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Effects of public investment in infrastructure on the Spanish economy

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EFFECTS OF PUBLIC INVESTMENT IN INFRASTRUCTURES ON THE SPANISH ECONOMY

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ABSTRACT

The objective of this paper is to evaluate the short and long term effects of public investment in infrastructure on aggregate output, labor and capital formation in the private sector. The problem is analyzed in a dynamic multivariate framework, which allows for explicit consideration of feedback among all the variables. This approach departs from the current literature, which relies on a single equation model to estimate production functions. The results suggest a positive long term effect of public investment on the private sector variables.

RESUMEN

El objetivo de este trabajo es evaluar los efectos a corto y a largo plazo de la inversión en infraestructuras sobre el crecimiento del producto agregado, el empleo y el stock de capital en el sector privado. A diferencia de otros estudios, donde se estiman funciones de producción en un marco univariacional, en este trabajo planteamos el problema en un contexto multivariacional dinámico, lo que permite tener en cuenta explícitamente la presencia de efectos de retroalimentación entre las variables consideradas. Los resultados sugieren que el efecto a largo plazo de la inversión pública en infraestructuras sobre las variables del sector privado es positivo.
1. Introduction

The effect of public investment in infrastructure on the growth of the private sector has been an issue of recent debate. The pioneer work of Aschauer (1989a, 1989b) suggests that while current public expenditure decreases productivity and economic growth, public investment in infrastructure increases private productivity, finding a positive net effect in the U.S. for the period 1952-1986. Using these results, Aschauer (1989b) explains the stagnation of the American economy during the seventies on a crumbling infrastructure base. This explanation has arised a wide debate on the subject and several studies have emerged showing evidence for and against the Aschauer effect. Among others, Munnell (1990a, 1990b), Munnell and Cook (1990) and García-Milá and McGuire (1992) support the Aschauer effect, while Aaron (1990), Eberts (1990), Tatom (1991) and Ford and Porel (1991) do not support it. For the Spanish economy, Bajo and Sosvilla (1993) and Argimón et al. (1993) find a positive and significant effect of public investment on the productivity of the private sector.

All these studies use an unequational approach within the framework of a neoclassical theory of production. They use a production function for the private sector, usually a Cobb-Douglas, in which public capital enters as an additional input. So that the debate of whether or not public capital is productive has focused on the size of the elasticity of output with respect to public capital. That elasticity measures the effect on production of a permanent unit change in the level of public capital stock, holding the level of employment and private capital constant.

This approach has a major shortcoming since it only considers explicitly one of the four dynamic relationships that may exist among the four basic variables in the production function. Therefore, if public capital stock in previous periods affects the current level of employment and/or the stock of current private capital, then the elasticity of output with respect to public capital, estimated through a production function, is not adequate to provide a conclusive answer to the question of whether or not public capital is productive. The same applies if output, employment and/or private capital in previous periods affect the current level of public capital.

In fact, if changes in public capital stock affect directly the other inputs and/or these, together with output, have feedback effects on public capital formation, then the elasticity of output with respect to public capital will be only a part of the total effect of public investment. Therefore, a value of that elasticity equal to zero or greater than zero may be compatible with all kind of total effects, since it only represents a measure of the proportion that the levels of output and public capital stock achieve in equilibrium.

To overcome the limitations of the traditional approach, in this paper we use a dynamic multiequational model. This framework allows for explicit consideration of all the dynamic relationships among the variables in order to obtain adequate estimates of the responses of each of the private sector variables to a shock in the stock of public capital. Moreover, this general formulation is particularly appropriate for dealing with other technical problems previously recognized in the literature [see Tatom (1991) and Munnell (1992)], such as: i) The endogeneity of labor and private capital, which may generate simultaneous equation biases and invalidate OLS estimates and ii) the inadequate treatment of the statistical properties of the time series, i.e. non stationarity and the possible existence of cointegration relationships.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 contains the empirical analysis for the Spanish economy during the period 1964-1992. The main conclusions are summarized in section 4.
2. THE THEORETICAL MODEL

We propose a conceptual model which has been adapted from Flores (1990) and Flores and Pereira (1993). In this model we consider the same type of economic variables as in the previous literature: private output $Y$, private employment $I$, stock of private capital $K$, and stock of public capital in infrastructure $PK$; (hereafter lower case letters will denote the logs of these variables).

We assume the existence of two sectors in the economy: the private sector and the public sector. The two sectors are different in that they have control over different variables. The private sector controls $Y$ and $K$, which in vector notation will be $z = (Y, K)'$, and the public sector determines $PK$. The behavior of the two sectors is the following:

**Private sector** - Each period the private sector determines the levels of $Y$, $I$, and $K$, using information on past values of all these variables as well as past and current values of $PK$. Formally:

$$z_t = \pi_1(B)\pi_t + \epsilon_t$$

where:
- $\pi_1(B)$ is a (3x1) vector of stable transfer functions (see Box and Jenkins (1970)):
- $\pi_1(B) = (\nu_1 B, \nu_2 B, \nu_1 B^2, ...)$ for $j = 1, \ldots, K$

**Public sector** - The public sector determines $PK$ using information about the past values of all the variables. Formally:

$$pk_t = \pi_p(B)z_t + \epsilon_p$$

where:
- $\pi_p(B)$ is a (1x3) vector of stable transfer functions:
- $\pi_p(B) = (\pi_p(B, B)\nu_p(B)\nu_p(B))$

$a_p$ is a white noise scalar with variance $\sigma_p^2$ and independent of the elements of $a_r$.

Notice that:

1. In equation (2), which describes the behavior of the public sector, we have: $\pi_p(0) = \pi_p(0) = \pi_p(0) = 0$. This restriction is a consequence of the assumption about the information set used by the public sector to determine $PK$. This information set does not include $z_t$ and therefore none of its components will affect the determination of $PK$. On the other hand, in equation (1) we allow $\pi_1(0)$ to be different from zero since the information set of the private sector may include $PK$.

2. The elements of the vector $a_p$ are assumed to be independent from $a_r$. That is, both the public sector and the private sector have complete control over their own variables. This control would not be complete if some of the elements of $a_p$ could affect $a_r$ in a systematic fashion (or vice versa).
iii) Equation (2) explicitly allows for feedback of the private sector variables to the public sector. Note that, if the public sector does not use information on previous values of the private sector variables, no feedback rules exist and public capital is truly an exogenous variable.

As we have already pointed in i) and ii), in the model formulation there are two basic assumptions: asymmetry and independence. These assumptions jointly represent sufficient conditions for the parameters of the theoretical model to be exactly identified.

Assumption 1: Asymmetry - Let \( \Omega_p \) and \( \Omega_p \) be respectively the information sets of the private and public sectors at \( t \). These sets are defined as:

\[
\Omega_p = \{ z_{ij}, p_k, j=1,2, \ldots \}
\]

\[
\Omega_p = \{ z_{ij}, p_k, j=1,2, \ldots \}
\]

This assumption can be interpreted as follows:

i) Both the public and the private sectors are assumed to know at the beginning of each period all the past values of all the variables determined in both sectors. Later, empirical results will tell us whether or not that information has been used by either the private or the public sectors in their decision making.

ii) In each period the private sector knows the current values of public capital, while the public sector does not know the current values of the variables determined in the private sector. This is the reason why this assumption is called asymmetry. This assumption is consistent with the fact that the public sector announces in advance, i.e. at the beginning of the period, what public capital expenditures will be during the period. Therefore, the information on public capital formation for the period is available when the private sector makes its decisions. On the contrary, the current values of the variables determined by the private sector cannot be included in the information set of the public sector.

iii) According to the previous assumption, the private sector clearly has information about the plans for public capital formation announced at the beginning of the period. We also assume that the public sector implements the plan that has been previously announced. Then, our strategy is to allow the empirical analysis to determine whether or not the private sector actually uses that information. Alternatively, current information could be excluded a priori and symmetry assumed. However, we believe that this is unnecessarily restrictive.

Assumption 2: Independence - Let \( \sigma_p, \sigma_a, \sigma_a \) and \( \sigma_a \) be the white noise errors associated with the equations for output, labor, private capital and public capital respectively. This assumption asserts that \( \sigma_a \) is independent from \( \sigma_p, \sigma_a \) and \( \sigma_a \).

This establishes that random shocks in the evolution of public capital are independent from shocks in the variables determined in the private sector. On the contrary, random shocks in the private sector variables \( \sigma_p, \sigma_p \) and \( \sigma_a \) can be contemporaneously correlated. This assumption of independence is directly related to the separation of functions between the private and public sectors. Indeed, it is not possible to consider two sectors with different tasks unless we also assume that the specific shocks in the two sectors are independent.

From an econometric perspective, it can be argued that omitted variables and measurement errors can result in contemporaneous correlation among the shocks of the different equations of a structural model. However, assuming contemporaneously correlated structural shocks is not the proper way to deal with those problems. In addition, contemporaneously correlated structural shocks lead to identification problems which are often solved by imposing a priori constraints on the parameters of the dynamic structure of the model. This approach would be particularly inappropriate in our context, since the main objective of this paper is to study the dynamic relationships among all the variables in the model.

Impulse response functions - Our objective is to analyze the reaction of the private sector variables to a shock in \( p_k \). From equations (1) and (2) the vector \( z_t \) can be written as:

\[
z_t = \Psi_j(B)\sigma_t + \Psi_j(B)\sigma_t
\]
where:

\[
\Psi_p(B) = (I - \psi_p(B)\eta_p(B))^{-1}\eta_p(B)\eta_p(B)^{-1}
\]

\[
= -\Phi_0 + \Phi_1B + \Phi_2B^2 + \ldots
\]

\[
\Psi_p(B) = (I - \psi_p(B)\eta_p(B))^{-1}\eta_p(B)^{-1}
\]

\[
= I + \Phi_1B + \Phi_2B^2 + \ldots
\]

The sequence of coefficients associated with the lag polynomial \(\Psi_p(B)\) of equation (4) is to be interpreted as the response function of \(z_t\) versus an impulse in \(a_{p,t}\), that is, \(\partial z_t/\partial a_{p,t}\) for \(j=0,1,2,\ldots\). This function measures the dynamic consequences for the private sector variables of a change in \(p_k\). Therefore, the estimation of this function is the key for describing the effects of public capital upon the performance of the private sector.

Notice that by assumption 2, \(a_{p,t}\) is independent from the shocks in equation (1). Therefore, the impulse response function of \(z_t\) with respect to \(a_{p,t}\) does not depend on the contemporaneous correlations among the variables in \(z_t\). Thus, in order to study the effects of changes in \(p_k\) on \(z_t\), it is not necessary to specify a whole structural model and the model given by equations (1) and (2), together with the underlying assumptions 1 and 2, is all that is needed.

The information of the impulse response function is complemented by the step response function. The value of this function at moment \(j\) is the sum of the cumulative effects from the initial moment \(t\) up to \(j\) of a transitory unit shock in \(a_{p,t}\). Therefore, it is obtained as the sum of the cumulative impulse responses.

Additionally, one may be interested in considering the effects of shocks to \(f_t\) and \(l_t\) on \(y_t\). As in the case of shocks to \(p_k\), this requires the use of orthogonalized impulse response functions. However, since we assume that the components of the vector \(a_t\) are contemporaneously correlated, the polynomial \(\Psi_p(B)\) of equation (5) does not have the same interpretation as \(\Psi_p(B)\). Comparing the effects of shocks in the different private sector variables is an interesting question, but it is beyond the scope of this paper.

Estimation strategy - The model in (1) and (3) can be written in matrix form as \(\Pi_p(B)w_t = a_{w,t}\), where \(\Sigma\) is the matrix of contemporaneous correlations of \(a_{w,t}\), or:

\[
\begin{bmatrix}
\pi_p(B) - \pi_p(B)\psi_p(B)
\end{bmatrix}
\begin{bmatrix}
\pi_p(B)
\end{bmatrix}
= \begin{bmatrix}
\psi_p(B) - \psi_p(B)\psi_p(B)^{-1}
\end{bmatrix}
\begin{bmatrix}
\psi_p(B)^{-1}
\end{bmatrix}
\begin{bmatrix}
\xi_t
\end{bmatrix}
= \begin{bmatrix}
a_{w,t}
\end{bmatrix}
\]

(6)

(7)

The stochastic multivariate model in (6) is not normalized, since:

\[
\Pi_p(0) = V = \begin{bmatrix}
I & -\psi_p(B)
\end{bmatrix}
\begin{bmatrix}
0
\end{bmatrix}
\]

(8)

where \(\psi_p = (\psi_{p1} \psi_{p2} \psi_{p3})\) is the vector of contemporaneous effects of \(p_k\) on \(z_t\).

However, the model can be easily normalized by premultiplying equation (6) by \(V^{-1}\):

\[
\Pi_p^*(B)w_t = a_{w,t}^*
\]

(9)

where:

\[
\Pi_p^*(B) = V^{-1}\Pi_p(B)
\]

\(a_{w,t}^* = V^{-1}a_{w,t}\)

In model (9), the contemporaneous covariance matrix of \(a_{w,t}^*\) is \(\Sigma^*\), which is given by:

\[
\Sigma^* = V^{-1}\Sigma V^{-1}
\]

(10)

Notice that the model in (9) and (10) is an exactly identified general multivariate stochastic model which is written in its VARMA representation (autoregressive infinite...
representation). This implies that the estimation of the theoretical model (6)-(7) can be performed as follows: we construct the empirical VARMA representation of (9)-(10) from the data using the methodology developed by Tiao and Box (1981) and Jenkins and Alavi (1981), suitably modified to account for the possible presence of cointegration relationships. After estimating equations (9) and (10), we estimate $V$ from equation (10) since $V$ can be obtained by multiplying the partition $(1,2)$ of matrix (10) by the estimated value of $1/\delta^2$. Then, we estimate $I_{L/B}$ by premultiplying equation (9) by the estimate of $V$. Finally, after estimating (6) and (7), the estimation of the impulse response functions is immediate from (4).

3. Empirical Analysis

The data - We use yearly data for the period 1964-1992. The variables are defined as follows:

$Y$: Private gross domestic product (GDP), measured in thousand million 1980 pesetas. This series has been computed by Molinas et al. (1990) as the difference between total real GDP at factor cost and public GDP.

$L$: Private employment, measured in thousand workers. It has been computed by García-Perea and Gómez (1993) from the Spanish survey of labor (Encuesta de Población Activa, EPA) as the difference between total employment and employment in the Public Administration. It should be noted that the series of private employment could underestimate the total occupation in this sector, since people working in both sectors have been only counted as public workers.

$PK$: Public capital stock in infrastructure, measured in thousand million 1980 pesetas. This variable includes public investment in infrastructure in transport and communications as defined in the National Accounting. This series has been computed by Argimón and Martín (1993) using a method of permanent stock, in which a constant rate of depreciation is assumed.

$K$: Private productive capital stock, also measured in thousand million 1980 pesetas. This series has been constructed by Corrales and Taguas (1989) also using a method of permanent stock.

Univariate Analyses - Table 1 shows the univariate and intervention models (Box and Jenkins (1970)) for the series $y$, $l$, $k$ and $pk$, as well as for the series of labor productivity ($y/l$), private capital productivity ($y/k$) and public capital productivity ($y/pk$). These models suggest that all the variables are I(1). This result casts doubt in the case of $pk$ due to the high value of the autoregressive parameter; however, since the series $y$ and $(y-pk)$ are truly I(1), this implies that $pk$ will necessarily be I(1). It is also important to note the permanent decrease in the rates of growth of output and labor productivity that takes place in 1975: 4.6 and 3 percentage points respectively. This fact is consistent with the downwards rigidity of employment, which does not fully adjust to the slow growth situation that starts in 1975. The rest of the variables do not show any important shift during the period, which suggests that these variables have adjusted fully to the slow growth situation that characterizes the period 1975-1992.
Cointegration - Table 2 shows the results of the Johansen (1988) test to determine the number of existing cointegration relationships. The first column shows the different null hypotheses that have been considered, the second column shows the computed values of the statistics and the third column contains the 95% critical values.

From this table we do not reject the existence of one cointegration relationship, which can be estimated by the following OLS regression:

\[ y_t = 0.43 + 0.34 I_t + 0.46 k_t + 0.21 ph_t + ecm_t \]  

(11)

This equation can be interpreted as a Cobb-Douglas production function with constant returns to scale in all the inputs, which implies decreasing returns to scale over private inputs. Equation (11) can also be considered as an equilibrium long run relationship, in which the term \( ecm_t \) represents a measure of the disequilibrium at each period \( t \).

Empirical model - Incorporating the above cointegration restriction in model (6) and using the methodology of Tiao and Box (1981) for constructing vector ARMA models, we obtain the following empirical model (standard deviations are shown in parenthesis):

\[
(1 - 1.15B + .45B^2) \cdot ecm_t = (.35 - .25B) \cdot V_l_{t-1} + (.32 - 0.31B) \cdot V_k_{t-1} + \sigma_e \\
(20) (17) (15) (12) (20) (19)
\]

\[
(1 - .54B) \cdot V_h_t = .34 \cdot ecm_{t-1} + \sigma_e \\
(15) (18)
\]

\[
(1 - .78B) \cdot ecm_t = (.41 - .24B) \cdot ecm_{t-1} + .20 \cdot V_p_{h,1} + \sigma_e \\
(06) (14) (13) (07)
\]

\[
(1 - .97B) \cdot V_p_{h_t} = .26 \cdot ecm_{t-1} + \sigma_e \\
(05) (15)
\]

These equations have been estimated by the exact maximum likelihood procedure developed by Hillmer and Tiao (1979). Residual averages and residual standard deviations for each equation are:

\[
\bar{\sigma}_e = -0.0003 \quad \bar{\sigma}_t = -0.0027 \quad \bar{\sigma}_h = -0.0002 \quad \bar{\sigma}_m = 0.0014
\]

\[ \sigma_e + 100 = 1.13 \quad \sigma_t + 100 = 1.43 \quad \sigma_h + 100 = 1.66 \quad \sigma_m + 100 = 1.53 \]

The estimated matrix of contemporaneous correlations for the error vector is:

\[
R = \begin{bmatrix}
1 \\
.34 & 1 \\
.55 & .41 & 1 \\
.27 & .37 & -.21 & 1
\end{bmatrix}
\]

The cross correlation matrices of orders 1, 2 and 3 for the error terms of the different equations are:

\[
\begin{bmatrix}
-0.05 & .25 & -0.07 & .19 \\
.26 & .25 & -0.07 & .25 \\
.06 & .35 & .03 & .32 \\
.09 & -.14 & .01 & .07
\end{bmatrix} \quad \begin{bmatrix}
.02 & .12 & -.13 & .07 \\
-.05 & -.19 & -.24 & .17 \\
-.08 & -.09 & -.39 & -.10 \\
-.07 & -.06 & .30 & -.03
\end{bmatrix} \quad \begin{bmatrix}
-.03 & -.03 & -.15 & -.01 \\
.08 & -.08 & -.01 & -.19 \\
.10 & -.14 & -.12 & -.32 \\
.10 & .18 & .17 & .02
\end{bmatrix}
\]

where each \((i,j)\) element is the cross correlation coefficient between each pair of error series when series \(i\) leads to series \(j\). These matrices show that no cross correlation is larger than two standard deviations (\(\pm 2/\sqrt{n} = \pm 0.40\)), so they are not statistically different from zero at the 95% level. Moreover, the likelihood ratio tests to examine the existence of autoregressive structure in the error series, presented in Table 3, indicates the absence of additional structure. These results suggest that the model (12)-(15) represents adequately the existing dynamic correlation structure.

Model (12)-(15) can also be written in compact notation as:
Equation (15) establishes the existence of lagged feedbacks from the private sector variables to public capital stock. In fact, this equation shows that the rate of growth of public capital formation responds slowly and positively to a disequilibrium situation in the previous period. That is, a positive value of $\text{ecm}_t$ (which occurs when output is above its equilibrium level) increases the rate of growth of public capital in the next period. This suggests that in the Spanish economy, the stock of public capital is procyclical: public investment is higher in periods of high productivity relative to periods of low productivity.

Response functions - As we have pointed in the previous section, the estimation of (1) and (2) permits the estimation of (4) and, therefore, the estimation of the impulse response functions.

Figures 1 and 2 show, respectively, the impulse response functions and the step response functions of the four variables to a transitory unit shock in the rate of growth of public capital stock. Tables 4.a and 4.b contain the values of these functions for the period 1993-2007.

Both figures show the slow convergence of all the variables to the equilibrium situation. For example, after 15 years (year 2007) the initial shock of 1 percentage point in $V_{pk}$ has decreased only to 0.3 points. This is essentially due to the presence of feedback effects between the public capital stock and the private sector variables.

For the period 1993-2000, the effects of an increase of 1 percentage point in $V_{pk}$ are:

i) An increase of 6.9 percentage points in the level of public capital stock.
ii) An increase of 3.1 percentage points in the level of private capital stock.
iii) An increase of 0.5 percentage points in the level of private employment.
iv) An increase of 2.8 percentage points in the level of output.

Whereas for the period 1993-2007, the effects are:

i) An increase of 9.8 percentage points in the level of public capital stock.
ii) An increase of 5.9 percentage points in the level of private capital stock.
iii) A decrease of 0.3 percentage points in the level of private employment.
iv) An increase of 4.7 percentage points in the level of output.
Specifically, an increase of one percentage point in $v_{PK}$ has a positive effect on the levels of $PK$, $K$ and $Y$ after both 8 and 15 years. However, the response of private employment is different. It is positive after 8 years but slightly negative after 15 years. This, together with the fact that the long run effect on employment is positive (as shown in Table 5.a), suggests that this variable presents a cyclical adjustment to the equilibrium level.

Table 5.a contains the long run equilibrium levels (after 97 years) for all the variables. The second column of this table shows the initial levels in 1992 (reference year), columns 3 contains the equilibrium levels to which the system would move in absence of future shocks (SO) and column 4 presents the equilibrium levels with a transitory unit shock in $v_{PK}$ in 1993 (SI). Table 5.b includes the same information for the inverse of factor productivities ($PK/Y$, $K/Y$, $L/Y$) and the inverse of public investment productivity ($P/I$). From these tables we can conclude that:

i) In absence of shocks (SO), the equilibrium levels of $PK$, $K$ and $Y$ are considerably higher than the (disequilibrium) values in 1992. However, the equilibrium level of $L$ is lower than the one registered in 1992. This reflects an inertia in the system to reduce the number of employments.

ii) In absence of shocks (SO), the system also tends to reduce public capital productivity and private capital productivity, whereas private labor productivity increases. This is consistent with the past evolution of these variables. Since 1964 public capital productivity and private capital productivity have been decreasing, while there has been a continued increase in the productivity of private labor.

iii) A transitory unit shock in the rate of growth of public capital stock (SI) leads to higher long run equilibrium levels of $PK$, $K$ and $Y$ than those obtained under the simulation SO. With respect to $L$, it should be noticed that although the equilibrium level of $L$ is lower than the value registered in 1992, it is higher than the level reached in absence of shocks. This result suggests that public investment has a substantial effect to reduce the number of employments that will be destroyed in the long run.

4. CONCLUSIONS

Using data on the Spanish economy for the period 1964-1992, we have analyzed the empirical effects of public investment in infrastructure on output, employment and capital stock in the private sector.

We have used a theoretical model which explicitly considers the dynamic relationships that may exist among the relevant variables. This framework is general enough to provide an adequate estimation of the effect of public investment on each of the private sector variables. It is important to note that this approach departs from previous analyses on this issue, which rely on the estimation of production functions and implicitly assume the absence of feedback relations.

Our main results are as follows:

i) The empirical analysis shows the existence of dynamic relationships among all the variables in the model. This stresses the inadequacy of the classic unequational framework.

ii) In fact, not only public capital formation affects the private sector variables, but also previous values of the private sector variables affect the current level of public capital stock. The response of the public capital stock is pro-cyclical; that is, it increases in periods in which previous values of output have been above the equilibrium level.

iii) There exists only one long term equilibrium relationship among the variables in the model. This equilibrium relationship can be interpreted as a Cobb-Douglas production function with constant returns to scale in all the inputs. This result is consistent with previous findings for the Spanish economy [see Bajo and Sosvilla (1993) and Argimón et al. (1993)].

iv) The response of the private sector variables to an increase in public capital stock is positive. In the long run, a transitory increase of one percentage point in the rate of growth of public capital stock implies a permanent increase in the levels of equilibrium of output, employment, private capital and public capital of 15, 0.5, 21 and 25 percentage points respectively. However, the response of
The system is very slow, since the equilibrium is reached almost one hundred years after the original shock. In the case of employment, the long term response is also cyclical.

v) The short run responses of the relevant variables are different depending on the definition of short run. After both eight and fifteen years the responses of output and private capital stock to a transitory unit shock in the rate of growth of the public capital stock are always positive. However, the response of labor may be either positive or negative, due to the fact that this variable presents a cyclical adjustment towards equilibrium.

REFERENCES


Argimón, I. and M. J. Martín (1993) "Series de stock de infraestructuras del estado y de las administraciones públicas en España", Servicio de Estudios del Banco de España, documento de trabajo n° 9315.


Table 1. Univariate and intervention models.

<table>
<thead>
<tr>
<th>Rates</th>
<th>$\mu$</th>
<th>$\phi$</th>
<th>$\sigma_0^2$</th>
<th>Q(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{V}_{1_r}$</td>
<td>-0.046</td>
<td>0.27</td>
<td>0.064</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.16)</td>
<td>(0.007)</td>
<td></td>
</tr>
<tr>
<td>$\bar{V}_{1_k}$</td>
<td>-</td>
<td>0.75</td>
<td>-</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{V}_{1_k}$</td>
<td>-</td>
<td>0.93</td>
<td>-</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{V}_{1_k}$</td>
<td>-</td>
<td>0.94</td>
<td>-</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{V}(y_{1-k})$</td>
<td>-0.03</td>
<td>-0.06</td>
<td>-0.056</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.15)</td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td>$\bar{V}(y_{1-k})$</td>
<td>-</td>
<td>0.65</td>
<td>0.023</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{V}(y_{1-k})$</td>
<td>-</td>
<td>0.56</td>
<td>0.019</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.010)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: (1) The model specification for all the variables is:

$$V_{1_t} = \omega_\phi + \eta_t$$

$$(1 - \phi B)[1 - \mu] = \sigma_0^2$$

(2) Standard deviations are in parenthesis, $\sigma_0^2$ is the residual standard deviation and Q(6) is the Ljung-Box statistic for six lags.
Table 2. Johansen test.

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>Statistic</th>
<th>critical (95%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>At most 3 cointegration relationships</td>
<td>0.49</td>
<td>8.08</td>
</tr>
<tr>
<td>At most 2 cointegration relationships</td>
<td>11.04</td>
<td>17.84</td>
</tr>
<tr>
<td>At most 1 cointegration relationship</td>
<td>27.14</td>
<td>31.26</td>
</tr>
<tr>
<td>0 relationship versus 1</td>
<td>37.66</td>
<td>27.34</td>
</tr>
</tbody>
</table>

Note: Specification AR(3) in levels with a constant.

Table 3. Likelihood ratio test.

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(0) versus AR(1)</td>
<td>10.48</td>
</tr>
<tr>
<td>AR(1) versus AR(2)</td>
<td>14.54</td>
</tr>
<tr>
<td>AR(2) versus AR(3)</td>
<td>13.00</td>
</tr>
</tbody>
</table>

Note: critical value at 95%: 26.3

Figure 1

IMPULSE RESPONSE FUNCTIONS
RATES OF GROWTH 1993-2007

Figure 2

STEP RESPONSE FUNCTIONS
LEVELS: 1993-2007
Table 4.a. Impulse response functions.

<table>
<thead>
<tr>
<th>Years</th>
<th>PK</th>
<th>K</th>
<th>L</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
<td>1</td>
<td>0</td>
<td>0.321</td>
<td>0.321498</td>
</tr>
<tr>
<td>1994</td>
<td>0.966</td>
<td>0.195</td>
<td>0.174068</td>
<td>0.459961</td>
</tr>
<tr>
<td>1995</td>
<td>0.957905</td>
<td>0.378699</td>
<td>0.12741</td>
<td>0.348363</td>
</tr>
<tr>
<td>1996</td>
<td>0.931147</td>
<td>0.46627</td>
<td>0.076883</td>
<td>0.338896</td>
</tr>
<tr>
<td>1997</td>
<td>0.879383</td>
<td>0.505644</td>
<td>0.014957</td>
<td>0.352026</td>
</tr>
<tr>
<td>1998</td>
<td>0.810375</td>
<td>0.520271</td>
<td>-0.04594</td>
<td>0.353374</td>
</tr>
<tr>
<td>1999</td>
<td>0.732212</td>
<td>0.517759</td>
<td>-0.09119</td>
<td>0.344079</td>
</tr>
<tr>
<td>2000</td>
<td>0.651498</td>
<td>0.503107</td>
<td>-0.12375</td>
<td>0.329568</td>
</tr>
<tr>
<td>2001</td>
<td>0.573578</td>
<td>0.480974</td>
<td>-0.14314</td>
<td>0.313014</td>
</tr>
<tr>
<td>2002</td>
<td>0.502524</td>
<td>0.455257</td>
<td>-0.14526</td>
<td>0.296253</td>
</tr>
<tr>
<td>2003</td>
<td>0.441077</td>
<td>0.428907</td>
<td>-0.13781</td>
<td>0.28052</td>
</tr>
<tr>
<td>2004</td>
<td>0.390713</td>
<td>0.404002</td>
<td>-0.12179</td>
<td>0.26607</td>
</tr>
<tr>
<td>2005</td>
<td>0.351801</td>
<td>0.381872</td>
<td>-0.10013</td>
<td>0.254924</td>
</tr>
<tr>
<td>2006</td>
<td>0.323827</td>
<td>0.363224</td>
<td>-0.07558</td>
<td>0.24557</td>
</tr>
<tr>
<td>2007</td>
<td>0.305627</td>
<td>0.348275</td>
<td>-0.05052</td>
<td>0.238415</td>
</tr>
</tbody>
</table>

Table 4.b: Step response functions.

<table>
<thead>
<tr>
<th>Years</th>
<th>PK</th>
<th>K</th>
<th>L</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
<td>1</td>
<td>0</td>
<td>0.321</td>
<td>0.321498</td>
</tr>
<tr>
<td>1994</td>
<td>1.966</td>
<td>0.195</td>
<td>0.495068</td>
<td>0.772481</td>
</tr>
<tr>
<td>1995</td>
<td>2.923905</td>
<td>0.573699</td>
<td>0.622478</td>
<td>1.120844</td>
</tr>
<tr>
<td>1996</td>
<td>3.855052</td>
<td>1.039959</td>
<td>0.699561</td>
<td>1.499741</td>
</tr>
<tr>
<td>1997</td>
<td>4.734435</td>
<td>1.545603</td>
<td>0.714318</td>
<td>1.811766</td>
</tr>
<tr>
<td>1998</td>
<td>5.54481</td>
<td>2.065874</td>
<td>0.67033</td>
<td>2.16514</td>
</tr>
<tr>
<td>1999</td>
<td>6.27022</td>
<td>2.583633</td>
<td>0.579169</td>
<td>2.592219</td>
</tr>
<tr>
<td>2000</td>
<td>6.92852</td>
<td>3.086739</td>
<td>0.455439</td>
<td>2.837878</td>
</tr>
<tr>
<td>2001</td>
<td>7.50298</td>
<td>3.567713</td>
<td>0.3314</td>
<td>3.151801</td>
</tr>
<tr>
<td>2002</td>
<td>8.004622</td>
<td>4.022971</td>
<td>0.168861</td>
<td>3.449854</td>
</tr>
<tr>
<td>2003</td>
<td>8.445509</td>
<td>4.451878</td>
<td>0.037854</td>
<td>3.728754</td>
</tr>
<tr>
<td>2004</td>
<td>8.836411</td>
<td>4.85588</td>
<td>-0.00975</td>
<td>3.995181</td>
</tr>
<tr>
<td>2005</td>
<td>9.188212</td>
<td>5.237752</td>
<td>-0.19088</td>
<td>4.250105</td>
</tr>
<tr>
<td>2006</td>
<td>9.512039</td>
<td>5.600976</td>
<td>-0.26646</td>
<td>4.495676</td>
</tr>
<tr>
<td>2007</td>
<td>9.817666</td>
<td>5.949251</td>
<td>-0.31698</td>
<td>4.734091</td>
</tr>
</tbody>
</table>

Table 5.a. Long run equilibrium in the year 2089 (variables in levels)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Initial Levels (1992)</th>
<th>Long Run Equilibrium Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>PK</td>
<td>1486.1</td>
<td>2253.9</td>
</tr>
<tr>
<td>K</td>
<td>23983.6</td>
<td>34954.7</td>
</tr>
<tr>
<td>L</td>
<td>10590</td>
<td>8347.1</td>
</tr>
<tr>
<td>Y</td>
<td>16768.7</td>
<td>21160.8</td>
</tr>
<tr>
<td>PI</td>
<td>196.3</td>
<td>115.5</td>
</tr>
</tbody>
</table>

Note: The numbers in parenthesis indicate the difference between the value to which they are referred (S1) and the equivalent value in column 3 (S0).

Table 5.b. Long run equilibrium in the year 2089 (inverses of the productivities)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Initial Levels (1992)</th>
<th>Long Run Equilibrium Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>PK/Y</td>
<td>0.089</td>
<td>0.107</td>
</tr>
<tr>
<td>K/Y</td>
<td>1.430</td>
<td>1.652</td>
</tr>
<tr>
<td>L/Y</td>
<td>0.632</td>
<td>0.394</td>
</tr>
<tr>
<td>PI/Y</td>
<td>0.012</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Note: The numbers in parenthesis indicate the difference between the value to which they are referred (S1) and the equivalent value in column 3 (S0).
SERIE DE DOCUMENTOS DE TRABAJO DEL ICAR

9301 "Análisis del Comportamiento de las Cotizaciones Reales en la Bolsa de Madrid bajo la Hipótesis de Eficiencia". Rafael Flores de Frutos. Diciembre 1992. (Versión final aceptada para publicación en Revista Española de Estadística Española)


