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FURTHER EVIDENCE ON FORECASTING INTERNATIONAL GNP

GROWTH RATES USING UNOBSERVED COMPONENTS

TRANSFER FUNCTION MODELS*

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ABSTRACT

Forecast of international GNP growth rates are computed using a novel, onobserved components model that allows for estimating the trend and the perturbational components in GNP data. The model is formulated in state space terms, and estimating using recursive methods of filtering and fixed interval smoothing. The decomposition crucially hinges on the choice of the Noise-Variance Ratio parameter. As any other signal extraction method, the choice of the relevant parameters affects the statistical characteristics of the estimated components. Here, we incorporate a priori beliefs on the values of the NVR parameter leading to a decomposition with reasonable business cycle properties. Throughout the paper, forecast comparisons are made with other Bayesian and non-Bayesian alternatives.

RESUMEN

En este trabajo presentamos predicciones de las tasas de crecimiento del PIB/PNB de un conjunto de países, utilizando un modelo de componentes no observables que permite la estimación de componentes de tendencia y perturbación de dichas variables. El modelo se formula en espacio de los estados y se estima mediante procedimientos recursivos de filtrado y de suavizado con la muestra completa. La descomposición se basa en la opción del parámetro Noise-Variance Ratio (NVR). Como en cualquier procedimiento de extracción de señales, la elección de los parámetros relevantes afecta a las características estadísticas de los componentes estimados. En este artículo, incorporamos supuestos apriorísticos sobre los valores del NVR que generan una descomposición fácilmente interpretable en términos del ciclo económico. A través del artículo se establecen comparaciones predictivas con otras alternativas Bayesianas y no Bayesianas.

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1. INTRODUCTION

The clear tendency to grow over time of most macroeconomic aggregates conflicts with the stationarity assumption of the standard statistical time series methods. That has spurred a great deal of attention to trend estimation in macroeconomic time series analysis which have departed from the old tradition of assuming deterministic trends. The Nelson and Plosser (1982) paper was crucial in concluding that most macroeconomic time series seem to be better represented by a difference stationary process, rather than by a trend-stationary one. Hence, the component specifications proposed by Beveridge and Nelson (1981), Harvey and Todd (1983), Watson (1986) and Stock and Watson (1988) all assume a random walk for the first or second order difference of the variable under consideration.

A proper characterization of the long-run trend in economic time series is important for a variety of reasons: it might well be the case that separate modelling of the different components leads to a better short-run forecasting performance. Second, characterizing the different behavior of an economic time series at its different frequency bands might lead to a better multicomponent specification of its interrelations with other variables. This, in turn, might again be most useful in improving its forecasting performance.

This latter point is important, because the literature on components estimation has taken an approach of extracting from the variable under study its 'noisy' or less interesting characteristics. A statistical model is then specified that relates the filtered variable to other 'exogenous' variables. We take a different view in considering that the relationship between two variables might be different at the different relevant frequency bands. Hence, if we are interested in decomposing two macroeconomic variables X_t and Y_t into their 'trend' and 'perturbation' components is because we might get a much better model by aggregating two relations: one between the trends of X_t and Y_t , and a different one between their perturbations.

This paper takes on previous work by the same authors [(García-Ferrer et al.(1993)], where we proposed a novel, unobserved components (UC) model which could be used to forecast a group of analogous annual macroeconomic time series, like the GNP data from a set of countries. That model is formulated in state space form, and estimated using recursive methods of filtering and fixed interval smoothing.

In our previous paper, we implemented our proposal for an initial sample of nine

countries, those considered in García-Ferrer et al.(1987), for comparison purposes. Here, we extend our own work to another nine countries, which were analyzed in Zellner and Hong(1989). We use their same sample interval, 1950-1984, again so that the forecasting results are comparable. But we introduce here some further steps of our analysis, which were not included in our previous paper.

We discuss a sequence of steps, each one of them using a different strategy for the estimated trend and perturbation components of each country's GNP. After describing in section 2 our UC model, sections 3 and 4 deal with univariate GNP forecasts: in section 3 we present the forecasting results with a simple Integrated Random Walk (IRW) model for the trend, added to a more sophisticated representation for the perturbational component. In section 4 we improve on our modeling of the trend, and show the results of forecasting each country's GNP with just its trend component. Sections 5 and 6 use a leading indicator, the money supply, already used in García-Ferrer et al.(1987), and in Zellner and Hong(1989), to try to improve GNP forecasts. In section 5 we use a transfer function between the estimated perturbational components of both variables, whereas in section 6 we add to that the forecasts obtained from a transfer function between the estimated trends for both variables. Section 7 contains a summary of the results as well as possible questions for further research.

2. THE THEORETICAL UNOBSERVED COMPONENTS MODEL.

The stochastic state-space model belongs to the class of unobserved components ARIMA (UC-ARIMA) models developed by Engle (1978) and Nerlove et al. (1979) that have been popular in the forecasting literature for some years. However, it has only been recently that papers which exemplify a time variable parameter estimation (TVP) approach [Harvey (1984), Kitagawa and Gersch (1984), Engle et al. (1988) and Ng and Young (1990)] have been utilized within the context of SS estimation. In particular, Young et al. (1989) use a novel spectral interpretation of the SS smoothing algorithms to decompose the series into various, quasi-orthogonal components, the models for which can be identified and estimated using recursive methods of estimation [Young(1984)] that can handle TVP models.

Following Young and Young(1990), we can write the "component" or "structural" model of a univariate time series Y_t as:

$$Y_t = T_t + P_t + \epsilon_t \quad (2.1)$$

where T_t is a low frequency or trend component, whereas P_t is a perturbational component around the long run trend with fairly general statistical properties¹; both T_t and P_t might be dependent upon the similar components of some exogenous (leading indicator) variables. Finally, ϵ_t is a zero mean, serially uncorrelated, white noise component with variance σ_ϵ^2 .

2.a The Models for the Components

It is assumed here that the low-frequency or trend component can be represented by a local linear trend model of the form,

$$\begin{aligned} T_t &= T_{t-1} + S_{t-1} + \eta_t \\ S_t &= S_{t-1} + \xi_t \end{aligned} \quad (2.2)$$

where S_t denotes the local slope or derivative of the trend, and η_t and ξ_t are zero mean, serially and mutually uncorrelated white noise inputs with variances σ_η^2 and σ_ξ^2 , respectively. It is further assumed that these noise inputs are statistically independent of the white noise observational errors ϵ_t in equation (2.1), and therefore: $E(\epsilon_t, \eta_t) = E(\epsilon_t, \xi_t) = E(\eta_t, \xi_t) = 0 \quad \forall t, s$.

This formulation assumes that the time-series can be characterized by a mean value whose time variation depends upon the nature of the model (2.2). Except in cases when there are sharp discontinuities of level or slope η_t can be safely constrained to be zero [Young and Ng(1989)], which we do in what follows. Then, the variance of ξ_t is the only unknown in (2.2) and it can be defined by the Noise Variance Ratio (NVR), which is the relation between σ_ξ^2 and the variance of the observational noise σ_ϵ^2 , that is:

¹ P_t might generally have zero mean, although that is not at all necessary; in frequently observed time series, P_t will include the seasonal characteristics of Y_t .

$$NVR = \frac{\sigma_\xi^2}{\sigma_\epsilon^2} \quad (2.3)$$

It is also assumed that the sum of the stochastic perturbation P_t and the white noise component ϵ_t allows an ARMA representation of the form:

$$P_t + \epsilon_t = \frac{\gamma(L)}{\phi(L)} a_t \quad (2.4)$$

where $\gamma(L)$ and $\phi(L)$ are polynomials of orders m and n in the lag operator L . No stationarity assumptions are necessarily imposed in (2.4), although in our analysis of international GNP data, we will estimate trend components that produce stationary time series² for $P_t + \epsilon_t$. We will also work with purely autoregressive (AR) forms of (2.4)³.

2.b Model Identification and Estimation.

Having defined the SS model structures for all the components, it is now straightforward to assemble them into an aggregate SS form, in which the state vector is composed of all the states from the different submodels, and the observation vector is chosen to extract from the state vector the structural components T_t and P_t [Young et al. (1989)]. However, the problems of structural identification and subsequent parameter estimation for the complete SS model are clearly non-trivial, and imposing a particular set of restrictions has generally been the way to achieve

² Under certain conditions, the model proposed in (2.1) to (2.4) can be written as an ARIMA($m, 2, m+2$) model for Y_t . So, our model implies that Y_t is I(2), which can be tested. Following Dickey y Patula (1987), we performed augmented Dickey-Fuller tests for the GNP series from the nine countries which we will later describe, and for the whole sample period, 1950-1984, obtaining ADF statistics ranging from -1.86 and -3.24, which allow for safely maintaining the I(2) hypothesis.

³ Alternatively, we assume that the rational lag polynomial in (2.4) can be well approximated by a finite order AR model. The presence of non trivial $\phi(L)$ polynomials in (2.4) will imply fairly long AR approximations to that equation.

identification in the statistical literature on signal extraction.

In order to estimate the proposed model, the most obvious approach is to formulate the problem in maximum likelihood (ML) terms. If the disturbances are normally distributed, the likelihood function for the observations can be obtained from the Kalman filter by "prediction error decomposition" [Harvey(1984); Harvey and Peters(1990)]. However, practical experience with this approach indicates that it can turn out to be rather complex, even for particularly simple structural models [García-Ferrer(1992)], with the likelihood function tending to be rather flat around the optimum⁴.

In this paper, however, we utilise a rather different 'manual tuning' approach based on the spectral filtering properties of the fixed interval smoothing (FIS) algorithms used in the state-space analysis [see Young(1988); Young(1993); Young and Tych(1993)]. To explain this approach, let us consider again the simplest version of equation (2.1); namely where Y_t is represented by a simple trend plus noise model, i.e.,

$$Y_t = T_t + e_t$$

in which T_t is assumed to evolve as a IRW process; i.e., in vector matrix terms,

$$\begin{bmatrix} T_t \\ S_t \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xi_t$$

This model can be written in the following alternative TF form,

$$Y_t = \frac{1}{(1-L)^2} \xi_t + e_t \quad (2.5)$$

⁴ Despite its problems, this ML approach has become the standard in recent years, following from the work of Harvey(1984), Kitagawa(1981) and others. More recently, Young and Tych(1993), have optimised the NVR values so that the logarithm of the pseudo-spectrum (pseudo- because the IRW model is non stationary) matches the logarithm of either the AR spectrum or the periodogram of the data, in a least squares sense.

so that the autocovariance generating function $g(L)$ for the model is defined by:

$$g(L) = g_T(L) + \sigma_e^2$$

where $g_T(L)$ is the autocovariance generating function for the IRW component alone, i.e.

$$g_T(L) = \frac{\sigma_\xi^2}{(1-L)^2 (1-L^{-1})^2}$$

Bell(1984) has shown that the classical Kolmogorov-Wiener-Whittle approach to filtering and signal extraction can be applied to nonstationary processes such as (2.5). Consequently, for a large sample size N , the optimal smoothing (signal extraction) filter for estimating T_t is given by the ratio of $g_T(L)$ to $g(L)$. In terms of the NVR this can be written simply as,

$$\hat{T}_{t|N} = \frac{NVR}{NVR + (1-L)^2 (1-L^{-1})^2} Y_t \quad (2.6)$$

where $\hat{T}_{t|N}$ is the optimally smoothed estimate of T_t for period t based on all N observations. This is a symmetric, two sided filter requiring only the specification of the NVR value. It is easy to verify that this is a lag-free, low-pass filter with a sharp cut off for smaller values of the NVR and excellent filtering properties which attenuate all higher frequency noise on the data. The associated FIS algorithm has been used for many years [see Jakeman and Young(1979)] in the various versions of the CAPTAIN and *micro*CAPTAIN programs [e.g. Young and Benner(1991) for a description of the latest version] where it is termed the "IRWSMOOTH" algorithm. Since, for large N , the asymptotically optimal smoothing filter (2.6) will yield the same results as the recursive IRWSMOOTH estimator (except for samples near the beginning and end of the series) the IRWSMOOTH estimates will naturally have similarly favourable properties.

Figure 1 shows the spectral densities associated with different NVR values. The bandpass

of the corresponding filter is reduced for lower values of the NVR⁵. It is clearly important that the smoothed estimate \hat{T}_{N} follows the low frequency components of Y_t , and excludes those components P_t and e_t , that can be considered as higher frequency cycles or noise. For macroeconomic data, it seems reasonable that the detrended data will need to explain business cycle effects, and so the NVR should be chosen so that the IRWSMOOTH algorithm removes the low frequency effects without affecting the higher frequency behaviour that may be associated with such business cycles. For annual data, an NVR = 0.1 yields a 50% cut-off frequency: $F_{50} = 0.09$ cycles/year, i.e., an associated cyclical period of $P_{50} = 11.2$ years and a 10% cut-off frequency: $F_{10} = 0.16$ cycles/year, i.e., an associated period: $P_{10} = 6.3$ years, so that the estimated trend will contain very little power at cycles with periods of 6 years or less, the ones which could be associated with business cycle behaviour.

The widely used filter (KP) in Hodrick and Prescott(1980) [see also Kydland and Prescott(1990)] is equivalent to an IRWSMOOTH algorithm with NVR = 0.000625 [see Jakeman and Young(1984)], and is obtained by solving a constrained optimisation problem. For quarterly data, that NVR value corresponds to a filter bandwidth with $P_{50} = 9.9$ years and $P_{10} = 5.7$ years, respectively, and so it is very similar to the one used here for our annual data.

In García-Ferrer et al.(1993), it was observed that for the logarithm of the USA GNP series, the periodogram of the estimated trend using NVR = 0.1 matches the periodogram of the original data exactly at the lower frequency range. The periodogram of the detrended data clearly shows how these low frequency components have been removed, leaving the components in the frequency range above 0.16 cycles virtually unaffected. That was the major justification for our choice of NVR = 0.1 for the analysis in that paper, as well as in the subsequent sections of this paper⁶. It is obviously relevant to our goal of producing a fairly automatic, good forecasting methodology, that the only parameter choice to be made can be taken to be the same for the whole set of 18 countries we have so far considered.

⁵ Included in Figure 1 is the spectral density for the filter used by Hodrick and Prescott (1980) and Kydland and Prescott(1990), which is an IRWSMOOTH-type filter for the *quarterly* data, with the NVR constrained to 0.000625.

⁶ Interestingly enough, this value tends to be confirmed by the optimisation approach of Young and Tych mentioned in footnote 4. For example, optimisation of this type based on an AR(15) spectrum of the US and UK GNP series yields NVR values of 0.14 and 0.079, respectively.

Equation (2.6) shows how the IRWSMOOTH filter provides an estimate of the trend which depends upon a centralised moving average (CMA) of the data either side of t [see Young and Tych(1993)]. The weighting pattern of the CMA for the FIS smoothing filter depends upon the value of the NVR parameter, but for our choice of NVR = 0.1, that moving average extends to four periods either side of a given data point, with rapidly decreasing weights.

3. ANALYSIS OF GNP/GDP ANNUAL DATA FOR NINE COUNTRIES.

In this section, we report on the results obtained from fitting the SS model described in the previous section to real gross national product for: Australia, Austria, Canada, Finland, Japan, Norway, Spain, Sweden and Switzerland. GNP/GDP data for these countries, taken from the International Monetary Fund's International Financial Statistics for the period 1950-1984, were first studied in a pioneering paper by Zellner and Hong (1989) and were subsequently analyzed in a sequence of papers⁷.

The steps we followed in our analysis were:

1. Real GNP data up to 1973 were used to obtain fitted values for T_t and P_t according to (2.2) and (2.4), based on an approach discussed in Section 2 and using the *microCAPTAIN* software package.
2. The fitted models were used to generate eleven one-step-ahead forecasts, to cover the same 1974-1984 period studied in Zellner and Hong(1989). The models were re-estimated using all past data prior to each forecast period. Conversion from the original forecasts to growth rates followed immediately.
3. The root mean squared errors (RMSE's) by country, as well as overall measures of forecasting precision, were obtained in order to appraise the forecasting performance of the different approaches considered.

⁷ Zellner and Hong also consider GNP/GDP data from Belgium, Denmark, France, West Germany, Ireland, Italy, the Netherlands, The U.K. and the U.S., but we already applied some of the methods in this paper to these countries. Further study of these countries GNP data, using all the procedures we describe in this paper is left for future research.

Figures 2.a to 2.i show the trend components estimated for GNP/GDP for the nine countries. Also included are the similar trends estimated for real balances, to which we will refer below. The estimates were obtained with the sample through 1984, using the Kalman filter and fixed interval smoother, as discussed in Section 2. Figures 3.a to 3.i present the detrended GNP (perturbational) data P_t . To model these series, we specified pure AR models, choosing their order on the basis of the Akaike Information Criterion (AIC). The adjusted coefficients of determination ranging between $R^2_T = .27$ and $R^2_T = .72$ [see Table 1 for a summary of the full sample identification and estimation results].

[INSERT FIGURES 2 AND 3]

[INSERT TABLE 1]

In Zellner and Hong(1989) and subsequent papers, several univariate forecasting models were investigated for this same GNP data set. First of all, the forecasting performance of three 'naive models'(NMI, NMII and NMIII) was analyzed in order to serve as a benchmark in evaluating the forecasting performance of more sophisticated and complex alternatives. NMI assumes that the GNP/GDP annual growth is equal to zero for each year in the sample. NMII assumes that the GNP/GDP growth rate is a random walk, whereas NMIII assumes that the growth rate of GNP is each year a weighted average of past growth rates. The implied RMSE's for each naive model, as calculated in Zellner and Hong(1989) for the sample period 1974-84, are reproduced in rows A to C of Table 2.

[INSERT TABLE 2]

The naive models weigh the past too heavily, not anticipating future changes. Their forecast errors are therefore rather large in the vicinity of turning points in the rate of growth of output, which generally occurred in 1974-75, 1979-80 and 1983-84. As a first attempt to improve upon these models, Zellner and Hong(1989) fitted an AR(3) to the growth rate of output. Such a model was chosen to allow for the possibility of the AR polynomial having two complex roots associated with a cyclical solution, plus one real root associated with a trend on the growth rate. The AR(3) RMSE's from the mentioned source are reproduced in row D of Table 2. These models did not produce a substantial improvement in forecasting performance, the random walk models (NMII) doing better in six of the nine countries.

Univariate, one-step-ahead forecasts for our unobserved component models (UCM) represented in equations (2.1), (2.2) and (2.4) can be obtained via the Kalman filter, by repeated application of the prediction equations developed in Young(1988) as a simple by-product of the recursive forecasting and smoothing algorithms. Their RMSE's are shown in line E of Table 2. They range from 1.42 percentage points for Sweden to 3.92 for Japan. Our UCM models had the smallest RMSE's among the previous univariate alternatives considered in Table 2 for Australia, Austria, Canada and Sweden. They also had the smallest median of the nine countries RMSE's [see Table 3], being 14% lower than that for the random walk model (NMII), although they showed again rather large forecasting errors in the vicinity of turning points. It seems obvious that both, the trend and the perturbation models need to be improved to avoid such errors. We start by considering in the next section a better univariate alternative to the trend model. Later on, we will consider in Section 5 using an additional variable (the money supply) as a leading indicator, to improve forecasting performance.

[INSERT TABLE 3]

4. THE TREND MODEL RECONSIDERED

On purely theoretical grounds, the reduced form equation corresponding to the IRW trend model would be an ARIMA(0,2,0). However, when we obtained the sample autocorrelation (acf) and partial autocorrelation (pacf) functions for the second differences (∇^2) of our estimated trends, the evidence was in all cases against the white noise hypothesis. On the contrary, as Table 4 shows, we identified and estimated an AR(2) structure in all countries. The estimation results for such these models are shown in Table 4, together with measures of statistical fit (R^2) and the Ljung-Box (LB) test for the estimated residuals⁸. The estimated lag polynomials are remarkably similar, as it was the case with the analysis in García-Ferrer et al.(1993). They all have complex roots, allowing for the possibility of a pseudo-cycle within the trend. The complex roots of the estimated AR polynomials produce the expected long term oscillatory behavior, but with such a long period that it has no economic interpretation. This reflects the fact that our NVR choice just allows for

⁸ The R^2 and LB statistics suggest that the AR(2) specifications is quite appropriate, except in Finland, where there seems to be a rather important residual autocorrelation left.

some cyclical effects to start showing up in the estimated trends. For a careful study of the periodicities of the cyclical fluctuations in this GNP data set, see Hong(1989).

[INSERT TABLE 4]

Under these sample results, the IRW model for the trend in (2.2) can now be extended in the following manner (see Young et al. 1992):

$$\begin{aligned} T_i &= T_{i-1} + S_{i-1} & (i) \\ S_i &= S_{i-1} + D_{i-1} & (ii) \\ \phi(L) D_i &= a_i & (iii) \end{aligned} \quad (4.1)$$

where $\phi(L)$ is an m -order lag polynomial, which has been taken equal to 2 for each model in Table 4. Equation (iii) above can be considered as a Double Integrated Autoregressive (DIAR) model that can easily be transformed into state space form [Young(1993)] and adjoined with (i) and (ii) to yield a complete SS model for the trend, which then replaces the simpler IRW model. Row F in Table 2 shows the forecasting results for real output growth rates using just the AR(2) models for the second difference of the trend. In spite of being just one of the components of the GNP series, it outperforms our own univariate models in five out of the nine countries. The TM model produced the lowest median RMSE among all the univariate alternatives.

5. TRANSFER FUNCTION MODELING BETWEEN ANNUAL GNP AND THE MONEY SUPPLY FOR THE NINE COUNTRIES.

An alternative option to try to achieve a better forecasting performance consists of using an additional variable as a leading indicator (LI). This was the strategy followed in García-Ferrer et al.(1987), as well as in Zellner and Hong(1989), where an autoregressive leading indicators (AR(3)LI) model was used to generate one-step-ahead forecasts for the 1974-1984 period for the nine countries considered in this paper. There, the possible leading indicators were: three lags of

output growth rates, two lags of the rates of return of each country's stock market, one lag of the world's stock rate of return, and one lag of the money supply growth rate, all in real terms. Further additional computations were performed to check the effects of using two types of Stein-like shrinkage techniques in forecasting, to which we refer below.

As it is the case in this paper, the search for a fairly automatic, low cost, forecasting system suggested the estimation of the same model for all countries, even though by doing a country specific analysis, the choice of appropriate leading indicators might differ among them. That search could only lead to a better forecasting performance, although at the cost of a more time demanding analysis.

[INSERT TABLE 5]

The results obtained by Zellner and Hong(1979), reported in line G of Table 5 do not show a clear improvement in forecasting performance. In spite of using more sample information, the random walk model for the rate of growth of output (NMII) was still better than the AR(3)LI model in five out of the nine countries. Even more surprisingly, within this sample and forecasting exercise, relative to the univariate AR(3) models, the use of the leading indicators improved the forecasts in just four of the nine countries. To the contrary, in all countries except Norway and Switzerland, our univariate trend model (TM) performed better than the AR(3)LI, which suggests the convenience of some country specific modelling¹⁰. On the other hand, the results obtained in Zellner and Hong by the two mentioned shrinkage procedures are reproduced in rows H.1 and H.2 of Table 5 and clearly show a significant forecasting improvement relative to their simpler AR(3)LI model.

In line with these models, we use in this paper the money supply as a potential leading indicator of real output growth. The nominal money supply and the consumer price index for each country had been obtained from the same source as the GNP data. Real money balances were

⁹ The median of the rates of return for the countries considered in that paper.

¹⁰ Even though we were specific just to the point of allowing a different order for the AR model for P_1 , and that is done quite automatically by microCAPTAIN.

calculated as the ratio of these two variables¹¹. We estimated the money supply perturbations (PMS) using the same detrending procedure as with the GNP data, and the same NVR = 0.1¹². We then identified and estimated individual country specific TF models between these two perturbations (PMS and PGNP), which took the following general form,

$$PGNP_t = \frac{B(L)}{A(L)} PMS_t + \frac{D(L)}{C(L)} e_t \quad \begin{matrix} i = 1, \dots, 9 \\ t = 1, \dots, T \end{matrix} \quad (5.1)$$

where $A(L)$, $B(L)$, $D(L)$ and $C(L)$ are polynomials in the backward operator L .

Since both, $PGNP_t$ and PMS_t are mean stationary variables, the identification and estimation of the TF models (5.1) can be obtained directly by application of the Simplified Recursive Instrumental Variable (SRIV) algorithm developed by Young(1985) and included in the input-output option of *microCAPTAIN*¹³. The cross-correlation functions between the $PGNP_t$ and PMS_t components generally showed important contemporaneous and first lagged values, suggesting a dynamic relationship between the two variables. Table 6 shows a summary of the identification and estimation results of (5.1): R_r^2 is the coefficient of determination based on the full TF model and YIC [see Young(1988)] which is an identification criterion based on balancing the degree to which the model explains the data with how well the model parameter estimates are statistically defined¹⁴.

[INSERT TABLE 6]

¹¹ We calculated real money supply data for the same sample periods as the ones studied for the GNP data, except in the case of Sweden, where the money supply was available for the Zellner and Hong work just until 1982.

¹² Notice that in obtaining the PMS data, we only need to specify the NVR for the trend, and there is no need to further identify its univariate model.

¹³ At present, the I-O mode available in *microCAPTAIN* does not have a forecasting option ready. We have, therefore, used the package mainly as a specification tool, while the estimation and prediction with the full model (the TF plus the noise model) have been carried out using the SCA statistical package.

¹⁴ As with other identification criteria, the YIC is aimed at identifying models which explain the data well within an efficient parameterization.

The analysis we performed showed that: a) there is evidence of simultaneity in several countries, b) the dynamics are not homogeneous across countries, and c) real balances are not always a leading indicator of future GNP. We have focused in this paper on the one-sided relationship for consistency with the above mentioned references, leaving the analysis of the feedback relation for future research. When the identified TF suggested a contemporaneous relationship between the two variables, we estimated an univariate ARIMA model for the input (PMS) and used their one-step-ahead forecasts to obtain future output values (PGNP) through the TF model (5.1).

The GNP perturbation forecasts obtained from the TF model (5.1) were added to those that we had already obtained from the univariate trend models TM analyzed in Section 4. As before, in obtaining the forecasts, the models were re-estimated every year using all past data up to the forecasting period. Conversion from output levels to growth rates forecasts follows immediately, and forecasts errors and RMSE's were computed for each country. The forecasting results are shown in row I of Table 5. As it was the case in García-Ferrer et al. (1993), the addition of the money supply perturbation transfer models improves the forecasting performance of the TM model by a 10% on the average. The TFUCM1 model RMSE's are lower than those of the TM model in all countries but Finland, although this improvement does not show up in the median value.

In terms of the median RMSE, the combined model (TFUCM1 in row I.2) essentially reproduces the result obtained with the trend model (TM) (2.31 versus 2.32). Relative to our own univariate model (UCM in Table 2) the reduction in the median RMSE is of 4.5% (2.31 versus 2.42). Our TFUCM1 model worked reasonably well in comparison with the previous multivariate alternatives in Table 5, having the lowest RMSE's in four out of the nine countries. It does better than the AR(3)LI, which uses more sample information, in all countries but Switzerland.

The better forecasting results of this section have been obtained working with the trend and perturbation components of GNP separately. However, they have received an asymmetric treatment, since the GNP perturbations have been related to those of the money supply in real terms, while the GNP trend has been forecasted from its univariate ARIMA model. This is quite in line with the standard practice of 'detrending' economic variables prior to establishing empirical relationships between them. Accepting that as a somewhat natural practice when working with trends which are deterministic functions of time, the view we adopt here is more general, accepting that each of the stochastic components (trend, perturbation, seasonal) may have its own model when

trying to specify the relationships between economic variables. Establishing the trends TF is the object of the next section.

6. TRANSFER FUNCTION MODELS BETWEEN THE ESTIMATED TREND COMPONENTS.

To close up on our use of the money supply as a leading indicator of GNP, we specified and estimated a transfer function model for the estimated trends of GNP and the money supply obtained in the previous sections. The resulting forecasts for the GNP trend for each country and year were added to the corresponding forecasts for the GNP perturbations from the TF (5.1), to obtain the GNP forecasts. Growth rates forecasts and the corresponding RMSE's were obtained as in previous instances.

The results in row J of Table 5 show that the use of the money supply trend as a leading indicator of the GNP trend produces a clear gain in forecasting performance, relative to the univariate trend model TM. The transfer function between the trends has a median RMSE of 2.26 versus a 2.32 median value in the TM model. However, the most important improvement is obtained when the trends transfer function forecasts from TFUCM2 are added to the ones from the perturbations transfer function TFUCM1, to obtain row K in Table 5 (which we label TFUCM3). The median RMSE for this latter model represents a 13% reduction (1.97 versus 2.26) relative to the transfer function between the trends (TFUCM2), and a 15% reduction (1.97 versus 2.31) relative to the transfer function between the perturbations (TFUCM1). Relative to our own univariate model UCM, the gain in forecasting performance as measured by the median RMSE values is of 19% (1.97 versus 2.42). Compared to the AR(3)LI model, the gain is still more important, in spite of using less sample information.

Summary forecasting performance measures for the TF models are presented in Table 7. Our TFUCM3 model produced the lowest median RMSE among the different alternatives in Table 6. Plots of the TFUCM3 forecasts and observed annual growth rates are presented in Figures 4.a to 4.i.

7. SUMMARY AND CONCLUDING REMARKS

We have continued in this paper with previous work by the same authors, trying to provide a quite automatic method to obtain a good forecasting performance when dealing with a large set of comparable macroeconomic variables. We apply our proposed model to GNP/GDP data for a set of nine countries which, together with those considered in our previous work, make up the full set of countries, and the same sample (1950-1984) that was analyzed in a series of papers by previous authors. That allows for comparisons between the forecasting results obtained with the different approaches.

Our proposal consists on a state-space formulation for the different components of an unobserved components model. In our proposal, the trend component is second order stationary. The recursive estimation of the components in our procedure crucially hinges on the choice of the noise-variance ratio for the trend model, but the same objective rules followed in our previous work, which are based on the spectral characteristics of the recursive smoothing algorithm, result in a choice common to all GNP countries.

That such a relevant choice may be common to all time series considered, is quite consistent with our intention that the statistical treatment that all the series in the data set receive be the same. In particular, no intervention analysis of any kind was performed, even though the evidence suggesting it in some series was quite clear. The same intention was behind the alternative modelling proposals with which we have established forecasting comparisons in this paper.

We first obtained in Section 3 univariate forecasts from our UC model for real output growth rates, assuming an integrated random walk representation for the trend. They showed a significant improvement in comparison with other univariate alternatives. In Section 4, we allowed for a more flexible representation for the trend component, which turned to admit an AR(2) representation in its second order differences. The estimated parameters in such representations were remarkable similar across countries. Estimates obtained from this trend component by itself turned out to show a better forecasting performance than the previous, complete univariate models.

In section 5 we incorporated a possible leading indicator, the money supply, in an attempt to improve upon the forecasts of the GNP perturbational component. This attempt did not lead to a significant improvement, possibly due to: a) the relation between both perturbations being rather simultaneous, and b) having already extracted most of the stochastic structure (signal) in the

estimated trend.

Finally, we specified and estimated in Section 6 a transfer function between the estimated trends for GNP and the money supply. The obtained trend forecasts were then added to the perturbation forecasts obtained in Section 5. This has been the first time we have incorporated this relationship in our forecasting model and, contrary to the results obtained with the perturbation model, the trend relationship led to a significant improvement in forecasting performance.

Some additional questions could result in interesting extensions of our current research:

a) some other variables, like the stock price index could be considered as a source of further improvement in forecasting performance, as it has already been done by other authors, b) the feedback dynamic relation between GNP and the money supply needs to be incorporated into our analysis, e.g. by introducing a VAR model between the GNP and money supply perturbations, c) given the visible correlations between all the GNP perturbations in Figure 4, a full multivariate model is clearly possible, d) median values through the countries considered, taken as world measures of economic performance, like output growth rates, real money growth or stock market rates of return, provided a considerable improvement in forecasting performance in work by other authors. These indicators could be easily incorporated into the analysis we have presented here.

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Country	AR Model order	AIC	R^2_t
Australia	7	-0.08	0.39
Austria	5	3.98	0.45
Canada	8	1.88	0.48
Finland	8	1.27	0.71
Japan 1952 - 83	6	15.47	0.27
Norway	8	1.09	0.50
Spain 1954 - 83	8	9.70	0.51
Sweden	7	2.22	0.68
Switzerland	7	9.25	0.72

Note: The estimation period was 1950-1983 for all countries, except where noted.

Model	Australia	Austria	Canada	Finland	Japan	Norway	Spain	Sweden ^b	Switzerland	Median
A. NMI ($y_t = 0$)	3.10	2.91	3.51	3.39	4.10	4.15	2.51	2.29	3.03	3.10
B. NMII ($y_t = y_{t-1}$)	2.90	2.82	4.13	2.34	3.12	1.98	1.86	1.87	3.91	2.82
C. NMIII ($y_t = \text{past average}$)	2.75	3.11	3.31	2.88	4.98	1.76	3.56	2.43	4.48	3.11
D. AR(3)	2.87	3.16	3.55	2.58	3.26	1.75	2.50	2.22	4.24	2.87
E. UCM (NVR = 0.1)	2.42	2.08	2.93	2.84	3.92	2.28	2.18	1.42	3.83	2.42
F. Trend Model (TM)	2.34	2.21	3.05	2.32	2.70	1.71	1.67	1.93	4.87	2.32

NOTES: a) Lines A to D are reproduced from Zellner and Hong(1989), Table 2, p.194.
b) 1974-1983 forecasting period.

Table 3			
Summary Forecasting Performance Measures for the Univariate Models			
Model	Largest Country RMSE	Smallest Country RMSE	Median of Nine Countries RMSE
Forecasting Period: 1974 - 1984 ^b			
A. NMI ($y_t = 0$)	4.15	2.29	3.10
B. NMII ($y_t = y_{t-1}$)	4.13	1.86	2.82
C. NMIII ($y_t = \text{past average}$)	4.98	1.76	3.11
D. AR(3)	4.24	1.75	2.87
E. UCM (NVR = 0.1)	3.92	1.42	2.42
F. TM	4.87	1.67	2.32

Notes: a) Based on information in Table 2.
b) Except for Sweden, 1974-1983.

Table 4			
ARIMA Models for the Estimated Trend of GNP for the Nine Countries			
Country	Model	R ²	LB _n , LB _n
Australia	$(1-1.52L+.70L^2) \nabla^2 T_t = a_t$ (.13) (.13)	0.89	1.4, 2.9
Austria	$(1-1.39L+.64L^2) \nabla^2 T_t = a_t$ (.13) (.13)	0.87	2.5, 4.5
Canada	$(1-1.64L+.83L^2) \nabla^2 T_t = a_t$ (.16) (.16)	0.89	0.4, 3.0
Finland	$(1-1.47L+.86L^2) \nabla^2 T_t = a_t$ (.10) (.10)	0.89	10.6, 12.1
Japan 1952 - 1983	$(1-1.64L+.80L^2) \nabla^2 T_t = a_t$ (.11) (.11)	0.93	1.5, 5.2
Norway	$(1-1.74L+.96L^2) \nabla^2 T_t = a_t$ (.10) (.10)	0.94	2.1, 6.3
Spain 1954 - 1983	$(1-1.65L+.79L^2) \nabla^2 T_t = a_t$ (.12) (.12)	0.94	1.1, 5.1
Sweden	$(1-1.38L+.59L^2) \nabla^2 T_t = a_t$ (.15) (.15)	0.83	2.8, 6.3
Switzerland	$(1-1.54L+.87L^2) \nabla^2 T_t = a_t$ (.09) (.09)	0.92	4.9, 5.7

Notes: a) LB_n denotes the Ljun-Box statistics for n degrees of freedom.
b) The estimation period was 1950-1983 in all cases, except where indicated.

Table 5

RMSE for the one-year-ahead TF Forecast Errors: 1974-81 and 1974-84
(Percentage Points)

Model	Australia	Austria	Canada	Finland	Japan	Norway	Spain	Sweden	Switzerland	Median
Forecast Period: 1974 - 1984										
G. AR(3)LI *	3.34	2.71	3.68	3.37	3.33	1.62	2.06	2.32	3.45	3.33
H. AR(3)LI *										
1. Shrinkage(1)	2.01	1.77	2.39	2.01	2.40	1.68	1.65	2.01	2.71	2.01
2. Shrinkage(2)	2.32	2.12	2.92	2.42	2.51	1.52	1.81	2.29	3.25	2.32
I. TFUCM1 (Perturbations)	2.31	2.11	3.00	2.40	2.33	1.51	1.32	1.75	4.01	2.31
J. TFUCM2 (Trends)	2.26	2.04	2.72	3.46	2.65	1.49	1.91	1.87	4.97	2.26
K. TFUCM3 (Trends+ Perturbations)	1.95	1.97	2.70	2.99	2.34	1.35	1.23	1.68	4.13	1.97

NOTE: a) Zellner and Hong (1989, Table 1, p.193). See also this reference for the definitions of the shrinkage(1) and shrinkage(2) models

Table 6
Transfer Function Models Identification

Country	GNP/Money Perturbations					GNP/Money Trends				
	Best Time Delay	A(L) order	B(L) order	YIC	R _T ²	Best Time Delay	A(L) order	B(L) order	YIC	R _T ²
Australia	0	2	1	-3.702	0.471	0	1	2	-9.727	0.989
Austria	1	2	1	-2.612	0.279	1	0	1	-10.913	0.935
Canada	0	0	1	-2.160	0.145	1	1	1	-9.525	0.984
Finland	0	2	1	-4.676	0.635	0	0	1	-11.539	0.953
Japan	2	1	1	-3.611	0.378	0	1	2	-3.748	0.875
Norway	1	2	1	0.018	0.196	1	2	3	-1.816	0.916
Spain	1	2	1	-4.735	0.756	1	2	3	-3.960	0.971
Sweden	1	2	1	-4.415	0.599	1	2	3	-0.588	0.861
Switzerland	2	1	1	-4.931	0.660	1	0	1	-13.693	0.980

Table 7			
Summary Performance Forecasting Measures for the TF Models			
Model	Largest RMSE	Smallest RMSE	Median of Nine Countries RMSE
Forecasting Period : 1974 - 1984 ^b			
G. AR(3)LI	3.68	1.62	3.33
H. AR(3)LI			
1. Shrinkage(1)	2.71	1.65	2.01
2. Shrinkage(2)	3.25	1.52	2.32
I. TFUCM1 (Perturbations)	4.01	1.32	2.31
J. TFUCM2 (Trends)	4.97	1.49	2.26
K. TFUCM3 (Perturbations+Trends)	4.13	1.23	1.97

Notes: a) Based on information in Table 6.
 b) Except for Sweden, 1974-1983.

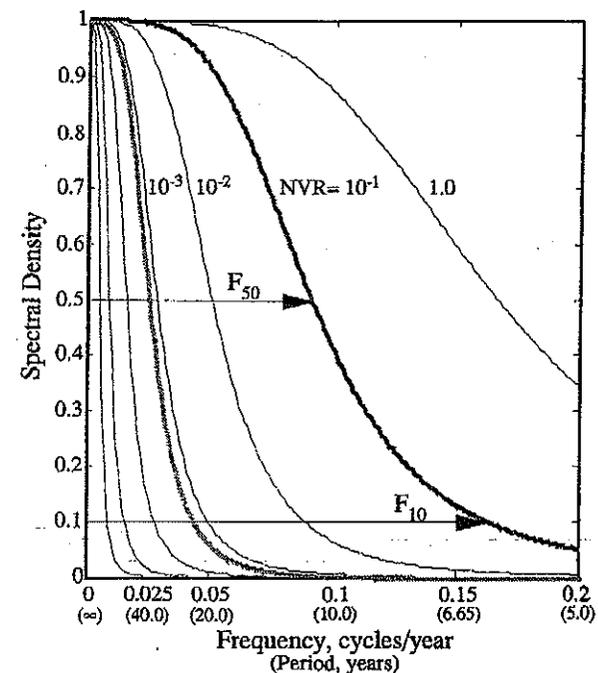


Fig. 1 Spectral characteristics of the IRWSMOOTH filter: the NVR=0.1 filter used in the present analysis is shown (thick dark) with the 50% and 10% bandwidths indicated by arrows; the filter characteristics for other NVR values (1.0, 10^{-2} , 10^{-3} , 10^{-4} , 10^{-5} , 10^{-6}) are shown by fine lines. Also shown (thick light) for comparison is the KP filter for quarterly data (see text), where the frequency (period) axis should be read in cycles/quarter (quarters).

Figure 2.a
AUSTRALIA: GNP AND REAL MONEY TRENDS

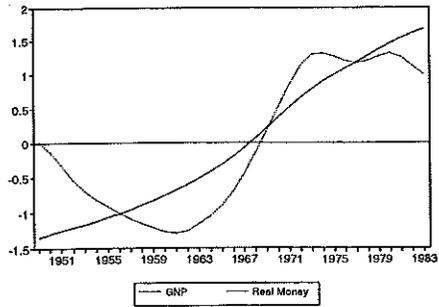


Figure 2.b
AUSTRIA: GNP AND REAL MONEY TRENDS

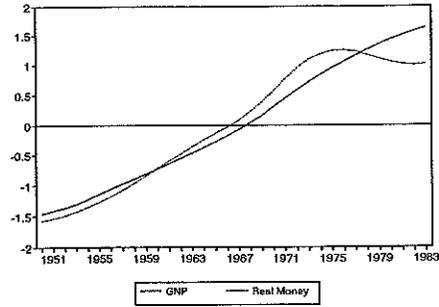


Figure 2.c
CANADA: GNP AND REAL MONEY TRENDS

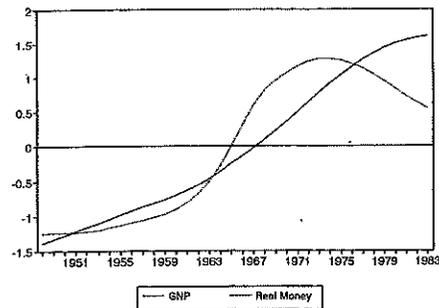


Figure 2.d
FINLAND: GNP AND REAL MONEY TRENDS

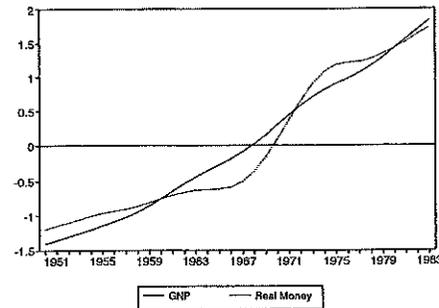


Figure 2.e
JAPAN: GNP AND REAL MONEY TRENDS

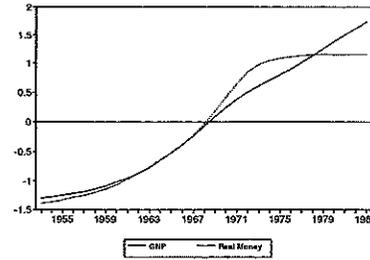


Figure 2.f
NORWAY: GNP AND REAL MONEY TRENDS



Figure 2.g
SPAIN: GNP AND REAL MONEY TRENDS

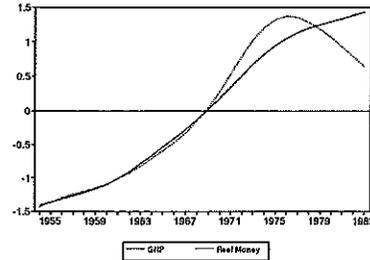


Figure 2.h
SWEDEN: GNP AND REAL MONEY TRENDS

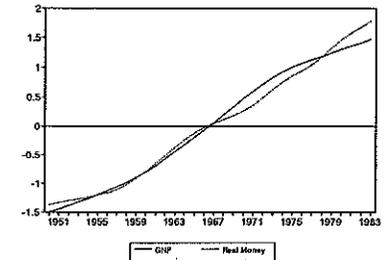


Figure 2.i
SWITZERLAND: GNP AND REAL MONEY TRENDS

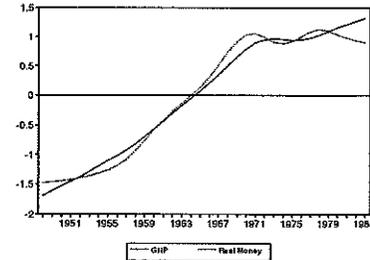


Figure 3.a
AUSTRALIA: GNP PERTURBATIONS

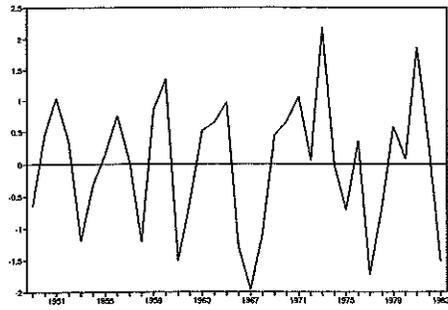


Figure 3.b
AUSTRIA: GNP PERTURBATIONS

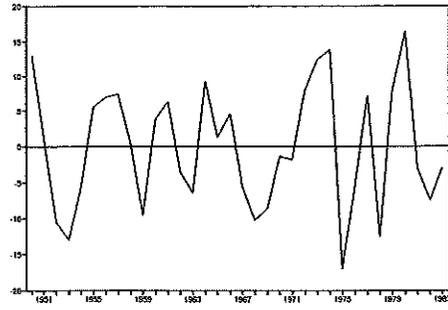


Figure 3.c
CANADA: GNP PERTURBATIONS

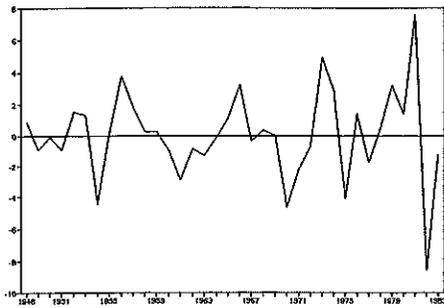


Figure 3.d
FINLAND: GNP PERTURBATIONS

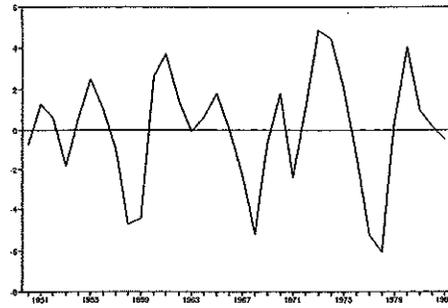


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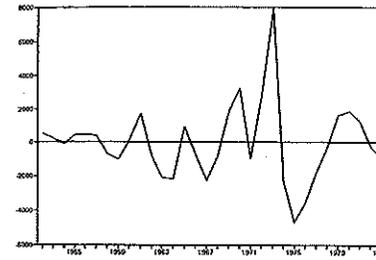


Figure 3.f
NORWAY: GNP PERTURBATIONS

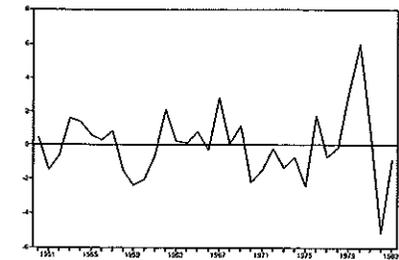


Figure 3.g
SPAIN: GNP PERTURBATIONS

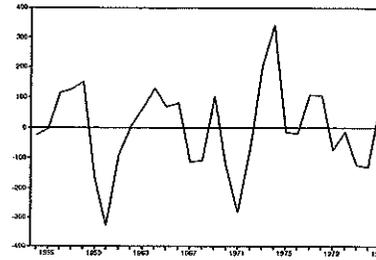


Figure 3.h
SWEDEN: GNP PERTURBATIONS

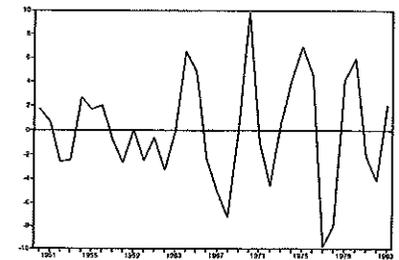


Figure 3.i
SWITZERLAND: GNP PERTURBATIONS

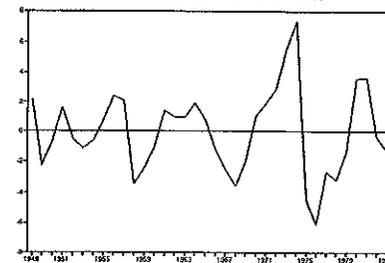


Figure 4.a
AUSTRALIA: GNP GROWTH RATES

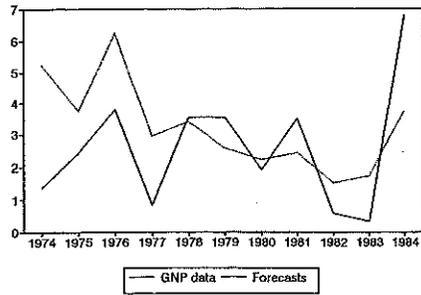


Figure 4.b
AUSTRIA: GNP GROWTH RATES

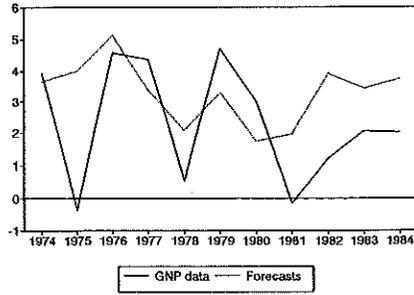


Figure 4.c
CANADA: GNP GROWTH RATES

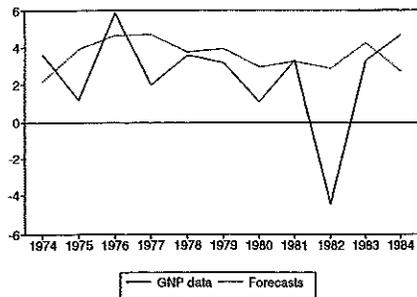


Figure 4.d
FINLAND: GNP GROWTH RATES

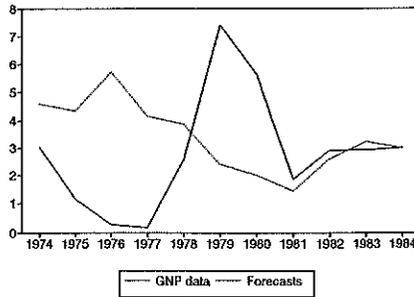


Figure 4.e
JAPAN: GNP GROWTH RATES

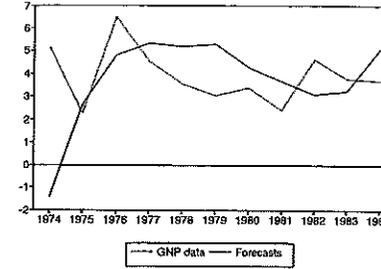


Figure 4.f
NORWAY: GNP GROWTH RATES

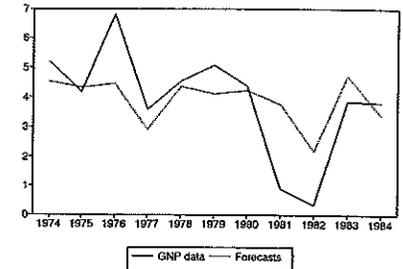


Figure 4.g
SPAIN: GNP GROWTH RATES

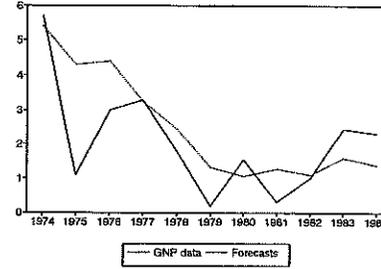


Figure 4.h
SWEDEN: GNP GROWTH RATES

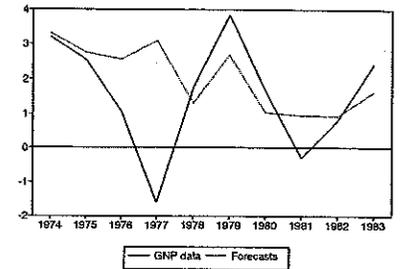


Figure 4.i
SWITZERLAND: GNP GROWTH RATES

