SOME RESULTS ON FUZZY SYSTEMS

F. J. Montano & J. Tajada

Dept. Estadística e I.O., Fac. C.C. Matemáticas
Universidad Complutense
28040 Madrid (Spain)

Generalization of classical structure
Function of a system is considered in order
to represent non-probabilistic uncertainties
attached to systems with degrees of
performance between perfect functioning and
failed. Some new concepts for static and non-
static fuzzy systems are developed.

INTRODUCTION

In traditional System Theory, a system with n
components is described by a binary structure
function \( \phi: \{0,1\}^n \rightarrow \{0,1\} \) in such a way that
the state \( \phi \) of the system is determined uniquely
by the states of its components. \( \phi \in \{0,1\} \).
Each component and the system itself is supposed to
be in either two states: perfect functioning "1" and
failed "0" (see [5]). Multistate systems of
multicate components has been developed when a
finite number of states is supposed (see [5]), but
in this paper we consider the whole range of levels
between "1" and "0", representing the degrees of
performance of functioning. In this context, new
concepts are suggested. Examples of fuzzy systems
can be easily found in electrical engineering. The
study of fuzzy systems will be relevant in the
analysis of complex systems (nuclear power plants,
social systems,...).

PERFORMANCE IN STATIC SYSTEMS

A fuzzy system can be denoted by \( (S, \phi) \), where
\( S = \{0,1\}^n \) is the set of components and
\( \phi: \{0,1\}^n \rightarrow \{0,1\} \) a fuzzy structure function.

- **Monotonic**: If \( \phi \) is non-decreasing.
- **Continuous**: If \( \phi \) is continuous.
- **Standard**: If \( \phi(0) = 0 \) and \( \phi(1) = 1 \).
- **Dichotomous**: If \( \phi(0) = 0 \) and \( \forall x \in [0,1] \phi(x) \in \{0,1\} \).

Let us denote \( (y, x) = \{y_1, \ldots, y_n\}, x_1, \ldots, x_N \|
\), given \( (x, y) \), a component \( i \) is '1'-influenced if
\( \phi(y, x) = \phi(1, x) \). If \( \phi(y, x) = \phi(0, x) \)
\( \forall y \in \{0,1\}, \forall i \in [1, N] \). A
monotonic standard system with no irrelevant
components is said coherent. Given a partition
\( A_1, \ldots, A_K \) of \( S \), a fuzzy system \( (S, \phi) \)
is constructed from the fuzzy subsystems \( (A_i, \phi_i) \)
\( i = 1, \ldots, K \), where \( K \) is a finite set of
integers such that \( \phi_i = \phi(0, x) \vee \phi(1, x) \), \( \forall x \in [0,1] \), \( i = 1, \ldots, K \), \( \vee \) being the state vector of components in \( A_1 \). In crisp theory, systems constructed from coherent systems are coherent. But in our context this property is not necessarily true, since the condition of relevance does not propagate without imposing some aditional hypothasis, like continuity of \( \phi \) and a more restricted concept of
relevance on \( [0,1] \); for example, the following
concepts will be useful in the next section: a fuzzy
system \( (S, \phi) \) is said to be of type 1 (type 2) if
\( \phi(0, x) \geq 0 \) (or \( \phi(1, x) \geq 0 \)) \( \forall x \in [0,1] \), \( \vee \) being \( \vee = (\vee(0), \vee(1)) \).

PERFORMANCE OF NON-STATIC SYSTEMS

In the previous section, performance has been
considered in terms of the degree of functioning at
a fixed time. In this section aging will be
considered. Fuzzy performance of a non-static system
can be understood as a mapping \( p(t): \mathbb{R} \times \Omega \rightarrow [0,1] \),
\( p(t) \) meaning the degree of performance at time \( t \)
\( \Omega \) the aging system is supposed to be connected at time
\( t_0 \). Standard components will be characterized by
a non-increasing performance \( x(t) \) such that \( x(0) = 1 \) and \( x(t) = 0 \). If \( x(t) \) represents
the performance of some component \( j \) at time \( t \), the
fuzzy performance of a system \( (S, \phi) \) with \( n \) components will be defined by
\( F(t) = \phi(x_1(t), \ldots, x_n(t)) \). But it is easy to prove
that coherent systems of standard components does not necessarily verify $F(t) = 0$. When performance is a non-increasing function being absolutely continuous, the concept of wear rate at time $t$ can be defined as follows: $\tau(t) = x'(t)/x(t)$, being $x'$ the derivative of $x$. Wear rates must not be understood as a stochastic conditional failure rate; it represents the intensity rate of the decreasing performance, usually due to mechanical wear.

Analogously to mathematical formulation of classical PDF and FDF distributions, we can propose the concepts of increasing wear rate and increasing wear rate in average performances. A performance function $x$ is said INRA (DNRA) when $x(t+h)/x(t)$ is non-decreasing (non-increasing) for each $h > 0$. As a consequence, we obtain that $\tau(t)$ is non-decreasing (non-increasing) in $t$. An INRA performance $x$ is characterized by the fact that $x_a(s,t) x_a(t)$ with $a \leq 0$, $\forall t$, a performance function $x$ will be INRA if and only if $x_a(s,t) x_a(t)$ with $a \geq 0$, $\forall t$. Obviously, an INRA (DNRA) performance function is INRA (DNRA), but the converse is not necessarily true. Then the following closure theorem holds:

**Theorem.** Let $(\emptyset, \mathfrak{d})$ be a monotone system of type 1 (type 2) such that each relevant component has an INRA (DNRA) performance function. Then the system itself has an INRA (DNRA) performance function.

In particular, coherent systems where $\emptyset$ is a fuzzy switching function (see [1]) with INRA (DNRA) components will be INRA (DNRA).

**Final Comments**

Some definitions relative to fuzzy systems have been translated from binary systems, but new concepts appear in our context. It must be remarked that performance function is not understood as a stochastic property. Moreover, many properties of finite multistate systems can be generalized. In particular, when performance is understood as a stochastic property, probabilistic sets (see [3]) can be considered in order to follow the properties of random cut vectors (see [2]).

**References**