APE Results of Hadron Masses in Full QCD Simulations. *

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We present numerical results obtained in full QCD with 2 flavors of Wilson fermions. We discuss the relation between the phase of Polyakov loops and the sea quarks boundary conditions. We report preliminary results about the HMC autocorrelation of the hadronic masses, on a 16^2 × 32 lattice volume, at β = 5.55 with k_{sea} = 0.1570.

1. THE SPATIAL POLYAKOV LOOPS PHASE.

Polyakov loops are responsible for the difference between quenched and unquenched finite size effects on the QCD mass spectrum [1][2]. The reason can be understood by looking at the valence quark hopping parameter expansion of the meson propagator [2]:

\[ G = \sum_C k^{(C)}_{val}(W(C)) + \sum_C k^{(C)}_{val} \sigma_{val}(P(C)). \]  

The sums extend over all closed paths (C) of length l(C). W(C) are standard Wilson loops, completely contained into the lattice, while P(C) are Polyakov type loops and (·) denotes gauge field average; the value of the index \( \sigma_{val} \) depends on the spatial boundary conditions on the valence quarks: \( \sigma_{val} = +1 \) for periodic boundary conditions, PBC, and \( \sigma_{val} = (-1)^n \) for antiperiodic boundary conditions, APBC, respectively, with n the number of windings around the lattice.

In full QCD the fermionic part of the action (both Wilson and staggered fermions) breaks explicitly the symmetry that, in quenched confined

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Figure 1. Histogram of the phase, \( \theta \), of the \( z \) component of the Polyakov loop, \( P^z_{AP} \), for APBC and \( P^z_{P} \), for PBC on sea quarks.
QCD guarantees $< P >= 0$. A simple model\cite{3} may be useful to clarify the situation. The presence of the fermionic part of the QCD action is analogous to the existence of a magnetic field $h$ which breaks the $Z_3$ symmetry in a model for a spin, $\Pi$, which can take the three possible $Z_3$ values. $\Pi$ is coupled to the external magnetic field via the Hamiltonian

$$H_h = h\Pi + h^\dagger \Pi^\dagger. \quad (2)$$

$H_h$ is not $Z_3$ invariant. The values taken by $\Pi$ are in correspondence with the expected phases of Polyakov loops, while the values of $h$ are in correspondence with the chosen sea quarks boundary conditions. Following this model we expect that with PBC on sea quarks the $e^{i2\pi/3}$ and the $e^{-i2\pi/3}$ phases of the Polyakov loops will be present with equal probability, while Polyakov loops are likely to be polarized in the $\Pi_3$ direction if APBC are used. Similarly we expect to be able to align the Polyakov loops along the $e^{i2\pi/3}$ (or $e^{-i2\pi/3}$) in the $Z_3$ space, if we choose $-e^{-i2\pi/3}$ (or $-e^{i2\pi/3}$ respectively) boundary conditions on the sea quarks.

We checked this suggestions with a simulation, performed on APE100, with 2 flavor Wilson fermions at $\beta = 5.3$ on a $8^3 \times 32$ lattice with $k_{sea} = 0.1670$. We carried out two different runs, one with fully PBC on the sea quarks and the other one with APBC in the spatial directions and PBC in the temporal one. Gauge configurations have been produced with APE100 using the HMC algorithm\cite{4}. We created a set of 440 (thermalization) + 1350 thermalized trajectories. We have performed a measurement, every 5 trajectories, of the hadronic correlation functions, of the $0^{++}$ and $2^{++}$ glueball correlation functions and of the Polyakov loop. For each trajectory we estimate the average value of the Wilson loop, $W_{1 \times 1}$, (the plaquette).

2. SIMULATIONS ON $16^3 \times 32$ AT $\beta = 5.55$.

We started with a run on a $16^3 \times 32$ lattice volume at $\beta = 5.55$ with $k_{sea} = 0.1570$. We use APBC on spatial directions on sea quarks. We created a set of 900 (thermalization) + 500 trajectories. The last 500 have $dt = 0.083$ and $N_{MD} = 60$ ($dt \times N_{MD} \sim 0.5$). We have performed a measure, every 10 trajectories, of the hadronic correlation functions, of the $0^{++}$ and $2^{++}$ glueball correlation functions and of the Polyakov loop. For each trajectory we estimate the average value of the Wilson loop, $W_{1 \times 1}$, (the plaquette). In

![Figure 2. Autocorrelation for the pseudoscalar, $\pi$, meson correlation functions at the Euclidean time $t = 16$. On the top we present the results from method (A) and on the bottom the ones from method (B).](image)

the calculation of the valence quark propagator we use a smeared source of size $7^3$, while for the glueball correlation functions we use 21 iterations of the smearing. The physical lattice size (estimated from ref.\cite{6}) is $L \sim 1.5$ fm.

The first goal that we want to reach is an understanding of the correlations between measures done on consecutive HMC trajectories. This study is important in order to estimate a correct value for the statistical error. We use two different
Table 1

Values of the hadronic masses, measured with
\( k_{\text{valence}} = k_{\text{sea}} = 0.1570 \), using APBC on spatial
directions on the sea quarks, PBC on the valence quarks and a \( 7^3 \) smeared source. We
report also the value for the square ratio of the pseudoscalar to the vector mesons and the value
for the plaquette.

<table>
<thead>
<tr>
<th>( \pi (\gamma_3) )</th>
<th>( \rho (\gamma_1) )</th>
<th>Nucleon</th>
<th>( N^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0.479(5) )</td>
<td>( 0.567(7) )</td>
<td>( 0.89(1) )</td>
<td>( 1.32(5) )</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>( \Delta^* )</td>
<td>( (\frac{m_{\pi}}{m_{\rho}})^2 )</td>
<td>( W_{1\times1} )</td>
</tr>
<tr>
<td>( 0.95(1) )</td>
<td>( 1.36(8) )</td>
<td>( 0.71(1) )</td>
<td>( 0.5635(1) )</td>
</tr>
</tbody>
</table>

methods: A) we study the behavior of the jacknife error \( \sigma^2 \) (calculated by dividing the measures
in \( N \) bins of size \( T \) ) as a function of the size \( T \), analyzing when the error reaches a plateau and, B), we
calculate the autocorrelation function \( C(t) \) at
time \( \tau_s \) (see eq.(2.19) of ref.[7]).

With both methods we obtain for the plaquette and the hadrons correlation functions, at Euclidean
time 16 (the middle of the temporal lattice), an autocorrelation time of about 50 HMC trajectories (see fig. 2). Starting from these
result we estimate our statistical errors with the jacknife method used on 10 bin of 5 measures (50 trajectories) each. Results are reported in Table
1. We can see from this table that the masses are estimated with a statistical error of about 1%.

In order to study the systematic errors coming from the finite lattice, we calculated the hadronic masses with both PBC and APBC on valence
quark. The difference between the values of masses obtained gives us an estimation of the contribution from \( < P > \neq 0 \) of eq. 1. From fig. 3, we can see an error of 3% on the estimation of the plateau of the \( \pi \) and of 6% for the \( \rho \). The systematic finite
size errors are greater than the statistical errors quoted in Table 1. This difference is lower than that found for staggered fermions in fig 2 of ref.[2]
as expected since our lattice is almost a factor 2 larger. However the effect is in the same direction: if the boundary conditions for sea quarks
are AP, the masses obtained with PBC on valence quarks are lower than those obtained with APBC on valence quarks, as expected from eq.

(1). This is true for both Wilson and staggered fermions.

We want to stress that the smearing procedure is crucial also in full QCD, as we have observed in the study of the baryons correlation functions.

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REFERENCES

4. S. Antonelli, M. Bellacci, A. Donini, R. Sarno "Full QCD on APE100 Machines". To appear in Int. Jour. of Mod. Phys. C.