

Ten Things You Should Know About DCC*

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March 2013

* For financial support, the second author wishes to acknowledge the Australian Research Council, National Science Council, Taiwan, and the Japan Society for the Promotion of Science.

Abstract

The purpose of the paper is to discuss ten things potential users should know about the limits of the Dynamic Conditional Correlation (DCC) representation for estimating and forecasting time-varying conditional correlations. The reasons given for caution about the use of DCC include the following: DCC represents the dynamic conditional covariances of the standardized residuals, and hence does not yield dynamic conditional correlations; DCC is stated rather than derived; DCC has no moments; DCC does not have testable regularity conditions; DCC yields inconsistent two step estimators; DCC has no asymptotic properties; DCC is not a special case of GARCC, which has testable regularity conditions and standard asymptotic properties; DCC is not dynamic empirically as the effect of news is typically extremely small; DCC cannot be distinguished empirically from diagonal BEKK in small systems; and DCC may be a useful filter or a diagnostic check, but it is not a model.

Keywords: DCC, BEKK, GARCC, Stated representation, Derived model, Conditional covariances, Conditional correlations, Regularity conditions, Moments, Two step estimators, Assumed properties, Asymptotic properties, Filter, Diagnostic check.

JEL Classifications: C18, C32, C58, G17.

1. Introduction

In the last decade, there has been substantial and growing interest in the analysis of dynamic covariances and correlations across investment instruments, with an emphasis on both financial assets (see Engle (2002) and Cappiello et al. (2006), among others, and the references cited in the surveys by Bauwens et al. (2006) and Silvennoinen and Terasvirta (2009), and energy finance, particularly oil (see, for example, Lanza et al. (2006), Chang et al. (2011), and Hammoudeh et al. (2013)). In this research stream, the most widely-used specification is Multivariate GARCH Dynamic Conditional Correlation (DCC), as introduced by Engle (2002). The baseline specification has been extended in several directions, dealing with the parameterization (Billio et al. (2006), Cappiello et al. (2006), and Franses and Hafner (2009), among others), the introduction of additional elements such as asymmetry (Cappiello et al. (2006) and Kasch and Caporin (2013), among others), and the proposal of alternative estimation methods (Engle et al. (2008) and Colacito et al. (2011), among others).

Despite the growing interest in DCC and its central role in the estimation of dynamic correlations, several important issues relating to the representation seem to have been ignored in the financial econometrics literature. These important issues include the derivation of DCC and its mathematical properties, and demonstration of the asymptotic properties of the estimated parameters (for a summary of these issues, see McAleer (2005)). In this respect, a useful contribution is Aielli (2013), who demonstrates the inconsistency of the two step estimator of the parameters of DCC. By way of contrast, there are several published papers dealing with dynamic correlations that do not discuss stationarity conditions or the asymptotic properties of the estimators at all.

Another critical element of DCC is associated with the construction of the dynamic conditional correlations. In fact, DCC seems to provide estimated dynamic correlations as a bi-product of standardization, and not as a direct result of the equation governing the multivariate dynamics. This will be clarified below. An alternative representation which avoids this last criticism, but nevertheless lacks any discussion of the mathematical properties or demonstration of the

asymptotic properties of the estimators, has been proposed by Tse and Tsui (2002). However, this representation seems to have attracted considerably less interest in the literature.

It should be mentioned that many empirical applications involving DCC and related representation show that the impact of news can be rather limited, thereby making the estimated conditional correlations similar to those implied by simple BEKK models (see Baba et al. (1985) and Engle and Kroner (1995), at least in small cross-sectional problems (for further details, see Caporin and McAleer (2008) and Franses and Hafner (2009)).

This paper highlights some critical issues associated with the use of the DCC and related representations to make the potential user aware of the inherent problems they might encounter. The main message is not against the use of DCC, which is the most popular representation of dynamic conditional correlations, but is intended to be cautionary, so that users can understand and appreciate the limits of DCC. In fact, we suggest that DCC be regarded as a filter or as a diagnostic check, as in the Exponentially Weighted Moving Average approach adopted in the first versions of the RiskMetrics (1996) methodology. When an equation has not been derived in a rigorous way, and for which we do not have any explicit details regarding the existence of moments, derivation of the stationarity conditions, or asymptotic properties of the estimators, it should not be considered as a model, but as a filter or a diagnostic check for estimating and forecasting dynamic conditional correlations. We will elaborate on this issue in the remainder of the paper after highlighting the critical aspects of the DCC framework.

The plan of the paper is to discuss ten caveats you should know about DCC. These are discussed in Section 2. Some concluding remarks are given in Section 3.

2. Ten Caveats About DCC

The DCC representation was introduced by Engle (2002) to capture the empirically observed dynamic contemporaneous correlations of asset returns. The representation can be given as

follows. Denote by r_t the vector containing the log-returns of k assets. The density of the returns is characterized by the absence of serial correlation in the mean returns, but the presence of time-varying second-order moments:

$$r_t | I_{t-1} \sim D(\mu, \Sigma_t) \quad (1)$$

where I_{t-1} denotes the information set up to time $t-1$, μ is the unconditional mean, which is generally equal, or very close, to zero, Σ_t is the conditional covariance matrix, and $D(\cdot, \cdot)$ is a generic multivariate density function depending on the mean vector and dynamic conditional covariance matrix. Following Engle (2002), the covariance matrix can be decomposed into the product of dynamic conditional standard deviations and dynamic conditional correlations:

$$\Sigma_t = D_t R_t D_t \quad (2)$$

where $D_t = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_k)$, $\text{diag}(a)$ is a matrix operator creating a diagonal matrix with the vector a along the main diagonal, and R_t is a correlation matrix. From equations (1) and (2), the marginal density of each element of r_t has time-varying conditional variances, and can be modelled as a univariate GARCH process.

The DCC representation focuses on the dynamic evolution of R_t in (2), and recovers that quantity by considering the dynamics of the conditional variance of the standardized residuals, which are defined as follows:

$$\eta_t = (r_t - \mu) D_t^{-1} \quad (3)$$

By construction, the standardized residuals have second-order unconditional moment equal to

$$E[\eta_t \eta_t'] = R \quad (4)$$

thereby motivating the focus on standardized residuals to recover the dynamics for the conditional correlations. In practice, the standardized residuals can be used to verify empirically the existence of dynamics in the conditional correlations, for instance, by means of rolling regression approaches. Moreover, if the data generating process of the returns is as given in equations (1) and (2), the dynamic conditional covariance of the standardized residuals is given as:

$$E\left[\eta_t \eta_t' \mid I_{t-1}\right] = R_t \quad (5)$$

Without distinguishing between dynamic conditional covariance and dynamic conditional correlation matrices, Engle (2002) presents the following equation based on the cross-products of the standardized residuals:

$$Q_t = (1 - \alpha - \beta) \bar{Q} + \alpha \eta_{t-1} \eta_{t-1}' + \beta Q_{t-1} \quad (6)$$

where (6) has scalar parameters, as in the most common DCC representation, \bar{Q} is assumed to be a positive definite matrix with unit elements along the main diagonal (which is alleged to be a conditional correlation matrix), the two scalar parameters satisfy a stability constraint of the form $\alpha + \beta < 1$, and the matrix Q_t purportedly drives the dynamics of the conditional correlations.

However, as the matrix Q_t does not satisfy the definition of a correlation matrix, Engle (2002) introduces the following standardization:

$$R_t = \text{diag}\left(\text{dg}(Q_t)\right)^{-0.5} Q_t \text{diag}\left(\text{dg}(Q_t)\right)^{-0.5} \quad (7)$$

where $\text{dg}(A)$ is a matrix operator returning a vector equal to the main diagonal of matrix A. [In discussing the equivalent of equation (7) above, namely equation (25) in Engle (2002), the exponent -0.5 is missing.] It is clear that equation (6) is a simple standardization, implying that

the quantity in which we are interested, namely the dynamic conditional correlation matrix, can be computed. However, to state the obvious, a dynamic conditional correlation matrix is a standardization of a dynamic conditional covariance matrix, but not every standardization such as (7) leads to a dynamic conditional correlation matrix, especially if it cannot be shown (as distinct from being stated) that (6) is a dynamic conditional covariance matrix.

Bearing these points in mind in mind, the following caveat should be considered before using the DCC representation.

(1) DCC is based on the conditional second-order moment of the standardized residuals, and hence does not directly yield conditional correlations.

The simple observation of equation (6) recognizes the structure of the scalar BEKK model of dynamic conditional correlations (see Baba et al. (1985) and Engle and Kroner (1995)), as we can write:

$$\eta_t = (r_t - \mu) D_t^{-1} \square D(0, Q_t) \quad (8)$$

wherein Q_t in (5) might be interpreted as a dynamic conditional covariance matrix. A dynamic conditional correlation matrix may be obtained only through the standardization in (7). However, we can also note an inconsistency between the dynamic conditional expectation reported in (5) and the way in which the dynamic conditional correlation matrix is obtained in (7). Such inconsistency causes further problems as Q_t is not the conditional covariance of η_t , as shown in (5), and is not the conditional correlation of η_t as it is just positive definite, but need not correspond to a dynamic conditional correlation matrix.

The last remark can easily be verified by visual inspection of the empirical estimates of Q_t , which are typically not considered in empirical analysis. However, by using several datasets, it is straightforward to show that the elements of $|Q_t|$ can be greater larger than 1 (see, for example,

McAleer et al. (2008)). As a consequence, it might be stated that Q_t is a convenient device for obtaining dynamic conditional correlations but, as it stands, has no proper interpretation as either a dynamic conditional covariance or correlation matrix.

This leads to the following caveat you should know about DCC.

(2) DCC is stated rather than derived.

From the previous comments, it clearly emerges that DCC is a stated representation, but it is not a derived model based on the relationship between the innovations to returns and the standardized residuals, or on the definition relating dynamic conditional correlations to dynamic conditional covariances. As such, the interpretation of DCC as a model providing dynamic conditional correlations is inherently flawed and misdirected. This also begs the question as to whether DCC is actually a model, namely a representation with explicit, and hence testable, mathematical properties and derivable statistical properties.

The fact that DCC is a derived representation, affects also the many specifications which are obtained as generalizations of DCC, including, among others, Billio et al. (2005), Cappiello et al. (2006), and Aielli (2013). Moreover, this has serious implications regarding the model structure and associated statistical properties.

(3) DCC has no moments.

This follows from the stated rather than derived properties of the representation (see McAleer et al. (2008) for further details). Therefore, there is no connection between univariate models of conditional variance, such as ARCH (Engle (1982)) and GARCH (Bollerslev (1986)), and multivariate models of conditional correlations. This is in marked contrast to the direct connection between the alternative univariate conditional volatility models and the BEKK multivariate model of dynamic conditional covariances (see Baba et al. (1985) and Engle and

Kroner (1995)), and the direct connection between univariate conditional volatility models and the GARCC multivariate model of dynamic conditional correlations (see McAleer et al. (2008)).

(4) DCC does not have testable regularity conditions.

This follows from point (3) above. In particular, Engle (2002, p. 342) refers to “reasonable regularity conditions” and “standard regularity conditions”, without stating them explicitly. Aielli (2013, pp. 10-11) assumes that the unstated regularity conditions, whatever they might be, are satisfied. Cappiello et al. (2006) develop an extension of DCC to incorporate asymmetries, but do not provide any explicit regularity conditions. With no testable regularity conditions, such as log-moment or second moment conditions, the internal consistency of the model cannot be checked. There is, therefore, no evidence as to whether the purported estimates of dynamic conditional correlations have any connection to the definition of dynamic conditional correlations.

The absence of explicit regularity conditions and of explicit moment conditions also affects the derivation of asymptotic properties of the estimators. Engle (2002) suggests the following “two-step” approach for estimating DCC parameters. Within a Quasi-Maximum Likelihood framework, we have the following Gaussian log-likelihood for one observation of the returns r_t :

$$l_t \square -\frac{1}{2} \ln |\Sigma_t| - \frac{1}{2} r_t' \Sigma_t^{-1} r_t'$$

Following the decomposition in (2), we have

$$\begin{aligned} l_t &\square -\frac{1}{2} \ln |D_t R_t D_t| - \frac{1}{2} r_t' D_t^{-1} R_t^{-1} D_t^{-1} r_t' \\ &= -\frac{1}{2} \ln |D_t|^2 - \frac{1}{2} \ln |R_t| \pm \frac{1}{2} r_t' D_t^{-1} D_t^{-1} r_t' - \frac{1}{2} r_t' D_t^{-1} R_t^{-1} D_t^{-1} r_t' \end{aligned}$$

$$\begin{aligned}
&= \left(-\frac{1}{2} \ln |D_t|^2 - \frac{1}{2} r_t' D_t^{-1} D_t^{-1} r_t' \right) + \left(-\frac{1}{2} \ln |R_t| + \frac{1}{2} \eta_t \eta_t' - \frac{1}{2} \eta_t R_t^{-1} \eta_t' \right) \\
&= l_t^V(\Theta_V) + l_t^C(\Theta_V, \Theta_C)
\end{aligned}$$

where it is shown that the single observation likelihood can be decomposed into two terms, the first as a function of the variance parameters only, Θ_V , while the second depends on both the variance and correlation parameters, Θ_V and Θ_C . Note that the first likelihood component is based on a correlation matrix set to the identity matrix, which is then used to recover the variance parameters only. The second likelihood component is used to estimate the correlation parameters, conditionally on the first stage likelihood estimated parameters ($\hat{\Theta}_V$).

Engle (2002) suggests that the first likelihood component can be further decomposed into the sum of univariate likelihoods representing the marginal contribution of each return series, under the assumption of independence. This is a first simplification imposed to deal with the curse of dimensionality that generally affects multivariate GARCH models (see Caporin and McAleer (2012) for further details). In addition, to simplify the computational burden associated with the maximization of the second stage likelihood $L(\hat{\Theta}_V, \Theta_C) = \sum_{t=1}^T l_t^C(\hat{\Theta}_V, \Theta_C)$, Engle (2002) suggests replacing the matrix \bar{Q} with the sample correlation matrix of the standardized residuals η_t , introducing therefore a “1.5” step.

The previously outlined approach entails a number of assumptions that are generally not satisfied by empirical data:

- (i) Marginal variances are assumed to be independent; which rules out any form of spillovers or feedback across variances and shocks of the various assets. This is related to the general idea of dependence across assets governed only by the correlations. However, this is not always the case, and shocks of different assets can affect the variance of a single asset.

- (ii) The sample correlation matrix is assumed to be an appropriate estimator for the matrix \bar{Q} , which is not necessarily a correlation matrix.
- (iii) The approach is called “two-step” estimation, although in reality it involves “three-step” estimation when sample correlations are used for \bar{Q} and is a proper “two-step” when the correlation likelihood $\sum_{t=1}^T l_t^C(\Theta_V, \Theta_C)$ is maximized with respect to the full parameter set Θ_C , and conditionally on the variance parameters Θ_V .

However, possible incompatibilities in the assumptions underlying the estimation approach described above do not prevent its use, which can be motivated and supported by its computational simplicity (a key issue about which users need to be aware). Nevertheless, the asymptotic properties of the “two-step” estimator are not discussed in Engle (2002), apart from a cryptic reference to Engle and Sheppard (2001), which remains an unpublished manuscript.

These points lead to the following caveat:

(5) DCC yields inconsistent “two step” estimators.

Engle (2002, p. 342) states that the standardized residuals in equation (6) are “a Martingale difference by construction” in suggesting how to estimate the parameters of DCC by resorting to standard ARMA methods. Moreover, the fact that standardized residuals are a martingale difference sequence allows recovery of the general results for Multivariate GARCH processes and, in particular, those associated with Engle and Kroner (2005). However, Aielli (2013) points out that DCC cannot be interpreted as a linear multivariate GARCH, and this leads to the inconsistency of the “two-step” DCC estimator discussed above. The inconsistency is generated by the fact that in equation (6) the matrix Q_t is not the expectation of the cross-products of the standardized residuals, so that it is not possible to obtain a martingale difference by rewriting equation (6) in a companion VARMA form.

The merit of Aielli (2013) is in highlighting the consistency problem, but the proposed solution still suffers from the same problematic issues that affect the DCC representation of Engle (2002). In fact, Aielli (2013) discusses targeting and a modification to DCC to enable consistent estimation. However, he assumes that the estimators of the modified DCC representation are asymptotically normal under “standard” regularity conditions, without stating what the conditions might be. Caporin and McAleer (2008, 2012) have shown that dynamic conditional correlations can be estimated by using an indirect DCC representation based on the BEKK model, but asymptotic normality has yet to be established.

(6) DCC has no desirable asymptotic properties.

McAleer et al. (2008), Caporin and McAleer (2012) and Aielli (2013) have shown that the estimated parameters of DCC under the standard two-step approach have no asymptotic properties. Moreover, the asymptotic properties of the joint maximum likelihood estimator (for all parameters in one step) are not known. In their extension of DCC, Cappiello et al. (2006) do not establish any asymptotic properties. In addition, in a more recent contribution, Engle et al. (2008) claim to prove consistency (Theorem 1), but not of the parameters of interest, and then assume consistency of the estimated parameters of interest in the proof of asymptotic normality (Theorem 2) (see Caporin and McAleer (2012) for further details). Therefore, not only are the purported proofs in Engle et al. (2008) based entirely on assumptions, but the lack of asymptotic properties also makes invalid the use of any inferences drawn from confidence intervals associated with estimation of the DCC representation.

(7) DCC is not a special case of GARCC, which has testable regularity conditions and standard asymptotic properties.

McAleer et al. (2008) derive the Generalized Autoregressive Conditional Correlation (GARCC) model based on the relationship between the innovations to returns and the standardized residuals, using a vector random coefficient autoregressive process. The scalar and diagonal

versions of BEKK are also shown to be special cases of a vector random coefficient autoregressive process. The GARCC model provides a motivation for dynamic conditional correlations, and hence can be shown to produce dynamic conditional correlations, with the estimated parameters being consistent and asymptotically normal.

(8) DCC is not dynamic empirically as the effect of news is typically extremely small.

Are the purported dynamic conditional correlations real or apparent, and do they arise solely from the standardization of the dynamic conditional covariances? In the context of the Box-Jenkins procedure, if an ARMA(1,1) is estimated when an ARMA(2,1) representation is correct, then the residuals would still be AR(1). Thus, estimating the residuals with an AR filter would capture the remaining dynamics.

Transposing the same argument into the GARCH framework, the conditional variance might be estimated as a GARCH(1,1) model, but the correct model might actually have asymmetry, leverage, jumps, thresholds and/or higher time-varying moments. Consequently, the parameter estimates could be biased. The standardized residuals, which are typically not checked for further conditional heteroskedasticity, as the common wisdom is that GARCH(1,1) should be sufficient for capturing time-varying conditional variances and covariances, may have some remaining heteroskedasticity, however mild. Fitting standardized residuals using GARCH(1,1), which is the diagonal term of the DCC formulation, will surely capture some of the underlying dynamics. Even if the conditional correlations happen to be constant, the conditional covariances across the standardized residuals may appear to be dynamic because of the misspecification. Therefore, standardization does not filter out the dynamics in the covariances due to the biases in the initial GARCH(1,1) estimates. As a result, the conditional correlations may appear to be dynamic (with significant parameter estimates) due to misspecification in the first step. However, to date no research seems to have followed this line of research.

(9) DCC cannot be distinguished empirically from diagonal BEKK in small systems.

Caporin and McAleer (2008) show that the estimates of the dynamic conditional correlations from a scalar BEKK model, which is effectively an indirect DCC representation, are very similar to those from DCC. This supports the argument that DCC can mimic dynamic conditional correlations, at least for small financial portfolios. Theoretical arguments to support this claim are presented in Caporin and McAleer (2012).

(10) DCC may be a useful filter or a diagnostic check, but it is not a model.

A significant problem in empirical practice is that many users seem to be under the misapprehension that DCC is a model when it is not. DCC has no obvious or desirable mathematical or statistical properties. Nevertheless, DCC may be a useful filter or a diagnostic check that can capture the dynamics in what are purported to be conditional “correlations”, even if they arise through possible model misspecification.

3. Conclusion

The paper discussed ten things potential users should know about the Dynamic Conditional Correlation (DCC) representation for estimating and forecasting time-varying conditional correlations. The reasons given for being cautious about the use of DCC included the following: DCC represents the dynamic conditional covariances of the standardized residuals, and hence does not yield dynamic conditional correlations; DCC is stated rather than derived; DCC has no moments; DCC does not have testable regularity conditions; DCC yields inconsistent two step estimators; DCC has no asymptotic properties; DCC is not a special case of GARCC, which has testable regularity conditions and standard asymptotic properties; DCC is not dynamic empirically as the effect of news is typically extremely small; DCC cannot be distinguished empirically from diagonal BEKK in small systems; and DCC may be a useful filter or a diagnostic check, but it is not a model. .

As DCC is the most popular representation of dynamic conditional correlations, potential users are advised to understand and appreciate the limits of DCC in order to be able to use it as a sensible filter or as a diagnostic check for estimating and forecasting dynamic conditional correlations. As it stands, the lack of asymptotic properties makes invalid the use of any inferences that might be drawn from confidence intervals associated with estimation of DCC.

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