represents a finite family of classes or states. We shall say that \( A \) is a Meta Fuzzy Set if a directed graph \((C, R)\) verifying all the above conditions \( B1, A2 \) and \( A3 \) has been defined, together with a logic \((\lor, \land, \neg)\) on \((C, R)\).

This logic should be defined taking into account the existence of a structure (see, e.g., [3]).

7 Final comments

Now, the next step in our research should be to check how key properties can be translated from Atanassov’s original model into this model, taking into account alternative valuation spaces, alternative structures and alternative logics (putting together the results obtained in [3] and [12]).

Nevertheless, even if we acknowledge that Atanassov’s model is not properly intuitionistic, and that it is mathematically equivalent to another pre-existent model, researchers should look ahead and realize from Atanassov’s arguments
that Zadeh’s original proposal is still heavily dependent on a binary view of reality. Representing knowledge requires more complex models, starting with a proper representation of ignorance. Moreover, Atanassov’s arguments point out the necessity of a global view to any question. In this way, some Atanassov’s modelling deficiencies have been bypassed by including the existence of a structure in potentially very different valuation states where ignorance plays a significative role.

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References


