Comment

Comment on "Creating artificial magnetic fields for cold atoms by photon-assisted tunneling" by Kolovsky A.R.

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Cold atom systems held in optical lattice potentials are proving to be a remarkably versatile means of investigating coherent quantum phenomena. A particularly interesting recent development has been the use of these systems to study synthetic gauge fields. In Ref. [1] a scheme was proposed to simulate a uniform magnetic field by applying a periodic driving to the optical lattice, and employing the effect known as "photon assisted tunneling" to generate the required Peierls phases. Here, however, we show that the analysis of this system was incorrect; contrary to the central claim of the paper, a driving of the kind described cannot produce a uniform synthetic magnetic field.

For simplicity, we begin by considering a particle hopping on a one-dimensional lattice, described by a tight-binding model

$$H_0 = -\frac{J}{2} \sum_m \left(|m\rangle\langle m+1| + H.c. \right) , \tag{1}$$

where $|m\rangle$ are Wannier states localized on sites m, and J is the hopping between nearest neighbors. Oscillating the position of the optical lattice, $x \to x + x_0 \cos(\omega t + \phi)$, produces an inertial force in the rest frame of the lattice $F_I = -M x_0 \omega^2 \cos(\omega t + \phi)$, which can be described in terms of a potential function

$$F_I = -\frac{\partial}{\partial x} V(x,t) . \tag{2}$$

This allows the driving to be included in the Hamiltonian as

$$\begin{aligned} H &= H_0 + V(x,t) \\ &= H_0 + a F_\omega \cos(\omega t + \phi) \sum_m |m\rangle m \langle m| , \end{aligned} \tag{3}$$

where a is the lattice spacing and $F_\omega = M x_0 \omega^2$. In addition to the oscillation the lattice can also be subjected to a constant acceleration [2], which adds a constant component F to the total lattice potential

$$\begin{aligned} H &= -\frac{J}{2} \sum_m \left(|m\rangle\langle m+1| + H.c. \right) \\ &+ a \left(F + F_\omega \cos(\omega t + \phi) \right) \sum_m |m\rangle m \langle m| . \end{aligned} \tag{4}$$

It can be readily seen that this Hamiltonian mimics that of a charged particle subjected to a uniform electric field, with F and F_ω playing the roles of the DC and AC components respectively.

When the static component matches the driving frequency, $F = \omega$, a straightforward Floquet analysis [3, 4] reveals that in the high-frequency limit the dynamics of the driven system can be described by an effective static Hamiltonian

$$\begin{aligned} H^{\text{eff}} &= -\frac{J^{\text{eff}}}{2} \sum_m \left(|m+1\rangle\langle m| e^{i\phi} + H.c. \right) \\ J^{\text{eff}} &= J \mathcal{J}_1(F_\omega/\omega) . \end{aligned} \tag{5}$$

where $\mathcal{J}_1(x)$ is the Bessel function of the first kind. The presence of the phase factors, $e^{\pm i\phi}$, in the effective Hamiltonian provides the motivation for attempting to use this kind of driving to generate synthetic gauge potentials.

A two-dimensional square lattice threaded by a uniform magnetic field can be described in the Landau gauge $\mathbf{A} = A(0, x, 0)$ by

$$\begin{aligned} H = &- \frac{J_x}{2} \sum_l \left(|l+1, m\rangle\langle l, m| + H.c. \right) + \\ &- \frac{J_y}{2} \sum_m \left(|l, m+1\rangle\langle l, m| e^{i\phi l} + H.c. \right) \end{aligned} \tag{6}$$

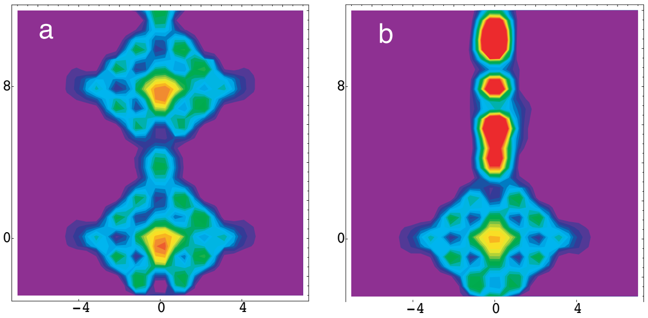

Fig. 1: Evolution of the initial state $\psi(x,y) = N\left(\exp[-2(x^2+y^2)] + \exp[-2(x^2-(y-8)^2)]\right)$. (a) Density distribution of the system at $t = 30T$ after evolving in the presence of a uniform magnetic field (Eq.6). (b) As in (a), but for a periodically-driven system. The peak centered on the origin shows a similar evolution to the exact case, but the other peak is instead compressed into a narrow strip parallel to the y-axis, due to the renormalization of J_x. Parameters are $\omega = 19.9$, $F/\omega = 1$, $F_\omega/\omega = 0.2$, and $\phi = \pi/3$.

where J_x/J_y are the tunneling amplitudes in the x/y directions, and (l,m) are the coordinates of the lattice sites. In Ref. [1] it was proposed that this pattern of Peierls phases could be mimicked by adding the periodic driving potential

$$V(x,y,t) = y\left(F + F_\omega \cos(\omega t - x\phi)\right) \tag{7}$$

to the two-dimensional tight-binding model. With this driving potential, the force experienced in the rest frame of the lattice ($\mathbf{F} = -\nabla V$) is given in the continuum limit by

$$\begin{aligned} F_x &= -\partial_x V = -\left[\phi F_\omega \sin(\omega t - x\phi)\right] y & (8)\\ F_y &= -\partial_y V = -\left[F + F_\omega \cos(\omega t - x\phi)\right] . & (9)\end{aligned}$$

Comparing (9) with Eqs.4 and 5 indicates that the tunneling in the y-direction will indeed acquire the correct phases to reproduce the Landau gauge. However, it is also apparent that tunneling in the x-direction will experience a periodic driving (8) too, and so will *also* be renormalized. This may appear unexpected, as this renormalization is occurring for tunneling perpendicular to the shaking of the lattice, but is a consequence of the explicit x-dependence of $V(x,y,t)$. This effect was not considered in Ref. [1], as F_x was implicitly assumed to be equal to zero. Consequently the results obtained in Ref. [1] are only valid for $F_x \simeq 0$, that is, in a sufficiently small region around $y = 0$.

Performing a Floquet analysis reveals that while J_y is renormalized as in Eq.5, the the effective x-hopping is given by

$$J_x^{\text{eff}} = J_x \mathcal{J}_0\left(2y \sin[\phi/2]\ \frac{F_\omega}{\omega}\right) . \tag{10}$$

Thus away from $y = 0$, J_x will be suppressed by the zeroth Bessel function. In Fig.1 we show the magnitude of this effect. The system is initialized in a superposition of two narrow Gaussians, one centered on the origin, and the other on $(0,8)$. Fig.1a shows the system evolving under the action of Eq.6 for a magnetic flux of $\phi = \pi/3$, while in Fig.1b we show the result of evolving the system under the time-dependent driving potential (7). We can see that the peak centered on the origin behaves similarly in both cases, indicating that near this point the driving indeed mimics a magnetic field with reasonable accuracy. The behavior of the other peak, however, is strikingly different. The suppression of tunneling produced by the Bessel function squeezes the wavepacket into a narrow band, with a very different dynamics to that of the true magnetic flux.

In summary we have shown that the scheme proposed in Ref. [1] cannot produce a uniform magnetic field. At best it can produce a field that is approximately constant over a finite number of lattice spacings, which severely limits its applications. We have demonstrated this by an explicit calculation of the effect of the driving potential. This result is also consistent with the basic observation that a vector field with a non-zero curl cannot be described by a conservative potential. More complicated experimental setups are required to avoid this drawback; for example introducing additional optical lattice potentials [5] to compensate for the transverse renormalization (10).

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