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Horizontal and Vertical Inequality in a Social Welfare Framework

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HORIZONTAL AND VERTICAL INEQUALITY IN A SOCIAL WELFARE FRAMEWORK**

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ABSTRACT

In this paper a new definition of horizontal inequality is adopted. It is defined in terms of the distributional change within intervals of similar households, produced by the Tax System. We believe that this definition is better suited for measurement of the comparative injustice that may be caused by the Tax System among similar households. In particular a within-group Atkinson inequality index applied to one minus the tax rates of similar households is proposed. It enables us to introduce this concept in a general social welfare framework together with efficiency and vertical equity redistribution considerations, where the horizontal equity and vertical redistribution are income-invariant measures. It contributes to a more appropriate evaluation of the desirability of tax reforms aimed at achieving greater horizontal equity.

RESUMEN

En este papel se define la desigualdad horizontal en términos del cambio distributivo intragrupos que el Sistema Fiscal produce en intervalos de hogares similares. Creemos que ésta es una mejor definición para evaluar los agravios comparativos del Sistema Fiscal entre hogares similares. A este respecto el índice de Atkinson de desigualdad intragrupos de uno menos los tipos medios de los hogares similares es adecuado. Una ventaja de este índice es que nos permite la introducción de este concepto en un marco de bienestar social junto con consideraciones de eficiencia y de redistribución vertical, donde la equidad vertical y la equidad horizontal son medidas invariantes ante cambios relativos de renta. El papel contribuye a una evaluación más adecuada de la deseabilidad de las reformas fiscales con consideraciones de equidad horizonal.

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1. INTRODUCTION.

The purpose of this paper is twofold: firstly, the use of a new horizontal inequality index is proposed, and secondly, an effort is made to reconcile the concepts of vertical and horizontal equity proposed within a general framework of social welfare.

The first aim is connected with the redefinition of the traditional concept of horizontal inequality. We redefine horizontal inequality in terms of the distributional change in intervals of similar households produced by the Tax System, in line with the idea proposed by Camarero, Herrero and Zubiri (1993). The classical view of horizontal inequality is based on the definition in terms of the reranking produced after tax. Unlike Camarero, Herrero and Zubiri, we modify the concept of Cowell's distributional change [e.g., Cowell (1980) and (1985)] proposed by these authors, and we define it in terms of the dispersion of one minus the relative changes in income within intervals of similar households, measured according to Atkinson's indexes. Both indexes have very similar properties[1], but the index we propose is best suited to our model.

Secondly, this horizontal inequality index will be introduced into a framework of general social welfare, to allow for the evaluation of the desirability of tax reforms aimed at achieving greater horizontal equity, in line with the idea proposed by King (1983). In contrast to King, however, we recommend the use of our horizontal inequality index, instead of a traditional reranking index.

The paper is organized as follows: in sections 2 and 3 we describe and define the concepts of vertical and horizontal inequality, within the proposed framework of social welfare; in section 4 we include an empirical estimate of the alternative horizontal inequality indexes, producing similar results; in section 5 we define the concept of vertical redistribution and in section 6 we present the conclusions. Finally, in the Appendix we provide a decomposition of the inequality indexes into their between and within group components.

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[1] Both are relative distributional change indexes defined for intervals of similar households, that is, they are zero-degree homogeneous indexes in the interval income variations and, in consequence, they measure "aggregate distances" with respect to proportional changes in income.

2. THE CONCEPT OF VERTICAL AND HORIZONTAL INEQUALITY.

Economists have generally distinguished between two types of inequaly, namely vertical and horizontal inequality, that provide an answer to two different kinds of questions. The first one refers to the degree of dispersion existing between dissimilar households (rich households with respect to their needs in contrast to poor households with respect to their needs), and the associated vertical redistribution indexes answer the question of to what extent the Public Sector corrects the vertical inequality observed between households in different brackets. In contrast, the second question relates to the extent to which the Public Sector grants similar treatment to uniform households [e.g., Feldstein (1976)]. This second question is different and extremely interesting since it enables us to observe, independently of vertical redistribution, the degree of injustice that may be caused by the Public Sector if it treats two similar households belonging to one single bracket differently.

In the relevant literature a large number of indexes, of a greater or lesser degree of appropriateness, have been proposed to measure each of these concepts. With regard to the vertical inequality indexes proposed, we may highlight the relative indexes holding the Lorenz principle of domination of income distributions, such as the Theil, Gini and Atkinson indexes. The (vertical) redistribution indexes are constructed in terms of reductions in the vertical after-tax inequality indexes produced by the Tax System [e.g., Lambert (1993)].

The traditional horizontal inequality indexes concentrate on the measurement of the rerankings in income distribution produced by the Tax System, as in the case of the Atkinson (1980), Plotnick (1981) or King (1983) indexes, that are based on the idea that if there is reranking between two similar households, different fiscal treatment ensues. However, reranking is the consequence and not the source of horizontal inequality, and as such it could be considered a sufficient, but not a necessary, condition for horizontal inequality. In this respect the distributive change indexes of intervals of similar households, such as those proposed by Camarero, Herrero and Zubiri, are more precise; in accordance with these indexes two similar households could be treated differently not only if there is reranking but also if their final dispersion increases after tax. They propose the use of Cowell's index of relative distributional change, among similar households. In this paper we propose a similar index, that is, the Atkinson index applied to one minus the average tax rates.

There is neither a simple nor a unique answer to which are the most appropriate indexes, since this depends on the normative judgements considered.

If all households were identical in income, characteristics and utilities, a consensus would be reached on fiscal treatment: that is, all households should pay the same amount of tax.

However, reality is much more complex, since households differ in both their income levels and their needs, and the latter depend on the family and demographic characteristic features of each household. In this respect, personal criteria on redistributive justice are also different, since each one of us has different opinions regarding the amount that each individual should pay. There are, however, two criteria that are essentially different, namely vertical redistribution criteria, establishing the degree of aversion to vertical inequality, and horizontal inequality criteria, establishing the degree of aversion to comparative injustices

between similar households.

However, the above definitions have a number of disadvantages, some of a practical nature and others of a deeper and conceptual nature. One of the most important practical disadvantages, noted by Feldstein (1976), is that the principle of equal treatment of equals has little applied interest. If we consider individuals to be similar when they have exactly the same utility, it is possible that we may never find two identical individuals for the purpose of comparison.

As a consequence, and for practical purposes, the analysis must be capable of being extended to groups of a greater or lesser size, and this factor also affects the traditional approach of measurement of horizontal inequality as reranking. The disadvantage arises upon extension of the analysis to wider groups, since the analysis is extended to individuals who are not genuinely similar, and who become less similar as the size of the subgroups increases. Nor is it clear that treatment of a large number of groups is the most appropriate procedure, since the problems generated between individuals on the contiguous limits of different subgroups are multiplied. This methodology opens up some new conceptual questions: Which criteria should be followed when selecting similar groups ? To what extent do the criteria chosen affect the results ? And so on. We will answer these questions below.

Furthermore, according to Feldstein's definition, horizontal inequality is associated with a Tax System. Unlike the concept of vertical inequality, the concept of horizontal inequality is not valid for a distribution in itself, but is associated with a distributive change produced by fiscal policy, and this implies that it must necessarily refer to the initial distribution. As a consequence, horizontal inequality is a concept that tries to measure comparative injustices in the status quo, that is, in the initial income distribution before tax. It is evident, therefore, that the initial distribution will have a certain influence on the results obtained.

Within this framework, the horizontal inequality in the income tax will not only be due to the treatment of the system of personal deductions, nor to the different fiscal treatment of uniform incomes (those obtained following application of the equivalence scales to households with different family characteristics), nor to the initial distribution, but rather to what is a new factor in this approach of measurement as distributional change within the intervals of similar households, that is, to different taxation among households in like circumstances. This new factor is important since it emphasises a possible overlap between the concepts of vertical redistribution and horizontal equity, even though it may influence with opposite signs. There is no such overlap between these concepts in the classical approaches measuring horizontal inequality by reranking of positions.

3. THE SOCIAL WELFARE FRAMEWORK.

We assume an individualistic social welfare function of the after-tax adult-homogeneous income levels Y_i and of the effective average tax rates t_i of all the households i = 1, 2,.... n:

$$W(Y_1,Y_2,\ldots,Y_n;(1-t_1),(1-t_2),\ldots,(1-t_n)) \qquad (1)$$

We assume this function to be symmetric, increasing and concave function in the household incomes Y_i and in one minus the average tax rates $(1\text{-}t_i)$. We define the vertically equally distributed equivalent income Y^*_v as the level of income that guarantees vertical equity. This is the equally distributed income level providing the same level of welfare as the after-tax present distribution, for given average tax rates:

$$W(Y^*_v,Y^*_v,\ldots,Y^*_v;(1-t_1),(1-t_2),\ldots,(1-t_n)) = W(Y_1,Y_2,\ldots,Y_n;(1-t_1),(1-t_2),\ldots,(1-t_n)) \qquad (2)$$

We define the vertical inequality index as the Atkinson (1970) index of Y_i :

$$I_v = 1-\frac{Y^*_v}{Y_M} \qquad (3)$$

Given the concavity of W in Y_i, Y^*_v is no greater than Y_M, the aggregate mean income. I_v indicates the proportion of total income that one would be prepared to sacrifice in order to eliminate all the existing vertical inequality.

Let Y_i^I and t_i^I denote the household income and the tax rate of the household i belonging to the interval I of similar households. We can express the social welfare function divided between the intervals I = 1, 2, ..., N; where n_I is the number of households in interval I, as:

$$W((Y^1_1,\ldots,Y^1_{n_1}),\ldots,(Y^I_i,\ldots,Y^I_{n_I}),\ldots;((1-t^1_1),\ldots,(1-t^1_{n_1})),\ldots,((1-t^I_1),\ldots,(1-t^I_{n_I}),\ldots)) \qquad (4)$$

We define t^{*I}_h , the equivalent tax rate guaranteeing horizontal equity in interval I, as the uniform income reduction of all the households within the interval, over the initial position before tax, that guarantees the same level of welfare as the initial distribution of different tax rates, for a given initial income distribution:

$$W^I(Y^I_1,Y^I_2,\ldots,Y^I_{n_I};(1-t^{*I}_h),(1-t^{*I}_h),\ldots,(1-t^{*I}_h)) = W^I(Y^I_1,Y^I_2,\ldots,Y^I_{n_I};(1-t^I_1),(1-t^I_2),\ldots,(1-t^I_{n_I})) \qquad (5)$$

We define the horizontal inequity index in interval I as the Atkinson index of $1\text{-}t_i$:

$$I^I_h = 1-\frac{1-t^{*I}_h}{1-t^I_M} \qquad (6)$$

Given the concavity of W^I in $(1\text{-}t^I_i)$, t^{*I}_h is no lower than t^I_M, the interval average tax rate. I^I_h denotes the proportion of income after tax that one would be prepared to sacrifice in order to eliminate all the horizontal inequality existing in this interval. It should be noted that, although the average tax rates are defined with respect to income before tax, I^I_h refers to the reduction over income after tax, since:

$$I^I_h = \frac{t^{*I}_h - t^I_M}{1-t^I_M} \qquad (7)$$

We denote the aggregate horizontal equity index by I_h. A between groups aggregation problem arises. By analogy with the vertical aggregate index, we define I_h as the proportion of after-tax income that one would be prepared to sacrifice in order to eliminate all the horizontal inequality existing within each interval. That is, the uniform income reduction of all the households that guarantees the same level of welfare as that obtained if all the households had the average tax rates of each group:

$$\begin{aligned} &W(Y_1,\ldots,Y_n;((1-t^1_1),\ldots,(1-t^1_{n_1})),\ldots,((1-t^N_1),\ldots,((1-t^N_{n_N}))) = \\ &W(Y_1,\ldots,Y_n;((1-t^1_M)(1-I_h),\ldots,(1-t^1_M)(1-I_h)),\ldots,((1-t^N_M)(1-I_h),\ldots,(1-t^N_M)(1-I_h))) \end{aligned} \qquad (8)$$

It should be noted that I_h corresponds to the within groups inequality index of one minus the average tax rates[2]. I_h may be expressed as:

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[2]It also corresponds to the Atkinson within groups index of before-tax income over the after-tax income one, which has similarities with the horizontal inequality index proposed in Aronson et al. (1993) and Lambert (1994) as the Gini within groups index of after-tax income, applied to groups with no before-tax inequality.

$$I_h = 1 - \frac{1-t_h^*}{1-t_M} \qquad (9)$$

where t^*_h is the aggregate tax rate that would guarantee horizontal equity within each group and t_M is the aggregate average tax rate.

Let t^* be the equivalent tax rate that guarantees equal tax rates for all the population

$$W(Y_1,...,Y_n;((1-t_1^1),...,(1-t_{n_1}^1),...,((1-t_1^N),...,((1-t_{n_N}^N)) = W(Y_1,...,Y_n;((1-t^*),...,(1-t^*)),...,((1-t^*),...,(1-t^*))) \qquad (10)$$

and t^*_B be the equivalent tax rate between groups that guarantees a level of welfare equal to that which would be obtained if all the households had the average tax rates of each group

$$W(Y_1,...,Y_n;((1-t_M^1)(1-I_h),...,(1-t_M^1)(1-I_h)),...,((1-t_M^N)(1-I_h),...,((1-t_M^N)(1-I_h))) = W(Y_1,...,Y_n;((1-t_B^*)(1-I_h),...,(1-t_B^*)(1-I_h)),...,((1-t_B^*)(1-I_h),...,(1-t_B^*)(1-I_h))) \qquad (11)$$

Then I_h may be expressed as[3]:

$$I_h = 1 - \frac{1-t^*}{1-t_B^*} \qquad (12)$$

Strictly speaking the aggregate index I_h depends on the income distribution, tax rates and degree of concavity of the welfare function with respect to $(1-t_i)$, which as we shall see defines within groups horizontal inequality aversion and in turn the between groups horizontal inequality aversion (or, if preferred, the kind of between groups aggregation of horizontal inequality).

[3] Let I be the Atkinson global inequality index of one minus the average tax rates in all the intervals. Then we can break down: $1 - I = (1-I_h)(1-I_B)$, where I_B is the between groups Atkinson index calculated as if all the households had the average tax group rate (that is, the amount of income that we would sacrifice in order to prevent between groups inequality) and I_h is the within groups Atkinson index (that is, the amount of income that we would sacrifice in order to prevent within each group inequality).

If we assume that W(.) is homothetic in Y and (1-t), then I_v and I_h are invariant to proportional changes in income and in one minus the average tax rates within each interval. In this case:

$$W = Y_M(1-I_v)(1-I_h) \qquad (13)$$

The advantage of an expression of welfare such as (13) is that it allows us to include, in an operative manner, the effects of efficiency (through Y_M) as well as the effects of vertical and horizontal inequality separately (through I_V and I_h).

In addition, if we assume that W(.) is additively separable into households[4]

$$W = \sum_{i=1}^n U(Y_i, 1-t_i) \qquad (14)$$

and that Y_i and $(1-t_i)$ are independent[5],

$$W = \sum_{i=1}^n V(Y_i) + \sum_{i=1}^n H(1-t_i) \qquad (15)$$

then homotheticity implies

$$V(Y) = \frac{Y^{1-\epsilon}}{1-\epsilon}, \qquad \epsilon \neq 1 \qquad (16)$$

$$V(Y) = LnY, \qquad \epsilon = 1 \qquad (17)$$

$$R(1-t) = \frac{(1-t)^{1-\gamma}}{1-\gamma}, \qquad \gamma \neq 1 \qquad (18)$$

[4] This implies that relative social valuation of the income or tax rates between two households does not depend on the other households.

[5] This implies that relative social valuation of the households' income does not depend on their tax rates, and vice versa.

$$R(1-t)=Ln(1-t), \qquad \gamma=1 \tag{19}$$

and the vertical equity index may be expressed as follows:

$$1-I_v=[\frac{1}{n}\Sigma_{i=1}^{n}[\frac{Y_i}{Y_M}]^{1-\epsilon}]^{\frac{1}{1-\epsilon}}, \qquad \epsilon\neq1 \tag{20}$$

$$1-I_v=\exp[\frac{1}{n}\Sigma_{i=1}^{n}Ln[\frac{Y_i}{Y_M}]], \qquad \epsilon=1 \tag{21}$$

where ϵ is the degree of aversion to vertical inequality, which is greater than zero if we require concavity.

In this case, the aggregate horizontal equity index can be expressed as follows:

$$1-I_h=[\frac{1}{n}\Sigma_{I=1}^{N}n_I[1-I_h^I]^{1-\gamma}]^{\frac{1}{1-\gamma}}, \qquad \gamma\neq1 \tag{22}$$

where:

$$1-I_h^I=[\frac{1}{n_I}\Sigma_{i=1}^{n_I}[\frac{1-t_i}{1-t_M^I}]^{1-\gamma}]^{\frac{1}{1-\gamma}}, \qquad \gamma\neq1 \tag{23}$$

and

$$1-I_h=\exp[\frac{1}{n}\Sigma_{I=1}^{N}n_I Ln[1-I_h^I]], \qquad \gamma=1 \tag{24}$$

where:

$$1-I_h^I=\exp[\frac{1}{n_I}\Sigma_{i=1}^{n_I}Ln[\frac{1-t_i}{1-t_M^I}]], \qquad \gamma=1 \tag{25}$$

where γ is the degree of aversion to horizontal inequality, which is greater than zero for concavity.

It would enter into the SWF in equation (13), defined for ϵ and γ parameters of aversion to vertical and horizontal inequality. We can also aggregate the two inequality indexes into one total inequality index I_t, with fixed values for ϵ and γ:

$$W=Y_M(1-I_v)(1-I_h)=Y_M(1-I_t) \tag{26}$$

where I_t would be the proportion of income after tax that one would be prepared to sacrifice in order to eliminate all the existing vertical and horizontal inequity. In the Appendix we provide a decomposition of the inequality indexes into population subgroups.

4. HORIZONTAL INEQUALITY IN THE SPANISH PERSONAL INCOME TAX.

We use the data base of the Spanish Personal Income Tax "expanded panel", consisting in simple annual random samples of individual Income Tax returns for the years 1982 to 1990. In 1988 and subsequent years the separate tax returns of married couples are added together to constitute one single item.

A serious difficulty arising in the analysis of horizontal inequality is that of the correct determination of adult-equivalent income. We use the tax base as the monetary household income, which we divide by the following equivalence scale, which is close to that proposed by the OECD[6]:

$$E=1+0.7A_1+0.5A_2+0.7A_3 \tag{27}$$

in which A_1 is one in those cases in which there is a spouse and zero in cases in which there is no spouse, A_2 is equal to the number of children and A_3 is equal to the number of ascendant relatives in the household earning no income.

As a tax variable, we have made use of the net tax liability recorded in the tax returns. Since some tax returns have negative tax bases, these have been modified to one peseta following

[6] The OECD scale adopts the value of one for the first adult in the household, plus 0.7 for the second adult, plus 0.5 for each child (under 14 years of age) in the household.

verification that this change, or elimination of these items, did not produce any significant differences in the indexes. This is done in order to eliminate non-positive arguments from the logarithms, while keeping at the same time the maximum possible amount of data and criteria uniformity for all the indexes.

Table 1 shows the indexes computed with the rates of change from the previous period; see Pazos, Rabadán and Salas (1994). Graph 1 shows the evolution of all the indexes.

The first two indexes are reranking indexes: the King's Scaled Order Statistics (1983) and the Atkinson's (1980) and Plotnick's (1981) Preordered Inequality Index. The last two indexes are aggregate relative distributional change indexes computed for similar households, such as the Camarero et al. (1993) proposal of use of the Cowell index (1985) or our proposed index. In these cases intervals of similar households are taken as centiles of adult-equivalent income levels.

It can be noticed that all the computed indexes show similar variations from one year to the next. Thus all the indexes rise between 1984 and 1988, and then fall again in 1990. We can say that 1988 is the year in which there is the greatest "maltreatment" with regard to the horizontal behaviour of taxation, since all the indexes reflect this trend. In 1990 horizontal inequality fell again, almost to the 1986 levels.

This evolution can be observed both in the indexes that only measure the horizontal inequity implied by reranking (the King and Plotnick indexes) as well as in the distributional change of similar households indexes.

Another important result is that the proposed index offers, as expected, a certain degree of sensitivity, even though stable, to the choice of intervals of similar income levels, which decreases as the number of intervals increases. In this case we have chosen to organize these intervals according to different income quantiles, finally opting in favour of centiles. Table 2 shows the variations in the proposed distributional change index according to the change in the number of intervals. These intervals are the 10th, 100th and 1000th quantiles, respectively.

In comparison with the results of Pazos et al. (1994), in which a linear additive aggregation in population is used, different from that of the equations (22) and (24) and more in line with that of Camarero et al., the results show little variation.

5. THE VERTICAL REDISTRIBUTION INDEX.

We define the concept of vertical redistribution as the effect of taxation on social welfare, by means of the relative reduction in vertical inequality. Given social welfare before tax:

$$W = Y_M(1 - I_v) \tag{28}$$

the tax system produces the following effect on social welfare:

$$W' = Y'_M(1 - I'_v)(1 - I_h) \tag{29}$$

The relative variation in welfare may be expressed as follows:

$$\frac{\Delta W}{W} = \frac{\Delta Y_M}{Y_M} + \frac{\Delta(1 - I_v)}{(1 - I_v)} - \frac{I_h}{W} \tag{30}$$

The three components into which the effect on welfare can be divided are: the rate of change in average income (the contribution to efficiency), the rate of change in vertical equity (the degree of vertical redistribution of the tax) and, finally, the rate of reduction in welfare due to the horizontal inequity of the system. With a minor transformation, the vertical redistribution index of the tax may be expressed as a relative reduction in the vertical inequality index[7]:

$$RV = \frac{I_v - I'_v}{(1 - I_v)} \tag{31}$$

6. CONCLUSIONS

In this paper a new horizontal inequality index has been derived and, in addition, we have reconciled the concepts of vertical and horizontal equity proposed within a general framework of social welfare.

The first aim is related to the redefinition of horizontal inequality in terms of the distributional change within intervals of similar households, produced by the Tax System, in line with the idea proposed by Camarero, Herrero and Zubiri (1993) and in contrast with the classical indexes based upon the reranking effects. Our index is defined in terms of the dispersion of one minus the relative variations in income in intervals of similar households, measured according to the Atkinson indexes, which best suits our model of general social welfare and which, in turn, allows for evaluation of the desirability of tax reforms aimed at achieving greater horizontal equity.

We believe that our index is better suited for the observation of the degree of injustice that may be caused by the Public Sector if it treats similar households differently. Empirical

[7] The classical absolute inequality reduction indexes IRI = I_v - I_v' could also be proposed, even though their interpretation would be somewhat different since they would measure the increase in absolute welfare.

evidence shows no significant divergence with respect to the evolution of alternative horizontal inequality indexes proposed in the literature.

Furthermore, important conceptual and practical disadvantages arise in the analysis of inequality of within groups of similar households. Conceptually, the analysis is necessarilly done for households who are not genuinely similar. This problem is also inherent to the traditional reranking indexes. Nor is it clear that treatment of a large number of groups is the most appropriate procedure, since the problems generated in the treatment of individuals on the contiguous limits of different subgroups are multiplied. We have made a sensibility exercise that shows a desirable stability of our index to changes in the number of groups.

APPENDIX.

DECOMPOSITION OF VERTICAL INEQUALITY AND REDISTRIBUTION BY POPULATION SUBGROUPS: WITHIN AND BETWEEN GROUPS.

We can decompose the vertical inequality index I_v into K population subgroups, and into a between groups index $I_{v,B}$ and a within groups index $I_{v,W}$, defined as follows:

$$I_{v,B}=1-\frac{Y^*_{v,B}}{Y_M} \qquad \textbf{(32)}$$

where $Y^*_{v,B}$ is the uniformly distributed equivalent income level providing the utility level equal to that which would be attained if all the individuals had the mean income of each group. It would therefore be the equivalent income level that would guarantee equality of the groups (between groups). In consequence, $I_{v,B}$ would indicate the proportion of income that we would be prepared to sacrifice to eliminate all inequality between groups. Given the fact that W is concave in Y_i, $Y^*_{v,B}$ is less than Y_M and, therefore, $I_{v,B}$ is positive. Furthermore, the within groups index is defined as follows:

$$I_{v,W}=1-\frac{Y^*_{v,W}}{Y_M} \qquad \textbf{(33)}$$

where $Y^*_{v,W}$ is the uniformly distributed equivalent income level that would guarantee equality within each group (within groups). Then, $I_{v,W}$ would indicate the proportion of income that we would be prepared to sacrifice for elimination of all inequality within groups. Given that W is concave in Y_i, $I_{v,W}$ is lower than the average of the population-weighted internal group indexes.

$I_{v,W}$ can be expressed in terms of the equivalent income that guarantees total vertical equality and that which only guarantees equality between groups:

$$I_{v,W}=1-\frac{Y^*_{v}}{Y^*_{v,B}} \qquad \textbf{(34)}$$

In this case we obtain the following decomposition:

$$(1-I_v)=(1-I_{v,B})(1-I_{v,W}) \tag{35}$$

If we assume that W(.) is homothetic in Y, then $I_{v,B}$ and $I_{v,W}$ are invariant to proportional changes in income. In this case:

$$W=Y_M(1-I_{v,B})(1-I_{v,W})(1-I_h) \tag{36}$$

The advantage of an expression of welfare such as (36) is that it allows us to include, in an operative manner, the effects of efficiency, the effects of vertical inequality both within and between groups, and the effects of horizontal inequality separately.

In addition, if we assume that W(.) is additively separable into individuals and that Y_i and $(1-t_i)$ are independent, then the vertical equity index can be expressed in terms of the individual indexes of each subgroup J, if ϵ is greater than zero and different from one, as follows:

$$1-I_{v,W}=[\frac{1}{n}\sum_{J=1}^{K} n_J[1-I_v^J]^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{37}$$

where I^J_v is the vertical inequality index of subgroup J, equal to:

$$1-I_v^J=[\frac{1}{n_J}\sum_{j=1}^{n_J}[\frac{Y_j}{Y_M^J}]^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{38}$$

If ϵ is equal to one:

$$1-I_{v,W}=\exp[\frac{1}{n}\sum_{J=1}^{K} n_J Ln[1-I_v^J]] \tag{39}$$

where:

$$1-I_v^J=\exp[\frac{1}{n_J}\sum_{j=1}^{n_J} Ln[\frac{Y_j}{Y_M^J}]] \tag{40}$$

Similarly, vertical redistribution can be subdivided into the effects of contribution within and between groups:

$$RV=\frac{I_{v,B}-I'_{v,B}}{(1-I_{v,B})}+\frac{I_{v,W}-I'_{v,W}}{(1-I_{v,W})} \tag{41}$$

as the sum of the rates of variation in vertical equity between and within groups produced by taxation.

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TABLE 1

EVOLUTION OF A NUMBER OF HORIZONTAL INEQUALITY INDEXES (x 1000). 1982 - 1990.

	1982	1984	1986	1988	1990
King I.	23,015	25,871	35,052	38,996	35,751
(%)		(12.41)	(35.49)	(11.25)	(-8.32)
Plotnick I.	1,070	1,198	2,124	2,875	2,111
(%)		(11.96)	(77.30)	(35.36)	(-26.57)
Camarero et al. I.	0,4622	0,5297	1,0258	1,6685	1,1018
(%)		(14.60)	(93.66)	(62.65)	(-33.96)
Ih	0,4371	0,5028	1,0061	1,5786	1,1226
(%)		(15.01)	(100.02)	(56.81)	(-28.84)
Size of sample	123599	132693	165069	181390	210457

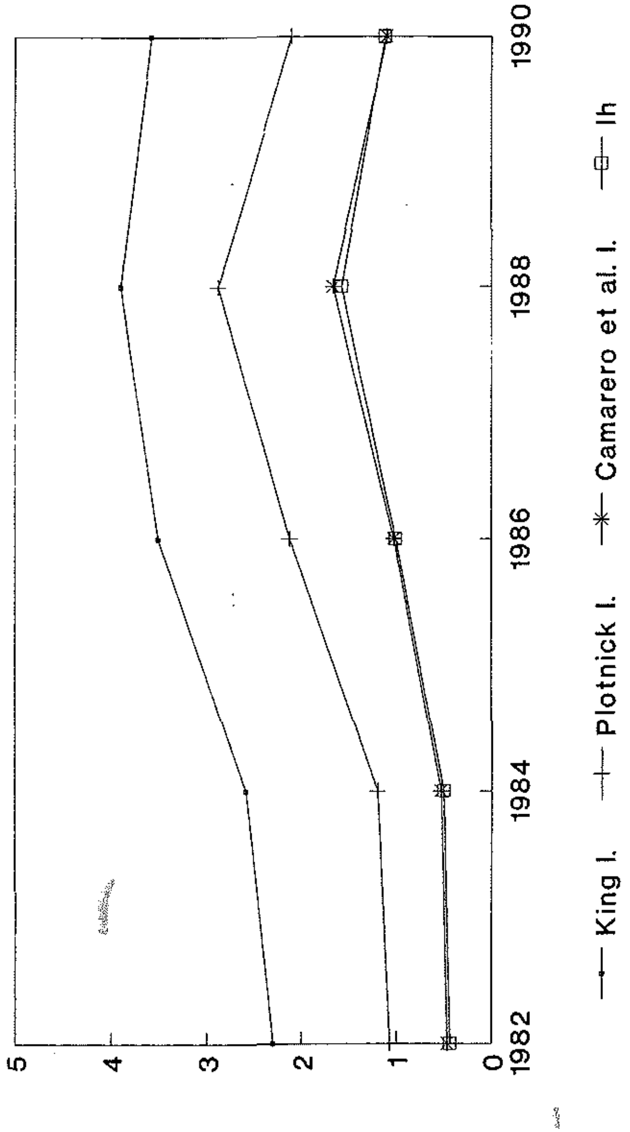
Note: All indexes are multiplied by 1000. Camarero et al I. is evaluated for $\alpha=1$ and Ih for $\gamma=1$ and both for centiles of adult-homogeneous income.

TABLE 2

I_h INDEXES (x1000) FOR DIFFERENT PARTITIONS.

QUANTILES	1982	1984	1986	1988	1990
10	0.4906	0.5818	1.0972	1.7102	1.2787
100	0.4371	0.5028	1.0061	1.5786	1.1226
1000	0.4226	0.4802	0.9818	1.5551	1.0941

GRAPH 1
HORIZONTAL INEQUALITY INDEXES



All indexes are mutiplied by 1000, except the King I. which is multiplied by 100.

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