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THE PERFORMANCE OF THE MONETARY MODEL
OF EXCHANGE RATES

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ABSTRACT

The paper tests the ability of the monetary approach to explain the long-run behavior of the exchange rate in the G-7 bilateral relationships during the floating period. I use Johansen's (1988,1991) approach to test for the existence of a cointegrating relationship between the exchange rate, the money supply and real output. Consistent with previous results I find little evidence in support of the monetary model in the full sample (1973-1991). Further analysis reveals that one of the principal building blocks of the monetary model, the money demand equation, exhibited a significant degree of instability during the period. I use the tests prepared by Hansen (1992) to test for parameter instability in the money demand equation and to examine potential break points. The estimated break points are then used to define stable subsamples for each bilateral relationship. The subsample cointegration results provide strong support for the monetary approach in most non US bilateral models, indicating that the exchange rate responds with a lag, primarily to monetary shocks, and that long-run exchange rate homogeneity cannot be rejected in most instances. In addition, I compare the predictive performance of the error correction model against the predictions of a random walk model. In four of the six bilateral models for which forecasting comparison can be undertaken the error correction model produces more accurate one-quarter-ahead forecasts.

RESUMEN

Este trabajo estudia la capacidad del enfoque monetario para explicar el comportamiento a largo de los tipos de cambio bilaterales entre los G-7 durante el periodo de tipos flexibles. Utilizo el método de Johansen (1988,1991) para verificar la existencia de una relación de cointegración ente el tipo de cambio, la oferta monetaria y la producción real. Al igual que en trabajos anteriores encuentro muy poco apoyo para el modelo monetario en la muestra total (1973-1991). Un análisis más profundo revela que uno de los componentes principales del modelo, la ecuación de demanda de dinero, presenta un alto grado de inestabilidad durante el periodo. Utilizo los tests propuestos por Hansen (1992) para contrastar la estabilidad de los coeficientes de la ecuación de demanda de dinero y para estimar puntos de corte potenciales. Los puntos de corte estimados son utilizados para definir submuestras más estables en cada relación bilateral. El análisis de cointegración en las submuestras es consistente con el modelo monetario en la mayoría de las relaciones bilaterales que no incluyen al Dolar, indicando que el tipo de cambio responde con retraso a shocks eminentemente monetarios, y que la homogeneidad nominal a largo plazo del tipo de cambio no puede ser rechazada en muchos casos. Además, comparo la capacidad predictiva del modelo de error de corrección con las de un modelo de camino aleatorio. En cuatro de los seis modelos bilaterales en los que es posible hacer tal comparación, el modelo de error de corrección produce predicciones a un trimestre más precisas.
I. INTRODUCTION

The monetary model is often viewed as the benchmark model of exchange rate determination. Grounded in the work of Frenkel (1976) and Frenkel and Johnson (1978) it has received considerable theoretical attention over the years. Early empirical successes, however, have failed to be replicated in more recent sample periods. In particular, as Meese and Rogoff (1983a, 1983b) pointed out early on, the model's out-of-sample predictive performance during the eighties has been unable to improve upon the predictions of a simple random walk model. The more recent empirical research has focussed on the efficacy of the monetary model as a long-run equilibrium relationship rather than a model for the short-run fluctuations of the exchange rate. Nevertheless, results are still inconclusive. The empirical failure of the model has led numerous authors to reexamine its theoretical foundations; in particular, the purchasing power parity (PPP) assumption, the uncovered interest parity assumption, and the assumption of money demand stability. This paper tests the validity of the monetary approach as an equilibrium exchange rate model in G-7 bilateral relationships during the floating period. In particular, I examine how the empirical performance of the model is affected by instabilities in the money demand.

The monetary model of exchange rate determination links movements in the exchange rate to fluctuations in the money market. Shocks to the money supply or to the money demand are eventually transmitted to the exchange rate via the current account, such as in the flexible-price version of the model, or via the capital account as in the sticky-price version of the model. The money supply and real income, the so-called fundamentals, generally account for most of the shocks to the money market. The empirical evaluation of the monetary model has evolved in two different directions.

One line of research has tested the in-sample and out-of-sample fit of reduced-form monetary exchange rate equations. In spite of some earlier successes, e.g. Woo (1985), the empirical performance of the monetary model is often surpassed by that of a simple random walk model. Another line of research has tested instead the rational expectations parameter restrictions of the model. In general, the results in this area are more

1 In the flexible-price monetary model a disequilibrium in the money market leads to an instantaneous adjustment in prices and through PPP to an adjustment in the exchange rate. However, in the sticky price version of the model the same money market shock induces an exchange rate adjustment through uncovered interest parity.

2 See MacDonald and Taylor (1992a) for a recent survey on the subject.
positive. For example, Hoffman and Schlagenhauf (1983) and Finn (1986) fail to reject the parameter restrictions implied by rational expectations although the failure to reject those restrictions is often attributed to the low power of the tests.

The empirical failure of the monetary model has instigated a reexamination of its theoretical foundations. For example, the notion of purchasing power parity (PPP) typically assumed in the flexible-price version of the model has often been cited as a major limitation since it is usually rejected in empirical tests. The uncovered interest parity assumption is another potential reason for the empirical breakdown of the model. It is essential to the rational expectations solution of the model but will fail in the presence of less than perfect capital mobility or asset substitutability, for example, if risk premia are present. Recently, however, some authors, e.g. Boughton (1988), have emphasized instead the pernicious effects that money demand instability may have on the performance of the monetary model. Even though the issue of money demand instability has been extensively debated and tested with no conclusive results yet (see, for example, Boughton and Taylor (1991) for a recent multicountry analysis on the subject), it is reasonable to expect such instability in the uncertain economic environment and drastic policy changes of the last two decades.

Given the poor predictive performance of standard structural models of exchange rate determination, recent empirical modelling of exchange rates has shifted from fundamental to nonfundamental determinants such as bubbles or atheoretical (irrational) expectations. For example, Taylor and Allen (1992) report that a significant number of foreign exchange dealers use some form of chart analysis in formulating their short-run trading decisions. However, they also find that dealers tend to rely more on fundamental analysis in formulating long-term forecasts. These findings suggest that an empirical examination of the monetary model ought to concentrate on its validity as a long-run equilibrium model rather than a model for the short-run fluctuations of the exchange rate. New developments in the econometric analysis of long-run relationships have made this task possible. Particularly useful are the techniques developed by Engle and Granger (1987) and Johansen (1988, 1991) to estimate and test cointegrating relationships. Ballilie and Servaes (1987), Ballell and Pedreschi (1991) and MacDonald and Taylor (1991, 1992b) have employed these cointegration techniques to test for a long-run equilibrium relationship between the exchange rate and the fundamentals implied by the monetary model. In general, results have not been very supportive of the monetary model. A potential reason for the empirical rejection of the monetary model is the presence of parameter instability in the long-run exchange rate equation possibly induced by the aforementioned instability in money demand.

In this paper I reexamine the validity of the monetary approach as a model for the long-run behavior of the exchange rate with particular attention paid to the effects that money demand instability may have on the empirical performance of the model. I develop a general two country monetary model that allows for short-run price stickiness and doesn't require PPP as an assumption. Instead, long-run PPP is an implication of the model if nominal shocks are the main source of uncertainty in the foreign exchange market. In the presence of price stickiness, the short-run and long-run dynamics of the exchange rate differ. However, I will focus exclusively on the long-run (equilibrium) behavior of the exchange rate. Because all the variables involved are integrated of order 1 the model implies a cointegrating relationship between the exchange rate and the fundamentals. Using quarterly bilateral data for the floating period, 1973–1991, I find little evidence of cointegration in all 21 bilateral relationships among G–7 countries in the full sample. Consequently I examine the validity of the money demand equation used in the paper. Even though it appears the proposed specification is sufficient to capture the long-run behavior of money demand, parameter stability tests designed by Hansen (1992) provide a strong indication of instability in the money demand equation. I use the estimated break points to define periods of monetary stability for each bilateral relationship and then the monetary model is reestimated in the stable subsamples. A large number of bilateral exchange rate equations show signs of cointegration and the exchange rate, which responds with a lag to shocks to the fundamentals, exhibits in most cases monetary homogeneity. Furthermore, real shocks, as measured by real output, seem to play a weak role in explaining the long-run behavior of the nominal exchange rate. However, the model does not seem to provide a good characterization of the US dollar exchange rates.

II. A TWO COUNTRY STICKY PRICE MONETARY MODEL

This section derives the benchmark long-run exchange rate equation that is later estimated in the empirical section. I use a sticky-price, rather than a flexible-price, monetary model to derive the equilibrium exchange rate equation because it does not
require PPP as an assumption. Furthermore it serves to highlight the potential differences between the short-run and long-run (equilibrium) dynamics of the exchange rate.

Consider the following two country version of Dornbusch’s (1976) sticky-price monetary model:

\[ m_t - c_0 p_t = c_1 y_t - c_2 t_t \]  
\[ m_t^* - c_0 p_t^* = c_1 y_t^* - c_2 t_t^* \]  
\[ d_t = \beta_1 (s_t + \beta_2 (p_t^* - p_t)) + \beta_3 y_t - \beta_4 t_t + \beta_5 y_t^* \]  
\[ d_t^* = -\beta_1 (s_t + \beta_2 (p_t^* - p_t)) + \beta_3 y_t^* - \beta_4 t_t + \beta_5 y_t \]  
\[ \Delta p_{t+1} = \pi (d_t - y_t) + \Phi \]  
\[ \Delta p_{t+1}^* = \pi (d_t^* - y_t^*) + \Phi^* \]  
\[ i_t = i_t^* + \Delta e_{t+1} \]  

where the different variables represent:

- m: the domestic money supply in levels.
- p: the domestic price level.
- y: the domestic real potential output level.
- t: the domestic nominal interest rate.
- d: the real demand for domestic goods.
- s: the nominal exchange rate expressed in domestic currency units per unit of foreign currency.
- \( \Phi \): the domestic expected long-run inflation rate.

All the variables except the interest rates are expressed in logarithms, and asterisks indicate foreign country values. Equations (1) and (2) capture the domestic and foreign country money market equilibrium conditions, where \( c_0 \), \( c_1 \) and \( c_2 \) represent, respectively, the price elasticity, the real income elasticity and the interest rate semi-elasticity of money demand. Equations (3) and (4) describe the demand for domestic and foreign goods where \( \beta_1 \) measures the real exchange rate elasticity of goods demand, \( \beta_2 \) its own income elasticity, \( \beta_3 \) is the interest rate semi-elasticity and \( \beta_4 \) is the demand elasticity with respect to the other country’s income. In the presence of non traded goods the real exchange rate may not be the appropriate variable to characterize the competitiveness of domestic tradables. The parameter \( \beta_5 \) is introduced to capture such possibility via the non price homogeneity of the real exchange rate. Equations (5) and (6) describe the adjustment of each country’s price level as a function of the excess demand in their corresponding goods markets and the expected long-run inflation rate. Finally, equation (7) represents the uncovered interest parity condition where expectations are assumed to be formed rationally, \( \Delta s_{t+1} = E_t [s_{t+1}] - s_t \) and \( E_t \) is the mathematical expectations operator conditional on information at time \( t \). For notational economy I assume a symmetric model with identical elasticities in both countries.

Using both countries money market equilibrium conditions together with uncovered interest parity, a conditional expression for exchange rate expectations is derived. In an analogous fashion, substituting the interest rate from the money market equilibrium condition into the goods market demand equation leads to an expression for each country’s price level adjustment as a function of current endogenous and fundamental variables. The dynamic system is represented as a multivariate first order expectation difference equation:

\[
\begin{pmatrix}
    p_{t+1} \\
    p_{t+1}^* \\
    E_t [s_{t+1}]
\end{pmatrix} =
\begin{pmatrix}
    1 - \pi (\beta_1 \beta_5 + \alpha_2 \alpha_1) & \pi \beta_1 \beta_5 / \alpha_2 & \pi \beta_1 / \alpha_2 \\
    \pi \beta_1 / \alpha_2 & 1 - \pi (\beta_1 \beta_5 + \alpha_2 \alpha_1) & -\pi \beta_1 / \alpha_2 \\
    \pi \beta_1 / \alpha_2 & -\pi \beta_1 / \alpha_2 & 1
\end{pmatrix}
\begin{pmatrix}
    p_t \\
    p_t^* \\
    s_t
\end{pmatrix} +
\begin{pmatrix}
    \pi / \alpha_1 \\
    \pi / \alpha_2 \\
    \pi / \alpha_2
\end{pmatrix}
\begin{pmatrix}
    \pi / \alpha_1 \\
    \pi / \alpha_2 \\
    \pi / \alpha_2
\end{pmatrix} \begin{pmatrix}
    m_t \\
    m_t^* \\
    y_t
\end{pmatrix} + \begin{pmatrix}
    \pi / \alpha_1 \\
    \pi / \alpha_2 \\
    \pi / \alpha_2
\end{pmatrix}
\begin{pmatrix}
    \pi / \alpha_1 \\
    \pi / \alpha_2 \\
    \pi / \alpha_2
\end{pmatrix} \begin{pmatrix}
    t_t \\
    t_t^* \\
    y_t^*
\end{pmatrix}.
\]

Equation (8) describes the joint dynamic behavior of prices and the exchange rate conditional on the behavior of the fundamentals (money supplies and real outputs). In a sticky-price environment, exchange rate dynamics can be interpreted as arising from two different sources: the dynamics of the fundamentals and the dynamics of price adjustment. The first source determines the equilibrium behavior of exchange rates while the second source determines its disequilibrium behavior. The flexible-price version of the monetary model can be interpreted as emphasizing the equilibrium dynamics of exchange rates while traditional versions of the sticky-price monetary model, e.g. Dornbusch (1976), can be interpreted as emphasizing the disequilibrium behavior.

The equilibrium path of the endogenous variables are derived from (8) by imposing...
Let $A$ be the following form:

$$A = \begin{pmatrix}
1 & 0 & \alpha_1 & \alpha_2 \\
0 & 1 & \alpha_3 & \alpha_4 \\
0 & 0 & 1 & \alpha_5 \\
0 & 0 & 0 & 1
\end{pmatrix},$$

$$B = \begin{pmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{pmatrix},$$

$$J = \begin{pmatrix}
J_1 \\
0
\end{pmatrix},$$

where $J_1$ and $J_2$ are $(2 \times 2)$ and $(1 \times 1)$ diagonal matrices respectively. For the system to have a unique solution, $J_1$ must contain all eigenvalues inside the unit circle and $J_2$ all of them on or outside. 4 Provided the uniqueness condition is satisfied the system's solution, for $t > 0$ is then (see Blanchard and Kahn (1980)):

$$p_t = B_{11} J_1 B_{11}^{-1} p_{t-1} + \Gamma_1 z_{t-1}$$

$$- A_{12} C_{22}^{-1} \sum_{i=0}^{\infty} z_{t-i} - (C_{21} \Gamma_1 + C_{22} \Gamma_3) E_{t-1}[Z_{t+1}^2],$$

(11)

and

$$s_t = C_{22}^{-1} C_{21} p_t$$

$$- C_{22}^{-1} \sum_{i=0}^{\infty} z_{t-i} - (C_{21} \Gamma_1 + C_{22} \Gamma_3) E_t[Z_{t+1}^2].$$

(12)

In order to obtain explicit solutions for equations (11) and (12) the dynamic behavior of the fundamentals has to be specified. For example, assume that $Z_t$ is appropriately characterized by an autoregressive process of order $p$:

$$Z_t = \Theta_1 + \Psi_1 Z_{t-1} + \Psi_2 Z_{t-2} + \ldots + \Psi_p Z_{t-p} + \epsilon_t.$$  

(13)

It is shown in the appendix that the infinite sums of conditional expectations in (11) and (12) will also be characterized by an AR($p$) process. Consequently, equations (11) and (12) can be rewritten in general form as:

$$p_t = A_{11} \Theta + A_{12} \Gamma_1 + A_{13} \Gamma_2 + A_{14} \Gamma_3 + \ldots + A_{1p} \Gamma_p + \epsilon_t,$$

(14)

$$s_t = A_{21} \Theta + A_{22} \Gamma_1 + A_{23} \Gamma_2 + A_{24} \Gamma_3 + \ldots + A_{2p} \Gamma_p + \epsilon_t,$$

(15)

where the parameter matrices ($\Theta, A, \Gamma$) are functions of the structural parameters in (1)–(7) and the autoregressive coefficients in (13). Finally, substituting (13) and (14)

3 Simulation exercises conducted in another paper, Peruga and Wong (1991), have shown that the system has a unique solution for all reasonable structural parameterizations of the model.
in the right hand side of equation (16) leads to an expression that characterizes the joint dynamic behavior of the vector of endogenous, exogenous and predetermined variables, $X_t$, as a multivariate VAR of order $p$:

$$X_t = \Theta + \Gamma_1 X_{t-1} + \Gamma_2 X_{t-2} + \ldots + \Gamma_p X_{t-p}. \quad (16)$$

If expectations are formed rationally, then system (16) satisfies a set of zero and cross equation parameter restrictions. Unfortunately, given that the set of zero and cross-equation restrictions are difficult to derive in single-equation models of exchange determination, I am unable to derive the parameter restrictions in the current multivariate framework. In addition, as shown in DeJong and Husted (1990), traditional tests for rational expectations parameter restrictions tend to exhibit very little power and are, therefore, of little practical interest. There are, however, some interesting consequences of equations (13)-(15) that can be tested and that distinguish the latter model from previous models.

First, system (16) captures the joint dynamic behavior of endogenous ($x_t$), exogenous ($m_t, m^*_t, y_t, g_t^*$) and predetermined variables ($p_t, p^*_t$) while system (9) represents the long-run equilibrium relationship between these same variables. Because all the variables in the model are nonstationary and likely to be integrated of order one the three equations in (9) can be interpreted as the three cointegrating relationships imbedded in system (16). Several authors, e.g. Baillie and Selover (1987), Baillie and

4 See Hoffman and Schmidt (1981) and Hoffman and Schlaghaus (1983) for a example on how to derive the rational expectations parameter restrictions.

5 Alternatively, the system can be formulated in terms of the interest rate variables rather than the price levels by substituting equations (1) and (2) into systems (8) and (9). In particular, the long-run equilibrium solution becomes:

$$\begin{pmatrix} \tilde{e}_t \\ \tilde{g}_t \\ \tilde{y}_t \end{pmatrix} = \begin{pmatrix} 0 & \frac{\beta_1 + \delta_1}{\delta_2} & \frac{\beta_1 + \delta_1}{\delta_2} \\ 0 & \frac{\beta_1 + \delta_1}{\delta_2} & \frac{\beta_1 + \delta_1}{\delta_2} \\ \frac{\beta_1 + \delta_1}{\delta_2} & \frac{\beta_1 + \delta_1}{\delta_2} & \frac{\beta_1 + \delta_1}{\delta_2} \end{pmatrix} \begin{pmatrix} \bar{e}_t \\ \bar{g}_t \\ \bar{y}_t \end{pmatrix}, \quad (9')$$

6 The somewhat restrictive conditions under which systems (9) and (16) are derived can be easily relaxed without loss of generality. For example, the cross-country symmetry assumption can be dropped, the price level in the money market equation can be replaced by a CPI expression weighting the domestic currency price of domestic and

Pescimino (1991) and, MacDonald and Taylor (1991, 1992b), have examined the long-run validity of the monetary model by testing for cointegration in systems similar to the one presented in (16). In general, their empirical results fail to uncover evidence of cointegration among the principal dollar exchange rates during the current floating period. Second, the flexible-price monetary model frequently used in the empirical literature assumes purchasing power parity and typically implies that foreign and domestic money supplies and outputs are sufficient information to describe both the short-run and long-run behavior of the exchange rates. However, in the sticky-price version presented in the paper prices (or interest rates) should be included in the specification of the exchange rate equation to help explain its short-run behavior even though predetermined variables contain no additional information regarding the long-run behavior of the exchange rate. Finally, if either PPP is not satisfied or if excess demand for goods play no role in price adjustment ($\pi = 0$), both the flexible and the sticky-price monetary models will imply that prices play a significant role in explaining the short and the long-run behaviors of the exchange rate.

7 Macdonald and Taylor (1991, 1992b) find evidence of at least one cointegrating vector in the model comprised by the nominal exchange rate and the domestic and foreign monies supplies and real outputs (ocasionally interest rates are also included). However, a significant cointegration result is not necessarily supportive evidence for the monetary model. First, the size of the test for cointegration may be seriously distorted given that the authors use asymptotic rather than adjusted critical values. The finite sample properties of Joansen's statistics can be very sensitive to the ECM's lag length. Second, the estimated cointegrating vector is nowhere to be found in their papers. Therefore, one can never evaluate how reasonable the estimates are. Third, no tests for the exclusion of variables is conducted in the unrestricted system. Consequently, it is not clear which subset of variables is responsible for the cointegration results. Finally, even if the nominal exchange rate is part of the cointegrating relationship, the estimated cointegrating vector may not represent an equilibrium monetary exchange rate equation. To make such a claim a further test of the endogeneity of the exchange rate in the cointegrated system is needed. The authors do not conduct such a test.

8 Equation (14) can always be inverted and substituted into (15) to yield an expression of the short-run dynamics of the exchange rate involving only current and past information about the fundamentals. However, that representation of the system will typically involve an infinite autoregressive process and thus, be much less parsimonious than the expression involving the fundamentals and the predetermined variables.
III. ECONOMETRIC METHODOLOGY

This section describes the econometric methodology used to estimate the long-run exchange rate equation (9) developed in the previous section. I use Johansen's (1988, 1991) approach to test for cointegration between the exchange rate, the money supplies, output levels and possibly the price levels (interest rates), of the G-7 countries during the floating period. Note that the equilibrium relationships in (9) and (9') imply the existence of three different cointegration relationships between the seven variables. To reduce the dimensionality of the problem I assume a perfectly symmetric world so that only four variables have to be considered: the exchange rate ($x_t$), the relative price ($p_t - p_{t-1}$) (or the interest rate differential ($i_t - i_{t-1}$)), the relative money supply ($m_{t-1} - m_t$) and the real output ($y_t$). I begin by analyzing the benchmark system commonly used in the literature involving the exchange rate and the two fundamentals, the relative money supply and the relative output level; that is, excluding the predetermined variables. In the event that cointegration between this subset of variables is rejected, then test whether or not prices (or interest rates) have a long-run relationship. This sequential procedure could potentially help discriminate between the alternative predictions of the monetary model. Finally, if cointegration is not found in the expanded system the simplifying symmetry constraints initially adopted are then tested.

Cointegration between the exchange rate and a set of variables is not necessarily evidence of an exchange rate determination equation since the long-run relationships could be interpreted in other ways. For example, such a relationship could be also interpreted as a money supply reaction function where the money supply responds to fluctuations in the exchange rate, perhaps in order to target the current account. To distinguish between the alternative interpretations one is required to examine the exogeneity of the exchange rate in the long-run relationship. If the exchange rate is weakly exogenous then the equation cannot be interpreted as supporting evidence for the monetary model of exchange rate determination. Fortunately, the issue of exogeneity is readily explored in the context of Johansen's approach.

Let $H_t$ be a $(k \times 1)$ vector of nonstationary I(1) variables whose dynamic behavior

The variables in $H$ are said to be cointegrated if there is a linear combination of them, $\beta H_t$, that is stationary ($I(0)$). $\beta$ is known as the cointegrating vector. If the system $H$ is cointegrated, then the rank($\Pi$) = $q$ < $k$, and there exist ($k \times q$) matrices $\alpha$ and $\beta$ such that $\Pi = \alpha \beta'$. $q$ is the dimension of the space of cointegrating vectors $\beta$, and the $\alpha$'s are the vectors of adjustment coefficients. In most cases $q$ will be 1 and there is a unique cointegrating vector representing a stable long-run relationship between the variables in $H$.

Johansen (1988) has developed a maximum-likelihood procedure based on canonical correlation theory to estimate the coefficients in $H$. He also provides the mathematical foundation to conduct statistical tests in nonstationary environments. The ordered eigenvalues $\lambda_1 > \ldots > \lambda_k$ from $\Pi$ play an important role in the testing for cointegration as well as in the testing of parameter restrictions. Two statistics are available to test for cointegration and help determine the rank of $\Pi$. The first statistic (MAXEIG) tests the unconditional significance of individual eigenvalues. The second statistic (TRACE) tests the conditional significance of the ordered eigenvalues, for example, $\lambda_1 > 0$ given $\lambda_{n+1} = \lambda_{n+2} = \ldots = \lambda_k = 0$. Asymptotic critical values for these statistics are available in Osterwald-Lenum (1990); however, because of the small sample sizes used in this empirical study and the potential size distortions in the cointegration statistics arising from the lag length choice in the ECM, appropriately adjusted critical values have been computed. Restrictions on the $\alpha$'s and $\beta$'s can also be easily tested comparing the eigenvalues from the constrained and unconstrained models through a likelihood

is captured by the following autoregressive model:

$$ H_t = \Pi_1 H_{t-1} + \Pi_2 H_{t-2} + \ldots + \Pi_p H_{t-p} + \epsilon_t, $$

where the $\epsilon$'s are IID$(0, \Sigma)$. The system can be written in first-differenced form as:

$$ \Delta H_t = \Pi_1 H_{t-1} + \Pi_2 \Delta H_{t-1} + \Pi_3 \Delta H_{t-2} + \ldots + \Pi_{p-1} \Delta H_{t-p+1} + \epsilon_t, $$

where

$$ \Pi_i = -(\Pi_{i+1} + \Pi_{i+2} + \ldots + \Pi_p). \quad (i = 1, \ldots, p - 1), $$

and

$$ \Pi = \Pi_1 + \Pi_2 + \ldots + \Pi_p. $$

The variables in $H$ are said to be cointegrated if there is a linear combination of them, $\beta H_t$, that is stationary ($I(0)$). $\beta$ is known as the cointegrating vector. If the system $H$ is cointegrated, then the rank($\Pi$) = $q$ < $k$, and there exist ($k \times q$) matrices $\alpha$ and $\beta$ such that $\Pi = \alpha \beta'$. $q$ is the dimension of the space of cointegrating vectors $\beta$, and the $\alpha$'s are the vectors of adjustment coefficients. In most cases $q$ will be 1 and there is a unique cointegrating vector representing a stable long-run relationship between the variables in $H$.

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where

$$ \Pi_i = -(\Pi_{i+1} + \Pi_{i+2} + \ldots + \Pi_p). \quad (i = 1, \ldots, p - 1), $$

and

$$ \Pi = \Pi_1 + \Pi_2 + \ldots + \Pi_p. $$

The variables in $H$ are said to be cointegrated if there is a linear combination of them, $\beta H_t$, that is stationary ($I(0)$). $\beta$ is known as the cointegrating vector. If the system $H$ is cointegrated, then the rank($\Pi$) = $q$ < $k$, and there exist ($k \times q$) matrices $\alpha$ and $\beta$ such that $\Pi = \alpha \beta'$. $q$ is the dimension of the space of cointegrating vectors $\beta$, and the $\alpha$'s are the vectors of adjustment coefficients. In most cases $q$ will be 1 and there is a unique cointegrating vector representing a stable long-run relationship between the variables in $H$.

Johansen (1988) has developed a maximum-likelihood procedure based on canonical correlation theory to estimate the coefficients in $H$. He also provides the mathematical foundation to conduct statistical tests in nonstationary environments. The ordered eigenvalues $\lambda_1 > \ldots > \lambda_k$ from $\Pi$ play an important role in the testing for cointegration as well as in the testing of parameter restrictions. Two statistics are available to test for cointegration and help determine the rank of $\Pi$. The first statistic (MAXEIG) tests the unconditional significance of individual eigenvalues. The second statistic (TRACE) tests the conditional significance of the ordered eigenvalues, for example, $\lambda_1 > 0$ given $\lambda_{n+1} = \lambda_{n+2} = \ldots = \lambda_k = 0$. Asymptotic critical values for these statistics are available in Osterwald-Lenum (1990); however, because of the small sample sizes used in this empirical study and the potential size distortions in the cointegration statistics arising from the lag length choice in the ECM, appropriately adjusted critical values have been computed. Restrictions on the $\alpha$'s and $\beta$'s can also be easily tested comparing the eigenvalues from the constrained and unconstrained models through a likelihood

is captured by the following autoregressive model:

$$ H_t = \Pi_1 H_{t-1} + \Pi_2 H_{t-2} + \ldots + \Pi_p H_{t-p} + \epsilon_t, $$

where the $\epsilon$'s are IID$(0, \Sigma)$. The system can be written in first-differenced form as:

$$ \Delta H_t = \Pi_1 H_{t-1} + \Pi_2 \Delta H_{t-1} + \Pi_3 \Delta H_{t-2} + \ldots + \Pi_{p-1} \Delta H_{t-p+1} + \epsilon_t, $$

where

$$ \Pi_i = -(\Pi_{i+1} + \Pi_{i+2} + \ldots + \Pi_p). \quad (i = 1, \ldots, p - 1), $$

and

$$ \Pi = \Pi_1 + \Pi_2 + \ldots + \Pi_p. $$

The variables in $H$ are said to be cointegrated if there is a linear combination of them, $\beta H_t$, that is stationary ($I(0)$). $\beta$ is known as the cointegrating vector. If the system $H$ is cointegrated, then the rank($\Pi$) = $q$ < $k$, and there exist ($k \times q$) matrices $\alpha$ and $\beta$ such that $\Pi = \alpha \beta'$. $q$ is the dimension of the space of cointegrating vectors $\beta$, and the $\alpha$'s are the vectors of adjustment coefficients. In most cases $q$ will be 1 and there is a unique cointegrating vector representing a stable long-run relationship between the variables in $H$.
ratio test. Furthermore, under the null of cointegration these statistics are distributed asymptotically as a \( \chi^2 \).

To test the lag specification of the error correction model I use a statistic proposed by Hosking (1980). This statistic is a multivariate extension of the better known Ljung-Box statistic applied to univariate time series. It is distributed asymptotically as a \( \chi^2 \), with the degrees of freedom being determined by the dimension of the system, \( k \), the number of lags estimated in the error correction model, \( p \), as well as the number of lagged correlation matrices used to compute the statistic (8 in the paper). Finally, to avoid an overparametrization of the ECM I have used whenever necessary a seasonal dummy rather than a seasonal lag. 10

While the theoretical foundations for the analysis of structural breaks in stationary environments have long been established, the theoretical foundations required to analyze structural breaks in non-stationary environments are still in their early stages. I use the approach recently developed by Hansen (1992) to test for structural breaks in a cointegrated relationship and to estimate the potential break point. Hansen derives the asymptotic distribution of four different Lagrange multiplier tests for parameter instability. The first statistic, \( F_{p1} \), tests the null of no structural breaks against the alternative of a known break point in the same spirit as the traditional Chow test. The second statistic, \( F_{p2} \), is the maximum of the \( F_{p1} \) statistics over all possible break points \( (\tau \in (0, 1)) \). In practice the range of potential break points is limited to a subset, e.g. \([.15,.85]\). Therefore, the \( F_{p2} \) statistic is designed to have power against the alternative of a unknown single break point. The other two statistics, \( F_{p3} \) and \( F_{p4} \), are designed to have power against the alternative that one or several parameters in the cointegrating vector follow a martingale process. This alternative more accurately captures parameter changes that take place over time rather than instantaneous changes. The interested reader is directed to Hansen's paper for a detailed derivation of the statistics and a sample of their tabulated asymptotic distributions. 11

10 A reduction in the dimensionality of the ECM model is always desirable since for a given sample size the power of the cointegration test will decrease with the dimension of the system.

11 Gregory and Nason (1991) have studied the size and power of Hansen's tests in a cointegrated linear quadratic adjustment model. The simulation experiments indicate a reasonable performance. I would like to thank them for providing me with the computer programs used in their paper.

IV.- EMPIRICAL RESULTS

The monetary model of exchange rate determination, in both the sticky and flexible price version, implies the existence of a stable long-run relationship between the nominal exchange rate and the fundamentals. I begin the empirical analysis by testing for cointegration in a benchmark symmetric monetary model that includes the logs of the nominal exchange rate, the relative money supply and the relative output level. A total of 21 bilateral models between the G-7 countries (the United States (US), Canada (CN), Japan (JP), France (FR), Italy (IT), the United Kingdom (UK) and the former West Germany (WG)) are studied using quarterly data from 1973 to 1991 with a total of 76 observations. In what follows I represent a bilateral relationship using the initials of the two countries involved. For example, JPFR represents the bilateral model of Japan and France. All the data was obtained from the IMF's International Financial Statistics (IFS). The nominal exchange rate (IFS line "...ag") is the end of the quarter quotation measured in US dollars per unit of foreign currency. The cross rate between two non US currencies is obtained from their dollar exchange rate using triangular arbitrage. The money stock measure is the seasonally adjusted M1 figure (IFS line "...a", except for France and the United Kingdom where it is unadjusted (IFS line "...a")). The seasonally adjusted real output measure is either GNP (IFS line "99a"," or GDP (IFS line "99b"," at constant prices, depending on availability. Finally, the CPI line (IFS line "...a") is used as a measure of the price level.

Table 1 presents the cointegration results for the benchmark monetary model. Of the 21 bilateral models only 6 show signs of cointegration and among these only two, USIT and CNIT, present (positive) money supply elasticities consistent with the theory. This absence of cointegration confirms previously reported results. In an attempt to evaluate the sources of potential failure of the monetary model researchers have tested separately each of the model's building blocks. For example, they have tested the PPP condition assumed in most flexible-price models, or the uncovered interest parity, or the specification of the money demand equation. Since PPP is not a building block in our model and evidence of a sizeable risk premium is weak I concentrate on the latter condition. There has been a considerable effort in the literature to estimate and test the existence of stable long-run money demand equation. For example, Johansen and Juselius (1990), Boughton and Tavlas (1991), and Hoffman and Rache (1991) are some
recent applications of cointegration to the analysis of money demand. These authors find strong evidence of cointegration between real balances, real incomes and short term interest rates.

Table 2 presents the cointegration results for the money demand equation (1) in each of the G-7 countries. The TRACE and MAXEIG statistics show evidence of cointegration in all 7 equations. However, in three of these equations, FR, UK and WG, income elasticities fall outside commonly accepted values. A possible explanation for the unsatisfactory results is the potential instability of money demand in a period that witnessed large shifts in inflation and interest rates, relatively large output fluctuations and frequent changes in monetary policy to accommodate such shocks. The instability of money demand during the floating period is evidenced in Table 2. The bottom part of the Table presents the results from a series of parameter stability tests for cointegrating relationships suggested by Hansen (1992). Four of the models, US, CN, JP and FR, show strong indications of parameter instability, while the Italian money demand equation shows only mild evidence of structural breaks. Table 2 also presents an estimate of the break point. 13

The five unstable equations were recomputed for the largest subsamples defined by the estimated break points. In addition the German money demand equation was reestimated excluding the interest rate variable since it appears to have no long-run explanatory power in the cointegration relationship. 14 The subsample results are also

12 The sample size for Germany extends only until the fourth quarter of 1989 because of the reunification. For the United Kingdom the sample extends only to the first quarter of 1989 when accounting procedures where changed.

13 The parameter instability detected by Hansen's tests seem to suggest slow rather than drastic changes in money demand; therefore, the break point estimates need not be associated with specific events. However, some of these estimates appear to have a reasonable interpretation. For example, M1 velocity in the US became progressively more unpredictable in 1985 and 1986 where actual growth rates notably diverged from the target rates eventually leading the Fed to abandon M1 targets in 1987. In the case of Canada it appears that a similar targeting problem took place between 1975 and 1981. In particular, two major financial innovations took place in 1975 and 1981, dates closely linked to the two estimated break points 1981:1 and 1987:2. The estimated break point for France appears to coincide with a change in accounting procedures in October of 1977. In the case of Italy the instability apparently arises as a consequence of high volatility in all the equation variables during the early and mid-seventies. Finally, the break point in the Japanese money demand equation is similar to the one obtained in Boughton and Taylor (1991).

14 A possible reason is that the Bundesbank has traditionally targeted only their M3 stock (close to the traditional M2 measure in other countries). This argument may also account for the instability of the US and UK money demands, where monetary authorities changed from an M1 to an M2 target during the 1980's.

15 The small sample properties of these tests have not been fully explored. Therefore, the subsample results should be interpreted with caution specially, since some of the estimated break points are in the sample margins.

Table 3 presents the cointegration results for the stable subsamples. The first column in Table 3 shows the estimated bilateral exchange rate model. It should be noted that none of the bilateral models involving the US dollar are included since they showed no evidence of cointegration or reasonable parameter estimates. 16 The second column defines the extent of the stable subsample. Occasionally, a particular break point was ignored to maintain the minimum subsample size of 40 quarterly observations as long as the parameter estimates and test statistics were consistent with the theory. The third and fourth columns present the estimates of \( \beta_1 \) and \( \beta_2 \), the relative money supply and relative income elasticities of the exchange rate. In many bilateral models only the estimate of \( \beta_1 \) is presented. This indicates that the relative output level was found to contain no relevant information regarding the long-run behavior of the exchange rate (the restriction \( \beta_2 = 0 \) could not be rejected) and nominal shocks alone appear to drive exchange rate movements in such models. This feature has important consequences for the possible stationarity of the real exchange rate.

The fifth and sixth columns present the estimate for the adjustment coefficient in

16 Given the strong instability of the US money demand equation and the international currency nature of the dollar the monetary approach may not be a suitable model to explain the behavior of US exchange rates, largely dominated by US rather than foreign factors.
the exchange rate equation, $a_t$, and a likelihood ratio test for its statistical significance, $\text{LRT}_a$. The adjustment coefficient measures the speed (in quarterly percentage rates) at which the exchange rate responds to deviations from its long-run equilibrium value. If the adjustment coefficient is negative and statistically significant it implies that the exchange rate is endogenous within the benchmark system and the cointegrating relationship can be appropriately interpreted as an exchange rate equation. A typical result in monetary models is the homogeneity of degree one of equilibrium exchange rate with respect to the money supply ($\beta_1 = 1$). The column labeled $\text{LRT}_b$ presents a likelihood ratio test for the null hypothesis $\beta_1 = 1$. Finally, the last two columns present the statistics $\text{MAEIG}$ and $\text{TRACE}$ proposed by Johansen to test the null of no cointegration. 16

Of the 15 non US bilateral models considered in the stable subsample study, 9 models now show strong evidence of cointegration while in another 2 models the statistics are significant at the 80% level. 19 In fact, UKWG is the only model for which no cointegration is clearly not rejected. In only 5 of the 15 bilateral models does the relative output level appear to contribute to the estimation of the long-run behavior of the exchange rate. Furthermore, the $\text{LRT}_b$ statistic can only reject the money supply homogeneity of exchange rates in 4 of the models. The adjustment coefficient in the exchange rate equation is always negative and statistically significant in 12 of the models. This coefficient indicates that each quarter exchange rates tend to close between 10 to 30 percent of the disequilibrium gap.

To conclude the analysis the Hansen parameter stability tests were computed for

17 Both $\text{LRT}_a$ and $\text{LRT}_b$ are asymptotically distributed $\chi^2_1$ under the null. See Johansen and Juselius (1990) for a derivation of the statistics. 18 Asymptotic critical values for these statistics are taken from Osterwald-Lenum (1990).

<table>
<thead>
<tr>
<th>nvar = 2</th>
<th>nvar = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>80%</td>
<td>90%</td>
</tr>
</tbody>
</table>

Tests for the null hypothesis of no cointegration traditionally exhibit low power against cointegrated alternatives.

The forecasting comparison results are presented in Table 5. In four of the bilateral

the exchange rate equations in the stable subsamples. There is strong evidence of parameter instability in 6 of the 15 models and weak evidence for the ITUK model. Three of these models involve the Canadian dollar (CNJP, CNFR and CNWG) while four involve the French franc (FRCN, FRFP, FRUK and FRWG). However, in most of the models the estimated break point lies in the sample edges, making the test results less reliable. Whenever enough data was available the models were reestimated for the appropriate subsamples. The cointegration results for five of these models are presented in Table 4 and reveal only minor improvements in the estimates. Noteworthy is the fact that the income variable becomes insignificant in the two models where it was initially present.

I now turn to the evaluation of the predictive power of the estimated error correction models. Ever since Meese and Rogoff's pioneer work, the main validation criteria for empirical models of exchange rate determination has been their ability to predict out-of-sample and, in particular, their ability to improve upon the predictions of a simple random walk model. Given the limited availability of data only one forecasting exercise is conducted. In this exercise the estimated ECM is used to generate a set of one-quarter-ahead forecasts for each of the 1991 quarters. That is, data up to 1990 is used to forecast the first quarter of 1991, then this observation is added to the data set and the ECM is reestimated to generate a forecast for the second quarter. The sequential procedure is performed for all four quarters. The ECM forecasts are compared to the forecasts from a random walk model (RW), that is, they are compared with the lagged exchange rate value.

Following Meese and Rogoff's analysis, I use three statistics to compare the forecasting accuracy of the two models: the mean error (ME), the mean absolute error (MAE) and the root mean square error (RMSE). In order to conduct the forecasting exercise two conditions have to be met. First, the stable subsample has to contain a minimum number of observations so that the precision of parameter estimates doesn't become the major factor in forecast errors. Second, the samples over which forecasts and parameter estimates are computed have to exhibit similar characteristics. Because of changes in data definition occur the United Kingdom (89:2) and Germany (91:1), models involving the two countries could not be used in the forecasting analysis, leaving only the six bilateral models involving Canada, Japan, France and Italy.

The forecasting comparison results are presented in Table 5. In four of the bilateral
models (CNJP, CNFR, CNIT and JPIT), the ECM forecasts outperform those from the random walk in terms of both the MAE and RMSE statistics. In the JPFR model the results are mixed while in the FRIT model the random walk provides superior estimates. The poor forecasting performance of the ECM in the JPFR and FRIT models can be possibly explained by looking back at Table 4. The results in Table 4 indicate that cointegration in the JPFR model weakens significantly when the last observations of the subsample are dropped, an indication of parameter instability. In addition, France and Italy are both in the EMS and their exchange rate fluctuates within a target zone that experienced no realignment during the forecasting period. Divergence in monetary growth between the two countries will lead to a currency realignment only in the long run. In the short run foreign exchange market intervention will maintain the exchange rate inside the target zone. In general, the ECM model will not exhibit good forecasting properties when adjustments towards equilibrium are discrete rather than continuous.

In summary, when the long-run exchange rate equation is estimated for subsamples where money demand is perceived to be stable in both countries, the empirical results are very much in accordance to the predictions of the monetary model. First, the variables in the benchmark model are cointegrated indicating that the fundamentals fully account for the long-run behavior of the exchange rate. Second, it appears that nominal shocks are the basic source of uncertainty in the foreign exchange market and in many instances the traditional monetary homogeneity of exchange rates cannot be rejected. Third, the estimated long-run relationship can be truly interpreted as an exchange rate equation since it appears that the exchange rate is endogenous as it slowly adjusts to close the gap between actual and equilibrium values. Finally, the forecasts from the estimated error correction model outperformed the forecasts from a random walk model in four of the six bilateral relationships where the forecasting comparisons could be conducted.

Two issues remain to be explored. The first issue relates to the finding that monetary shocks are the preeminent source of uncertainty in exchange rate markets. This result has important implications in regard to the long-run behavior of the real exchange rate. If the monetary model, the real exchange rate is expected to exhibit a stationary behavior when nominal shocks are the only source of uncertainty. This is in effect an empirically testable implication that deserves further analysis. Second, the long-run exchange rate equation estimated in the paper is consistent with a variety of versions of the monetary model. In principle, both the flexible-price and sticky-price versions are compatible with the empirical findings. It is thus of potential interest to investigate which of the two versions is more empirically plausible. That issue can be explored by comparing the short and long-run dynamics of the exchange rate in the error correction model.
REFERENCES


Appendix I

Assume the dynamics of the fundamentals are appropriately captured by a p-th order autoregressive model:

\[ z_t = \theta + \psi_1 z_{t-1} + \psi_2 z_{t-2} + \ldots + \psi_p z_{t-p} + \varepsilon_t, \]

where \( z_t \) is a k-dimensional vector. We are interested in obtaining an explicit solution for the convergent infinite sum of expectations,

\[ \sum_{t=1}^{\infty} \Lambda^{t+1} E_{t-1} [z_{t+t}]. \]

Note that the expression in the text is of the form \( \sum_{t=1}^{\infty} \Lambda^{t+1} E_{t-1} [X_{t+t}] \), where \( \Lambda \) is a matrix conforming with the dimensions of both \( \Lambda \) and \( X_t \). However, the former expression is readily transformed to the one treated in this appendix by substituting \( Z_t \) for \( \Pi X_t \). First, expand the sum

\[ \sum = \Delta \left( z_{t-t} + \Delta E_{t-1} [\varepsilon_t] + \Delta^2 E_{t-1} [\varepsilon_{t+1}] + \ldots \right), \]

and substitute the variable inside the expectations brackets for their dynamic expression

\[ \Delta^{-1} \sum = \Delta \left( z_{t-t} + \Delta E_{t-1} [\varepsilon_t] + \Delta^2 E_{t-1} [\varepsilon_{t+1}] + \ldots \right) \]

where the expected values of future innovations to the fundamentals are set to zero. Next, I collect all the terms on the constant and lagged values of the fundamentals and add and subtract some terms necessary to complete summations
\[
\Delta^{-1} \sum = \Delta[I + \Delta + \Delta^2 + \ldots] \Theta_i \\
+ [I + \Delta \psi_1 + \Delta^2 \psi_2 + \ldots + \Delta^p \psi_p] Z_{t-1} \\
+ \Delta[\psi_1 + \Delta \psi_2 + \ldots + \Delta^{p-1} \psi_{p-1}] Z_{t-2} \\
+ \Delta[\psi_1 + \Delta \psi_2 + \ldots + \Delta^{p-2} \psi_{p-2}] Z_{t-3} \\
+ \ldots + \Delta \psi_{p-1} Z_{t-p} \\
\pm \Delta \psi_1 Z_{t-1} \pm \Delta^2 \psi_2 Z_{t-1} \pm \ldots \pm \Delta^p \psi_p Z_{t-1} \\
+ \Delta^2 \psi_1 E_{t-1}[Z_1] + \Delta^2 \psi_2 E_{t-1}[Z_2] + \ldots + \Delta^{p+1} \psi_p E_{t-1}[Z_p] \\
+ \Delta^2 \psi_1 E_{t-1}[Z_{t+1}] + \Delta^2 \psi_4 E_{t-1}[Z_{t+4}] + \ldots + \Delta^{p+2} \psi_p E_{t-1}[Z_{t+p}] \\
+ \ldots + \Delta \psi_{p-1} Z_{t-p}. \\
\]

This expression can be further simplified noting that the terms not included in the first five rows can be grouped by columns as functions of the summation to be computed

\[
\Delta^{-1} \sum = (I - \Delta)^{-1} \Theta_i + Z_{t-1} \\
+ \Delta[I - \Delta^2 \psi_2 + \ldots - \Delta^p \psi_p] Z_{t-2} \\
+ \Delta^2[I - \Delta^3 \psi_3 + \ldots + \Delta^{p-1} \psi_{p-1}] Z_{t-3} \\
+ \ldots + \Delta \psi_{p-1} Z_{t-p} \\
+ \psi_1 \sum \Delta \psi_2 + \ldots + \Delta \psi_p \psi_i. \\
\]

Solving explicitly for the summation term gives finally

\[
\sum = \left[I - \Delta \psi_1 - \Delta^2 \psi_2 - \ldots - \Delta^p \psi_p \right]^{-1} X \\
(\Delta[I - \Delta]^{-1} \Theta_i + Z_{t-1} + \Delta[I - \Delta]^{-1} \Theta_i + \ldots + \Delta^{p-1} \psi_{p-1} Z_{t-1} \\
+ \Delta[I - \Delta]^{-1} \Theta_i + \ldots + \Delta^{p-2} \psi_{p-2} Z_{t-2} \\
+ \Delta[I - \Delta]^{-1} \Theta_i + \ldots + \Delta^2 \psi_2 Z_{t-2}). \\
\]

which can be expressed in compact form as

\[
\sum \Delta^{-1} E_{t-1}[Z_{t+1}] = (I - \Delta \Pi_i)^{-1} \Delta \\
\Theta (\Delta[I - \Delta]^{-1} \Theta_i + \Delta(I_2 Z_{t-2} + \ldots + I_2 Z_{t-2} + \ldots + I_2 Z_{t-2})) \\
\]

with

\[
\Pi_i = \psi_1 + \Delta \psi_{i+1} + \ldots + \Delta^p \psi_{i+p}, \quad i = 1 \ldots p. \\
\]

**TABLE 1**

Long Run Exchange Rate Equation
\[ S = \beta_0(M1 - M1') + \beta_1(RY - RY') \]
Cointegration Results

<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \text{MAX} \text{EIG} )</th>
<th>( \text{TRACE} )</th>
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</thead>
<tbody>
<tr>
<td>US - CN</td>
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<td>12.55</td>
<td>18.55</td>
<td></td>
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</tbody>
</table>

* statistically significant at the 90% level.

** statistically significant at the 95% level.

\( \text{TRACE} \) is a cointegration test for the joint significance of the eigenvalues from \( \Pi = 0 \).

\( \text{MAX} \text{EIG} \) is a cointegration test for the individual significance of the largest eigenvalue from \( \Pi = 0 \).
TABLE 2

Long Run Money Demand Equation

| M1 - P = α1R + α2ST |

Cointegration and Parameter Stability Tests

<table>
<thead>
<tr>
<th>Full Sample: 73:1-91:4</th>
<th>α1</th>
<th>α2</th>
<th>MAXEIG</th>
<th>TRACE</th>
<th>F Emp</th>
<th>Break</th>
<th>P Mean</th>
<th>L0</th>
</tr>
</thead>
<tbody>
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<td>US</td>
<td>-0.53</td>
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<td>30.25*</td>
<td>49.79**</td>
<td>85:4</td>
<td>24.38**</td>
<td>1.09**</td>
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<tr>
<td>CN</td>
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<td>-0.45</td>
<td>31.87**</td>
<td>49.48*</td>
<td>29.05**</td>
<td>78:1</td>
<td>13.79**</td>
<td>0.02**</td>
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<td>-3.30</td>
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<td>30.05**</td>
<td>29.05**</td>
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<td>1.02**</td>
</tr>
<tr>
<td>FR</td>
<td>-0.09</td>
<td>-17.1</td>
<td>32.72**</td>
<td>31.41*</td>
<td>15.05**</td>
<td>77:4</td>
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<td>0.74**</td>
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<tr>
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<td>-0.51</td>
<td>-2.14</td>
<td>20.22</td>
<td>28.81*</td>
<td>12.84</td>
<td>76:2</td>
<td>4.29</td>
<td>0.15</td>
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<td>UK</td>
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<td>-0.31</td>
<td>36.79**</td>
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<td>8.53</td>
<td>77:7</td>
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<td>0.44</td>
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<td>WG</td>
<td>-2.24</td>
<td>-0.36</td>
<td>21.65**</td>
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<td>7.51</td>
<td>87:1</td>
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</table>

Subsample Results

<table>
<thead>
<tr>
<th>α1</th>
<th>α2</th>
<th>MAXEIG</th>
<th>TRACE</th>
<th>F Emp</th>
<th>Break</th>
<th>P Mean</th>
<th>L0</th>
</tr>
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<tbody>
<tr>
<td>US 73:1-85:4</td>
<td>-0.32</td>
<td>-6.01</td>
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<td>31.90**</td>
<td>36.01**</td>
<td>62:2</td>
<td>25.52**</td>
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<td>CN 78:1-91:4</td>
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<td>-7.12</td>
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<td>36.05**</td>
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<td>-3.45</td>
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<td>42.07**</td>
<td>0.74</td>
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<td>-6.66</td>
<td>23.94**</td>
<td>31.25*</td>
<td>36.08**</td>
<td>60:3</td>
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<td>37.09**</td>
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<td>17.86</td>
<td>5.28</td>
<td>86:4</td>
<td>1.14</td>
</tr>
</tbody>
</table>

* statistically significant at the 90% level.
** statistically significant at the 55% level.

TRACE is a cointegration test for the joint significance of the eigenvalues from Π = αφ.
MAXEIG is a cointegration test for the individual significance of the largest eigenvalue from Π = αφ.

TABLE 3

Long Run Exchange Rate Equation

S = β1(M1 - M1') + β2(RY - RY')

Stable Subsamples

Cointegration Results

<table>
<thead>
<tr>
<th>β1</th>
<th>β2</th>
<th>αφ</th>
<th>LRTαφ</th>
<th>LRTαφ</th>
<th>MAXEIG</th>
<th>TRACE</th>
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<tbody>
<tr>
<td>CN - JP</td>
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<td>15.72**</td>
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<td>CN - IT</td>
<td>78:1-91:4</td>
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<td>22.50**</td>
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<tr>
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<td>78:1-89:4</td>
<td>+1.87</td>
<td>-0.15</td>
<td>6.79**</td>
<td>1.27</td>
<td>10.78</td>
</tr>
<tr>
<td>CN - WG</td>
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<td>-0.08</td>
<td>3.57**</td>
<td>1.84</td>
<td>19.35**</td>
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<td>-0.35</td>
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<td>0.77**</td>
<td>22.66**</td>
</tr>
<tr>
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<td>0.97</td>
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<tr>
<td>IT - WG</td>
<td>76:1-91:4</td>
<td>+0.59</td>
<td>-0.19</td>
<td>12.34**</td>
<td>27.53**</td>
<td>23.66**</td>
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<td>UK - WG</td>
<td>73:1-89:4</td>
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Parameter Stability Tests

<table>
<thead>
<tr>
<th>F Emp</th>
<th>Break</th>
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<th>L0</th>
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<tr>
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<td>86:4</td>
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<td>58:1</td>
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<td>58:3</td>
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<td>82:3</td>
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<td>JP - UK</td>
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<td>76:4</td>
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<td>JP - WG</td>
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<td>4.26</td>
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<td>FR - UK</td>
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<td>76:2-90:4</td>
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<td>88:3</td>
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<tr>
<td>UK - WG</td>
<td>73:1-89:4</td>
<td>5.31</td>
<td>77:1</td>
</tr>
</tbody>
</table>

* statistically significant at the 90% level.
** statistically significant at the 95% level.
αφ is the exchange rate adjustment coefficient.
LRTαφ is a likelihood ratio test for the individual significance of αφ.
LRTαφ is a likelihood ratio test for the long-run money supply homogeneity of the exchange rate.
TRACE is a cointegration test for the joint significance of the eigenvalues from Π = αφ.
MAXEIG is a cointegration test for the individual significance of the largest eigenvalue from Π = αφ.
Cointegration Results

Subsamples

<table>
<thead>
<tr>
<th></th>
<th>$\beta_1$</th>
<th>$\alpha_0$</th>
<th>LRT$_{\alpha_0}$</th>
<th>LRT$_{\alpha_0}'$</th>
<th>MAXEIG</th>
<th>TRACE</th>
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</thead>
<tbody>
<tr>
<td>CN – JP</td>
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<td>0.31</td>
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<tr>
<td>CN – WG</td>
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<td>3.95**</td>
<td>4.59</td>
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<td>1.76</td>
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<td>17.53**</td>
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<td>12.06**</td>
<td>2.33</td>
<td>22.43**</td>
<td>28.58**</td>
</tr>
</tbody>
</table>

* statistically significant at the 99% level.
** statistically significant at the 95% level.
$\alpha_0$ is the exchange rate adjustment coefficient.
LRT$_{\alpha_0}$ is a likelihood ratio test for the individual significance of $\alpha_0$.
LRT$_{\alpha_0}'$ is a likelihood ratio test for the long-run money supply homogeneity of the exchange rate.
TRACE is a cointegration test for the joint significance of the eigenvalues from $R = 0$.
MAXEIG is a cointegration test for the individual significance of the largest eigenvalue from $R = 0$.

TABLE 5

Forecasting Performance: ECM vs. Random Walk
One Quarter Ahead Forecast

Forecasting Error Statistics

<table>
<thead>
<tr>
<th></th>
<th>ECM ME</th>
<th>RW ME</th>
<th>ECM MAE</th>
<th>RW MAE</th>
<th>ECM RMSE</th>
<th>RW RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CN – JP</td>
<td>0.0093</td>
<td>0.0185</td>
<td>0.093</td>
<td>0.143</td>
<td>0.053</td>
<td>0.046</td>
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<tr>
<td>CN – FR</td>
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<td>0.038</td>
<td>0.025</td>
<td>0.073</td>
<td>0.079</td>
<td>0.080</td>
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<tr>
<td>CN – IT</td>
<td>-0.03</td>
<td>0.058</td>
<td>0.045</td>
<td>0.090</td>
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<td>0.082</td>
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<tr>
<td>JP – FR</td>
<td>0.04</td>
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<td>0.049</td>
<td>0.076</td>
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<td>JP – IT</td>
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<td>FR – IT</td>
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<td>0.008</td>
<td>0.008</td>
<td>0.000</td>
<td>0.003</td>
</tr>
</tbody>
</table>

ME is the Mean Error.
MAE is the Mean Absolute Error.
RMSE is the Root Mean Square Error.
SERIE DE DOCUMENTOS DE TRABAJO DEL ICATE

9301 "Análisis del Comportamiento de las Cotizaciones Reales en la Bolsa de Madrid bajo la Hipótesis de Eficiencia". Rafael Flores de Frutos. Diciembre 1992. (Versión final aceptada para publicación en "Estadística Española").


9401  "Contrastes de momentos y de la matriz de información". Teodosio Pérez Amaral. Junio 1994. (Versión final aceptada para publicación en Quaderns Econòmics del ICE)


