Use of Popular Science Materials for Teaching Topology

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***Abstract*—**We describe the teaching experience of use of popular science articles, books, and comics, for university teaching of topology for undergraduate students in Mathematics.

***Keywords*—**Experience teaching, materials, topology, university teaching.

I. Introduction

In undergraduate studies in Mathematics in Spain for over 10 years is a required course of Introduction to Topology, which is a compulsory subject for all students enrolled this grade. In the previous curriculum, this situation was not. The topology only course students of the specialties of fundamental mathematics and Methodology.

The current situation creates a serious problem of failure among students, as in the classroom explaining topology there are students from 2nd or 3rd year degree, which then will be inclined, some to pure mathematics, but other to things such as statistics, Computational Mathematics, Astronomy, ... where it is quite doubtful the usefulness of the topology. In addition, the abstract nature of the concepts that appear in the subject, coupled with the plethora of new concepts presented to students throughout the course, they are overwhelmed.

Already in the first weeks of course have seen a drop in attendance Topology by many students, since it is something that, in many cases, do not understand and that are far from practical applications.

In last years, I began to take direct action to motivate students to study the topology, regardless of the mathematical specialty, they want to choose.

Currently, on the 3rd year of undergraduate studies there exists the possibility of spending some of the practical classes to workshop. For this reason, I have taken the decision to include in these workshops the exhibition and comment on popular science materials on topology for this level.

Although it might, at first, seem strange, this type of material (books, articles, comics) is relatively abundant, beautiful and motivating for students who start the study of topology.

The practical procedure that can be followed in the classroom may be the prior distribution among students of popular science materials (articles or book chapters), for example through the virtual campus and later in the classroom, clarify or comment on the blackboard the concepts that appear in this materials.

The existing topology information material does not cover all topics of the subject, but there is enough variety to illuminate some of the concepts on topological spaces which appear throughout the course.

Specifically, we present interesting material on quotient spaces, such as the *Moebius strip* and *Klein bottle*, we motivate the definition of topological curve showing previously the *Peano curve*, *Sierpinski carpet* and the *Menger sponge*, we give approximations to the classification of compact surfaces.

I do not include data on outcomes achieved by students in exams, because that (in Spain) is not significant, since the percentage of undergraduates who do not show up exams is very high (about 50%) in all subjects degree. Similarly, there are a very high percentage of undergraduate students who do not attend class, even if they are provided with information material on certain subjects. This may be because the students for university were not selected according to their abilities to one degree or another. Therefore, in Spain there are students who choose to enroll in Mathematics race simply because they have learned from the newspapers that among graduates in mathematics there are few who are unemployed. These things are an illustrative commentary on how the situation of university education in Spain.

Then we will show the various materials used.

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II. MATERIALS USED IN THIS COURSE

A. *On the Moebius strip and Klein bottle*

Here, I used mainly items made by Macho-Stadler [7-10] and the comic "Le Topologicon" of J.-P. Petit [11] of which, for example, I presented a page.

[Annexes: archives
• Moebius.pdf
• Moebius2.pdf
• Klein.pdf
• Topo _the_world_eng [1].pdf]

B. *On the concept of curve*

Here I used primarily the book [1] and the materials developed by two professors at the Polytechnic University of Madrid for a course of fractals in internet.
[Annexes: curves.pdf file]

C. *Classification of compact surfaces*

Popular materials on topological classification of compact surfaces can be found in the aforementioned book by Boltyanskii & Efremovich [1] and [7], with didactic utility drawings on this subject.

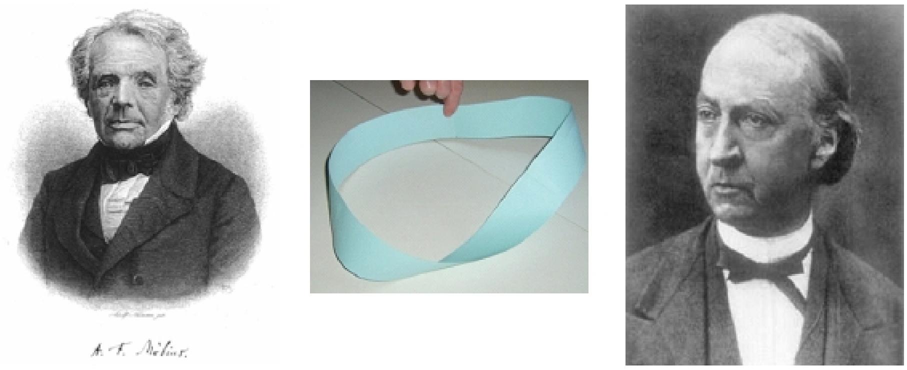
D. *Topology Overview*

The comic "Le Topologicon" or “Topo The world” [11] initiates the student to the applications of topology to other branches of mathematics. Other books with attractive pretty pictures for students, are [2], [3], [4], [12], [13].

This material is motivating undergraduate students to study the topology and, if possible, get them excited to continue with more advanced topological studies.

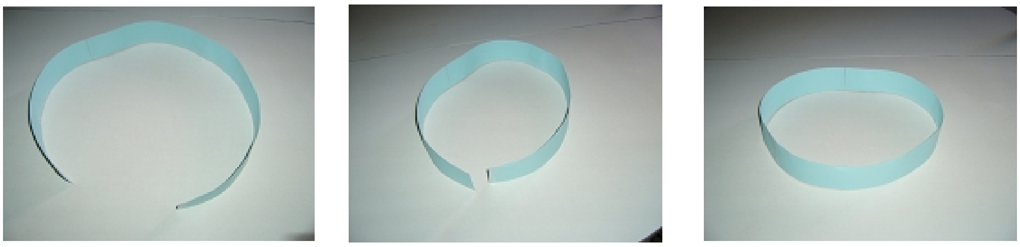
ANNEX 1

Un Paseo por la Geometría

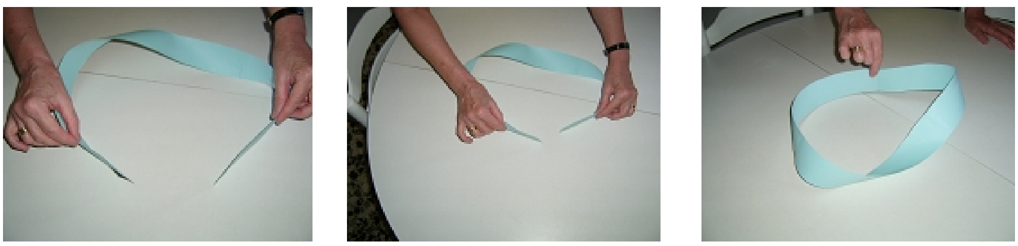


Möbius, la banda y Listing

Si se toma una tira de papel y se pegan los extremos como muestra la figura de debajo, se obtiene un cilindro, es decir, una superficie que tiene como bordes dos circunferencias disjuntas y dos lados (la cara interior y la exterior de la figura).



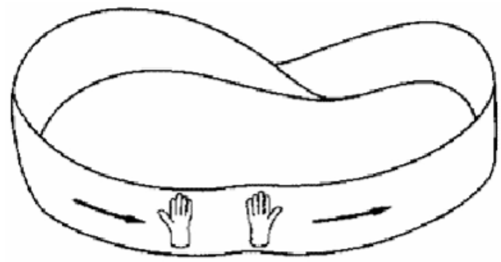
Si se hace lo mismo, pero antes de pegar los extremos se gira uno de ellos 180 grados, el objeto que se obtiene es una banda de Möbius.



La banda de Möbius posee dimensión dos –como el cilindro–, pero sorprendentemente tiene un único borde (el doble de largo: su longitud es la suma de las de las dos circunferencias que forman el borde del cilindro) y una única cara. Para comprobarlo, basta con pasar un dedo por el borde de la cinta, hasta verificar que se ha recorrido todo sin levantarlo en ningún momento; y pasar un lápiz por la cara de la banda, constatando que al regresar al punto de partida, las supuestas dos caras del objeto han quedado marcadas.

ANNEX 1

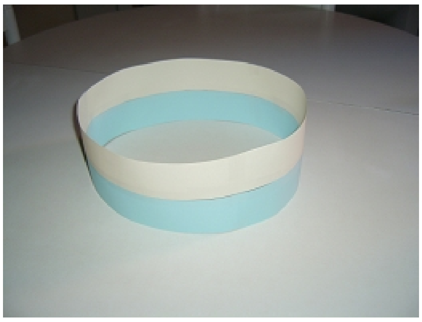
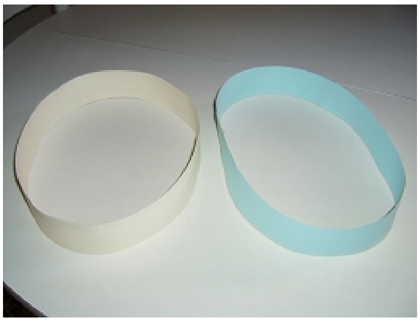
Listing, Möbius y su famosa banda



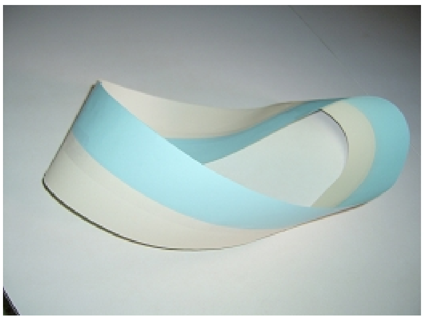
La banda de Möbius es no orientable: todas sus propiedades singulares (y de cualquier otro objeto que "la contenga") se derivan de esta última propiedad. En efecto, si se dibuja una mano sobre la banda y se mueve a lo largo de su única cara, al regresar al punto de partida, ¡la mano ha cambiado de sentido!

¿Qué sucede si antes de pegar los extremos de la banda de papel se gira uno de ellos 360 grados? Se obtiene (*topológicamente*) un cilindro, ya que este objeto y el obtenido al pegar sin realizar ningún giro son *homeomorfos*. Es fácil comprobar que, de hecho, sólo hay dos posibilidades al pegar una banda por dos de sus extremos opuestos: o bien se obtiene un cilindro (si antes de pegar los extremos, se gira uno de ellos un múltiplo par de 180 grados) o bien una banda de Möbius (en caso contrario).

Vamos a hacer un par de experimentos de extraño resultado. Si cortamos por la altura mitad un cilindro, se obtienen dos cilindros, la mitad de altos que el cilindro original:

Si se hace lo mismo con la banda de Möbius, parece que lo lógico sería obtener dos bandas de Möbius más pequeñas. Pero, no... se obtiene una única cinta, que es un cilindro, pues posee dos caras.




ANNEX 2

Se pueden realizar algunas experiencias con la banda de Möbius, que dan resultados paradójicos:

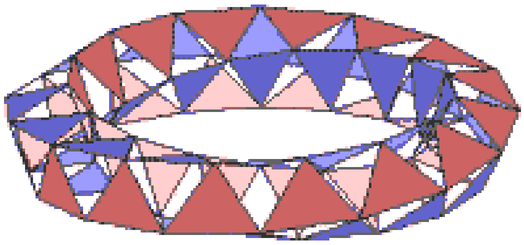
- Si se corta la banda por su mitad, como muestra la Figura 41, aparece una cinta el doble de larga, que contiene 4 semivueltas, dos caras y dos bordes, luego no es una banda de Möbius, sino un cilindro;
- Si se corta la banda de Möbius a la altura 1/3, se obtienen dos bandas de Möbius entrelazadas, una más larga que otra: la altura 1/2 es la única ***especial***.

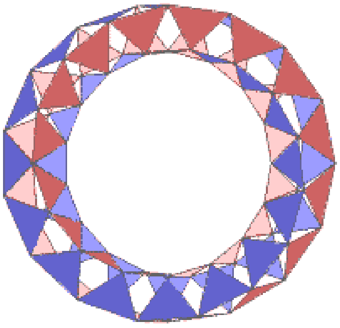
La banda de Möbius ha inspirado a artistas y científicos, como muestran los ejemplos que siguen.

El dibujante e ilustrador Jean Giraud *Möbius* (1938-), y su autocaricatura, portada del libro *Mi doble y yo*.

Las hormigas de Escher.

Elisabeth Zimmermann y sus bufandas de Möbius.





10.2. La botella de Klein

La *botella de Klein* es una superficie obtenida al identificar los lados de un cuadrado como muestra la Figura 45:

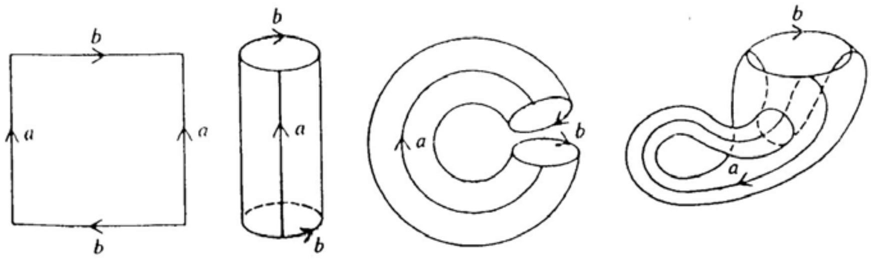


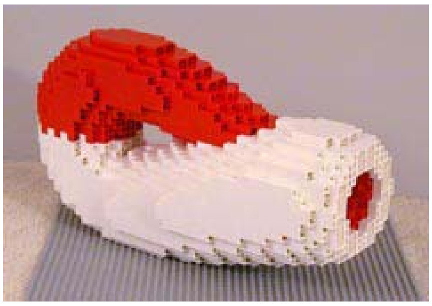
Figura 45.

Esta figura no puede construirse en el espacio de dimensión tres sin autointersecarse, pero sí que está contenida en el espacio de dimensión cuatro.

La botella de Klein presenta varias propiedades paradójicas: posee ***un*** solo lado (no tiene cara interior ni cara interior) y ***no*** tiene borde: de hecho, puede obtenerse esta superficie a partir de dos bandas de Möbius, y por ello hereda sus ***extrañas*** propiedades.

La botella de Klein ha servido de modelo para muchas construcciones extraordinarias, como muestran las imágenes que aparecen a continuación.

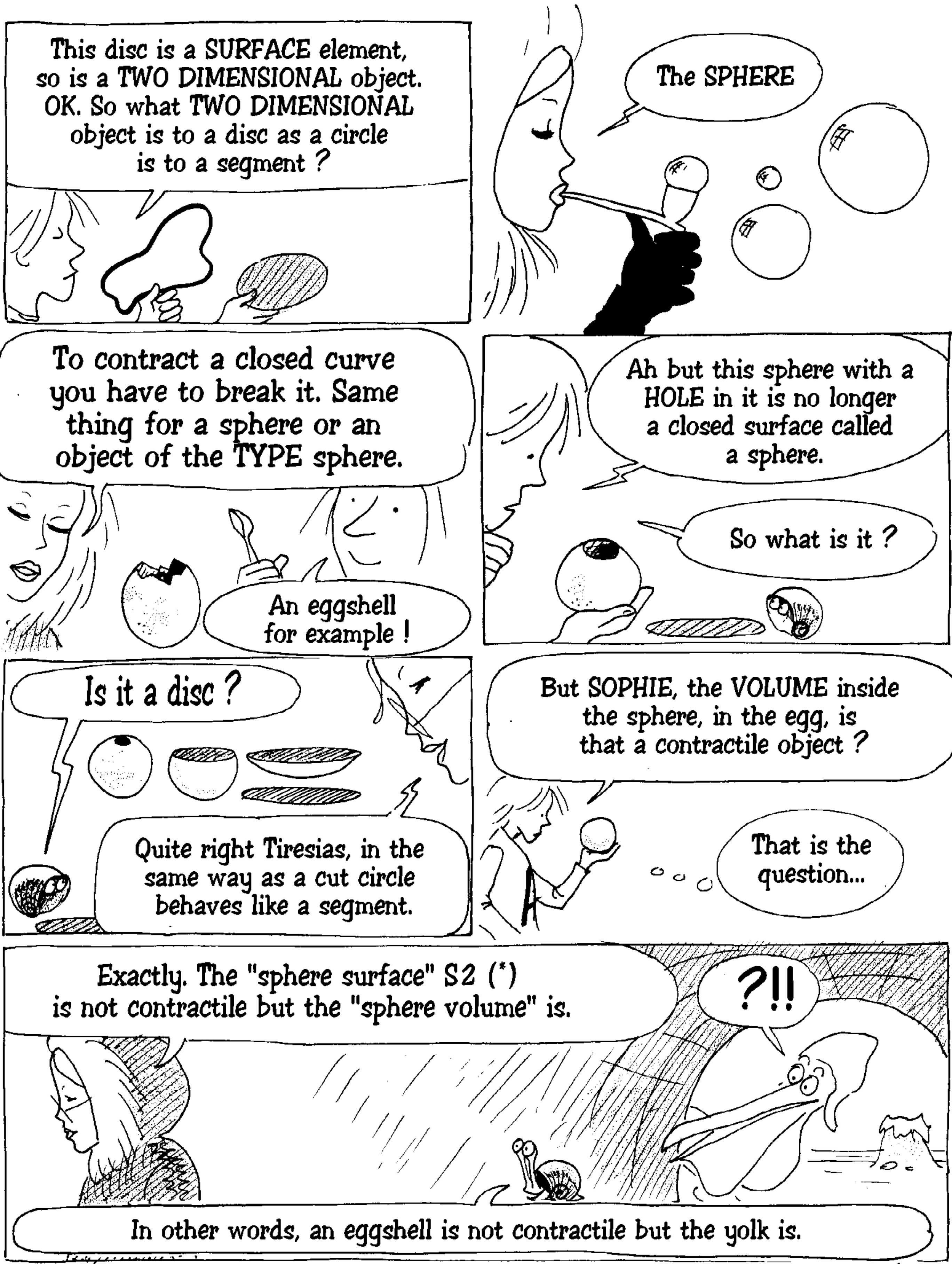
ANNEX 3



La botella de Klein de LEGO de Andrew Lipson.

La botella de Klein de origami de Robert Lang.

En la figura aparecen algunas botellas de Klein de Cliff Stoll, de la *Acme Klein Bottle*.



(*) See: HERE'S LOOKING AT EUCLID.

ANNEX 5

Example 18 The Polish mathematician Sierpiński constructed an interesting curve. We divide a square into nine equal squares and discard the central square (Figure 39a). Then we divide each of the remaining squares into nine equal squares and again discard the central square (Figure 39b). After one more such operation we arrive at the figure shown in Figure 39c. In the limit we obtain a one-dimensional figure C, i.e., a curve (known as the *Sierpiński carpet*).

The figure C is a *universal* plane curve: *If a curve l can be embedded in the plane, then it can be embedded in the Sierpiński carpet*, i.e., there is a curve $l' \subseteq C$ that is homeomorphic to l. It is clear that a curve that cannot be embedded in the

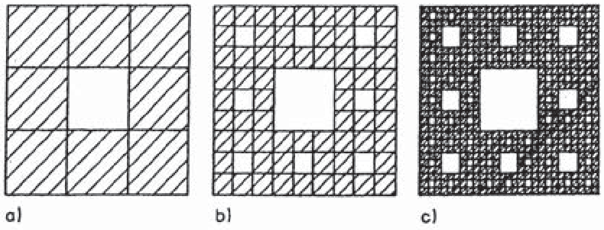


FIGURE 39.

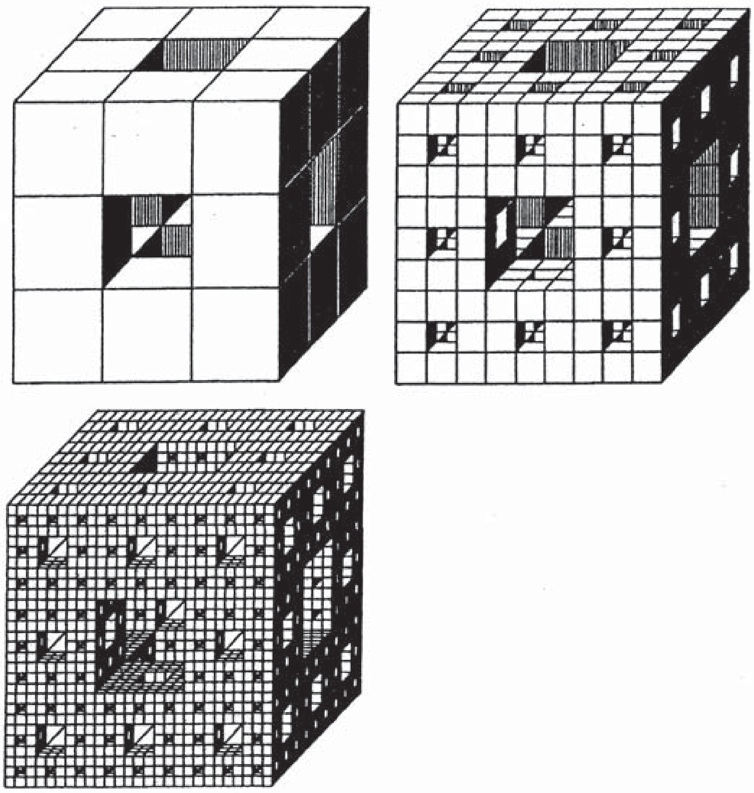


FIGURE 40.

plane cannot be embedded in the Sierpiński carpet either. But as was shown by the Austrian mathematician Menger, there is a curve in space, analogous to the Sierpiński carpet (Figure 40), in which any curve can be embedded.

ANNEX 5

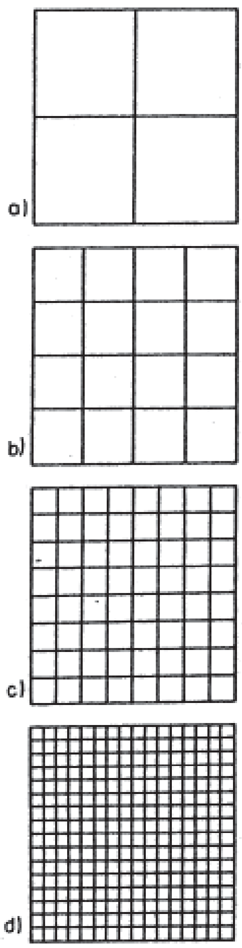


FIGURE 42.

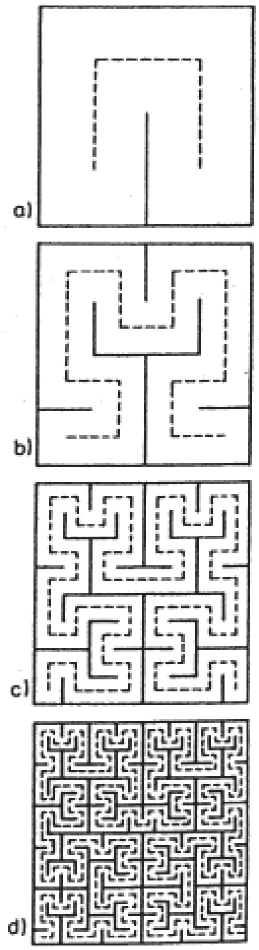


FIGURE 43.

To obtain a Peano curve we construct in a square Q ever more winding strips and take the limit of their midlines. This is a shorthand description of the following procedure: We divide the square into 4, 16, 64, ..., 4^n, ... congruent squares (Figure 42). At each stage of subdivision we remove some of the sides of the "subsquares" of the square. The leftover sides form non-removable partitions that determine the successive strips. In each strip we introduce its midline. (The first few winding strips and their midlines are shown in Figure 43; the midlines are the broken lines.) The limit of these midlines is a path that fills the whole square, i.e., a Peano curve.

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Francisco Gallego Lupiáñez has born in 1958, in Madrid, Spain. He received the M.S. and Ph.D. degrees of the University Complutense, of Madrid, in 1980 and 1985, respectively.

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Prof. F.G. Lupiáñez is author of more than forty scientific papers on topology and fuzzy mathematics, and is member of the editorial board of twelve journals.