LIQUIDITY AND GROWTH TRAPS: A FRAMEWORK FOR THE ANALYSIS OF MACROECONOMIC POLICY IN THE ‘AGE’ OF CENTRAL BANKS

By

Alfonso Palacio-Vera♣
Universidad Complutense de Madrid, Spain
Visiting Scholar
PERI, University of Massachusetts, Amherst (MA), USA

Abstract: Conventional explanations of how a growing potential output generates an equi-proportional increase in aggregate demand in the long run usually rely on the real balance effect. Yet this mechanism has a negligible size and an uncertain sign. We present a theoretical framework for the analysis of the power of conventional monetary policy to take the economy down its potential output path. We develop a simple model that predicts the behavior of the ‘neutral’ interest rate and the ‘pseudo-warranted’ interest rate in the wake of different types of shocks. We identify several different scenarios according to whether the behavior of the ‘neutral’ real interest rate enhances or weakens the power of conventional monetary policy. Likewise, we identify several regimes depending on whether a rise in the target rate of inflation in steady growth yields faster or slower output growth when the ‘natural’ rate is not (fully) exogenous. In addition, we provide a formal definition of the concept of the ‘growth trap’ which complements the notion of the ‘liquidity trap’. Finally, we propose a taxonomy of monetary policy regimes.

JEL Classification: E310, E490, E590

Key words: Neutral and warranted real interest rate, liquidity trap, growth trap, inflation, monetary policy
1.- Introduction

A core proposition in modern macroeconomic theory holds that market economies possess strong self-regulation mechanisms which guarantee that any expansion of potential output eventually generates an equi-proportional increase in the level of aggregate demand so that the latter adjusts passively to the former in the long run. This proposition is an implication of Say’s law, i.e., the notion that supply creates its own demand. Presumably, this proposition also holds that, in steady growth, the rate of growth of output equals its ‘natural’ rate, the latter being equal to the sum of the rate of growth of labor productivity and labor force. The mechanism through which this adjustment process takes place is a crucial area of macroeconomic theory yet it is frequently overlooked. In modern economic analysis, such mechanism usually comes in the form of the Scitovszky-Pigou-Haberler-Patinkin effect or ‘real balance effect’. However, a number of scholars have cast serious doubts into its practical relevance. For instance, Greenwald and Stiglitz argue that:

‘The enormous attention that the real balance effect has received over the years hardly speaks well for the profession. Quantitatively, it is surely an nth order effect; one calculation put it that, even at the fastest rate at which prices fell in the Great Depression, it would take more than two centuries to restore the economy to full employment. And in the short run even its sign is ambiguous, as intertemporal substitution effects may (depending on expectations) more than offset the wealth effects’ (Greenwald and Stiglitz, 1993a, p.36).
We will not discuss here the shortcomings of the real balance effect. We may note, though, that some authors appear to be leaving it behind and resort to other (though not necessarily more reliable) self-adjustment mechanisms. For instance, proponents of the so-called ‘modern’ view of macroeconomics (Clarida et al., 1999, pp. 1665-66; Taylor, 2000, p.91) use an aggregate demand function of the form \( y = -ar + u \), where \( y \) is the output-gap, \( r \) is the real interest rate and \( u \) is a stochastic component with zero mean. We have argued elsewhere that this relation does not hold unless it is assumed that aggregate demand shocks of the same sign and similar magnitude increase (decrease) the level of aggregate demand whenever a favorable (unfavorable) shock raises (lowers) potential output (Palacio-Vera, 2005). The mechanism through which increases in the latter lead to equivalent increases in aggregate demand is explained as follows:

‘Shocks to potential output also do not force a short-run trade-off. But they require a quite different policy response. Thus, e.g., a permanent rise in productivity raises potential output, but it also raises output demand in a perfectly offsetting manner, due to the impact of permanent income. As a consequence, the output gap does not change. In turn, there is no change in inflation. Thus, there is no reason to raise interest rates, despite the rise in output’ (Clarida et al., 1999, pp. 1675)

Apparently, these authors assume that an increase in potential output is ‘perfectly’ observed by individuals and ‘interpreted’ as leading to an equivalent rise in expected lifetime wealth. In turn, insofar as individuals tend to smooth consumption overtime the expectation of higher expected lifetime wealth leads them to consume more in the present
and in the future. However, this self-adjustment mechanism is unrealistic and, if it exists at all, it is likely to be negligible. Instead, the institutional framework that characterizes most, if not all, OECD economies is one where the Central Bank (hereafter CB) fine-tunes the economy through changes in short-term interest rates and attempts to keep the rate of growth of aggregate demand in pace with potential output.

We don’t wish to deny that there may well be other self-adjustment mechanisms at work. First, we recognize that the foreign sector may play a significant role through the effect that changes in potential output have – owing to changes in the rate of inflation – on the real exchange rate and this, in turn, on aggregate demand. Second, there is a role for changes in income distribution. For example, Skott (1989) provides an extensive theoretical analysis of the role of income distribution as a self-adjustment mechanism. Similarly, within the Marxian tradition, an increase in the rate of unemployment brings about a reduction in real wages which, in turn, increases the rate of profit and raises the rate of accumulation (Goodwin, 1967). Finally, there is also a role for (non-discretionary) fiscal policy through the effect of automatic stabilizers. This is because increases in the rate of unemployment and slow output growth usually prompt a rise in the government budget deficit which, in turn, increases aggregate demand and vice-versa. The current empirical relevance of these alternative mechanisms is extremely difficult to assess. However, we believe their relative relevance was probably higher in previous historical periods when CBs either did not exist or played a less prominent role in macroeconomic stabilization. Therefore, an implicit assumption of this study is that all these mechanisms currently play a less important role so the bulk of the adjustment of aggregate demand to
potential output actually occurs through the impact on aggregate demand of conventional monetary policy.

The purpose of this paper is to analyze the constraints conventional monetary policy is subject to when faced with the problem of generating a level of aggregate demand that keeps pace with a growing potential output. By conventional monetary policy we mean the regular actions that characterize the day-to-day setting of short-term interest rates by CBs with a view to achieving price stability. For that purpose, we set up a hypothesized economy represented by a simple model in which the CB uses short-term interest rates to hit an inflation target. Initially, the model is utilized to analyze the case of a closed economy without a government sector and, at a second phase, it is expanded to account for the case of an open economy with a government sector. Central to our discussion are two distinct analytical concepts; the ‘neutral’ interest rate and the ‘pseudo-warranted’ interest rate. The former is defined in the usual way, i.e., the real interest rate that makes ex-ante saving at full employment equal to ex-ante investment. It is closely associated to the notion of the ‘liquidity trap’ (hereafter LT). The latter complements the former. We define it as the real rate of interest that yields a rate of growth of current output equal to the ‘natural’ rate of growth of output for a given level of capacity utilization and inflation rate. It is linked to the notion of the ‘growth trap’ (hereafter GT) to be explained subsequently. When the current level of capacity utilization equals the level desired by firms the ‘pseudo-warranted’ rate becomes the ‘warranted’ rate. Both the LT and the GT are defined below. These two concepts allow us to analyze under what conditions the CB will be able to stabilize the economy and to determine the size of the shocks that will push the economy into either a LT or a GT. This analysis is coupled with
a discussion over the type and size of shocks that will take the economy out of a LT and a GT.

According to us the main contributions of this study are the following. First, we provide a framework for the analysis of the ability of conventional monetary policy to take the economy down its potential output path in the absence of any self-regulating mechanism other than the (modest) stabilization provided by the foreign sector and (fiscal) automatic stabilizers. Second, we provide a formal definition of the concept of the LT that, we claim, helps clarifying recent discussions in the literature. Likewise, we coin the concept of the GT. It complements the notion of the LT and provides some insights into the effectiveness of conventional monetary policy. Third, we develop a simple theoretical model that allows us to explain the behavior of the ‘neutral’ real interest rate and the ‘pseudo-warranted’ real interest rate in the wake of different types of shocks. As some recent studies highlight, the former may undergo relatively large fluctuations thus undermining the effectiveness of conventional monetary policy. Fourth, we analyze the contribution of the foreign and government sector to the effectiveness of the latter as the possible destabilizing nature of conventional monetary policy. Fifth, we determine the magnitude of changes in current real interest rates required to offset the impact of various types of shocks. Sixth, we identify several steady-state regimes depending on whether a rise in the target rate of inflation yields faster or slower output growth when the ‘natural’ rate of growth of output is not fully exogenous. Lastly, we propose a taxonomy of monetary policy regimes.

The structure of the paper is as follows. Section 2 below provides a background discussion on the topic. In particular, we discuss the way in which the adjustment of
aggregate demand to potential output is dealt with in mainstream macroeconomics. Section 3 presents a model for a closed economy without a government sector. The steady-state properties of the model are obtained and discussed. We also define the concepts of the ‘neutral’ and ‘pseudo-warranted’ interest rate as well as the associated ones of the LT and the GT. Next, we then analyze the behavior of the ‘neutral’ and ‘pseudo-warranted’ interest rate in the aftermath of inflation and aggregate demand shocks. Section 4 replicates the analysis for the case of an open economy with a government sector. Section 5 presents a taxonomy of monetary policy regimes that builds on the results in sections 3 and 4. Finally, section 6 concludes.

2.- Some preliminary considerations

This section contains a background discussion on the self-adjustment mechanism that guarantees the adjustment of aggregate demand to potential output in the long run in modern macroeconomics. For that purpose, we present a general analytical framework in the context an imperfectly competitive economy. Then we show that this framework highlights some conceptual shortcomings in the treatment of this topic in the mainstream literature. In order to keep the discussion concrete we take Lindbeck (1992) as the most representative contribution to this subject. Second, we discuss some further aspects related to the literature on monetary policy rules.

2.1.- Is the equilibrium rate of unemployment dynamically stable?

A general type of organizational framework for macroeconomic analysis that is shared by adherents to several schools of thought is represented by a labor market where
there is a price-setting curve (hereafter p-curve) and a wage-setting curve (hereafter w-curve) drawn up in real wage-employment space. The p-curve implies a real wage/employment relationship that is derived from the price/output decisions of firms in the context of imperfect competition. The w-curve implies a real wage-employment relationship that may be based on a labor supply curve or, alternatively, may be derived from considerations of collective bargaining and/or efficiency wages. A possible real wage/employment relationship for each curve is drawn in Figure 1 below. We wish to make the following five considerations (see Sawyer, 1995 for an detailed presentation and discussion of this general framework). First, movements along the p-curve are generated by variations in the level of aggregate demand. Second, under imperfect competition there may be sections of the p-curve that are horizontal or even positively sloped. As pointed out in Sawyer (1995, p. 95), this arises from the possibility of (imperfectly competitive) firms operating along the decreasing portion of their cost curves and/or the mark-up varying inversely with the level of output. A crucial implication of this is that, in general, there is no reason why there should a negative relationship between the real wage and employment. Such a negative relationship only applies to the special (and much less relevant) case of perfect competition. Third, firms meet demand at the prices that they set so that there can not be excess demand for output along the p-curve. Fourth, an implication of the former is that the real wage is de facto set by the firms when determining the output price. As a result of it, the real wage is an endogenous variable and there is not a causal relationship running from the real wage to the level of employment at the aggregate level. Fifth, the wage and price dynamics implied by the combination of the p-curve and the w-curve do not necessarily take the
economy to its equilibrium rate of unemployment. The dynamics are shown in Figure 1 below. We identify four different zones. In zone A nominal wages increase less (fall more) than expected prices and prices increase more (fall less) than expected nominal wages. In zone B nominal wages increase less (fall more) than expected prices and prices increase less (fall more) than expected nominal wages. In zone C nominal wages increase more (fall less) than expected prices and prices increase less (fall more) than expected nominal wages.

Finally, in zone D nominal wages increase more (fall less) than expected prices and prices increase more (fall less) than expected nominal wages. It is clear from this that there will be a tendency for the real wage to converge to the equilibrium real wage $w/p^*$.
only when the initial position of the economy is at zones A and C. By contrast, in zones B and D, the realized real wage may drift away from \( w/p^* \) in the absence of an additional self-adjustment mechanism that is superimposed to the price and wage dynamics exposed above\(^5\). A detailed algebraic analysis of the stability of equilibrium point \( b \) in Figure 1 – corresponding to the equilibrium level of employment \( N^* \) and the equilibrium real wage \( w/p^* \) - in the context of perfect competition is in Lindbeck (1992, 1993). The perfect competition assumption implies that the \( p \)-curve is downward-sloping. All the other elements remain the same as in the framework presented above. According to Lindbeck (1992, p. 220), dynamic stability requires that the real wage falls to the left, and rises to the right, of point \( b \). He argues that this assumption is reasonable as wage inflation tends to rise when the rate of unemployment falls and price inflation tends to rise when there is higher excess demand in the product market. However, as pointed out above this does not guarantee \textit{per se} that the economy converges to a point like \( b \) in Figure 1. Adding Phillips curve type wage and price dynamics (adjusted to account for the special case of perfect competition) he goes on to show algebraically that point \( b \) is a \textit{stable node}\(^6\) (Lindbeck, 1993, Appendix B). The reduced-form equations of his model are as follows (for ease of comparison we use the same numbers as originally presented in Lindbeck, 1993):

\[
\dot{\omega} = \omega \cdot F(\omega, P) \equiv f(\omega, P) \tag{B.3b}
\]

\[
\dot{P} = P \cdot G(\omega, P) \equiv g(\omega, P) \tag{B.4b}
\]

where \( \omega \) is the real wage, \( P \) is the price level, \( f_\omega < 0, \ f_p > 0, \ g_\omega > 0, \ g_p < 0 \) and \( \dot{\omega} \) and \( \dot{P} \) are time derivatives. The negative sign of \( f_\omega \) is justified on the basis that a higher real wage will (under perfect competition) reduce the actual level of employment and
this, in turn, will reduce the rate of growth of nominal wages for a given (expected) rate of price inflation. The positive sign of $f_p$ is based on the assumption that a higher price level will reduce excess demand in the product market thus reducing the rate of inflation for a given (expected) rate of wage inflation. According to Lindbeck (1992), this occurs partly through the operation of the real balance effect and partly through changes in the degree of international competitiveness. He claims though that the latter effect is much more important than the former. The positive sign of $g_\omega$ is due to the notion that a higher real wage will reduce the actual level of employment thus increasing the degree of excess demand in the product market. Finally, the negative sign of $g_p$ is based on the argument that a higher price level will, through the channels mentioned above, reduce the degree of excess demand in the product market. Thus, equilibrium point $b$ of the static system is a stable node iff:

$$\frac{\partial f}{\partial \omega} + \frac{\partial g}{\partial P} < 0 \quad (B.5)$$

and

$$\frac{\partial f}{\partial \omega} \cdot \frac{\partial g}{\partial P} - \frac{\partial g}{\partial \omega} \cdot \frac{\partial f}{\partial P} > 0 \quad (B.6)$$

At this juncture, the author argues that this will be the case under the assumption that each of the own-market effects on real wages and prices ($\partial f / \partial \omega$ and $\partial g / \partial P$) is large relative to the cross-market effects ($\partial f / \partial P$ and $\partial g / \partial \omega$) (Lindbeck, 1993, p.177). However, as highlighted above, $\partial f / \partial \omega < 0$ will only be necessarily the case under perfect competition so that B.5 may or may not hold. Likewise, B.6 only holds if $\partial N / \partial \omega < 0$ and this, again, will only be necessarily the case under perfect competition. In an imperfectly competitive economy B.5 and, particularly B.6, may not hold and, as a result of it, point $b$ may not be a stable node. For instance, if B.6 does not hold - and this will certainly be the
case if equilibrium point \( b \) happens to be either on an upward-sloping or on a horizontal section of the \( p \)-curve - then the former will be a *saddle-point* and its dynamic stability will depend on the initial position of the system\(^8\). More generally, and as stressed in the discussion above, real wages are effectively set by firms and this undermines the practical relevance of a theory where *exogenous* variations in the real wage clear the labor market.

### 2.2.- Macroeconomic stabilization and monetary policy

The equilibrium condition in the goods market for an open economy with a government sector when current output equals potential output is:

\[
s(r)Y^p = I(r) + G(r) - T + NX(r)
\]

where \( s \) is the saving ratio, \( Y^p \) is potential output, \( I \) is the (gross) level of investment, \( G \) is government spending, \( T \) is government revenue and \( NX \) is net exports. The real interest rate that results from (4) is the ‘neutral’ real interest rate or \( r^n \), i.e., the interest rate that makes ex-ante saving at potential output equal to the sum of ex-ante gross investment, the government budget deficit and the foreign sector current account balance. It is better thought of as a medium-term interest rate. If \( Y^p \) is the level of output that keeps inflation constant in the absence of inflation shocks (hereafter ISs) we have that inflation will rise (fall) when \( r < r^n (r > r^n) \). This is the Wicksellian approach to inflation dynamics (Wicksell, 1936). As a result, \( r^n \) represents a critical benchmark for the setting of interest rates.

As pointed out in the introduction, we believe that the most important regulation mechanism at work in modern economies comes in the form of a CB who brings about changes in real interest rates so as to achieve its ultimate policy objectives. The most
important objective of modern CBs as far as macroeconomic stabilization is concerned is price stability. However, CBs can only control the path of short-term nominal interest rates. As long as nominal interest rates remain above the zero lower bound the CB will also be able to move real interest rates in the desired direction. In turn, if the level of aggregate demand is a negative function of the real interest rate then the CB can, under normal circumstances, manipulate real interest rates so as to generate a rate of expansion of aggregate demand that matches a growing potential output. Let us present the following simple model:

\[ \dot{\pi} = f_1(r - r^n) \]  
\[ r - r^n = f_2(\pi - \pi^d) \]

Substituting (3) into (2) yields:

\[ \dot{\pi} = f_3(\pi - \pi^d) \]

where \( f_1' < 0, f_2' > 0, f_3' < 0 \), \( \pi \) is the inflation rate, \( \pi^d \) is the target inflation rate and \( r \) is the current real interest rate. Differential equation (2) shows the dynamics of inflation in a Wicksellian fashion. Equation (3) is a Taylor-like monetary policy rule. It can be seen that – since \( f_3' < 0 \) - the rate of inflation will converge to \( \pi^d \) in the long run. Thus as long as the CB interest rate rule is governed by (3), monetary policy will succeed in hitting the inflation target. Needles to say, CBs face a good deal of difficulties when implementing a rule like (3). A first problem is how to set nominal interest rates in order to push ex-ante current real interest rates in a given direction and by a certain magnitude. This is because there is some degree of uncertainty as to the rate of inflation expected by the public. A second and more important problem stems from the fact that CBs do not
actually know the value of $r^*$. Nevertheless, there may be circumstances when, even if a CB knows both the value of $r^*$ and the rate of inflation expected by the public, it may not be able to vary real interest rates as indicated by expression (3) above. If so the economy may stagnate. This will occur when the CB needs to yield a negative real rate of interest to stimulate aggregate demand but it can’t do so because the nominal interest rate is already at the zero lower bound. This situation is usually referred to as a LT⁹.

A number of authors has recently analyzed the causes of the existence of a zero lower bound on nominal interest rates as well as the policy options that will minimize the probability that the zero lower bound becomes a binding constraint on monetary policy and remove this constraint in case the economy gets into a LT. It is not our purpose to review this literature. However, we can safely highlight two points on which there seems to be an emerging consensus. First, the existence of a zero lower bound on nominal interest rates can, as a minimum, engender a moderate deterioration in macroeconomic stability as the inflation target approaches zero and therefore it is an important constraint on how conventional monetary policy can operate in a low inflation environment (Fuhrer and Madigan, 1997; Reifschneider and Williams, 2000; Mussa, 2000). Second, CBs should set a low and positive inflation target (let’s say 2 per cent). Conversely, there is no emerging consensus on whether unconventional monetary policy options can take the economy out of a LT should it be necessary. Several authors have proposed a number of different policy options for a CB to deal with the zero lower bound. More important for our discussion, they suggest that these options or a suitable combination of them will push the economy out of a LT if needed. These options include setting a carry tax on currency and vault cash as well as on electronic reserves, open market operations on
long-term bonds, foreign exchange intervention and price-level targeting (Goodfriend, 2000; Buiter, 2003; Ito and Mishkin, 2004). By contrast a different set of authors remains skeptical about the power of unconventional monetary policy to spur activity through any channel when nominal interest rates are at zero\(^{10}\) (Freedman, 2000; Blinder, 2000; Mussa, 2000).

3.- The case of a closed economy without a government sector

This section considers the case of a closed economy without a government sector. We consider a one-sector economy with two inputs, labor and capital and we assume that the production function has fixed coefficients. Potential output \(Y^p\) is determined by:

\[ Y^p = \lambda \cdot \bar{N} \leq \nu \cdot K \]  

(5)

where \(\bar{N}\) is the level of employment that keeps the rate of inflation constant and \(\lambda\) and \(\nu\) are respectively the productivity of labor and capital when the factors are fully utilized. The current rate of capacity utilization is:

\[ \varphi = \frac{Y}{\nu \cdot K} \leq 1 \]  

(6)

We assume that firms have a desired rate of capacity utilization \(\varphi < 1\) so they expand capacity when \(\varphi > \overline{\varphi}\) and stop expanding capacity when \(\varphi < \overline{\varphi}\)\(^ {11}\). As a result, the desired rate of capacity utilization when \(Y = Y^p\) is\(^ {12}\):

\[ \overline{\varphi} = \frac{Y^p}{\nu K} = \frac{\lambda}{\nu} \cdot \frac{\bar{N}}{K} \leq 1 \]  

(7)

We denote by \(\overline{e}\) the employment ratio corresponding to the non-accelerating inflation rate of unemployment (NAIRU) and by \(L\) the total labor force. For simplicity, we assume that \(\overline{e}\) is constant. Hence, we have that:
\[
\bar{N} = \bar{\sigma} \cdot L
\]  
(8)

In turn, the dynamics of the rate of inflation are given by:

\[
\hat{\pi} = \xi (e - \bar{e}) \quad \xi > 0
\]  
(9)

Since we are concerned with the case of a closed economy without a government sector, expression (1) now becomes:

\[
s \cdot Y'' = I
\]  
(10)

If we divide through (10) by the capital stock \( K \) and denote the rate of capital accumulation by \( g \) and the rate of depreciation of physical capital by \( \psi \) we get:

\[
s \cdot \frac{Y''}{K} = g + \psi
\]  
(11)

and inserting (7) into (11) yields:

\[
s \cdot \frac{\nu \cdot \bar{\phi}}{\bar{\nu}} = g + \psi
\]  
(12)

The actual real wage \( w/p \) is determined by firms’ profit-maximization objectives so that:

\[
\frac{w}{p} = \frac{\lambda}{m}
\]  
(13)

where \( m > 1 \) is one plus the (average) mark-up. If we assume that the average mark-up set by (imperfectly competitive) firms is constant, then the joint assumption of a fixed coefficients technology and a constant mark-up implies that the p-curve is horizontal in real wage-employment space so that condition B.6 in Lindbeck’s model above does not hold in this case. The ‘natural’ rate of growth of output is:

\[
g_n = l + a
\]  
(14)

where \( l \) and \( a \) are respectively the growth rate of labor force and labor productivity.
We now turn our attention to functions $s$ and $g$. We assume that the saving ratio $s$ is a function of the rate of inflation $\pi$, the rate of growth of output $\dot{y}$, the real interest rate $r$ and a measure of exogenous shocks $\varepsilon$, or:

$$s = s(\pi, \dot{y}, r, \varepsilon)$$

(15)

where $s_\pi > 0$, $s_\varepsilon < 0$, $s_r \geq 0$ and $\varepsilon$ is a stochastic variable with zero mean. The positive sign of $s_\pi$ is based on the life cycle hypothesis of saving. The latter establishes a positive relationship between $s$ and $\dot{y}$ in the short and the long run (Modigliani and Brumberg, 1980; Modigliani, 1986). The non-negativity of $s_r$ is due to the fact that although households are, on average, net lenders and substitution and income effects move in opposite directions for individual households who are net lenders, yet wealth effects operate in the same direction as the substitution effect thus making $s_r < 0$ a very unlikely scenario. The sign of $s_\varepsilon$ requires some clarification. In a study of the US economy by Pollin (1985), the author shows that the stability of the total outstanding debt ratio $S_t$ of the economy’s non-financial sectors has displayed essentially no trend throughout the post-World War II period. Using the formula $S_t = h_t(1 + \dot{Y})/\dot{Y}$ derived in Gurley and Shaw (1957) where $h_t$ is the marginal propensity of the aggregate non-financial sector to issue net new debt and $\dot{Y}$ is the rate of growth of nominal GNP, the author argues that the stability of $S_t$ throughout the postwar period, and especially since the 1960’s, has resulted from rising trends for $\dot{Y}$ and $h_t$ as well as a declining trend for the rate of growth of real GNP. As a result, the ratio $(1 + \dot{Y})/\dot{Y}$ has fallen correspondingly over this period and $h_t$ has risen along with $y$ in order for $S_t$ to remain constant. According to
Pollin (1985), the divergent patterns of $S_t$ and $h_t$, are due to the asymmetric impact of inflation on the two ratios. As for $S_t$, its numerator, the stock of debt, remains fixed in nominal terms regardless of variations in the price level (relative to trend) whereas its denominator, nominal $GNP$, varies in nominal terms directly with the price level. As a result, in an inflationary environment, the nominal value of the debt stock remains fixed while $GNP$ rises, so that $S_t$ is biased downwards. Conversely, with $h_t$, current-period flow values are in both numerator and denominator, and thus the impact of inflation on the ratio is neutral. Because of this asymmetry, an increasing reliance on debt by the non-financial sectors, i.e. a rising $h_t$, may not engender increases in their debt burdens.

Next, we have that for a given $h_t$, a fall in the rate of inflation will increase net borrowers’ real debt burden and vice-versa. In turn, this will increase the general level of bankruptcy risk. In the case of net borrower households the increase in the real level of indebtedness will lead to a fall in consumption (Bernanke, 1981). In the meantime, the increase in net borrowers’ real debt burden will be coupled by a rise in net lenders’ real financial wealth and, for the same reason, this will tend to increase net lender households consumption demand. Nevertheless, it is reasonable to assume that net borrowers’ marginal propensity to consume out of wealth is, on average, higher than net lenders’. In addition, bankruptcy imposes net social costs so that, as the general level of bankruptcy rises (owing to rising real debt burdens by net borrowers), the level of spending of net lenders will rise by less than the fall in the level of spending by net borrowers. Hence we assume that $s_x < 0$. The size of $s_x$ will be directly proportional to the size of the debt ratio and to the degree of dispersion of balance sheet positions across households. Next, we define the rate of capital accumulation, $g$ as:
\[ g = g(\pi, \hat{y}, r, \varepsilon) \]  

where \( g_{\pi} > 0, \ g_{\hat{y}} > 0, \ g_{r} < 0 \) and \( \varepsilon \) is a stochastic variable with zero mean which captures exogenous shocks to \( g \). The positive sign of \( g_{\pi} \) stems from the same considerations made above as to the likely stabilizing effect of inflation on consumption demand\(^{15} \). Since firms’ are on average, net borrowers a rise in the rate of inflation will, on average, reduce firms’ debt burden in real terms and vice-versa. In turn, this will stimulate investment (Bernanke, 1981; Caskey and Fazzari, 1987; Greenwald and Stiglitz, 1993b). The size of \( g_{\pi} \) is determined by the same factors affecting the size of \( s_{\pi} \). The negative sign of \( g_{r} \) reflects conventional cost of capital as well as access to (external) finance considerations. Finally, the positive sign of \( g_{\hat{y}} \) reflects the accelerator effect for a desired rate of capacity utilization \( \bar{\varphi} < 1 \). In steady growth, we have that \( g_{\hat{y}} = 1 \) but \( g_{\hat{y}} \) may be either above or below one when the economy is not in steady growth. Therefore, expression (12) can be formulated as:

\[ s(\pi, \hat{y}, r, \varepsilon) \cdot \varphi \cdot \bar{\varphi} = g(\pi, \hat{y}, r, \varepsilon) + \psi \]  

3.1.- Steady-state analysis

In steady growth we have that \( \hat{y} = g_{n} \), \( \varphi = \bar{\varphi} \) and \( \varepsilon_{n} = \varepsilon_{g} = 0 \) so that expression (17) above can be split into the following two equations:

\[ g(\pi, g_{n}, r) = g_{n} \]  

\[ s(\pi, g_{n}, r) \cdot \varphi \cdot \bar{\varphi} = g_{n} + \psi \]  

Equation (18) tells us that in steady growth the rate of accumulation must equal the ‘natural’ rate of growth as long as \( \varphi = \bar{\varphi} \). Equation (19) represents the equilibrium
condition in the goods market when \( Y = y^p \). Thus we have a system of two equations and two unknowns \( \pi^* \) and \( r^* \). In order to get explicit solutions for \( \pi^* \) and \( r^* \) we need to assume that functions \( s \) and \( g \) adopt a linear form so that in steady growth (18) and (19) become:

\[
\begin{align*}
g &= \bar{g} + g_{\bar{y}} \cdot g_n + g_{\pi} \cdot \pi + g_r \cdot r = g_n \\
s \cdot v \cdot \bar{\varphi} &= (\bar{s} + s_{\bar{y}} \cdot g_n + s_{\pi} \cdot \pi + s_r \cdot r) \cdot v \cdot \bar{\varphi} = g_n + \psi
\end{align*}
\]

where \( \bar{s} \) and \( \bar{g} \) are the exogenous components of \( s \) and \( g \) respectively. Recalling that in steady growth \( g_{\bar{y}} = 1 \), we can obtain explicit solutions for \( \pi^* \) and \( r^* \) by solving the system of two linear equations made up by (20) and (21) or:

\[
\pi^* = \frac{s_{\bar{r}} \cdot \bar{g} - \psi \cdot \bar{\varphi} + \left[ s_{\bar{y}} - \frac{1}{v \cdot \bar{\varphi}} \right] \cdot g_n}{s_{\bar{r}} \cdot g_{\pi} - s_{\pi} g_r}
\]

and

\[
r^* = -\left( \frac{\bar{g} + g_{\pi} \pi^*}{g_r} \right) = \frac{\left[ \psi \cdot \bar{\varphi} - s_{\bar{y}} g_n + \frac{g_n}{v \cdot \bar{\varphi}} \right] \cdot g_{\pi}}{s_{\bar{r}} g_{\pi} - s_{\pi} g_r}
\]

The expression for the steady-state ‘neutral’ real interest rate above shows that it is independent of the ‘natural’ rate of growth of output and that, instead, it is determined by all the factors embedded in \( \bar{g} \) and by the steady-state rate of inflation\(^{16} \). As for \( \pi^* \), we can see that its sign is ambiguous. Further, and more important, it is clear that there exists a steady-state inflation rate \( \pi^* \) which differs from the inflation target \( \pi^d \). To fix ideas, the model suggests that if and only if \( g_n \) is exogenously determined, the CB can do nothing to affect the rate of inflation in the long run and that, as a result of it, there is no reason why the CB should be held accountable for the behavior of inflation. No wonder,
this result runs against most propositions in mainstream monetary theory. Nevertheless, our results can be reconciled with conventional wisdom in the field by dropping the assumption that \( g_n \) is fully exogenous. For instance, we may assume that the rate of technical progress is demand-pull so that its rate of change exhibits some degree of endogeneity, i.e., it rises when the rate of growth of output increases and vice-versa\(^{17}\) (Schmookler, 1966; Brouwer and Kleinknecht, 1999; Geroski and Walters, 1995). As a result, we can express the ‘natural’ rate of growth as:

\[
g_n = g_n^e + \gamma \cdot \hat{y}
\]

(22)

where \( g_n^e > 0 \) is the exogenous component of \( g_n \) and \( 0 < \gamma < 1 \).

If we set the steady-state rate of inflation equal to target inflation \( \pi^d \), insert (22) and \( \pi^d \) into (20) and (21) and rearrange, we get respectively the rate of growth of output and the ‘neutral’ real interest rate in steady growth or:

\[
\hat{y}^* = \frac{-\frac{S_r \cdot g}{g_r} + \left[ -\frac{1}{v \cdot \bar{\phi}} + s_\gamma \right] \cdot g_n^e - \frac{\psi}{v \cdot \bar{\phi}} + \left[ s_\pi - \frac{s_r \cdot g_\pi}{g_r} \right] \cdot \pi^d}{\gamma \cdot \left[ \frac{1}{v \cdot \bar{\phi}} - s_\gamma \right]}
\]

and

\[
r^* = \frac{-(g + g_\pi \cdot \pi^d)}{g_r}
\]

Thus, the latter is a positive function of \( \pi^d \) as long as \( \pi^d > 0 \). This result runs against the notion of a steady-state neutral interest rate that is exclusively determined by ‘real forces’ that pervades most discussions of this topic in the literature\(^{18}\). Furthermore, since the denominator in \( \hat{y}^* \) is positive for plausible values of \( v \) and \( s_\gamma \), we get:

21
\[ \frac{\partial \hat{y}^*}{\partial \pi^d} = \left[ s_\pi - s_r \cdot g_\pi \right] \gamma \left[ \frac{1}{v \cdot \varphi} - s_\gamma \right] > 0 \iff \frac{g_\pi}{s_\pi} < \frac{g_r}{s_r} \quad (23) \]

and

\[ \frac{\partial r^*}{\partial \pi^d} = -\frac{g_\pi}{g_r} > 0 \quad (24) \]

Hence, we can’t say \emph{a priori} whether or not an increase in \( \pi^d \) will result in an increase in \( \hat{y}^* \). However, we know that an increase in \( \pi^d \) will lead to an increase in \( r^* \) as long as \( \pi^d > 0 \). This is because an increase in the steady-state rate of inflation will, since \( s_\pi < 0 \) and \( g_\pi > 0 \), increase the level of aggregate demand. The ambiguous result in (23) suggests the existence of several different regimes in steady growth. A first regime corresponds to the case when \( \partial \hat{y}^* / \partial \pi^d > 0 \) and we refer to it as the ‘inflation-led growth’ regime. A second regime corresponds to the case when \( \partial \hat{y}^* / \partial \pi^d < 0 \) and we refer to it as the ‘disinflation-led growth’ regime. A third and last regime corresponds to the case when \( \partial \hat{y}^* / \partial \pi^d = 0 \) and we can refer to it as the ‘inflation-neutral growth’ regime. Their basic properties are summarized in Table 1 below. The significance of these regimes is that a rise in \( \pi^d \) will lead to an increase in \( r^* \) thus making the economy less vulnerable to shocks that may push it into a LT. Yet, in the case of the ‘disinflation-led growth’ regime, the rise in \( r^* \) will come at the expense of a lower \( \hat{y}^* \). In this case, the CB will face a trade-off. The notion of the existence of several different regimes in steady growth stems from the joint assumption that: (i) the ‘natural’ rate of growth exhibits some degree of endogeneity and (ii) that increases (decreases) in the rate of inflation lead to a rise (fall) in aggregate demand owing to the balance sheet considerations discussed above.
Arguably, these effects may be weak. In addition, they may only be relevant as long as inflation remains within moderate levels (let’s say below 10 per cent). As inflation reaches higher levels, it may well have large adverse supply-side effects which offset any favorable demand-side effect on economic growth.

3.2.- The behavior of the economy in the short run and the growth trap

We now focus on the behavior of the economy in the short run. The equilibrium condition in the goods market is:

\[(\bar{s} + \hat{s} \hat{y} + s_{\pi} \pi + s_{r} r) \cdot v\varphi = \bar{g} + g_{\hat{s}} \hat{y} + g_{\pi} \pi + g_{r} r + \psi\]  

(25)

and, as a result, the rate of growth of output is:

\[\hat{y} = \frac{\bar{g} - \bar{s} v\varphi + (g_{\pi} - s_{\pi} v\varphi) \pi + (g_{r} - s_{r} v\varphi) r + \psi}{s_{\hat{s}} v\varphi - g_{\hat{s}}}\]  

(26)

If we assume, for consistency purposes, that \(s_{\hat{s}} v\varphi - g_{\hat{s}} \geq 0\), we have that:

\[\frac{\partial \hat{y}}{\partial r} = \frac{(g_{r} - s_{r} v\varphi)}{(s_{\hat{s}} v\varphi - g_{\hat{s}})} \times 0\]  

(27)

and

\[\frac{\partial \hat{y}}{\partial \pi} = \frac{(g_{\pi} - s_{\pi} v\varphi)}{(s_{\hat{s}} v\varphi - g_{\hat{s}})} \times 0\]  

(28)

We may justify the assumption that \(s_{\hat{s}} v\varphi - g_{\hat{s}} \geq 0\) by arguing that \(s_{\hat{s}}\) is much higher than \(g_{\hat{s}}\) in the short run owing to the presence of relatively large decision and execution lags in investment. Rearranging (25), we can obtain the expression for the real rate of interest that clears the goods market when \(\hat{y} = 0\) and \(\hat{y} = g_{\pi}\) or:

\[r_{\hat{s}=0} = \frac{\bar{s} v\varphi - \bar{g} - (g_{\pi} - s_{\pi} v\varphi) \cdot \pi - \psi}{g_{r} - s_{r} v\varphi}\]  

(29)
\[ r^g = \frac{-3\nu\varphi - \bar{g} - (g_x - s_x\nu\varphi) \cdot \pi + (s_y\nu\varphi - g_y) \cdot g_n - \psi}{g_v - s_x\nu\varphi} \]  

Expression (29) corresponds to point A in Figure 2 below. We can think of \( r^g \) as a pseudo-warranted real rate of interest in the sense of being the rate of interest that makes \( \hat{y} = g_n \) for a given \( \varphi, \pi \) and \( g_y \). Of course, the warranted real rate of interest \( r^w \) will be the rate of interest that makes \( \hat{y} = g_n \) when \( \varphi = \bar{\varphi} \) and \( g_y = 1 \). Therefore, the difference between the ‘warranted’ rate and the steady-state ‘neutral’ rate is that in the former it will generally be the case that \( \pi \neq \pi^* \). Similarly, when \( r = 0 \) we obtain point B in Figure 2 or:

\[ \hat{y}^{r=0} = \frac{-\bar{g} - 3\nu\varphi + (g_n - s_x\nu\varphi)\pi + \psi}{s_y\nu\varphi - g_y} \]

The determination of \( r^g \) is graphically illustrated in Figure 2 below. We measure the real interest rate \( r \) in the vertical axis and the rate of growth of output \( \hat{y} \) in the horizontal one. The line denoting the rate of growth of output for every value of the real rate of interest is downward-sloping under the assumption that (27) holds. We refer to it as the dynamic aggregate demand line or DAD line. In general it will not be a line owing to the presence of \( \varphi \) and \( \pi \) in expression (26) above. We will impose the assumption of linearity for presentational purposes. Its position is determined by the current inflation rate. Under the assumption that \( \partial r^g / \partial \pi > 0 \) an increase in the current rate of inflation from \( \pi_0 \) to \( \pi_1 \) will shift the DAD line upward from DAD\(_0\) to DAD\(_1\) thereby leading to a rise in \( r^g \). This will certainly be the case in a closed economy without a government.
sector as expression (35) below shows. However, it may not be the case in an open economy with a government sector\textsuperscript{19}.

**Figure 2: The determination of the pseudo-warranted real rate of interest**

If we now denote by $\omega$ the minimum (ex-ante) real interest rate that the CB can actually set, we can define a GT as a situation where:

$$r^{8*} < \omega$$

(32)

If we further assume that the minimum nominal interest rate the CB can set is zero and that the expected rate of inflation is roughly equal to the current rate of inflation, we have that\textsuperscript{20}:

$$\omega = 0 - \pi^e = -\pi^e = -\pi$$

(33)
As a result, we say that an economy is in a GT whenever:

\[ r^{g_s} < -\pi \quad \text{or} \quad r^{g_s} + \pi < 0 \]  \hspace{1cm} (34)

where

\[ \frac{\partial r^{g_s}}{\partial \pi} = -\frac{(g_s - s_r \psi \phi)}{g_r - s_r \psi \phi} > 0 \]  \hspace{1cm} (35)

Therefore, an IS will lead the economy into a GT if:

\[ d\pi < -\frac{(r^{g_s}_0 + \pi_0)}{1 + \frac{\partial r^{g_s}}{\partial \pi}} \]  \hspace{1cm} (36)

where \( \pi_0 \) and \( r^{g_s}_0 \) are the initial inflation rate and \( r^{g_s} \) respectively. We may wonder how the output-gap will vary when the economy gets into a GT. There are two possibilities. If \( g_n \) is fully exogenous, then the economy will exhibit either a rapidly narrowing positive output-gap or a rapidly widening negative one. By contrast, if \( g_n \) exhibits some degree of endogeneity, the positive (negative) output-gap will admittedly narrow (widen) at a lower pace. The latter will depend negatively on the degree of endogeneity exhibited by \( g_n \). In turn, the behavior of \( y^* \) when the economy is in a GT ultimately depends on the behavior of the rate of inflation. In a closed economy without a government sector, the faster it falls, the deeper the slump will be as a result of the impact of a falling rate of inflation on the real interest rate. Next, the variation of \( r^{g_s} \) resulting from a change in the current rate of capacity utilization has an ambiguous sign since:

\[ \frac{\partial r^{g_s}}{\partial \varphi} = \frac{(s + s_\pi + s_i g_n) \cdot v \cdot (g_r - s_r \psi \phi) + s_r \cdot v \cdot [3 \psi \phi - g - (g_s - s_\pi \psi \phi) \cdot \pi + (s_i \psi \phi - g_i) \cdot g_n]}{(g_r - s_r \psi \phi)^2} \]  \hspace{1cm} (37)

As a result, an (unfavorable) aggregate demand shock (hereafter DS) will lead the economy into a GT if:
000

\[ \pi_0 + r_{g*} + \frac{\partial r_{g*}}{\partial \varphi} < 0 \]  

(38)

where \( r_{g*} \) is the initial pseudo-warranted real rate of interest. Thus, expression (35) tells us that ISs will lead to changes in \( r_{g*} \) of the same sign as the initial shock whereas expression (37) tells us that the impact on \( r_{g*} \) of DSs is ambiguous. We will return to this point in sections 4 and 5 below.

3.3.- The behavior of the ‘neutral’ interest rate and the liquidity trap

A LT is usually defined as a situation in which conventional monetary policies have become impotent because nominal interest rates are at or near zero. In turn, this will be the case ‘when desired saving exceeds desired investment at full employment, even at a zero short-term interest rate’ (Krugman, 1998, p.171). One way of thinking of a LT is to define it as a situation in which \( r^n \leq \omega \). If we assume again that the minimum nominal interest rate that the CB can set is zero and that the expected rate of inflation is equal to the current rate of inflation we will say that an economy has got into a LT if:

\[ \pi - \pi + \pi \leq 0 \]  

(39)

Expression (39) above tells us that the lower \( r^n \) and \( \pi \) are, the more likely it is that the economy gets into a LT in the aftermath of either a favorable IS or an unfavorable DS. This suggests that, in the context of a closed economy without a government sector, the setting of an inflation target well above zero reduces the probability of getting into a LT. However, expression (39) also suggests that the focus on (too) low inflation targets as the single cause of an economy falling into a LT misses the fact that a negative \( r^n \) is as problematic as the setting of a very low inflation target.
Conventional wisdom regarding the LT seems to be that the latter may arise under certain (exceptional) circumstances as a result of various types of large shocks. For instance, if the rate of inflation is already very close to zero, then either a large favorable IS or a large unfavorable DS may push the rate of inflation below zero. Since the nominal interest rate may already be very close to the zero lower bound, the CB may be unable to push real interest rates down. The dominant view is that this is a very unlikely scenario as long as target (measured) inflation lies at or above 2 per cent\(^23\) (De Long, 1999). Yet the literature on the LT tends to sidestep the fact that at least in theory the economy may get into a LT even if the rate of inflation is positive. As condition (39) above shows, it is not only the rate of inflation that matters. If \( r^n < 0 \) the economy may be technically in a LT even if the rate of inflation is well above zero. Finally, expression (39) above suggests that a way out of a LT is, as suggested initially in Krugman (1998) and more recently in Eggertsson and Woodford (2003), the creation of inflationary expectations. However, insofar as the expected rate of inflation tracks the current rate of inflation, there is no way the CB can overcome the so-called ‘inverted credibility’ problem\(^24\). We believe CBs can only raise inflationary expectations by generating inflation but they cannot generate it as long as the economy is in a LT. Hence, in the absence of effective unconventional monetary policy options, only discretionary fiscal policy (especially when financed through money creation) can do the job of taking the economy out of a LT.

### 3.3.1.- Inflation shocks

We turn to the analysis of the behavior of \( r^n \) in the aftermath of ISs and to the analysis of the conditions that will push the economy into a LT. It is clear from (17)
above that $r^n$ is a function of both $\pi$ and $\hat{y}$. Its explicit solution can be obtained from (25) by making $\varphi = \bar{\varphi}$. However, we are here interested in determining how it varies in response to various types of shocks. Differentiating (17) and rearranging we obtain:

$$d r^n = \frac{(g_x - s_x \bar{\varphi}) \cdot d\pi + (g_y - s_y \bar{\varphi}) \cdot d\hat{y}}{s_y \bar{\varphi} - g_r}$$  \hspace{1cm} (40)$$

If we initially assume that $d\hat{y} = 0$, we have that (40) becomes:

$$d r^n = \Pi_0 \cdot d\pi$$  \hspace{1cm} (41)$$

where $\Pi_0 = \frac{(g_x - s_x \bar{\varphi})}{s_y \bar{\varphi} - g_r} > 0$

As long as condition (39) does not hold, if the CB seeks to offset an IS through changes in interest rates, the minimum size of the variation in the current real interest rate required to offset the shock is:

$$d r \succ (r^n_0 - r_0) + \Pi_0 \cdot d\pi$$  \hspace{1cm} (42)$$

where $r^n_0$ and $r_0$ are respectively the initial value of the neutral and current real interest rate. Since $\Pi_0 > 0$, a first result is that ISs per se induce changes of $r^n$ in the same direction as the rate of inflation. Insofar as the CB needs to lower (raise) current real interest rates following a fall (rise) in the current rate of inflation $r^n$ will tend to move in a destabilizing fashion. As a result, any given CB-induced change in current real interest rates will be less effective in affecting output and inflation than if $r^n$ remained constant.

If we retrieve expression (39) above we may conclude that in the absence of changes in $\hat{y}$ the economy will fall into a LT in the wake of a favorable IS ($d\pi < 0$) if:

$$d\pi \leq \frac{-(r^n_0 + \pi_0)}{(1 + \Pi_0)}$$  \hspace{1cm} (43)$$

29
It is clear from this that in the absence of changes in \( \dot{y} \) and for given values of \( \pi_0 \) and \( r_0^* \), the likelihood that the economy gets into a LT depends on the magnitude of the initial IS (\( d\pi \)) and the size of \( \Pi_0 \). For a given \( d\pi \), the larger \( \Pi_0 \) is the more likely it is that the economy will get into a LT following a favorable IS. Under an inflation-targeting regime an IS will tend to trigger a monetary policy response by the CB. Thus, we need to specify how current real interest rates will vary in the aftermath of changes in the (current) rate of inflation. We assume that, as long as inflation remains positive, the CB raises real interest when inflation rises and vice-versa. Conversely, when inflation becomes zero or negative, real interest rates will fall (rise) whenever inflation rises (falls). If we further assume that the response of the CB occurs before the economy gets into a LT, we have that:

\[
\frac{dr}{d\pi} = \delta \begin{cases} 
> 0 & \text{if } \pi > 0 \\
< 0 & \text{if } \pi \leq 0 
\end{cases}
\] (44)

Finally, if we assume that a rise (fall) in the current real interest rate has a negative (positive) effect on output growth in the short run and only inflation in the long run, we have that \( d\dot{y}/dr = \phi \prec 0 \). Inserting \( \phi \) and (44) into (40), we obtain:

\[
dr^n = \Pi_1 \cdot d\pi
\] (45)

where \( \Pi_1 = \left[ \frac{(g_{\pi} - s_{\pi}v\phi) + (g_{\dot{y}} - s_{\dot{y}}v\phi) \cdot \phi \cdot \delta}{s_r v\phi - g_r} \right] \) and \( d\dot{y} = \frac{d\dot{y}}{dr} \cdot \frac{dr}{d\pi} \cdot d\pi \)

Hence, the minimum size of the variation in the current real interest rate required to offset the IS is:

\[
\frac{dr}{dr^n} = (r_0^n - r_0) + \Pi_1 \cdot d\pi
\] (46)

and, therefore, expression (43) becomes:
\[
d \pi \leq \frac{-\left(r^n_0 + \pi_0\right)}{(1 + \Pi_1)}
\] (47)

where the sign of \( \Pi_1 \) is ambiguous. Looking at (47) above we have that, if \( \Pi_1 > \Pi_0 \), the rise (fall) in \( \hat{\pi} \) brought about by a fall (rise) in the (current) real interest rate will reinforce the direct effect on \( r^n \) of the fall (rise) in the rate of inflation so that it will become more likely that a favorable IS leads the economy into a LT. If \( 0 \leq \Pi_1 < \Pi_0 \), the CB-induced rise (fall) in \( \hat{\pi} \) will offset either partially or fully the impact on \( r^n \) of the initial fall (rise) in the rate of inflation. Finally, if \( \Pi_1 < 0 \) the CB-induced increase (fall) in \( \hat{\pi} \) will more than offset the impact on \( r^n \) caused by the fall (rise) in the rate of inflation. This latter case is the most favorable one for those authors who argue that a LT is a very unlikely scenario. We can also think of this case as the most favorable scenario for a CB since favorable ISs will lead to a rise in \( r^n \) and vice-versa thereby enhancing the stabilizing power of the CB. Indeed, if \( \Pi_1 \) is negative, it may be the case that the CB does not need to vary real interest rates at all to safely head the economy towards its potential output time-path. This is because \( r^n \) would then forcefully move in the opposite direction to the rate of inflation, i.e., the behavior of \( r^n \) would make the economy largely self-stabilizing. In the opposite extreme, if \( \Pi_1 \) is positive and large enough, \( r^n \) will move in the same direction as the rate of inflation and, as a result, the CB may be unable to stabilize the economy owing to the large variations of \( r^n \) in response to ISs. This second case resembles Harrod’s knife-edge instability model (Harrod, 1939) except for the fact that instability in Harrod’s model is two-sided but is only one-sided here. Thus, one way of addressing the existence of unstable growth paths in the economy is to analyze the behavior of the ‘neutral’ real rate of interest. Returning to expression (45) above, when
\( \pi > 0 \) we have that \( \delta > 0 \). As a result, \( r^n \) will move in the most destabilizing fashion when:

\[ g_y < s_y \bar{v} \bar{\phi} \quad (48) \]

Hence, a second result is that an economy’s ability to forestall the occurrence of a LT in the aftermath of ISs and, more generally, its ability to self-stabilize depends critically on the sign of (48) as well as on its size relative to the size of ISs and the term \( g_\pi - s_\pi \bar{v} \bar{\phi} \). Further, since the magnitude of \( \delta \) when \( \pi > 0 \) is determined \textit{de facto} by the CB, we have that, in our hypothesized economy, the CB-induced pro-cyclical behavior of real interest rates \textit{will prove destabilizing} – \textit{in the sense of leading to a fall in} \( r^n \) \textit{when inflation falls and vice-versa} - \textit{when condition (48) holds but will prove stabilizing otherwise}, \textit{i.e.}, when \( \Pi_1 < \Pi_0 \). Since the CB does not know \textit{a priori} the sign let alone the magnitude of (48), it will be sensible to change interest rates cautiously and at a speed that is inversely proportional to the degree of uncertainty about the sign and magnitude of (48). This result is reminiscent of Brainard’s principle (Brainard, 1967). What are the chances that the economy escapes from a LT if favorable DSs are not forthcoming? If \( \pi \leq 0 \), we have that \( \delta < 0 \) and any further favorable IS will raise \( r^n \) provided \( \Pi_1 < 0 \). However, as (47) above shows, this will not be enough for the economy to get out of a LT. As a result, once the economy gets into a LT, either large unfavorable ISs or, alternatively, large favorable DSs will be required to push it out of it.

### 3.3.2.- Aggregate demand shocks

In a closed economy without a government sector, DSs initially affect either \( s \) or \( g \) and, therefore \( r^n \) before affecting \( \dot{y} \). We restrict the analysis to the case of shocks hitting
the rate of capital accumulation, \( g \ (\varepsilon_g \neq 0) \). Notwithstanding, results are qualitatively similar for shocks affecting the saving ratio \( s \). If we differentiate (17) and rearrange, we get:

\[
\frac{d \ln r^*}{d \varepsilon_g} = \Phi \Phi(\hat{\varepsilon}) (49)
\]

where \( g \varepsilon \) is the partial derivative of \( g \) with respect to shock \( \varepsilon_g \). If we initially assume that \( d\pi/d\hat{\varepsilon} = 0 \), we obtain:

\[
\frac{d \ln r^*}{d \varepsilon_g} = \Pi_2 \cdot \varepsilon_g
\]

(50)

where \( \Pi_2 = \frac{(g_{\hat{\varepsilon}} - s_{\hat{\varepsilon}} \varepsilon) \cdot \frac{d\hat{\varepsilon}}{dg} + 1}{s_r \varepsilon g - g_r} \), \( \frac{d\hat{\varepsilon}}{dg} > 0 \) and \( g \varepsilon > 0 \).

Expression (50) tells us that the impact on \( r^* \) of a shock to \( g \) when \( d\pi/d\hat{\varepsilon} = 0 \) depends on the sign of \( \Pi_2 \). Since the denominator in \( \Pi_2 \) is positive, its sign ultimately depends on the sign and magnitude of (48) above. Next, if (39) does not hold, then the minimum size of the variation in the current real interest rate required to offset the DS is:

\[
dr > (r_0^* - r_0) + \Pi_2 \cdot \varepsilon_g
\]

(51)

In turn, the economy will fall into a LT in the aftermath of an adverse shock \((\varepsilon_g < 0)\) if:

\[
\varepsilon_g \leq -\frac{(r_0^* + \pi_0)}{\Pi_2}
\]

(52)

More generally, the behavior of \( r^* \) will prove destabilizing when \( \Pi_2 > 0 \) and vice-versa. In contrast to the case of ISs, when condition (48) holds, the destabilizing nature of DSs is mitigated rather than accentuated. However, as long as \( d\hat{\varepsilon}/dg > 0 \) we
have that, whether or not condition (48) holds, it is unlikely that $\Pi_2 < 0$ so that, as long as $d\pi/d\hat{y} = 0$, DSs will tend to prove destabilizing in the sense used in this study. In any case, if we compare expressions (43) and (47) on the one hand and expression (52) on the other hand, it can be seen that, for the same magnitude of shocks, $\pi_0$, $r_0^n$ and model parameters values (and even if $d\pi/d\hat{y} > 0$), the question whether or not ISs are more likely than DSs to lead the economy into a LT ultimately hinges on the magnitude of the (direct) impact of the latter on $r^n$, e.g., on the size of $g_\varepsilon$. The larger the (direct) impact on $r^n$ is, the more likely it is that DSs will prove relatively more destabilizing than ISs in terms of the chances of leading the economy into a LT. Next, if $d\pi/d\hat{y} \neq 0$ we have that:

$$d\pi = \Pi_1 \cdot \varepsilon_g$$

where

$$\Pi_1 = \left[ \frac{(g_x - s_x \hat{\varphi}) \cdot \frac{d\pi}{d\hat{y}} + (g_y - s_y \hat{\varphi}) \cdot \frac{d\hat{y}}{dg} + 1}{s_r \hat{\varphi} - g_\varepsilon} \right] \cdot g_\varepsilon$$

In $\Pi_1$ above, the sign of the first term in the inner bracket of the numerator is ambiguous. This is because inflation rises when $Y > Y^p$ and vice-versa and, as a result, the sign of $d\pi/d\hat{y}$ is uncertain. If $d\pi/d\hat{y} > 0$, the behavior of the rate of inflation will tend to reinforce the destabilizing impact on $r^n$ of a shock to $g$. Further, if (39) does not hold, the minimum size of the variation in the current real interest rate required to offset the shock to $g$ is:

$$dr \succ (r_0^n - r_0) + \Pi_1 \cdot \varepsilon_g$$

and the economy will get into a LT if:

$$\varepsilon_g \leq \frac{-(r_0^n + \pi_0)}{\Pi_1}$$
Therefore, we have that the higher $\Pi_2$ and $\Pi_3$ are, the more likely it is that an unfavorable DS leads the economy into a LT and that a favorable DS pushes the economy out of it.

4.- The case of an open economy with a government sector

This section replicates the analysis in section 3.1 above for the case of an open economy with a government sector. As in section 3, we first obtain the steady-state properties of the model and then analyze the behavior of $r^g$ and $r^n$ in the short run. In addition, we analyze and discuss the contribution of the foreign and government sector to monetary policy’s stabilization capacity. If we divide expression (1) through by the capital stock $K$ we get:

$$s \cdot \frac{Y^p}{K} = g + \psi + f + x$$

(56)

where $f$ is the ratio of the government budget deficit to capital and $x$ is the ratio of net exports to capital. We assume that $f$ and $x$ are given by:

$$f = f(\hat{y},r,\varepsilon_f) \quad \text{and} \quad x = x(\pi,\hat{y},r,\varepsilon_x)$$

where $\hat{y} < 0, f_r > 0, f_\varepsilon > 0, x_\pi < 0, x_\hat{y} < 0, x_r < 0, x_\varepsilon > 0$ and $\varepsilon_f$ and $\varepsilon_x$ are random shocks with zero mean to $f$ and $x$ respectively. The negative sign of $f_\hat{y}$ and the positive sign of $f_r$ stem from the fact that higher output growth raises government revenue and thus lowers the government budget deficit for a given level of government spending whereas higher real interest rates increase interest payments by the government to holders of government debt and thus increases the budget deficit. The negative sign of $x_\pi$ is due to the negative impact on net exports of a fall in the level of international competitiveness.
following an increase in $\pi$ relative to the rate of inflation prevailing in other trading economies\textsuperscript{27}. The negative sign of $x_{\hat{y}}$ results from the fact that a higher $\hat{y}$ will lead to an increase in imports relative to exports other things being equal. Finally, the negative sign of $x_r$ is due to the currency real appreciation – and the resulting adverse impact on net exports - that stems from higher real interest rates. If we assume that the rest of partial derivatives in $s$ and $g$ have the same sign as in section 3, we can rewrite (56) above as:

$$s(\pi, \hat{y}, r, \varepsilon) \cdot v \cdot \bar{p} = g(\pi, \hat{y}, r, \varepsilon) + \psi + f(\hat{y}, r, \varepsilon) + x(\pi, \hat{y}, r, \varepsilon)$$

and we further need to assume that:

$$\left| -s_v v \bar{p} + g_r + x \right| > \left| f_r \right|$$

4.1.- Steady-state analysis

In steady growth we have that $\hat{y} = g_n$, $\varphi = \bar{p}$ and $\varepsilon = \varepsilon_g = \varepsilon_f = \varepsilon_r = 0$ so that (57) above can be split into:

$$g(\pi, g_n, r) = g_n$$

and

$$s(\pi, g_n, r) \cdot v \cdot \bar{p} - \psi - f(g_n, r) - x(\pi, g_n, r) = g_n$$

Hence, we have a system of two equations and two unknowns. In order to get explicit solutions for $\pi^*$ and $r^*$ we make the same simplifying assumptions as in section 3 and assume that functions $s$, $g$, $f$ and $x$ adopt a linear form. Hence, in steady growth (18b) and (59) become respectively:

$$(\bar{g} + \tilde{g}_y g_n + g_{\pi} \pi + g_r r) = g_n$$

$$((\tilde{s} + s_y g_n + s_{\pi} \pi + s_r r) \cdot v \bar{p} - \psi - (\tilde{f} + f_y g_n + f_r r) - (\tilde{x} + x_{\pi} \pi + x_y g_n + x_r r) = g_n$$

Recalling that $g_{\hat{y}} = 1$ in steady growth and solving the system made up by (21b) and (60) yields:
Looking at $\pi^*$ above we have that, since the sign of both the denominator and the numerator is ambiguous, we can not reach any conclusion as to its sign. As for $r^*$ all the considerations made for the case of a closed economy without a government sector carry over to the current context. As before, we may drop the assumption that $g_n$ is fully exogenous and hence allow for the existence of a long-run rate of inflation $\pi^d$ that is set by the CB. If we assume that $g_n$ is given by expression (22) above, insert (22) and $\pi^d$ into (60) and (21b) and rearrange, we have the steady-state neutral real interest rate is the same as in section 3 and, in addition, we have that:

$$\hat{y}^* = \frac{B_0 + B_1 g_n^{ex} + B_2 \pi^d}{B_3}$$

where

$$B_0 = \left[ \bar{s}_v \bar{\phi} - \bar{\psi} - \bar{d} - \bar{x} - \frac{(s_v \bar{v} \bar{\phi} - f_r^e - x_r) \cdot \bar{g}}{g_r} \right]$$

$$B_1 = [s_{\bar{y}} \bar{v} \bar{\phi} - f_{\bar{y}} - x_{\bar{y}} - 1]$$

$$B_2 = \left[ s_{\pi} \bar{v} \bar{\phi} - x_{\pi} - \frac{(s_v \bar{v} \bar{\phi} - f_r^e - x_r) \cdot g_{\pi}}{g_r} \right]$$

$$B_3 = \gamma \left[ - s_{\bar{y}} \bar{v} \bar{\phi} + f_{\bar{y}} + x_{\bar{y}} + 1 \right]$$

so that:

$$\frac{\partial \hat{y}^*}{\partial \pi^d} = \frac{B_2}{B_3}$$
Table 1: Monetary policy regimes in steady growth

Since the sign of \( B_2 \) and \( B_3 \) is ambiguous, and as in section 3, we have that the impact on \( \dot{y}^* \) of a variation in \( \pi^d \) is uncertain whereas an increase in the latter will lead to an increase in \( r^* \) as long as \( \pi^d > 0 \). Therefore, as in section 3 we can distinguish between an ‘inflation-led growth’ regime, a ‘disinflation-led growth’ regime and an ‘inflation-neutral growth’ regime. These results are summarized in the second and fourth column of Table 1 above.

4.2.- The behavior of the economy in the short run and the growth trap

The equilibrium condition in the goods market for an open economy with a government sector is:

\[
(\bar{s} + s_y)\dot{y} + s_y\pi + s_y r)\cdot v\phi - \bar{f} - f_y\dot{y} - f_r r - \bar{x} - x_x x - x_y y - y - r = g_x + g_y \dot{y} + g_x x + g_y r + v\psi
\]  

(62)

and, as a result, the rate of growth of output is given by:

\[
\dot{y} = \frac{-(\bar{s} v\phi - \bar{f} - \bar{x} - \bar{g} - v\psi) - (s_x v\phi - x_x - g_x)\pi - (s_y v\phi - f_y - x_y - g_y)r}{s_y v\phi - f_y - x_y - g_y}
\]  

(63)
If we assume, for consistency purposes that \( s_\gamma v\varphi - f_\gamma - x_\gamma - g_\gamma \succ 0 \) we have that, for plausible parameters:

\[
\frac{\partial \hat{y}}{\partial r} = \frac{(g_r + f_r + x_r - s_r v\varphi)}{(s_\gamma v\varphi - f_\gamma - x_\gamma - g_\gamma)} < 0 \tag{64}
\]

and

\[
\frac{\partial \hat{y}}{\partial \pi} = \frac{(g_\pi + x_\pi - s_\pi v\varphi)}{(s_\gamma v\varphi - f_\gamma - x_\gamma - g_\gamma)} \tag{65}
\]

where (65) has an ambiguous sign. The negative sign of (64) indicates that the DAD line in Figure 2 above is also downward-sloping in an open economy with a government sector. Rearranging (62), we can obtain the expression for the real rate of interest that clears the goods market when \( \hat{y} = 0 \) (corresponding to point A in Figure 2 above) and when \( \hat{y} = g_n \) or:

\[
r_{\hat{y}=0} = -\frac{(s_\gamma v\varphi - g - f - x - \varphi) - (s_\pi v\varphi - x_\pi - g_\pi)\pi}{-g_r - f_r - x_r + s_r v\varphi} \tag{66}
\]

\[
r_{g_\pi} = -\frac{(s_\gamma v\varphi - g - f - x - \varphi) - (s_\pi v\varphi - x_\pi - g_\pi)\pi - (s_\gamma v\varphi - f_\gamma - x_\gamma - g_\gamma)g_n}{-g_r - f_r - x_r + s_r v\varphi} \tag{67}
\]

where

\[
\frac{\partial r_{g_\pi}}{\partial \pi} = -\frac{(s_\pi v\varphi - x_\pi - g_\pi)}{-g_r - f_r - x_r + s_r v\varphi} \tag{68}
\]

has an ambiguous sign and hence we can not know in advance how the DAD line in figure 2 will move as the rate of inflation changes. As before, we can think of \( r_{g_\pi} \) as a pseudo-warranted real rate of interest in the sense of being the rate of interest that yields \( \hat{y} = g_n \) for a given \( \varphi, \pi \) and \( g_\gamma \). Again, the warranted real rate of interest \( r_{\varphi=\bar{\varphi}} \) will be the rate that yields \( \hat{y} = g_n \) when \( \varphi = \bar{\varphi} \) and \( g_\gamma = 1 \). Setting \( r \) equal to zero in expression (63) will yield \( y^{r=0} \) for an open economy with a government sector, i.e., a point like B in
Figure 2. Next, a favorable IS will lead the economy into a GT if expression (36) holds whereas and unfavorable DS will lead the economy into a GT if expression (38) holds. Finally, the variation in \( r^g \) following a change in \( \phi \) also has an ambiguous sign since:

\[
\frac{\partial r^g}{\partial \phi} = \frac{B_4 + B_5}{B_6}
\]

where:

\[
B_4 = (-\psi - s \pi - s \pi g_n) \cdot \nu \cdot (-g_r + s \nu \phi - f_r - \chi_r)
\]

\[
B_5 = s \nu \cdot [\psi + \bar{\pi} + \bar{\gamma} + \psi + (s \nu \phi - x_r - g_r) \pi + (s \nu \phi - f \hat{\pi} - \chi \hat{\pi} - g \hat{\pi}) g_n]
\]

and

\[
B_6 = (s \nu \phi - f \hat{\pi} - \chi \hat{\pi} - g \hat{\pi})^2
\]

<table>
<thead>
<tr>
<th>Inflation shocks</th>
<th>Aggregate demand shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed economy without government sector</td>
<td>+</td>
</tr>
<tr>
<td>Open economy with government sector</td>
<td>?</td>
</tr>
</tbody>
</table>

Note: A + means that the shock will lead to an increase in the pseudo-warranted rate whereas a ? means that the shock will have an impact of uncertain sign on the pseudo-warranted rate.

**Table 2: Impact of unfavorable ISs and favorable DSs on the pseudo-warranted real rate of interest**

4.3- The behavior of the ‘neutral’ real rate of interest and the liquidity trap

In this section we discuss the behavior of \( r^n \) in the aftermath of ISs and DSs for the case of an open economy with a government sector. We also discuss the contribution of the foreign and the government sector to the stabilizing properties of conventional monetary policy. Again, an explicit solution for \( r^n \) can be obtained by setting \( \phi = \phi \) in (62) above and rearranging. It can be easily seen that \( r^n \) continues to be a function of \( \pi \) and \( \hat{\pi} \) in an open economy with a government sector.
4.3.1.- Inflation shocks

Differentiating (57) and rearranging we have that:

\[ d\pi^n = \frac{(g_\pi + g_\pi - s_\pi v\phi) \cdot d\pi + (g_\tilde{\pi} + f_\tilde{\pi} + x_\tilde{\pi} - s_\tilde{\pi} v\phi) \cdot d\delta}{s_\pi v\phi - g_\pi - f_\pi - x_\pi} \]  \tag{70}

and if we initially assume that \( d\delta = 0 \) then (70) becomes:

\[ d\pi^n = \Pi_4 \cdot d\pi \]  \tag{71}

where \( \Pi_4 = \frac{(g_\pi + x_\pi - s_\pi v\phi)}{s_\pi v\phi - g_\pi - f_\pi - x_\pi} \gg 0 \) for plausible parameter values.

As long as (39) does not hold, the minimum size of the variation in the current real interest rate required to offset an IS is:

\[ dr \succ (r^n_0 - r_0) + \Pi_4 \cdot d\pi \]  \tag{72}

and the economy will get into a LT in the aftermath of a favorable IS if:

\[ d\pi \leq \frac{- (r^n_0 + \pi_0)}{(1 + \Pi_4)} \]  \tag{73}

If the change in the rate of inflation triggers a response by the CB in the form of a change in current real interest rates which in turn has an impact on output then we can not assume that \( d\delta = 0 \) and we have that:

\[ d\pi^n = \Pi_5 \cdot d\pi \]  \tag{74}

where \( \Pi_5 = \frac{[(g_\pi + x_\pi - s_\pi v\phi) + (g_\tilde{\pi} + f_\tilde{\pi} + x_\tilde{\pi} - s_\tilde{\pi} v\phi) \cdot \phi \cdot \delta]}{s_\pi v\phi - g_\pi - f_\pi - x_\pi} \]

and \( d\hat{\pi} = \frac{d\hat{\pi}}{dr} \cdot \frac{dr}{d\pi} \cdot d\pi \)

The considerations made in section 3 above as to the possible destabilizing role played by conventional monetary policy when \( d\hat{\pi} \neq 0 \) carry over to the current context.
Conventional monetary policy contributes to stabilizing the economy if $\Pi_5 < \Pi_4$ and is destabilizing if $\Pi_5 > \Pi_4$. Hence, when $d\hat{y} \neq 0$ expressions (72) and (73) become:

$$dr > (r_0^n - r_0) + \Pi_5 \cdot d\pi$$

(75)

and

$$d\pi \leq -\frac{(r_0^n + \pi_0)}{(1 + \Pi_5)}$$

(76)

If we look at $\Pi_4$ and $\Pi_5$ above, we can analyze the role played by the foreign and government sector in the aftermath of ISs. When $d\hat{y} = 0$ the role of the government sector is determined by the sign and magnitude of $f_r$ in the denominator of $\Pi_4$. Since $f_r$ is negative and the other terms in the denominator are positive, we have that the existence of a government sector contributes to a rise in the value of $\Pi_4$ thereby playing a destabilizing role. This is because, as inflation falls (rises) after a shock, the CB lowers (raises) current real interest rates and this leads to a fall (rise) in the budget deficit owing to lower (higher) interest payments on government debt. In turn, this translates into a fall (rise) in $r^n$. The size of $f_r$ depends positively on the amount of government debt in the hands of the private sector and negatively on the (average) length of maturity of the debt\(^{28}\). When $d\hat{y} \neq 0$, the role of the government sector is determined by the sign and magnitude of $f_r$ and $f_{\hat{y}}$ in the denominator and numerator of $\Pi_5$ respectively. In this second case, both $f_r$ and $f_{\hat{y}}$ contribute to a higher value of $\Pi_5$ thus making the government sector be clearly destabilizing. This is because the negative sign of $f_{\hat{y}}$ - which results from the regular operation of automatic stabilizers - tends to increase the value of the numerator of $\Pi_5$ as long as $\pi > 0$. Thus, in the case of ISs the government
sector plays a destabilizing role and, in addition, the latter tends to be stronger the more forceful the response of the CB is, i.e., the larger is the value of $\delta$.

As for the foreign sector, when $d\hat{y} = 0$, its role is determined by the sign and magnitude of $x_r$ and $x_\pi$ in the denominator and numerator of $\Pi_4$ respectively. Since $x_r$ and the other terms in the denominator (except $f_r$) are positive, we have that it contributes to a fall in the value of $\Pi_4$. The same result holds for $x_\pi$ in the numerator. Thus, when $d\hat{y} = 0$, the foreign sector plays unambiguously a stabilizing role. This is because as inflation falls (rises) after a shock the CB lowers (raises) current real interest rates and this leads to a currency depreciation (appreciation) in real terms which, in turn, raises (lowers) net exports and contributes to a rise (fall) in $r^n$. The negative sign of $x_\pi$ in the numerator reinforces this process. When $d\hat{y} \neq 0$, the role of the foreign sector is determined by the sign and magnitude of $x_\pi$ and $x_\pi$ in the numerator and by $x_r$ in the denominator of $\Pi_5$ respectively. In this case, the role of the foreign sector is ambiguous since the negative sign of $x_\pi$ increases (as long as $\pi > 0$) the value of $\Pi_5$. Thus, in the case of ISs the role of the foreign sector can be stabilizing or destabilizing depending on the relative values of $x_\pi$ and $x_r$ on the one hand and $x_\pi$ on the other hand. The higher the relative value of the former is vis-à-vis the value of latter the more likely it is that the CB will be able to stabilize the economy in the aftermath of ISs. Lastly, the joint operation of the foreign and government sectors will be stabilizing in the aftermath of ISs if $\Pi_4 < \Pi_0$ when $d\hat{y} = 0$ and if $\Pi_5 < \Pi_4$ when $d\hat{y} \neq 0$. These results are presented in Tables 3 and 4 below.
Finally, when the economy is in a LT $\delta$ becomes negative and the size of $\Pi_5$, other things being equal, will fall and may even become negative. However, as in the case of a closed economy without a government sector, this will not be enough to push the economy out of a LT in the absence of either unfavorable ISs and/or favorable DSs. Table 3 above summarizes the discussion on the contribution of the cyclical behavior of $r^*$ to the stabilization power of conventional monetary policy in the wake of ISs and DSs. The behavior of the former reinforces the ability of conventional monetary policy to offset any type of shocks if $\Pi_i < 0$ and vice-versa. Finally, the latter contributes to stabilizing the economy if $\Pi_5 < \Pi_4$ ($\Pi_i < \Pi_0$ for the case of a closed economy without a government sector).

<table>
<thead>
<tr>
<th>$d\hat{\gamma} = 0$</th>
<th>$\Pi_0 &lt; 0 / \Pi_0 &gt; 0^*$</th>
<th>$\Pi_4 &lt; 0 / \Pi_4 &gt; 0^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\hat{\gamma} \neq 0$</td>
<td>$\Pi_i &lt; 0 / \Pi_i &gt; 0$</td>
<td>$\Pi_5 &lt; 0 / \Pi_5 &gt; 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>CMP contributes to stabilization if</th>
<th>CMP contributes to stabilization if</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Pi_i &lt; \Pi_0$</td>
<td>$\Pi_i &lt; \Pi_4$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$d\pi / d\hat{\gamma} = 0$</th>
<th>$\Pi_2 &lt; 0 / \Pi_2 &gt; 0^*$</th>
<th>$\Pi_6 &lt; 0 / \Pi_6 &gt; 0^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\pi / d\hat{\gamma} \neq 0$</td>
<td>$\Pi_3 &lt; 0 / \Pi_3 &gt; 0$</td>
<td>$\Pi_7 &lt; 0 / \Pi_7 &gt; 0$</td>
</tr>
</tbody>
</table>

Note: The position of the asterisk * highlights either the actual or likely sign of $\Pi_i$ for plausible parameter values. The absence of an asterisk means that the sign of $\Pi_i$ is ambiguous. CMP is the abbreviation for conventional monetary policy.

Table 3: Contribution of the ‘neutral’ real interest rate to the stabilization capacity of conventional monetary policy in the aftermath of ISs and DSs
4.3.2.- Aggregate demand shocks

In an open economy with a government sector, and in addition to shocks hitting the saving ratio and the rate of accumulation, shocks may potentially arise from the foreign sector as well as from discretionary fiscal policy. However, as in the case of a closed economy without government sector, we assume that the economy is hit by a shock to the rate of capital accumulation, \( g \), albeit results are qualitatively similar if the source of the shock is different. If we differentiate (57) above and rearrange, we have that the change in the ‘neutral’ real rate of interest is:

\[
d r^n = \frac{\left( g_x + x_{\pi} - s_{\pi} v_{\phi} \right) \cdot d \pi + \left[ \left( g_{\ddot{y}} + f_{\ddot{y}} + x_{\ddot{y}} - s_{\ddot{y}} v_{\phi} \right) \cdot \frac{d \hat{y}}{dg} + 1 \right] \cdot g_{\varepsilon} \cdot \varepsilon_{g}}{s_{r} v_{\phi} - g_{r} - f_{r} - x_{r}}
\]  

(77)

If we initially assume that \( d \pi / d \hat{y} = 0 \), we obtain:

\[
d r^n = \Pi_6 \cdot \varepsilon_{g}
\]

(78)

where \( \Pi_6 = \frac{\left[ \left( g_{\ddot{y}} + f_{\ddot{y}} + x_{\ddot{y}} - s_{\ddot{y}} v_{\phi} \right) \cdot \frac{d \hat{y}}{dg} + 1 \right] \cdot g_{\varepsilon}}{s_{r} v_{\phi} - g_{r} - f_{r} - x_{r}} \) > 0 for plausible parameter values.

If condition (39) does not hold, the minimum size of the change in the current real interest rate required to offset a shock to \( g \) is:

\[
dr \succ (r^n_0 - r_0) + \Pi_6 \cdot \varepsilon_{g}
\]

(79)

and the economy will get into a LT following an adverse DS (\( \varepsilon_{g} \prec 0 \)) if:

\[
\varepsilon_{g} \leq \frac{- (r^n_0 + \pi_0)}{\Pi_6}
\]

(80)

Finally, when \( d \pi / d \hat{y} \neq 0 \) the variation in \( r^n \) following a shock to \( g \) is:

\[
d r^n = \Pi_7 \cdot \varepsilon_{g}
\]

(81)
where $\Pi_7 = \left( \frac{\left( g_x + x_r - s_x v\bar{\phi} \right) \cdot \frac{d\pi}{d\bar{y}} + \left( g_{\tilde{y}} + f_{\tilde{y}} + x_{\tilde{y}} - s_{\tilde{y}} v\bar{\phi} \right) \cdot \frac{d\bar{y}}{dg} + 1 \right) \cdot g_x}{s_r v\bar{\phi} - g_r - f_r - x_r}$

has an ambiguous sign. Hence, expressions (80) and (81) then become:

$$dr > (r_0^\delta - r_0) + \Pi_7 \cdot \epsilon_g$$  \hspace{1cm} (82)

and

$$\epsilon_g \leq - \frac{(r_0^\delta + \pi_0)}{\Pi_7}$$  \hspace{1cm} (83)

If we look at $\Pi_6$ and $\Pi_7$ above, we can analyze the role played by the foreign and government sectors in the aftermath of DSs. As for the government sector, its role is determined by the sign and magnitude of $f_r$ and $f_{\tilde{y}}$ in the denominator and numerator of $\Pi_7$ respectively. In the case of $f_r$ the comments in the previous section carry over to this section. Conversely, the negative sign of $f_{\tilde{y}}$ tends to reduce the size of $\Pi_7$ thereby exerting a stabilizing role through the operation of automatic stabilizers. In this case, the role of the government sector depends on the relative size of $f_{\tilde{y}}$ vis-à-vis $f_r$. Thus, one further result is that the stabilizing role of the government sector in the aftermath of DSs depends positively on the relative strength of automatic stabilizers and the average length of maturity of the government’s debt – for reasons explained above - and negatively on the amount of government debt in the hands of the private sector.

As for the foreign sector, when $d\pi / d\bar{y} = 0$ its role is determined by the sign and magnitude of $x_r$ and $x_{\tilde{y}}$ in the denominator and numerator of $\Pi_6$ respectively. Since $x_r$ and the other terms in the denominator except $f_r$ are positive, we have that it contributes to a fall in the value of $\Pi_6$. The same result holds for $x_{\tilde{y}}$ in the numerator. This result is reinforced when $d\pi / d\bar{y} > 0$ and is (at least partially) offset when $d\pi / d\bar{y} < 0$ owing to
the negative sign of $x_\pi$ in the numerator of $\Pi_7$. Hence, in general, the foreign sector plays a stabilizing role in the case of DSs. Finally, the joint operation of the foreign and government sectors is stabilizing in the aftermath of DSs if $\Pi_0 < \Pi_2$ when $d\pi / d\hat{y} = 0$ and if $\Pi_7 < \Pi_3$ when $d\pi / d\hat{y} \neq 0$. These results are summarized in Tables 3 and 4.

<table>
<thead>
<tr>
<th>Inflation shocks</th>
<th>Aggregate demand shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\hat{y} = 0$</td>
<td>$d\pi / d\hat{y} = 0$</td>
</tr>
<tr>
<td>FS is stabilizing</td>
<td>FS is stabilizing</td>
</tr>
<tr>
<td>GS is destabilizing</td>
<td>GS’s role is ambiguous</td>
</tr>
<tr>
<td>($\Pi_4 &lt; \Pi_0$)</td>
<td>($\Pi_6 &lt; \Pi_2$)</td>
</tr>
<tr>
<td>$d\hat{y} \neq 0$</td>
<td>$d\pi / d\hat{y} \neq 0$</td>
</tr>
<tr>
<td>GS is destabilizing</td>
<td>FS is stabilizing</td>
</tr>
<tr>
<td>FS’s role is ambiguous</td>
<td>GS’s role is ambiguous</td>
</tr>
<tr>
<td>($\Pi_6 &lt; \Pi_4$)</td>
<td>($\Pi_7 &lt; \Pi_3$)</td>
</tr>
</tbody>
</table>

Note: FS and GS are respectively abbreviations for foreign and government sector. The terms in parentheses show, for each scenario, the condition that will hold should FS and GS jointly play a stabilizing role in the economy.

Table 4: Contribution of the foreign and government sector to monetary policy’s stabilization power when $\pi > 0$

5.- Monetary policy regimes: an attempt at generality

The analysis of growth and liquidity traps performed in sections 3 and 4 above yields some insights about the different situations policy-makers may face. We identify four regimes. Their basic features are shown in Table 5 below. The first column in Table 5 shows the formal condition that denotes each regime as regards the value of $r^*$ and $r^s*$ vis-à-vis the value of $\omega$. The second and third columns indicate the trap or traps the economy has got into each regime if it has got into any trap at all. Finally, the last column contains some comments as to the nature of each regime. We explain each regime below.

The dynamics of the economy in the first two regimes are complex and depend on the particular values that model parameters take. Thus we do not claim generality for the stories we tell below. Rather, we will attempt to describe the dynamic processes in a
succinct way. The first regime corresponds to a situation where the economy is in a GT but not in a LT. The former means that \( \hat{y} < g_n \) whereas the latter implies that the CB can initially set interest rates low enough so as to yield a positive output-gap and, therefore, a rising rate of inflation. If the CB acts accordingly, inflation will rise. In turn, this will push \( r^g \) either upwards or downwards\(^{29}\). However as long as \( r^g < \omega \) the (positive) output-gap will gradually narrow and, as a result of it, the rate of inflation will tend to decelerate. Furthermore, as inflation keeps rising, and as long as \( \Pi_3 > 0 \), the neutral interest rate \( r^n \) will rise eventually allowing the CB to fuel if it keeps real interest rates down. Yet, the behavior of \( r^n \) is also uncertain as the ambiguous sign of \( \Pi_3 \) highlights.

If \( r^g \) rises fast enough and the (negative) growth-gap \( \hat{y} - g_n \) is closed before \( r^n < \omega \), then the economy will abandon the GT. Otherwise, the economy will fall into a LT before escaping the GT and will thus end up in the worst possible scenario. Hence, in this first regime, the higher are \( \partial r^n / \partial \pi \), \( \partial r^g / \partial \pi \) and upward price and wage flexibility the more likely it is that the economy will escape the traps. The monetary policy implication that emerges from this first regime is that the CB should set interest rates as low as possible in order to increase the chances that the economy avoids falling into the traps. Notwithstanding, under an inflation-targeting strategy, the CB will tend to keep interest rates up as long as there is a positive output-gap and current inflation is above target inflation. Finally, this first regime is transitory since the economy will inevitably escape or fall into the traps.

The second regime corresponds to a situation where the economy is in a LT but not in a GT. The former means that \( \hat{\pi} < 0 \) whereas the latter implies that the CB can set
interest rates low enough so as to yield a rate of growth of output \( \dot{y} > g_n \). If it happens that \( \partial r^g_\ast / \partial \pi > 0 \) then, as inflation falls, \( r^g_\ast \) will also fall thus gradually narrowing the (positive) growth-gap. The opposite will be true if \( \partial r^g_\ast / \partial \pi < 0 \). Likewise, a falling rate of inflation will push \( r^n \) downwards (upwards) if \( \Pi_5 > 0 (\Pi_5 < 0) \). If \( r^n \) falls, this will speed up the deflationary process. As a result, if \( r^g_\ast \) rises or does not fall fast enough so as to allow the (negative) output-gap to be closed before \( r^g_\ast < \omega \), then the economy will escape the LT. Otherwise, the economy will fall into a GT before escaping the LT. Hence, in this second regime, the lower are \( \partial r^n / \partial \pi \) and \( \partial r^g_\ast / \partial \pi \) - if they happen to be positive - the more likely it is that the economy escapes the traps. Conversely, if \( \partial r^n / \partial \pi \) and \( \partial r^g_\ast / \partial \pi \) happen to be negative, the opposite is true. As opposed to the first regime, the lower is downward price and wage flexibility, the more likely it is that the economy escapes the traps. Nevertheless, the monetary policy implication that results from this state of affairs is that the CB should keep interest rates down in order to retard the fall in the rate of inflation and the narrowing of the (positive) growth-gap. Finally, this second regime is also transitory.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Liquidity trap</th>
<th>Growth trap</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r^n &gt; \omega &gt; r^g_\ast )</td>
<td>No</td>
<td>Yes</td>
<td>Transitory regime</td>
</tr>
<tr>
<td>( r^g_\ast &gt; \omega &gt; r^n )</td>
<td>Yes</td>
<td>No</td>
<td>Transitory regime</td>
</tr>
<tr>
<td>( r^g_\ast &lt; \omega, r^n &lt; \omega )</td>
<td>Yes</td>
<td>Yes</td>
<td>Deflation + Stagnation</td>
</tr>
<tr>
<td>( r^g_\ast &gt; \omega, r^n &gt; \omega )</td>
<td>No</td>
<td>No</td>
<td>Tranquil regime</td>
</tr>
</tbody>
</table>

Table 5: Monetary policy regimes
The third regime corresponds to a situation where $p^8 < \omega$ and $p^\pi < \omega$ so that the economy finds itself in both a LT and a GT. As a result, it will exhibit stagnant growth as well as a falling (and possibly negative) rate of inflation. No wonder this is the worst possible scenario that a CB can face. This regime captures well the features exhibited by the Japanese economy in the last decade, i.e., deflation, stagnant growth and, arguably, a large negative output-gap$^{30}$. Finally, the last regime corresponds to a situation of tranquility in which the CB can set interest rates at the level required to yield a rate of growth of aggregate demand that matches an expanding potential output at the desired rate of inflation.

6.- Conclusion

Modern macroeconomic analysis has tended to skip the question through which mechanism a growing potential output level generates an equi-proportional increase in aggregate demand in the long run. Conventional stories rely on some kind of real balance effect to explain it. Yet this mechanism faces a well-known number of shortcomings notably its negligible size and, even worse, its uncertain sign. Alternative mechanisms are available but also tend to be unreliable. This study has presented a general theoretical framework for the analysis of the ability of conventional monetary policy to take the economy down its potential output path in the absence of any self-regulating mechanism other than the (modest) stabilization provided by the foreign sector and fiscal automatic stabilizers. For that purpose, we developed a simple theoretical model that explains the behavior of the ‘neutral’ interest rate and the ‘pseudo-warranted’ interest rate in the wake of different types of shocks. We defined the latter as the real interest rate that yields a rate
of growth of output equal to its ‘natural’ rate for a given level of capacity utilization and rate of inflation. The model covered both the case of a closed economy without a government sector and the case of an open economy with a government sector.

We identified several different scenarios according to whether the short-run behavior of the ‘neutral’ interest rate enhances or weakens the stabilization power of conventional monetary policy and we determined under what conditions conventional monetary policy may be destabilizing. The analysis of the steady-state properties of the model allowed us to identify several regimes depending on whether a rise in the target rate of inflation yields a faster or slower rate of growth of output when the ‘natural’ rate of growth of output is not fully exogenous. The model also allowed us to discuss the contribution of the foreign and government sector to the effectiveness of conventional monetary policy. In addition, we provided a formal definition of the notion of the liquidity trap and coined a complementary concept: the growth trap. Finally, and based on the previous concepts, we proposed a taxonomy of monetary policy regimes. Perhaps the most important question that we need to answer is how likely it is that conventional monetary policy can not yield a rate of growth of output along its potential path. This study has no answer for that question. We hope that the framework provided here and a good deal of empirical work will help answer that question in the future.

References


Freedman, C. (2000); ‘Comment on Overcoming the Zero Lower Bound on Interest Rate Policy’, Journal of Money, Credit and Banking, 32(4), November, pp. 1051-1057.


Laubach, T. and J.C. Williams (2001); ‘Measuring the Natural Rate of Interest’, Board of Governors of the Federal Reserve System, November.


Skott, P. (1989); *Conflict and effective demand in economic growth*, Cambridge: Cambridge University Press.


---

* This study was carried out while the author was a Visiting Scholar at the Political Economy Research Institute (PERI) at the University of Massachusetts, Amherst from October 2004 to January 2005. The author wishes to thank all members of PERI for providing an excellent research atmosphere. He owes special thanks to Gerald Epstein and Judy Fogg. This research also benefited from financial support without which it might not have been possible. In particular, the research project received a grant from the Spanish Ministry of Education, Culture and Sport through the ‘Programa Nacional de Ayudas para la Movilidad de Profesores de Universidad’ (Reference: PR-2004-0401).

1 Notwithstanding its survival in macroeconomic theory, the real balance effect is missing from the FRB/US model.
The classical exposition of the unreliability of the real balance effect as a self-adjustment mechanism is in Tobin (1975). A recent discussion on the shortcomings of this effect can be found in Palacio-Vera (2005) and a lengthy analysis of the consequences of deflation is in Palley (2004). In contrast, some authors are supportive of the power of the real balance effect to stabilize the economy. For instance, Sims (2000) argues that there is a potential for a large real balance effect in a deflationary environment and that its presence ‘makes it extremely unlikely that we get into a liquidity trap’.

Nevertheless he shows that the economy exhibits an unstable steady growth path albeit one that does not need to lead to cumulative divergence but may instead lead to cyclical fluctuations.

For instance, in the study by Laubach and Williams (2001), the ‘natural’ real rate of interest is estimated using quarterly U.S. data over the period 1961 to 2000. In all the model specifications displayed, the authors find substantial variations in its estimated value throughout that period. For instance, in the baseline specification, the minimum value of the natural interest rate is found to be as low as 1.28 per cent whereas the maximum value is found to be 4.52 per cent. In addition, the authors concede there is sizeable uncertainty around most of their estimates of the natural rate of interest, the trend growth rate and potential output so the actual variation could be much larger.

As argued in Sawyer (2001, p. 238), ‘the overall effect on realized real wages clearly depends on the relative size of the wage inflation and price inflation (and could depend on the responsiveness of wages and prices to unemployment and capacity utilization respectively and on the accuracy of wage and price expectation formation).’

A node is a singular point such that all integral curves pass through it (Gandolfo, 1997, p. 348).

In particular, the complete expression for B.6 is:

$$\frac{\partial f}{\partial \omega} - \frac{\partial g}{\partial P} \frac{\partial f}{\partial \omega} \frac{\partial g}{\partial P} = \frac{(1 - \gamma)}{(1 - \gamma \cdot a \cdot P)} \frac{\partial Y}{\partial Q} \frac{\partial X}{\partial N} \frac{\partial N}{\partial \omega} \frac{\partial N}{\partial \omega}$$

where $\gamma < 1$, $\gamma \alpha < 1$, $\partial Y / \partial Q > 0$, $\partial Q / \partial P < 0$, $\partial X / \partial N > 0$ and the sign of $\partial N / \partial \omega$ depends on the slope of the p-curve in the neighborhood of equilibrium point $b$.

A saddle point is a singular point through which only two integral curves pass, which are asymptotes to all remaining integral curves. Whatever the direction of the movement along an integral curve, the motion is always away from the equilibrium point except for the one along one of the two asymptotes, which is called the stable arm of the saddle (Gandolfo, 1997, p. 348).

Although not explicitly referred to as a LT situation, a brief discussion of this very same problem in the context of a simple model capturing the basic features of the so-called ‘New Consensus View’ in macroeconomics is undertaken in Arestis and Sawyer (2005). The authors conclude that CBs are unlikely to be able to offset a large adverse shock to investment demand through nominal interest rate cuts.

For this second group of authors, the words pronounced by Kazuo Ueda at the 1999 JMCB Conference in Woodstock (Vermont) may serve to summarize their position: ‘Don’t put yourself in the position of zero interest rates. You’ll have to face a lot of difficulties. I can tell you it will be a lot more painful than you can possibly imagine’ (Ueda, 2000, p.1109).

We assume firms use excess capacity to deter entry in an industry by making it unprofitable to potential new entrants (Spence, 1977).

We implicitly assume that $\varphi = \overline{\varphi}$ when $Y = Y^\varphi$.

It is not possible to say a priori how the debt service in real terms will vary since this ultimately depends on the behavior of real interest rates. However, our argument only applies to the impact on aggregate demand of changes in the rate of inflation for a given real interest rate.
This scenario changes considerably when inflation becomes negative. In a deflationary environment the redistribution of net worth away from net borrowers and towards net lenders will be larger and, in addition, there will be a redistribution of income in the same direction stemming from high (and possibly rising) real interest rates.

Let us note that households are the ultimate owners of firms.

As for the first result, our model suggests that factors like the ‘new economy’ do not affect, at least directly, to the steady-state neutral real rate of interest. As for the second result, one element embedded in the exogenous component of $g$ may be the process of financialisation experienced by some OECD economies over the past decades. Financialisation is the term coined to refer to the shareholder revolution and the development of a market for corporate control which has shifted power to shareholders and changed management priorities leading to a reduction in the desired growth rate (see Stockhammer, 2004).

Alternatively we may assume that the rate of growth of labor force is positively affected by the rate of growth of output through its effect on participation rates and/or the pace of immigration.

See, for instance, the excellent presentation of this issue in Kohn (1981).

In particular see expression 68 below.

If we think of $r$ as a medium-term interest rate, the minimum nominal interest rate the CB can set will be positive. This is because investors require a (time-varying) liquidity premium to purchase longer term financial assets. In turn, this suggests that considerations of liquidity preference may affect the likelihood the economy falls into a LT. In particular, expression (34) below would adopt the more general form

$$r > \pi < \mu$$

where $\mu > 0$ is determined by the liquidity premium required by investors.

By an inflation shock we mean a one-time shock that affects the price level without affecting directly either the level or the rate of growth of potential output. In turn, the price level shock leads to a change in the current rate of inflation owing to the operation of adaptive expectations.

See comment in endnote 20 above.

However some authors have attributed the recent woes of the Japanese economy to the existence of a liquidity trap (Krugman, 1998).

Krugman (1998) argues that if a CB can credibly commit to pursue inflation and ratify inflation when it comes, it should be able to increase inflationary expectations despite the absence of any traction on the economy by means of conventional monetary policy. Eggertsson and Woodford (2003) present a more fully dynamic analysis of the problem. To them, a commitment to create subsequent inflation is presented as a commitment to keep interest rates low for some time in the future.

Admittedly this is not a precise description of the behavior of short-term interest rates. This is because, for a given change in the rate of inflation, the postulated change in the real interest rate will depend on the initial value of the nominal interest rate being above the (zero) lower bound when inflation is positive but right at the lower bound when inflation is not positive. We are leaving aside those situations when inflation is positive but nominal interest rates are already at the zero lower bound as well as those ones when the rate of inflation is negative but nominal interest rates are still above the zero lower bound. In addition, under an inflation-targeting approach the CB will tend to raise interest rates when current inflation is above target and vice-versa. In order to make (44) fit this behavior we need to further assume that the initial rate of inflation equals target inflation when the rate of inflation is positive.
26 We may think of this case and similar ones as the case relevant for a time span long enough to allow the CB to observe the IS, adopt the appropriate policy response and let it have an impact on current output.

27 We implicitly assume that purchasing power parity theory does not hold in the short run and probably either in the long run.

28 This is because changes in (overnight) nominal interest rates induced by the CB tend to have a larger impact on short and medium-term rates than on long-term rates.

29 It will push it upwards in a closed economy without a government sector but the sign of the effect is ambiguous for an open economy with a government sector.

30 Krugman (1998) argues that the negative output-gap exhibited by the Japanese economy in the late 1990s is largely underestimated in official statistics. He adds that it could be as large as 8 per cent of GDP.