Charged-Current Inclusive Neutrino Cross Sections in the SuperScaling Model

M.V. Ivanov\textsuperscript{1,2,a)}, G.D. Megias\textsuperscript{3}, R. González-Jiménez\textsuperscript{4}, O. Moreno\textsuperscript{5}, M.B. Barbaro\textsuperscript{6}, J.A. Caballero\textsuperscript{3}, T.W. Donnelly\textsuperscript{5}, A.N. Antonov\textsuperscript{1}, E. Moya de Guerra\textsuperscript{2} and J.M. Udías\textsuperscript{2}

\textsuperscript{1}Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria
\textsuperscript{2}Grupo de Física Nuclear, Departamento de Física Atómica, Molecular y Nuclear, Facultad de Ciencias Físicas, Universidad Complutense de Madrid, Madrid E-28040, Spain
\textsuperscript{3}Departamento de Física Atómica, Molecular y Nuclear, Universidad de Sevilla, 41080 Sevilla, Spain
\textsuperscript{4}Department of Physics and Astronomy, Ghent University, Proeftuinstraat 86, B-9000 Gent, Belgium
\textsuperscript{5}Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
\textsuperscript{6}Dipartimento di Fisica, Università di Torino and INFN, Sezione di Torino, Via P. Giuria 1, 10125 Torino, Italy

\textsuperscript{a)}Corresponding author: martin.inrne@gmail.com

Abstract. SuperScaling model (SuSA) predictions to neutrino-induced charged-current $\pi^+$ production in the $\Delta$-resonance region are explored under MiniBooNE experimental conditions. The SuSA charged-current $\pi^+$ results are in good agreement with data on neutrino flux-averaged double-differential cross sections. The SuSA model for quasielastic scattering and its extension to the pion production region are used for predictions of charged-current inclusive neutrino-nucleus cross sections. Results are compared with the T2K experimental data.

INTRODUCTION

New measurements of inclusive charged-current (CC) neutrino-nucleus scattering cross sections, where only the outgoing lepton is detected, have been recently performed by the T2K [1] collaboration. For neutrino energies around 1 GeV (T2K) the main contributions to the cross sections are associated with quasielastic (QE) scattering and one pion ($1\pi$) production. In the present work we evaluate the CC neutrino inclusive cross sections within the SuperScaling approach (SuSA), introduced in [2] to describe neutrino-nucleus scattering by using electron scattering data instead of relying on specific nuclear models.

The properties of neutrinos, particularly the parameters of their oscillations, are being studied with increasing interest as these may carry important information about the limits of the Standard Model. In most neutrino experiments, the interactions of the neutrinos occur with nucleons bound in nuclei. Model predictions for these reactions involve many different effects such as nuclear correlations, interactions in the final state, possible modification of the nucleon properties inside the nuclear medium, that presently cannot be computed in an unambiguous and precise way. This is particularly true for the channels where neutrino interactions take place by means of excitation of a nucleon resonance and subsequent production of mesons. The data on neutrino-induced charged-current (CC) charged pion production cross sections on mineral oil recently released by the MiniBooNE collaboration [3] provide an unprecedented opportunity to carry out a systematic study of double differential cross section of the processes: $\nu_\mu \ p \rightarrow \mu^- \ p \ \pi^+$ and $\nu_\mu \ n \rightarrow \mu^- \ n \ \pi^+$ averaged over the neutrino flux.

One way of avoiding model-dependencies is to use the nuclear response to other leptonic probes, such as electrons, under similar conditions to the neutrino experiments. The analyses of the world data on inclusive electron-nucleus scattering [4, 5] confirmed the observation of superscaling and thus justified the extraction of a universal nuclear response to be also used for weak interacting probes. However, while there is a number of theoretical models...
that exhibit superscaling, such as for instance the relativistic Fermi gas (RFG) [6, 7], the nuclear response departs from the one derived from the experimental data. This showed the necessity to consider more complex dynamical pictures of finite nuclear systems – beyond the RFG – in order to describe the nuclear response at intermediate energies. SuSA predictions are based on the phenomenological superscaling function extracted from the world data on quasielastic electron scattering [8]. The model has been extended to the Δ-resonance region [2] where the response of the nuclear system proceeds through excitation of internal nucleonic degrees of freedom. Indeed, a non-quasielastic cross section for the excitation region in which nucleon excitations, particularly the Δ’s, play a major role was obtained by subtracting from the data QE-equivalent cross sections given by SuSA [9, 10]. This procedure has been possible due to the large amount of available high-quality data of inelastic electron scattering cross sections on 12C, including also separate information on the longitudinal and transverse responses, the latter containing important contributions introduced by effects beyond the impulse approximation (non-nucleonic).

We have extended the analysis to CC pion production cross-section measured at MiniBooNE [11], that from the theoretical point of view can be seen as more challenging. For instance, Δ properties in the nuclear medium, as well as both coherent and incoherent pion production for the nucleus should be considered in any theoretical approach, while in the SuSA procedure they are included phenomenologically extracted from the electron scattering data. All what is assumed within SuSA approach is the nuclear response to be factorized into a single-nucleon part and a ‘nuclear function’ accounting for the overall interaction among nucleons. As mentioned before, the SuSA assumptions have been tested against a great deal of electron-nucleus scattering data with fair success. The factorization assumption allows one to apply the same nuclear responses derived from electron scattering to neutrino-induced reactions, with a mere use of the adequate single-nucleon terms for this case. To show the importance of nuclear interaction effects as predicted within SuSA, as a reference, we also show results obtained within the RFG, with no interactions among nucleons, for which the scaling function in the Δ-domain is simply given as $f_{\text{RFG}}(\psi_\Delta) = \frac{3}{4}(1 - \psi_\Delta^2)\theta(1 - \psi_\Delta^2)$ with $\psi_\Delta$ the dimensionless scaling variable extracted from the RFG analysis that incorporates the typical momentum scale for the selected nucleus [2, 12]. In Figure 1 we compare the Δ-region SuSA [2] and RFG scaling functions, which we use in our study.

![Figure 1](image_url)

**THEORETICAL SCHEME AND RESULTS**

**π⁺ Production in the MiniBooNE Experiment**

The charged-current neutrino cross section in the target laboratory frame is given in the form [2]:

$$
\frac{d^2\sigma}{d\Omega dk'} = \frac{(G \cos \theta_k)^2}{2\pi^2} \left(1 - \frac{|Q|^2}{4\epsilon e'}\right) F^2,
$$

where $\Omega$, $k'$ and $e'$ are the scattering angle, momentum and energy of the outgoing muon, $G$ is the Fermi constant and $\theta_k$ is the Cabibbo angle. The function $F^2$ depends on the nuclear structure through the $R$ responses and can be written as [2, 13]:

$$
F^2 = \tilde{V}_{CC}R_{CC} + 2\tilde{V}_{CL}R_{CL} + \tilde{V}_{LL}R_{LL} + \tilde{V}_{TR} + 2\tilde{V}_{TR}R_T.
$$

that is a generalized Rosenbluth decomposition having charge-charge (CC), charge-longitudinal (CL), longitudinal-longitudinal (LL) and two types of transverse ($T$, $T'$) responses ($R$’s) with the corresponding leptonic kinematical factors ($\tilde{V}$’s). The nuclear response functions in Δ-region are expressed in terms of the nuclear tensor $W_{\mu\nu}$ in the corresponding region. The basic expressions used to calculate the single-nucleon cross sections are given in [2]. These involve the leptonic and hadronic tensors as well as the response and structure functions for single nucleons. A convenient parametrization of the single-nucleon $W^+n \rightarrow \pi^+$ vertex is given in terms of eight form-factors: four vector ($C^{\nu}_{3,4,5,6}$) and four axial ($C^A_{3,4,5,6}$) ones. We use two different parameterizations: the one given in [14] where deuteron
FIGURE 2. (Color online) The double-differential cross section averaged over the neutrino energy flux as a function of the muon kinetic energy $T_\mu$ obtained by SuSA and RFG $\Delta$-region scaling functions. In each subfigure the results have been averaged over the corresponding angular bin of $\cos \theta$. For vector and axial form-factors two parameterizations, “PR1” [14] and “PR2” [15], are used. Effects were evaluated (authors estimated that the latter reduce the cross section by 10%), denoted as “PR1”, and the one from [15], called “PR2”. With these ingredients, we evaluate the cross section for CC $\Delta^+$ and $\Delta^-$ production on proton and neutron, respectively. Once produced, the $\Delta$ decays into $\pi N$ pairs. For the amplitudes $A$ of pion production the following isospin decomposition applies:

$$A(\nu_l p \rightarrow l^- p \pi^+)=A_3, \quad A(\nu_l n \rightarrow l^- n \pi^+)=\frac{1}{2}\sqrt{3} A_3 + \frac{2}{3} A_1, \quad A(\nu_l n \rightarrow l^- \pi_0)=\frac{3}{2\sqrt{2}} A_3 - \frac{3}{2} A_1,$$

with $A_3$ being the amplitude for the isospin $3/2$ state of the $\pi N$ system, predominantly $\Delta$, and $A_1$ the amplitude for the isospin $1/2$ state that is not considered here.

The double-differential cross section for CC neutrino-induced $\pi^+$ production averaged over the neutrino energy flux as a function of the muon kinetic energy $T_\mu$ is presented in Figure 2. Each panel corresponds to a bin of $\cos \theta$. PR1 and PR2 parametrizations have been considered. Results with the PR1 parameterization are about 5% higher, that is a measure of the degree of uncertainty that we expect from the choice of the single-nucleon response for this reaction.

We compare the predictions of SuSA and RFG with the MiniBooNE data [3]. Here we show that SuSA predictions are in good agreement with the MiniBooNE experimental data for $\pi^+$ cross-section in the case of the flux averaged data.

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In Figure 3 we show the CC inclusive $\nu_\mu -^{12}$C double-differential cross section per nucleon versus the muon momentum, $p_\mu$, for different angular bins, folded with the T2K flux. The QE curve (dashed line) corresponds to the results obtained using SuSA scaling function in the QE-region [2] and Pauli blocking effects in the scaling function introduced in Ref. [16]. The standard value of the axial mass $M_A = 1.03$ GeV is used in the QE calculations. The resonant pion production curve ($1\pi$) is derived with the SuSA scaling function in the $\Delta$-region $f^\Delta(\varphi_\Delta)$ (Figure 1). The band corresponds to the two different parametrizations, PR1 and PR2, described in the previous Section. We observe that the model yields good agreement with the T2K data. SuSA model fails to reproduce all T2K data just at the bin in muon angle $0.00 < \cos \theta_\mu < 0.84$. This could be due to ingredients that are missing in the considered theoretical model and would improve the agreement with the T2K data. All our calculations are based on the impulse approximation, i.e., they do not include effects beyond the one-body approach, for example, 2p-2h contributions induced by meson exchange currents (MEC).

CONCLUSIONS

The SuSA approach provides neutrino-nucleus cross section predictions, based on the observed nuclear response to electron projectile and the universal character of the scaling function. We show that SuSA predictions are in good agreement with the MiniBooNE experimental data for pionic cross section in the case of the flux averaged data. We conclude that the idea of the SuSA approach for the QE- and $\Delta$-regions (extracted from electron scattering experiments), when being extended to neutrino processes, proves to be successful in describing $\nu_\mu$ inclusive charged-current cross sections. MEC contributions have yet to be included, that might be expected to improve the agreement with the T2K data.
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