MONETARY POLICY WITH PRIVATE INFORMATION:
A ROLE FOR MONETARY TARGETS*
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ABSTRACT
This paper examines the optimal design of a set of legislative rules which balances credibility in monetary policy when the central banker has some private information. The main result is that the best legislative package should include a monetary target set by Congress, a targeting horizon consisting of one period of time and a small punishment on the central banker if it deviates from the target. Moreover, it is shown that both the discretionary and the average targeting approaches to monetary policy are nested into our more comprehensive approach.

RESUMEN
Este artículo examina el diseño óptimo de un paquete legislativo que equilibre credibilidad y flexibilidad en la política monetaria cuando el banco central dispone de información privada. El principal resultado es que tal paquete debería incluir un objetivo monetario fijado por el poder legislativo, un anuncio del objetivo para un único período de tiempo y una pequeña penalización sobre el banco central si se desvía del objetivo. Además, se muestra que nuestro enfoque engloba tanto políticas monetarias discrecionales como políticas basadas en objetivos medios.

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The classic time-inconsistency problem in monetary policy arises when the market-determined output level is deemed suboptimal by a discretionary central banker. As a consequence, its attempt to surprise rational and forward-looking agents with high inflation will cause the economy to suffer from an inflationary bias without any additional gain in output (Finn Kydland and Edward Prescott, 1977; Robert J. Barro and David B. Gordon, 1983b).

Although solutions to this problem have been proposed via reputation (Barro and Gordon, 1983a) or delegation (Kenneth Rogoff, 1985) mechanisms, such proposals are weakened if the central banker has some private information because the agents cannot verify that the monetary authority has not intentionally invalidated their expectations (Matthew B. Canzoneri, 1985).

As alternative resolutions, three legislative approaches have been suggested. First, Canzoneri’s (1985) average targeting procedure specifies that Congress should pass legislation requiring that the average money growth rate over a given time horizon equal the desired inflation rate. Second, Rogoff’s (1985) flexible targeting procedure suggests that Congress should impose a cost on the central banker if money growth deviates each period from the desired inflation rate. Third, Torsten Persson and Guido Tabellini’s (1993) approach suggests that Congress and the central banker should sign a performance-based contract specifying a linear punishment on the central banker for any percentage point of realized inflation (or money growth). Since these procedures do not depend on the central banker’s private information, they are operational.

This paper examines the optimal design of a monetary policy set of legislative rules which balances credibility and flexibility in monetary policy when there is private information. In particular, we analyze a legislated monetary policy package consisting of a monetary target, a punishment for deviation from the target, a targeting horizon, and a target setter. The main result of the model is that the best package should include a
target set by Congress, a targeting horizon consisting of just one period of
time and a small punishment on the central banker if it deviates from the
target. Moreover, it is shown that both the discretionary solution and the
average targeting solution are special cases of our more comprehensive
approach.

I. The basic model

Consider a standard rational expectations supply function
\[ Y_t = y_n + \sigma(w_t - w^*_t) \]
where \( Y_t \) is the log of output in period \( t \), \( y_n \) denotes the log of the natural
rate of output, \( w_t \) is the actual inflation rate in \( t \), and \( w^*_t \) is the wage
setters’ prediction of the inflation rate conditional on information available
at the end of period \( t-1 \).

Equation (1) incorporates the basic properties of an expectational
Phillips curve in which only unexpected inflation creates, for a time, a
deviation from the natural rate of output. This may due to the existence in
the economy of nominal labor contracts, firms that hire workers according to
their marginal productivity curve, and some labor market imperfections that
keep real wages above the real market-clearing wage (see Alex Cukierman, 1992,
pp. 38-42, for details).

The price level arises from a simple quantity equation
\[ P_t = m_t + v_t - y_n \]
where \( m_t \) and \( p_t \) are the logs of the money stock and the price level,
respectively, and \( v_t \) is a money demand disturbance realized at the end of
period \( t \). Suppose that \( v_t \) follows a random walk.

First differencing, (2) yields
\[ w_t = \omega_t - \delta_t \]

where \( \omega_t = (v_t - v_{t-1}) \) and \( \delta_t \) is the growth rate of the money stock and the
monetary policymaker’s instrument; \( \delta_t \) is a white noise innovation in the money
demand with a finite variance, \( \sigma_t^2 \).

After period-t wages are set, the monetary policymaker chooses its
policy, \( g_t \). Suppose further that the wage setters do not see \( \delta_t \) at the time
they have to specify nominal wages. However, the monetary policymaker does
have a private forecast \( (e_t) \) of \( \delta_t \) at the time it conducts monetary policy.
Such a forecast has a white noise error \( (e_t) \) with finite variance \( \sigma_e^2 \) and
uncorrelated with \( e_t \). Hence, \( \delta_t = e_t + e_t \) and \( \sigma^2 + \sigma_e^2 = \sigma_t^2 \). Although the wage
setters observe \( \delta_t \) and \( w_t \) after \( g_t \) is set, they cannot distinguish the
forecast, \( e_t \), from the forecast error, \( e_t \).

In our economy there exists a government that carries out certain
administrative and legislative duties. Since one of the government’s
administrative duties is to conduct monetary policy, we will refer to the
branch of government that performs administrative tasks as the central banker.
The branch of government that carries out legislative duties will be referred
as Congress.

We assume that the government as a whole has preferences over two policy
outcomes: a desirable output goal, \( k_n \), and a desirable inflation rate, \( \pi^* \), as
reflected in the utility function
\[ U_t = -(y_t - k_n)^2 - s(\pi_t - \pi^*)^2 \]

Assume that the government wants to maximize its expected N-period
average utility
\[ U = \frac{1}{N} E_0 \left( \sum_{t=1}^{N} u_t \right) \]

where \( N \) is the number of periods in the time horizon contemplated by the
government, whereas \( E_0 \) is an expectations operator conditional on period \( t = T \)
information. Discounting is ignored for parsimony.

The government's expected average N-period utility can be rewritten as

\[ U = \frac{1}{N} \mathbb{E}_0 \left\{ \sum_{t=1}^{N} u_t \right\} \]

Thus the central banker's problem is to choose the path of money stock rates of growth to maximize the expected value of its average N-period utility. Because of the stationary nature of the model, this maximization problem reduces to a sequence of one-period problems, in which the central banker chooses \( \delta_t \) to maximize its expected one-period utility for each period \( t \). If it could adhere to a fully state-contingent rule while truthfully revealing its private information, we would obtain the ideal solution

\[ \delta_t^* = \alpha^* + \epsilon_t \]

This solution does provide the desired inflation rate without changing the average rate of output (because the predictable part of the money demand shock, \( \epsilon_t \), is fully accommodated).

However, Canzoneri (1985) shows that if the central banker's forecast of money demand is private information, direct verification by the wage setters of the central banker's adherence to the ideal policy is not possible. Therefore, if the central banker lacks this type of commitment technology, the discretionary solution emerges as the equilibrium outcome

\[ \delta_t | P = \alpha^* + \epsilon_t + \frac{\alpha^*}{2} \]

characterized by an inflationary bias without a systematic higher output.

As a consequence, Canzoneri (1985) explores several types of resolutions to the inefficiency of the inflationary bias when the central banker cannot credibly reveal its private information. First, he centers upon a reputational approach along the lines of Edward J. Green and Robert H. Porter (1984). Specifically, he shows that the existence of a trigger strategy on the part of the wage setters can mitigate the time inconsistency problem partially, even in the presence of private information. However, this reputational solution is not feasible for a range of parameters such that \( \alpha^*/f \leq 2 \).

Second, he considers a legislative approach. In particular, Congress could legislate a two-period average targeting procedure requiring that the average money growth rate per two periods equal the socially desired inflation rate. This approach has recently been pursued by Michelle R. Garfinke and Seonghwan Oh (1993) by deriving the optimal (from Congress' viewpoint) length of the average targeting horizon. Although such rules are incentive compatible, it will be shown below that they are too rigid to provide a good resolution to the credibility problem.

A third approach is to add private information to Rogoff's (1985) perverse policymaker solution. In this scenario, monetary policy is delegated to a fully independent central banker with an (assumed) proclivity towards anti-inflationary policies. Although this type of resolution works when the central banker's information is verifiable, Canzoneri (1985) shows that it is not effective when the central banker has private information concerning the realization of the stochastic variables that constrain its choices. Moreover, the same ineffectiveness applies to the delegation of monetary policy to a partially independent conservative central banker proposed by Susanne Lohmann (1992).

A fourth approach has been suggested by Persson and Tabellini (1993). These authors propose that Congress should penalize the central banker through a linear punishment \( k \) for any percentage point of realized inflation. In this case, the central banker objective function will be

\[ U|_{PT} = \frac{1}{N} \mathbb{E}_0 \left\{ \sum_{t=1}^{N} [u_t - k \epsilon_t] \right\} \]
Maximizing this objective function, one obtains

\[ R_{\text{opt}} = \pi^* + e_t + \frac{1}{2} k \]

As a consequence, if Congress sets \( k = 2\pi^* \), then the ideal solution would be obtained.\(^1\)

Finally, we consider Rogoff's (1985) legislative approach. Congress could legislate a system of rewards and punishments through which the central banker's incentives are altered so that it places some direct weight on achieving a low rate of growth for a nominal variable that is observed by all market participants (e.g., the inflation rate or the money growth rate). In particular, Congress could legislate that a given (finite or infinite) punishment will be imposed on the central banker if such a nominal variable does not hit a prespecified target. In Rogoff's (1985) analysis, this target is fixed by the central banker itself and since his model considers a one-shot game, the targeting horizon is just one period.

Here we extend Rogoff's (1985) legislative approach in two directions: on the one hand, to consider the possibility that the target to meet can be specified not only by the central banker, but also by Congress; and, on the other hand, to explore the time horizon the target must be specified for. This new approach will permit us to identify an optimal monetary policy package from Congress' viewpoint consisting of a) the optimal target, b) the optimal target setter, c) the optimal punishment, and d) the optimal targeting horizon.

Although Persson and Tabellini (1993) claim that their approach is a targeting procedure, it should be clear that it cannot be considered so. The reason is that a targeting procedure involves a punishment for deviations from a prespecified target. Since in Persson and Tabellini's analysis the central banker is punished for any percentage point of inflation, not for any percentage point of deviation from a target, the existence of such a target becomes irrelevant. In other words, their results are the same irrespective of a target being announced or not. To my knowledge, the real world institutions that would fit Persson and Tabellini's inflation contract are fixing the budget of the central bank or the renumeration of its governors in nominal terms.

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We will center upon a money supply growth targeting procedure because of two reasons. First, since the central banker cannot control the inflation rate perfectly, it will be difficult to see a central banker willing to be punished if it does not hit an inflation target. Second, we will be able to compare our targeting approach to Canzoneri (1985) and Garfinkel and Oh's (1998) average targeting procedure.

II. Monetary Targeting in a Multiperiod Framework

Assume that Congress enters legislation punishing the central banker if it fails to hit an average period money growth target, \( \pi_a \), over a prespecified time horizon, \( N \). That is, the central banker chooses its policy to maximize

\[ U^* = \frac{1}{N} E_0 \left[ \sum_{t=1}^{N} \pi_t - h \left( \sum_{t=1}^{N} \pi_t - \pi_a \right)^2 \right] \quad 1 \leq N < \infty, \quad 0 \leq h \leq \infty \]

It is assumed that Congress can choose a cost \( h \), which the central banker incurs when the target is not met on average over the targeting horizon. The size of \( h \) determines the tightness of the targeting procedure. If \( h = \infty \), Congress imposes an infinite cost on the central banker if the average period money growth target is not met. However, such a cost might be lower and it does not exist if \( h = 0 \).

An important issue here is who specifies the target to be met. There are two potential candidates: Congress and the central banker. As it will be shown below, the optimal target for Congress differs from that for the central banker. In this respect, our work departs from Rogoff (1985) and Canzoneri (1985) in that we optimally derive the target depending upon the target setter. On the contrary, they assume that the target to be hit is the socially optimal inflation rate, \( \pi^* \). Later, we will demonstrate that such a target is optimal for the central banker but not for Congress.

Our monetary targeting model consists of four stages.
1) At the beginning of period 1, Congress sets the cost, $h$, and the targeting horizon, $N$.

2) Then, Congress (or the central banker itself) chooses the monetary target, $g_c$.

3) Each period from 1 to $N$ the following occurs: a) the wage setters form their money growth expectations on the basis of their knowledge of the central banker's utility function, the target setter's identity, the cost $h$, the monetary target, $g_c$, and the targeting horizon; b) the money demand shock $\delta_t$ is realized; c) the central banker sets the money growth rate.

4) At the end of period $N$, the central banker is punished if the target is not met on average.

In stage three the wage setters and the central banker play a non-cooperative game which provides the paths of money growth rates and of money growth expectations over the whole $N$-period. In stage two the target setter optimally chooses the monetary target in the light of such paths. Finally, in stage one, Congress optimally sets both the punishment and the targeting horizon in order to maximize its expected average utility. As a consequence, the model is solved by backward induction.

Proposition 1 characterizes the equilibrium of the non-cooperative game in stage three.

**Proposition 1:** For given $h$, $N$, and $g_c$, the paths of money growth rates and of money growth expectations are given by the following expressions:

- $g_c = \frac{f(N-h)}{1 + f(N-h)} + \frac{\delta_t}{1 + f(N-h)}$ for $t = 1, 2, \ldots, N$.

(see Appendix for the proof).

Once obtained the expressions for $g_c$ and $g_t$, it is necessary to find the optimal target. Here it is important to know who sets the target because the central banker's utility function differs from Congress' utility function since the former one contains the punishment for deviating from the target.

**Proposition 2:** For given $h$ and $N$, the optimal monetary targets for the central banker and Congress are given by the following expressions:

- $g_c^* = \frac{f(N-h)}{1 + f(N-h)} + \frac{1}{N} \frac{1}{f(N-h)}$
- $g_t^* = \frac{f(N-h)}{1 + f(N-h)} + \frac{1}{N} \frac{1}{f(N-h)}$

(see Appendix for the proof).

These results suggest that strategic considerations involve the election of an optimal monetary target. This finding contrasts with the approach adopted by Rogoff and Canzoneri. In particular, we show that using the desired inflation rate, $\pi^*$, as a target is optimal from the central banker viewpoint but Congress would not choose such a target. Only if the punishment $h$ on the central banker for deviating from the target is infinite or the targeting horizon is infinite, Congress would target $\pi^*$.

Since the optimal monetary targets for the central banker and Congress differ, it is clear that the paths of money growth rates will be different depending upon the target setter. Proposition 3 presents the existing relation between both paths of money growth rates.

**Proposition 3:** The path of money growth is higher when the central banker sets the monetary target:
These paths of money growth are related with those resulting from the average targeting solution proposed by Canzoneri (1985), and Garfinkel and Oh (1993), as shown in proposition 4.

**PROPOSITION 4:** The average targeting resolution can be reduced to a special case of our approach when the punishment on the central banker for deviating from the target is infinite.

**(Proof):** As h approaches infinity, it is easy to see that both $g^n_t$ and $g^c_t$ reduce to:

$$g^n_t = g^c_t + \frac{1}{N-t} \left[ \frac{1}{h} \right]$$

(see Appendix for the proof).

Moreover, we can see that the gross inflationary bias decreases as h rises; that is, the higher the punishment for not hitting the target the more inflation conscious the central banker will be.

Despite the existence of a gross inflationary bias, monetary targeting has a somewhat offsetting effect: because the target constraints monetary policy there must be some reversals of previous inflationary biases, as given by the term $\frac{1}{h} \left( \frac{1}{h} \right)$. As t approaches N, the deflationary reversal is higher, so that the net inflationary bias falls and eventually becomes negative. This may be so despite a finite punishment h originates a positive average inflationary bias over the targeting horizon (with the exception of the case where N=1).

At the same time, the deflationary reversal grows as h rises. Then, a higher punishment provokes both a lower gross inflationary bias and a higher deflationary reversal. As a consequence, it reduces the net inflationary bias and therefore the average inflationary bias over the whole N-period.

Moreover, $g^c_t$ contains an additional inflationary term, $\frac{1}{N-t} \left[ \frac{1}{h} \right]$. The reason behind this additional bias is that Congress chooses the target to maximize its utility function whereas the central banker maximizes its own utility function (which contains a punishment for deviating from the target). Therefore, there exists an incentive for the central banker to accommodate (to a certain extent) the target $g^c_t$ to $g_t$. Since $g^c_t$ contains an average inflationary bias (provided h < 1) the target set by the central banker will be higher than that set by Congress. This explains why both targets are different and why monetary policy will be more inflationary if the central banker sets the target.

As N rises the deflationary character of the target set by Congress is lessened, so that both targets are more similar. When N=1, Congress sets its
most deflationary target \((g^c = \pi^* - \frac{\nu^c}{\pi})\). This target completely offsets the inflationary character of monetary policy, the result being that on average actual inflation and desired inflation coincide. This finding is important, because it implies that Congress could legislate a set of incentive compatible rules (a positive punishment on the central banker, the target, and a one-period targeting horizon) that fully eliminates the inflationary bias in monetary policy.

For a given punishment \(h\) the target is less binding to the central banker at any given period \(t\) as \(N\) grows, so that the incentive for the central banker to create inflation is higher at the beginning of the targeting horizon. In addition, the deflationary reversals must be higher at the end of the targeting horizon. Accordingly, the variability of the net inflation bias around the desired inflation rate increases as \(N\) grows. However, when \(N=1\), there is no scope for flexibility in monetary policy over time so that Congress gains if the target is set as to fully eliminate the inflationary bias. If \(N > 1\), Congress is aware of the flexibility afforded to the central banker over time and, in turn, of its incentive to inflate more at the start of the targeting horizon (which produces variability of the actual inflation rate over the desired one). Hence, at the time Congress sets the target, it must accommodate to a certain extent that incentive on the part of the central banker in order to mitigate the variability of inflation around \(\pi^*\). To do so, as \(N\) grows Congress reduces the deflationary character of its target \(g^c\) by approaching it to \(\pi^*\) and provoking a positive net inflationary bias on average.

The existence of monetary targeting has a cost arising from a partial accommodation of the current shock in money demand. The reason is that accommodating a shock generates a deviation between actual money growth, \(g^c\), and the target, \(g^c\), implying a reduction in the central banker's utility. As a consequence, there will be variability of both inflation and output. This partial accommodation of \(e_t\) is given by the term \(\frac{(1+t)(1+(N-1)h)}{(1+t)(1+(N-t)h)} + \frac{hf}{h^t}\). This term decreases as \(h\) rises reflecting the fact that for a given deviation between \(g^c\) and \(g^c\) a higher punishment decreases the central banker's utility. Hence, the optimal central banker's reaction is to reduce accommodation. Moreover, that term raises a \(N-t\) increases, implying that the longer the remaining targeting horizon, the central banker will accommodate the shock to a higher extent because there will be more time to reverse the accommodation and therefore to try to hit the target.

The reversals of the partial accommodation of \(e_t\) in each of the remaining \(N-t\) periods are given by the term \(\frac{h(1+t)}{h(1+t)(1+(N-t)h)} + \frac{hf}{h^t}\). This term increases as \(h\) rises and decreases as \(N-t\) rises.

Overall, any shock \(e_t\) will be accommodated -on average- to a certain extent over the targeting horizon, unless the punishment \(h\) is infinite. It is worth noting that despite the accommodation of \(e_t\) in period \(t\) affects both inflation and output, the subsequent reversals of such accommodation in periods \(t+1, ... , N\) do not provoke variability of output because the wage setter's expectations incorporate these reversals. Nevertheless, they affect the variability of inflation around \(\pi^*\).

IV. Expected Average Utilities

In summary, we can find disutility arising from the targeting procedures due to variability of inflation and output, and due to the inflation biases.

Congress' expected average utility if Congress itself sets the target is given by the following expression:
Similarly, Congress' expected average utility if the central banker chooses the target to be hit is:

\[
U_b = -(y^*)^2 - (1+f)\sigma_e^2
\]

\[
\frac{f}{N} \sum_{t=1}^{N} \left[ \frac{f}{(1+f)(1+f)(N-t+1)} \right] - \frac{f}{N} \sum_{t=1}^{N} \left[ \frac{h}{(1+f)(1+f)(N-t+1)} \right] \sigma_e^2
\]

\[
= \frac{f}{N} \sum_{t=1}^{N} \left[ \frac{f}{(1+f)(1+f)(N-t+1)} \right] - \frac{f}{N} \sum_{t=1}^{N} \left[ \frac{h}{(1+f)(1+f)(N-t+1)} \right] \sigma_e^2
\]

It is easy to see that both (12) and (13) converge to the expected average utility under an average targeting procedure if \( h = 0 \). Moreover, both expressions converge to the expected discretionary utility if \( N \to \infty \).

The optimal targeting horizon and the optimal punishment that Congress jointly chooses in stage one must maximize (12) if Congress sets the target and (13) if the target is chosen by the central banker.

In order to obtain the optimal -from Congress' viewpoint- monetary policy package (consisting of punishment, horizon, target, and target setter) it will be necessary to simulate the above expected average utilities since it is not possible to analytically derive them.

Our model presents two key parameters: a) the ratio of the weight attached to inflation stability related to output stability, \( \sigma_e \), to the squared elasticity of output with respect to unanticipated inflation, \( \sigma_e \); this parameter has been denoted by \( f \); b) the weighted difference between output goals in the economy relative to the predictable part of the money demand shock, denoted by \( y^*/\sigma_e \).

A. Simulations when the central banker sets the target

We have performed several simulations that are shown in table 1 when the central banker sets the monetary target. In each cell, the figure on the left shows the optimal punishment \( h^* \) whereas the figure on the right denotes the optimal length of the targeting horizon \( N^* \). For instance, the first cell in the table says that for \( f=0.1 \), \( y^*/\sigma_e=0.1 \), the optimal combination of monetary policy instruments is given by \( h^*=0.13 \) and \( N^*=1 \). Since we have performed simulations up to \( N=100 \), the second cell in the table, for instance, is showing that for \( f=0.5 \), \( y^*/\sigma_e=0.1 \), the optimal monetary policy combines a punishment \( h^*=0.01 \) and a targeting horizon consisting of \( N^*=100 \) periods. The optimal combinations in the table provide a higher expected average utility for Congress than the discretionary solution \((h=0 \text{ or } N=\infty)\) and the average targeting solution \((h=0, N^*)\).

**Table 1. Optimal Punishments and Targeting Horizons**

<table>
<thead>
<tr>
<th>( y^*/\sigma_e )</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.13, 1</td>
<td>0.01, 100</td>
<td>0.01, 100</td>
</tr>
<tr>
<td>0.5</td>
<td>0.45, 1</td>
<td>0.01, 97</td>
<td>0.03, 100</td>
</tr>
<tr>
<td>1</td>
<td>0.79, 1</td>
<td>1.02, 1</td>
<td>0.04, 96</td>
</tr>
</tbody>
</table>

Table 1 shows that, in general, for a given \( f \), the optimal targeting horizon becomes shorter and the optimal punishment becomes larger as \( y^*/\sigma_e \) increases since in that case the benefits of flexibility provided by the targeting procedure are less worth.

In addition, table 1 reveals that, in general, for a given \( y^*/\sigma_e \), the
optimal targeting horizon becomes longer and the optimal punishment becomes smaller as \( f \) increases. However, we can find some cases for which the optimal punishment rises as \( f \) grows. To explain why it can be so, let us consider as an example the case where the optimal \( h^* \) grows from 0.79 to 1.02 and the optimal length \( N^* \) is 1. In this case the expected utility provided by a targeting procedure when the central banker sets the target is given by

\[
U_b = -\frac{f}{(f+h)^2}(y^*+y'^2 - \frac{h^2}{(1+f)(1+f+h)^2} - (y^*)^2 - (1+f)e_c^2
\]

The first term measures the disutility arising from the inflationary bias whereas the second term relates to the partial accommodation of money demand shocks. Both terms depend positively and negatively upon the value of \( f \), so that -as can be seen through their corresponding partial derivatives- they can either increase or decrease as \( f \) rises depending on the values of \( h \), \( y^* \), and \( e_c^2 \). With our particular parameter values we find that the disutility from the inflationary term rises and the disutility from money demand shocks decreases as \( f \) goes from 0.1 to 0.5. Hence, the overall utility rises by reducing the inflationary bias and by accommodating money demand shocks to a lesser degree.

Both effects are obtained by setting a higher punishment \( h \).

### B. Simulations when Congress sets the target

All simulations performed when Congress sets the target provide a common result: the optimal monetary policy combines a punishment \( h^* = 0.01 \) and a length \( N^* = 1 \). The reason for this result can be observed in the expression for the expected social utility when \( N = 1 \):

\[
U_c = -\frac{f}{(1+f+h)^2} - \frac{h^2}{(1+f)(1+f+h)^2} - (y^*)^2 - (1+f)e_c^2
\]

Equation (13) shows that if the targeting horizon is just one period, Congress sets the target in a fashion that completely removes the inflationary bias in the economy. Since we are left with the disutility arising from the less than full accommodation of money demand shocks, it is optimal to set a small punishment in order to reduce such a disutility. If \( N > 1 \), there will be an additional inflationary bias -on average- and additional variability of inflation around \( \pi^* \). Thus, targeting horizon longer than one period is not optimal.

### C. The Optimal Target Setter

Once obtained the optimal values of \( h \) and \( N \) for the two possible scenarios, we must decide between them in order to maximize expected Congress' utility. Table 2 presents the utility differences existing between the optimal monetary policy when Congress sets the target (\( h^* = 0.01, \ N^* = 1 \)) and the optimal monetary policy when the central bank sets the target (as given in Table 1).

<table>
<thead>
<tr>
<th>( f )</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y^*/e_c ) = 0.1</td>
<td>0.28%</td>
<td>0.66%</td>
<td>0.70%</td>
</tr>
<tr>
<td>( y^*/e_c ) = 0.5</td>
<td>0.05%</td>
<td>0.03%</td>
<td>0.05%</td>
</tr>
<tr>
<td>( y^*/e_c ) = 1</td>
<td>0.03%</td>
<td>0.01%</td>
<td>0.01%</td>
</tr>
</tbody>
</table>

For instance, the first cell in the table shows that the disutility generated by the optimal monetary policy when the target is set by Congress is just 0.28% of the same disutility when the target is set by the central bank.

Our results suggest that an improved Rogoff's (1985) flexible approach to
monetary targeting (in terms of optimizing not only the punishment but also both the length of the targeting horizon and the identity of the target setter) provides a better resolution to the time inconsistency problem that the average targeting approach of Canzoneri (1985) and Garfinkel and Oh (1993) in presence of private information on the part of the central banker.

V. Concluding Remarks

This paper studies the efficacy of monetary targeting to mitigate the classic time-inconsistency problem in monetary policy if the monetary authority's forecast of money demand is private information. In particular, we have analyzed the effects of a legislated monetary policy package consisting of a monetary target, a punishment for deviation from the target, a targeting horizon, and a target setter. Our results show that the best package should include a target set by Congress, a targeting horizon consisting of just one period of time and a small punishment on the central banker if it deviates from the target.

The analysis might be extended to include persistence of the central banker's private information and the possibility that the central banker could reveal, at least partially, its private information by making use of noisy announcements as in Garfinkel and Oh (1990) or Jeremy C. Stein (1989).

Another interesting task for future research is the choice of an optimal monetary targeting package in a multisector economy, along the lines of Christopher J. Waller (1992).

APPENDIX

PROOF OF PROPOSITION 1:

In period \( t = N \), the central banker maximizes the following program

\[
\begin{align*}
\text{max} \ E_{n} & \left\{ \sum_{t=1}^{N} \left[ -(\delta - \sigma \cdot \delta - \eta \cdot \eta)^2 - f(\delta - \delta - \eta \cdot \eta)^2 \right] - h \left[ \sum_{t=1}^{N} \delta_{t} - N_{\alpha} \right]^2 \right\} \\
\end{align*}
\]

The expectation of the first-order condition, conditional on \( \delta_{n} \), provides the central banker's decision rule as a function of the target, \( \delta_{n} \), the wage setter's action, \( \eta_{n} \), and previous period's money growth, \( \delta_{t-1} \).

\[
\begin{align*}
\text{max} \ E_{n} & \left\{ \sum_{t=1}^{N} \left[ -(\delta - \sigma \cdot \delta - \eta \cdot \eta)^2 - f(\delta - \delta - \eta \cdot \eta)^2 \right] - h \left[ \sum_{t=1}^{N} \delta_{t} - N_{\alpha} \right]^2 \right\} \\
\end{align*}
\]

(A2) \( \delta_{n} = \frac{1}{1+\delta_{n}} \left[ \eta_{n} + \sigma_{n} + \frac{1}{1+\delta_{n}} \left[ \delta_{t-1} \cdot \delta_{t} - N_{\alpha} \right] + (1+\delta_{n}) \right] \\

Assuming rational expectations on the part of the wage setters when setting \( \delta_{n} \), we obtain

(A3) \( \delta_{n} = \frac{f}{1+\delta_{n}} \delta_{n} + \frac{1}{1+\delta_{n}} \eta_{n} + \frac{1}{1+\delta_{n}} \left[ \delta_{t-1} \cdot \delta_{t} - N_{\alpha} \right] + 1+\delta_{n} \right] \\

In period \( t = N-1 \), the central banker solves the problem:

\[
\begin{align*}
\text{max} \ E_{n-1} & \left\{ \sum_{t=1}^{N} \left[ -(\delta - \sigma \cdot \delta - \eta \cdot \eta)^2 - f(\delta - \delta - \eta \cdot \eta)^2 \right] - h \left[ \sum_{t=1}^{N} \delta_{t} - N_{\alpha} \right]^2 \right\} \\
\end{align*}
\]

Substituting \( \delta_{n} \) and \( \delta_{n} \) into (A4) and taking into account that \( E_{n-1}(\delta_{n}) = 0 \), the expectation of the first-order condition, conditional on \( \delta_{n-1} \), provides:

\[
\begin{align*}
\text{max} \ E_{n-1} & \left\{ \sum_{t=1}^{N} \left[ -(\delta - \sigma \cdot \delta - \eta \cdot \eta)^2 - f(\delta - \delta - \eta \cdot \eta)^2 \right] - h \left[ \sum_{t=1}^{N} \delta_{t} - N_{\alpha} \right]^2 \right\} \\
\end{align*}
\]

(A5) \( \delta_{n-1} = \frac{1}{1+\delta_{n-1}} \left[ \delta_{n-1} + \eta_{n-1} + \frac{1}{1+\delta_{n-1}} \left[ \delta_{t-1} \cdot \delta_{t} - N_{\alpha} \right] + (1+\delta_{n-1}) \right] \\

Using \( \delta_{n-1} \), money growth in \( t=N-1 \) is
In period $t = N-2$, the central bank solves:

\[ \begin{aligned}
    (A6) \quad & \delta_{N-2} = \frac{g_t}{f(N-2)h} + \frac{1}{f(N-2)h^2} \left[ \delta_{N-1} - \delta_{N-2} \right] + \frac{1}{f(N-2)h^2} \delta_{N-2} \\
\end{aligned} \]

In period $t = N-2$, the central bank solves:

\[ \begin{aligned}
    (A7) \quad & \max E_{N-2} \left[ \delta_{N-2} + \mathbb{E}_t \left[ g_t \right] - g_t \left( \delta_{N-1} - \delta_{N-2} \right) \right] \\
\end{aligned} \]

Substituting $\delta_{N-2}, \delta_{N-1}, \delta_{N},$ and $\delta_{N-2},$ and taking into account that $E_{N-2}(\delta_t) = 0,$ the expectation of the first-order condition, conditional on $\delta_{N-2},$ provides:

\[ \begin{aligned}
    (A8) \quad & \delta_{N-2} = \frac{1}{f(N-2)h^2} \left[ \delta_{N-2} + \mathbb{E}_t \left[ g_t \right] - g_t \left( \delta_{N-1} - \delta_{N-2} \right) \right] + \frac{1}{f(N-2)h^2} \delta_{N-2} \\
\end{aligned} \]

Using $\delta_{N-2},$ money growth in $t = N-2$ is given by:

\[ \begin{aligned}
    (A9) \quad & \delta_{N-2} = \frac{1}{f(N-2)h} \left[ \delta_{N-2} + \mathbb{E}_t \left[ g_t \right] - g_t \left( \delta_{N-1} - \delta_{N-2} \right) \right] + \frac{1}{f(N-2)h} \delta_{N-2} \\
\end{aligned} \]

Repeating this sequence of maximization programs for periods $t = N - \mu,$ the general solution for money stock growth is given by:

\[ \begin{aligned}
    (A10) \quad & \delta_{N-\mu} = \frac{1}{f(N-\mu)h} \left[ \delta_{N-\mu} + \mathbb{E}_t \left[ g_t \right] - g_t \left( \delta_{N-1} - \delta_{N-\mu} \right) \right] + \frac{1}{f(N-\mu)h} \delta_{N-\mu} \\
\end{aligned} \]

for $\mu = 1, 2, \ldots, N-1.$

Using $\mu = N-t,$ we have the following expression for money growth in period $t$:

\[ \begin{aligned}
    (A11) \quad & \delta_t = \frac{1}{f(N-t+1)h} \left[ \delta_t + \mathbb{E}_t \left[ g_t \right] - g_t \left( \delta_{t+1} - \delta_t \right) \right] + \frac{1}{f(N-t+1)h} \delta_t \\
\end{aligned} \]

for $t = 1, 2, \ldots, N.$

Now, we have to eliminate past $g_t$ so as to express optimal monetary policy in terms of current $e_t$ and past $e_t,$ the parameters of the model, and the target $g_t.$ To do so we make use of (A11) to find $g_1$ and sequentially $\delta_2, \delta_3, \ldots, \delta_N.$

When $t = 1,$ we have:

\[ \begin{aligned}
    (A12) \quad & g_1 = e_t + \frac{h}{1+f(N-2)h} \left[ \mathbb{E}_t \left[ g_t \right] - g_t \left( \delta_{t+1} - \delta_1 \right) \right] + \frac{1}{1+f(N-2)h} \delta_1 \\
\end{aligned} \]

Substituting (A12) into $g_2,$ we obtain:

\[ \begin{aligned}
    (A13) \quad & \delta_2 = \frac{1}{f(N-1)h} \left[ \delta_2 + \mathbb{E}_t \left[ g_t \right] - g_t \left( \delta_{t+1} - \delta_2 \right) \right] + \frac{1}{f(N-1)h} \delta_2 \\
\end{aligned} \]

Substituting (A12) and (A13) into $g_3,$ we obtain:

\[ \begin{aligned}
    (A14) \quad & \delta_3 = \frac{1}{f(N-2)h} \left[ \delta_3 + \mathbb{E}_t \left[ g_t \right] - g_t \left( \delta_{t+1} - \delta_3 \right) \right] + \frac{1}{f(N-2)h} \delta_3 \\
\end{aligned} \]

Repeating this sequence of substitutions until period N, one can verify that the general solution is the expression given in the text. Moreover, (A11) shows that, at time $t,$ past money disturbances, $e_{t-1},$ are fully revealed to the wage setters upon their observing past $g_t.$

**Proof of Proposition 1:**

The central banker chooses the target, $\delta_t,$ to maximize

\[ \begin{aligned}
    (A15) \quad & \max E_0 \left[ \right. \delta_t = \frac{h}{f(N-t+1)h} \left[ \delta_{t+1} - \delta_t \right] \left. \right] \\
\end{aligned} \]
following:

\[
\sum_{t=1}^{N} \left( \frac{Nh}{f} + \frac{Nh}{f} + \frac{Nh}{f} \right) + \left[ \frac{f + (N - t)h}{f} - \frac{h}{f} \right] y^* \left[ \frac{f + (N - t + 1)h}{f} - \frac{h}{f} \right] \right] = 0
\]

Rearranging, we obtain the optimal target for the central bank, \( g^B = n^* \).

Congress chooses the target, \( g^C \), to maximize

\[
\max \mathbb{E}_0 \left\{ \sum_{t=1}^{N} u_t \right\}
\]

subject to the expressions for \( g_L \) and \( g^* \) in proposition 1. The expectation of the first-order condition is:

\[
\sum_{t=1}^{N} \left( \frac{Nh}{f} + \frac{Nh}{f} + \frac{Nh}{f} \right) + \left[ \frac{f + (N - t)h}{f} - \frac{h}{f} \right] y^* \left[ \frac{f + (N - t + 1)h}{f} - \frac{h}{f} \right] \right] = 0
\]

It is easy to see that the following equality holds:

\[
g^C = n^* + \frac{f + (N - t)h}{f} \left[ \frac{f + (N - t + 1)h}{f} - \frac{h}{f} \right] y^* \left[ \frac{f + (N - t + 1)h}{f} - \frac{h}{f} \right] = \frac{1}{N} \sum_{t=1}^{N} y^* \left[ \frac{f + (N - t + 1)h}{f} - \frac{h}{f} \right]
\]

Rearranging (A18) and using (A19), we obtain the expression for \( g^C \) in proposition 2.

**Proof of Proposition 3:**

Substituting \( g^B = n^* \) into the expression for \( g_L \) in proposition 1 one obtains the following

\[
g^\pi = n^* + \frac{f + (N - t)h}{f} \left[ \frac{f + (N - t + 1)h}{f} - \frac{h}{f} \right] y^* \left[ \frac{f + (N - t + 1)h}{f} - \frac{h}{f} \right]
\]

Similarly, substituting the expression for \( g^C \) into \( g_L \) we obtain

\[
\sum_{t=1}^{N} \left( \frac{Nh}{f} + \frac{Nh}{f} + \frac{Nh}{f} \right) + \left[ \frac{f + (N - t)h}{f} - \frac{h}{f} \right] y^* \left[ \frac{f + (N - t + 1)h}{f} - \frac{h}{f} \right] = 0
\]
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