Currency Hedging Strategies Using Dynamic Multivariate GARCH

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Abstract

This paper examines the effect on the effectiveness of using futures contracts as hedging instruments of: 1) the model of volatility used to estimate conditional variances and covariances, 2) the analyzed currency, and 3) the maturity of the futures contract being used. For this purpose, daily data of futures and spot exchange rates of three currencies, Euro, British pound and Japanese yen, against the American dollar are used to analyze hedge ratios and hedging effectiveness resulting from using two different maturity currency contracts, near-month and next-to-near-month contract. Following Tansuchat, Chang and McAleer (2010), we estimate four multivariate volatility models (CCC, VARMA-AGARCH, DCC and BEKK) and calculate optimal portfolio weights and optimal hedge ratios to identify appropriate currency hedging strategies. Hedging effectiveness index suggests that the best results in terms of reducing the variance of the portfolio are for the USD/GBP exchange rate. The results show that futures hedging strategies are slightly more effective when the near-month future contract is used for the USD/GBP and USD/JPY currencies. Moreover, CCC and AGARCH models provide similar hedging effectiveness although some differences appear when the DCC and BEKK models are used.

Keywords: Multivariate GARCH, conditional correlations, exchange rates, optimal hedge ratio, optimal portfolio weights, hedging strategies.

JEL Classifications: G32, G11, G17, C53, C22.

1. Introduction

With the rise of the capital market liberalization and globalization, foreign currency denominated assets circulate rapidly in the world. With increasing internationalization of financial transactions, the foreign exchange market has been profoundly transformed and became more competitive and volatile. This places the accurate and reliable measurement of market risks in a crucial position for both investment decision and hedging strategy designs.

Foreign exchange rate markets are the largest and most liquid of all asset markets. Developments in these markets influence national trade and monetary policies and the competitiveness of nations. Foreign exchange markets are also important for the increasing number of companies engaged in cross-border trade and investment. The foreign exchange business is naturally risky, because it deals primarily in measuring, pricing, and managing risk. The success of an institution trading in the foreign exchange market depends critically on how well it assesses prices and manages the inherent risk, on its ability to limit losses from particular transactions, and to keep its overall exposure under control.

The fact in managing currency risk is to control the volatility of the portfolio. The volatility of a portfolio includes variances and correlation coefficients of, and among, individual positions. Great losses may be yielded from holding this portfolio without a time-varying consideration of its variance and correlation parts simultaneously. If investors can sense the interacting dynamics among markets in advance, then adjusting and hedging activities will be implemented in time. Successful and profitable performances can therefore be made.

The aim of hedging is to use derivatives to reduce a particular risk. A relatively inexpensive and reliable strategy for hedging foreign exchange risk involves the use of foreign currency futures markets. Hedging with futures contracts is perhaps the simplest method for managing market risk arising from adverse movements in the foreign exchange market. Hedgers usually short an amount of futures contracts if they hold the long position of the underlying currency and vice versa. The question is how many futures contracts should be held for each unit of the underlying currency, as well as the effectiveness measure of that ratio. The hedge ratio provides information on how many futures contracts should be held, whereas its effectiveness evaluates the hedging performance and the usefulness of the strategy. In addition, hedgers may use the effectiveness measure to compare the benefits of hedging a given position from many alternative contracts.

Generally speaking, when the market trend is stable, the hedge ratio will become smaller, whereas if a big fluctuation of the market takes place it will get bigger. Several distinct approaches have been developed to estimate the optimal hedge ratio (OHR), also known as the minimum-variance hedge ratio.

The static hedging model with futures contracts (Johnson [32], Stein [56], Ederington, [21]) assumes that the joint distribution of spot and futures returns is time-invariant and therefore the OHR, defined as the optimal number of futures holdings per unit of spot holdings, is constant over time. The minimum variance OHR use to be derived form the ordinary-least squares (OLS) regression of spot price changes on future price changes. There is wide evidence that the simple OLS method is inappropriate to estimate hedge ratios since it suffers from the problem of serial correlation in the OLS residuals and the heteroscedasticity often encountered in spot and futures price series (Herbst et al. [31]).

Therefore, the underlying assumption of the static hedging model of time-invariant asset distributions has been changed. The Autoregressive Conditional Heteroscedastic (ARCH) framework of Engle [23] and its extension to a generalized ARCH (GARCH) structure by Bollerslev [8] have proven to be very successful in modelling asset price second-moment movements. Bollerslev [9], Bailie and Bollerslev [3], and Diebold [19] have shown that the GARCH (1,1) model is effective in explaining the distribution of exchange rate changes. However, Lien et al. [39] compared Ordinary Least Squares (OLS) and constant-correlation vector generalized autoregressive conditional heteroscedasticity (VGARCH) and claimed that the Ordinary List Squares (OLS) hedge ratio performs better than the VGARCH one. CHAN [16] proposed a dynamic hedging strategy based on a bivariate GARCH-jump model augmented with autoregressive jump intensity to manage currency risk. The collective evidence shows that the GARCH-modelled dynamic hedging strategies are empirically appropriate but the risk-reduction improvements over constant hedges vary across markets and may be sensitive to the sample period employed in the analysis.

Regarding foreign currencies, different results are provided. Kroner and Sultan [35] demonstrated that GARCH hedge ratios produce better hedging effectiveness than conventional hedge ratios in currency markets. Chakraborty and Barkoulas [15] employed a bivariate GARCH model to estimate the joint distribution of spot and futures currency returns and they constructed the sequence of dynamic (time-varying) OHRs based upon the estimated parameters of the conditional covariance matrix. The empirical evidence strongly supports time-varying OHRs but the dynamic model provides superior out-of-sample hedging performance, compared to the static model, only for the Canadian dollar. Ku et al. [37] applied the dynamic conditional correlation (DCC)-GARCH model of Engle [24] with error correction terms to investigate the optimal hedge ratios of British and Japanese currency futures markets and compare the DCC-GARCH and OLS model. Results show that the dynamic conditional correlation model yields the best hedging performance.

Given the distinct theoretical advantages of the dynamic hedging method over the static one, a great number of studies have employed the multivariate GARCH framework to examine its hedging performance for various assets. To evaluate the impact of model specification on the forecast of conditional correlations, Hakim and McAleer [29] analyze whether multivariate GARCH models incorporating volatility spillovers and asymmetric effect of negative and positive shocks on the conditional variance provide different conditional correlations forecasts. Using three multivariate GARCH models, namely the CCC model (Bollerslev, [10]), VARMA-GARCH model (Ling and McAleer, [41]), and VARMA-AGARCH model (McAleer et.al., [45]) they forecast conditional correlations between three classes of international financial assets (stock, bond and foreign exchange rates). The paper suggested that incorporating volatility spillovers and asymmetric of negative and positive shocks on the conditional variance does not affect forecasting conditional correlations.

To estimate time-varying hedge ratios using multivariate conditional volatility models Chang et.al. [17] examinned the performance of four models (CCC, VARMA-GARCH, DCC and BEKK) for the crude oil spot and futures returns of two major international crude oil markets (BRENT and WTI). The calculated OHRs form each multivariate conditional volatility model presented the time-varying hedge ratios and recommended to short in crude oil futures, with a high portion of one dollar long in crude oil spot. The hedging effectiveness indicated that DCC (BEKK) was the best (worst) model for OHR calculation in terms of the variance of portfolio reduction.

This paper extends Chang et.al. [17] to currency hedging. To evaluate the impact of model specification on conditional correlations forecasts, this paper calculates and compares the correlations between conditional correlations forecasts resulted from four different multivariate models (CCC, VARMA-AGARCH, DCC and BEKK) to estimate the returns on spot and futures (analyzing two sets of futures depending on their maturity) of three currency prices (USD/GBP, USD/EUR and USD/JPY). The purpose is to calculate the optimal portfolio weights and OHRs ratio from the conditional covariance matrices in order to achieve an optimal portfolio design and hedging strategy, and to compare the performance of OHRs from estimated multivariate conditional volatility models by applying the hedging effectiveness index. One of the main contributions of this study is that allows us to compare whether the results are different depending on the volatility model, currency and maturity of the futures contract selected. We have found no evidence of these three items considered together for currency hedging in prior literature.

The remainder of the paper is organized as follows. In Section 2 we discuss the multivariate GARCH models used, and the derivation of the OHR and hedging effective index. In section 3 the data used for estimation and forecasting and the descriptive statistics are presented. Section 4 analyses the empirical estimates from empirical modeling. Section 5 presents some conclusions.

2. Econometric Models

2.1. Multivariate Conditional Volatility Models

Following Chang et.al. [17] this paper considers the CCC model of Bollerslev [10], VARMA-AGARCH model of McAleer et al. [45], the DCC model of Engle [24] and BEKK model of Engle and Kroner [25]. Constant conditional correlations are assumed in the first two models while dynamic conditional correlations are taken in the last two models.

Considering the CCC multivariate GARCH model of Bollerslev [10]:

$$y_1 = E\left(y_t / F_{t-1}\right) + \varepsilon_t, \varepsilon_t = D_t \eta_t$$
$$\text{var}\left(\varepsilon_t / F_{t-1}\right) = D_t \Gamma D_t \quad \textbf{(1)}$$

Where $y_t = \left(y_{1t}, ..., y_{mt}\right)'$, $\eta_t = \left(\eta_{1t}, ..., \eta_{mt}\right)'$ is a sequence of independent and identically distributed random vectors, F_t is the past information available at time *t*, $D_t = diag\left(h_1^{1/2}, ..., h_m^{1/2}\right)$, *m* is the number of assets (see, for example, McAleer [43] and Bauwens et al. [6]). As $\Gamma = E\left(\eta_t \eta_t' / F_{t-1}\right) = E\left(\eta_t \eta'\right)$, where $\Gamma = \left\{\rho_{ij}\right\}$ for *i, j* = 1,…, *m*, the constant conditional correlation matrix of the unconditional shocks, η_t, is equivalent to the constant conditional covariance matrix of the conditional shocks, ε_t, from (1), $\varepsilon_t \varepsilon_t' = D_t \eta_t \eta_t' D_t, D_t = \left(diag Q_t\right)^{1/2}$, and $E\left(\varepsilon_t \varepsilon_t' / F_{t-1}\right) = Q_t = D_t \Gamma D_t$, where Q_t is the conditional covariance matrix.

The CCC model of Bollerslev [10] assumes that the conditional variance for each return, h_{it}, *i* =1, …, *m*, follows a univariate GARCH process, that is

$$h_{it} = \omega_i + \sum_{j=1}^{r} \alpha_{ij} \varepsilon_{i,t-j}^2 + \sum_{j=1}^{s} \beta_{ij} h_{i,t-j}, \quad \textbf{(2)}$$

where α_{ij} represents the ARCH effect, or short run persistence of shocks to return *i*, β_{ij} represents the GARCH effect, and $\sum_{j=1}^{r} \alpha_{ij} + \sum_{j=1}^{s} \beta_{ij}$ denotes the long run persistence.

The CCC model assumes that negative and positive shocks of equal magnitude have identical impacts on the conditional variance. McAleer et al. [45] extended the VARMA-GARCH to accommodate the asymmetric impacts of the unconditional shocks on the conditional variance, and proposed the VARMA-AGARCH specification of the conditional variance as follows:

$$H_t = W + \sum_{i=1}^{r} A_i \vec{\varepsilon}_{t-i} + \sum_{i=1}^{r} C_i I_{t-i} \vec{\varepsilon}_{t-i} + \sum_{j=1}^{s} B_j H_{t-j,} \quad \textbf{(3)}$$

Where C_i are $m \times m$ matrices for $i = 1,...r$ with typical element γ_{ij}, and $I_t = diag\left(I_{1t},...,I_{mt}\right)$, is an indictor function, given as

$$I\left(\eta_{it}\right) = \begin{pmatrix} 0, & \varepsilon_{it} > 0 \\ 1, & \varepsilon_{it} \leq 0 \end{pmatrix} \quad \textbf{(4)}$$

If m=1 (3) collapses to the asymmetric GARCH (or GJR) model of Glosten et al. [28]. If C_i = 0 and A_i and B_j are diagonal matrices for all i and j, then VARMA-AGARCH reduces to the CCC model. The structural and statistical properties of the model, including necessary and sufficient conditions for stationarity and ergodicity of VARMA-AGARCH, are explained in detail in McAleer et al. [45]. The parameters of model (1) to (3) are obtained by maximum likelihood estimation (MLE) using joint normal. We also estimate the models using Student's t distribution, in this case the appropriate estimator is QMLE.

The assumption that the conditional correlations are constant may seen unrealistic so, in order to make the conditional correlation matrix time dependent, Engle [24] proposed a dynamic conditional correlation (DCC) model, which is defined as

$$y_t \left| \Im_{t-1} \sim (0, Q_t), t = 1, 2, ..., n \right. \quad \textbf{(5)}$$

$$Q_t = D_t \Gamma_t D_{t,} \quad \textbf{(6)}$$

where $D_t = diag\left(h_1^{1/2},...,h_m^{1/2}\right)$ is a diagonal matrix of conditional variances, and $\Im_t$ is the information set available at time t. The conditional variance, h_{it}, can be defined as a univariate GARCH model, as follows:

$$h_{it} = \omega + \sum_{k=1}^{p} \alpha_{ik} \varepsilon_{i,t-k} + \sum_{l=1}^{q} \beta_{il} h_{i,t-l} \quad \textbf{(7)}$$

If η_t is a vector of i.i.d. random variables, with zero mean and unit variance, Q_t in (8) is the conditional covariance matrix (after standardization, $\eta_{it} = y_{it} / \sqrt{h_{it}}$). The η_{it} are used to estimate the dynamic conditional correlations, as follows:

$$\Gamma_t = \left\{(diag(Q_t)^{-1/2}\right\} Q_t \left\{(diag(Q_t)^{-1/2}\right\} \quad \textbf{(8)}$$

where the $k \times k$ symmetric positive definitive matrix Q_t is given by

$$Q_t = (1-\theta_1-\theta_2)\bar{Q} + \theta_1 \eta_{t-1}\eta'_{t-1} + \theta_2 Q_{t-1}, \quad \textbf{(9)}$$

in which θ_1 and θ_2 are scalar parameters to capture the effects of previous shocks and previous dynamic conditional correlations on the current dynamic conditional correlation, and θ_1 and θ_2 are non-negative scalar parameters. When $\theta_1 = \theta_2 = 0, \bar{Q}$ in (9) is equivalent to CCC. As Q_t is conditional on the vector of standardized residuals, (9) is a conditional covariance matrix, and $\bar{Q}$ is the $k \times k$ unconditional variance matrix of η_t. DCC is not linear, but may be estimated simply using a two-step method based on the likelihood function, the first step being a series of univariate GARCH estimates and the second step being the correlation estimates.

An alternative dynamic conditional model is BEKK, which has the attractive property that the conditional covariance matrices are positive definite. However, BEKK suffers from the so-called “curse of dimensionality” (see McAleer et al. [45] for a comparison of the number of parameters in various multivariate conditional volatility models). The BEKK model for multivariate GARCH (1,1) is given as:

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B, \quad \textbf{(10)}$$

Where the individual element for the matrices *C*, *A* and *B* matrices are given as

$$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, B=\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, C=\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

with $\alpha_{ii}^2+\beta_{ii}^2<1, \quad i=1,2$ for stationary. In this diagonal representation, the conditional variances are functions of their own lagged values and own lagged returns shocks, while the conditional covariances are functions of the lagged covariances and lagged cross-products of the corresponding returns shocks. Moreover, this formulation guarantees H_t to be positive definite almost surely for all *t.* A comparision between BEKK and DCC can be found in Caporin and McAleer [12].

2.2 Optimal Hedge Ratios and Optimal Portfolio Weights

Market participants in futures markets choose a hedging strategy that reflects their attitudes toward risk and their individual goals. Consider the case of exchange rates, the return on the portfolio of spot and futures position can be denoted as:

$$R_{H,t}=R_{S,t}-\gamma R_{F,t,} \quad \textbf{(11)}$$

Where $R_{H,t}$ is the return on holding the portfolio between *t-1* and *t*, $R_{S,t}$ and $R_{F,t}$ are the returns on holding spot and futures positions between *t* and *t-1*, and γ is the hedge ratio, that is, the number of futures contracts that the hedger must sell for each unit of spot commodity on which price risk is borne.

According to Johnson [32], the variance of the returns of the hedged portfolio, conditional on the information set available at time *t-1* is given by

$$\operatorname{var}\left(R_{H,t}\left|\Omega_{t-1}\right.\right)=\operatorname{var}\left(R_{S,t}\left|\Omega_{t-1}\right.\right)-2\gamma\operatorname{cov}\left(R_{S,t},R_{F,t}\left|\Omega_{t-1}\right.\right)+\gamma_t^2\operatorname{var}\left(R_{F,t}\left|\Omega_{t-1}\right.\right), \quad \textbf{(12)}$$

Where $\operatorname{var}\left(R_{S,t}\left|\Omega_{t-1}\right.\right)$, $\operatorname{var}\left(R_{F,t}\left|\Omega_{t-1}\right.\right)$, and $\operatorname{cov}\left(R_{S,t},R_{F,t}\left|\Omega_{t-1}\right.\right)$ are the conditional variance and covariance of the spot and futures returns, respectively. The OHRs are defined as the value of γ_t which minimizes the conditional variance (risk) of the hedged portfolio returns, that is $\min_{\gamma_t}\left[\operatorname{var}\left(R_{H,t}\left|\Omega_{t-1}\right.\right)\right]$. Taking the partial derivate of (12) with respect to γ_t, setting it equal to zero and solving for γ_t, yields the OHR_t conditional on the information available at *t-1*(see, for example, Baillie and Myers [4]):

$$\gamma_t^* \Big| \Omega_{t-1} = \frac{\operatorname{cov}\left(R_{S,t}, R_{F,t} \,|\Omega_{t-1}\right)}{\operatorname{var}\left(R_{F,t} \,|\Omega_{t-1}\right)} \tag{13}$$

where returns are defined as the logarithmic differences of spot and futures prices.

From the multivariate conditional volatility model, the conditional covariance matrix is obtained, such that the OHR is given as:

$$\gamma_t^* \Big| \Omega_{t-1} = \frac{h_{SF,t}}{h_{F,t}}, \tag{14}$$

where $h_{SF,t}$ is the conditional covariance between spot and futures returns, and $h_{F,t}$ is the conditional variance of futures returns.

In order to compare the performance of OHRs obtained from different multivariate conditional volatility models, Ku et al. [37] suggest that a more accurate model of conditional volatility should also be superior in terms of hedging effectiveness, as measured by the variance reduction for any hedged portfolio compared with the unhedged portfolio. Thus, a hedging effective index (HE) is given as:

$$HE = \left[\frac{\operatorname{var}_{unhedged} - \operatorname{var}_{hedged}}{\operatorname{var}_{unhedged}}\right], \tag{15}$$

where the variances of the hedge portfolio are obtained from the variance of the rate of return, $R_{H,t}$, and the variance of the unhedged portfolio is the variance of spot returns (see, for example, Ripple and Moosa [50]). A higher HE indicates a higher hedging effectiveness and larger risk reduction, such that hedging method with a higher HE is regarded as a superior hedging strategy.

Alternatively, in order to construct an optimal portfolio design that minimizes risk without lowering expected returns, and applying the methods of Kroner and Ng [33] and Haqmmoudeh et.al. [30], the optimal portfolio weight of exchange rate spot/futures holding is given by:

$$w_{SF,t} = \frac{h_{F,t} - h_{SF,t}}{h_{S,t} - 2h_{SF,t} + h_{F,t}} \qquad \textbf{(16)}$$

and

$$w^*_{SF,t} = \begin{cases} 0, & if \ \mathrm{w_{SF,t}} < 0 \\ w_{SF,t}, & if \ 0<\mathrm{w_{SF,t}} < 1 \\ 1, & if \ \mathrm{w_{SF,t}} > 1 \end{cases} \qquad \textbf{(17)}$$

Where $\mathrm{w^*_{SF,t}}\left(1\text{-}\mathrm{w_{SF,t}}\right)$ is the weight of the spot (futures) in a one dollar portfolio of exchange rates spot/futures at time *t*.

3. Data

We used daily closing prices of spot (S) and futures for three foreign exchange rate series, the value of the US dollar to one European Euro (USD/EUR), one British Pound (USD/GBP) or one Japanese Yen (USD/JPY). The 3,006 observations from 3 January 2000 to 11 July 2011 are obtained from the Thomson Reuters-Ecowin Financial Database. The perpetual series of futures prices derived from individual futures contracts. These contracts call for a delivery of a specified quantity of a specified currency, or a cash settlement, during the months of March, June, September and December (the "March quarterly cycle"). Selected contracts are available with two future position continuous series. The futures price series for First Position Future (FUT1) is the price of the near-month delivery contract and the Second Position Future (FUT2) is the price of the next-to-near-month delivery contract. For example, in 1 February 2011, FUT 1 is the price of the contract that expires in March 2011, while FUT 2 is the price of the contract that expires in June 2011.

[Insert Table 1]

[Insert Table 2]

[Insert Table 3]

The returns of currency *i* at time *t* are calculated as $r_{i,t} = \log\left(P_{i,t} / P_{i,t-1}\right)$, where $P_{i,t}$ and $P_{i,t-1}$ are the closing prices of currency *i* for days *t* and *t-1* respectively. In Tables 1, 2 and 3 we show the descriptive statistics for the return series of EUR, GBP and JPY. The mean is close to zero in all cases. For the EUR and JPY currencies the standard deviation of the futures returns is larger than that of the spot returns, indicating the futures market is more volatile than the spot market for these currencies. The exchange rate return series display high kurtosis and heavy tails. Most of them, except EUR, present negative skewness statistics that signify increased presence of extreme losses than extreme gains (longer left tails). The Jarque-Bera Lagrange Multiplier test rejects the null hypothesis of normally distributed returns for every exchange rate series.

[Insert Figure 1]

Figure 1 presents the plot of spot and futures daily returns for each currency. Extremely high positive and negative returns are evident from September 2008 onward, and have continued well into 2009. Therefore, an increase in volatility during the financial crisis is perceived, however, is lower than in other assets (see, f.e., Mc Aleer et al. [46]). In the same way, the plots indicate volatility clustering. Spot and futures returns move in the same pattern suggesting a high correlation (the highest one is between FUT1 and FUT2 for all currencies). Correlations between the returns in European markets (EUR and GBP) are higher than the correlations between these and JPY which is hardly surprising.

[Insert Figure 2]

The volatilities of exchange rate returns are showed in figure 2. These volatilities are calculated as the square of the estimated returns and seem to support the stated above.

The plots are similar in all returns and the volatility of the series appears to be high during the early 2000s, followed by a quiet period from 2003 to the beginning of 2007. Volatility increases dramatically after August 2008, due in large part to the worsening global credit environment.

4.- Empirical Results

Estimation Results

We estimate four multivariate models (CCC, VARMA-AGARCH, DCC and BEKK) for each error distribution, currency and two different futures. The estimate underlying parameters are reported in tables 4-7. Table 4 shows the estimates for the CCC model. The volatility persistence, as measure by the sum of $\alpha + \beta$, in either spot or futures markets for each currency is significantly high, ranging from 0.978 to 0.9976, indicating long memory processes. All markets satisfy the second moment and long moment condition, which is a sufficient condition for the QMLE to be consistent and asymptotically normal (see McAleer et.al. [44]) The ARCH and GARCH estimates of the conditional variance are statistically significant. The ARCH estimates are generally small (less than 0.04) and the GARCH effects are generally close to one, finding lower values for the JPY in both spot and futures prices (0.949 and 0.944 against 0.961 and 0.962 for the EUR). There are not big differences among the constant conditional correlation estimates ranging from 0.799 for the EUR to 0.811 for the JPY.

[Insert Table 4]

[Insert Table 5]

Table 5 reports the estimates of the conditional mean and variance for the AGARCH models. The ARCH and GARCH effects are statistically significant in all markets and similar to the estimates for the CCC model without asymmetric effects. The asymmetric impact of the unconditional shocks on the conditional variance estimates, γ, are weak for the three currencies, in particular are not statistically significant for the JPY.

[Insert Table 6]

The DCC model developed by Engle [24] is employed to capture dynamics conditional correlations. Table 6 summaries the results of the DCC models estimated for all spot and futures markets. Regarding the conditional variance, estimates of all parameters are statistically significant and satisfy the condition $\alpha + \beta < 1$. The estimates of the DCC parameters, $\hat{\theta}_1$ and $\hat{\theta}_2$, are statistically significant for all the currencies. These results suggest that the conditional correlation is not constant over time. The short run persistence of shocks on the dynamic conditional correlation is greater for the JPY at 0.051, although it shows the lower long run persistence of shocks to the conditional correlation 0.913 (0.051+0.862). The EUR shows the lowest short run persistence (0.024) and the greatest long run persistence 0.986 (0.024+0.962). The time-varying conditional correlations between spot and futures returns are given in figure 3. An apparent change in the conditional correlation appeared upon the bankruptcy of Lehman Brothers in New York on 15 September 2008. Due to an increase in the volatility of spot and futures exchange rates, the conditional correlations seem to change in all the currencies. The GFC caused an apparent decline in the conditional correlation between the spot and both FUT1 an FUT2.

[Insert Table 7]

Table 7 reports the estimates for the BEKK model. We have restricted the bivariate BEKK model to the reduced form of the diagonal BEKK. The elements of the covariance matrix depend only on past own squared residuals, and the covariances depend only on past own gross products of residuals. The estimates show that the mean of the returns is not statistically significant. The elements of the diagonal matrices, A and B, are statistically significant. From the empirical results we conclude a time variation in market risk, a strong evidence of GARCH effect and the presence of a weak ARCH effect. The results for the covariance equations are similar, indicating that there is a statistically significant covariation in shocks, which depends more on its lag than on past innovations. These results clearly mean that market shocks are influenced by information which is common to spot and future markets, and as a result of this we have statistically significant covariance in the variance-covariance equations. Model estimations for FUT2 contracts has been done (available upon request) with similar results for the parameters estimates.

Hedging Performance

With the estimated underlying parameters in the models, we first generate in-sample daily time series of variance and covariance of the spot and futures returns for each currency. Subsequently, we calculate OHRs and optimal portfolios weights given by equations (14) and (16) respectively.

[Insert Table 8A]

[Insert Table 8B]

[Insert Table 8C]

Tables 8A-8C report the average OHR values, the hedge effectiveness, the variance of the portfolio, the hedging effectiveness along with the average value of the optimal portfolio weights for the three currencies using FUT1 and FUT2 contracts when both student's t and normal error distribution are assumed. We show the results for the four multivariate variance models.

Tables 8A-8C show that hedging is effective in reducing the risks for every model, currency and maturity. In particular, we find that the average OHR using FUT2 contracts are slightly higher than when FUT1 contracts are used, except for the GBP. The highest average OHR value is 0.854 for the USD/JPY when FUT2 contracts are used, meaning that in order to minimize risk, a long (buy) position of one dollar in such a currency should be hedged by a short (sell) position of $0.854 in JPYFUT2 contracts. Additionally, when using Gaussian error distribution tables 8A-8C report lower average OHR values for the three currencies analyzed. The average OHRs from each model are not particularly different, slightly smaller for the DCC and BEKK models when the *Student's t* is used but bigger for the GBP and JPY when using Gaussian distribution. Apparently, the average OHR values are higher for the USD/JPY exchange rate. On the contrary, hedging effectiveness is higher for the DCC and BEKK models.

For the GBP and JPY we notice that hedging effectiveness is slightly higher when a FUT1 contract is used, as opposed to showing a higher hedging effectiveness when the EURFUT2 contract is used. We find a hedging effectiveness that falls between a maximum of 66.3% for the USD/GBP currency and a minimum of 62.5% for the USD/EUR. It seems that hedging effectiveness is slightly higher for the USD/GBP currency.

[Insert Figure 3]

[Insert Figure 4]

Figure 3 shows the DCC estimates between spot and futures exchange rates for both future contracts. The volatility of the dynamic correlations increases during GFC and, as expected, during turbulent periods correlations decreases. This is why OHR volatility increases during the Global Financial Crisis (GFC). Figure 4 represents the calculated time-varying OHRs from every multivariate conditional volatility model. There are clearly time-varying ratios. It is interesting to look at the optimal hedging ratios during the GFC, for all the models but DCC optimal hedging ratios seem to increase in average.

As showed in the optimal portfolio weight columns in Tables 8A-8C, there are not big differences among models. For example, the largest average value corresponds to a portfolio including the JPYFUT1 contract, which spot currency weight is calculated using the DCC model assuming normal error distribution. The value 0.566 would imply that investors should have more spot currency than futures contracts in their portfolio in order to minimize risk without lowering expected returns. In particular, the optimal holding of one USD/JPY spot/future portfolio is 56.6 cents for spot and 43.4 cents for futures. When Gaussian distribution is used we find higher optimal portfolio weights. For both USD/EUR and USD/JPY spot/futures portfolios the optimal holding of spot currencies is higher when hedging with FUT1 contracts than when FUT2 are used. This is the opposite of what happen for USD/GBP spot/futures portfolios. Estimates suggest holding spot more than GBPFUT1, whereas they suggest holding spot less than GBPFUT2 on one dollar spot/future portfolio.

Summarising estimates based on both OHR and optimal weight values recommend holding more FUT2 than FUT1 contracts for USD/EUR and USD/JPY spot/futures portfolios, meaning that we should increase the percentage of futures contracts for longer term portfolios when these currencies are used.

5. Conclusions

This study sheds light on the importance of measuring conditional variances and covariances when hedging daily currency risk using futures. The findings are of importance to currency hedgers who require taking futures positions in order to adequately reduce the risk. In this paper, we use four multivariate GARCH models, CCC, VARMA-AGARCH, DCC, and BEKK, to examine the volatilities among spot and two distinct maturity futures, near-month and next-to-near-month contracts. The estimated conditional covariances matrices from these models were used to calculate the optimal portfolios weights and optimal hedge ratios.

The empirical results in this paper reveal that there are not big effectiveness differences when either the near-moth or the next-to-near-month contract is used for hedging spot position on currencies. They even reveal that hedging ratios are lower for near-month contract when the USD/EUR and USD/JPY exchange rates are analyzed. This result is explained in terms of the higher correlation between spot prices and the next-to-near-month futures prices than that with near-month contract and additionally because of the lower volatility of the long maturity futures.

Finally, CCC and VARMA-AGARCH models provide similar results in terms of hedging ratios, portfolio variance reduction and hedging effectiveness. Some differences appear when the DCC and BEKK models are used. Hedging ratios seem to decrease during the GFC as opposed to increasing ratios when CCC and VARMA-AGARH models are consider for calculating the covariance matrix. Future research should be done to investigate the effects of the GFC on the conditional correlation between spot and futures contracts as well as its impact on hedging effectiveness.

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Figure 1. Spot and futures daily returns

USD/EUR	USD/GBP	USD/JPY

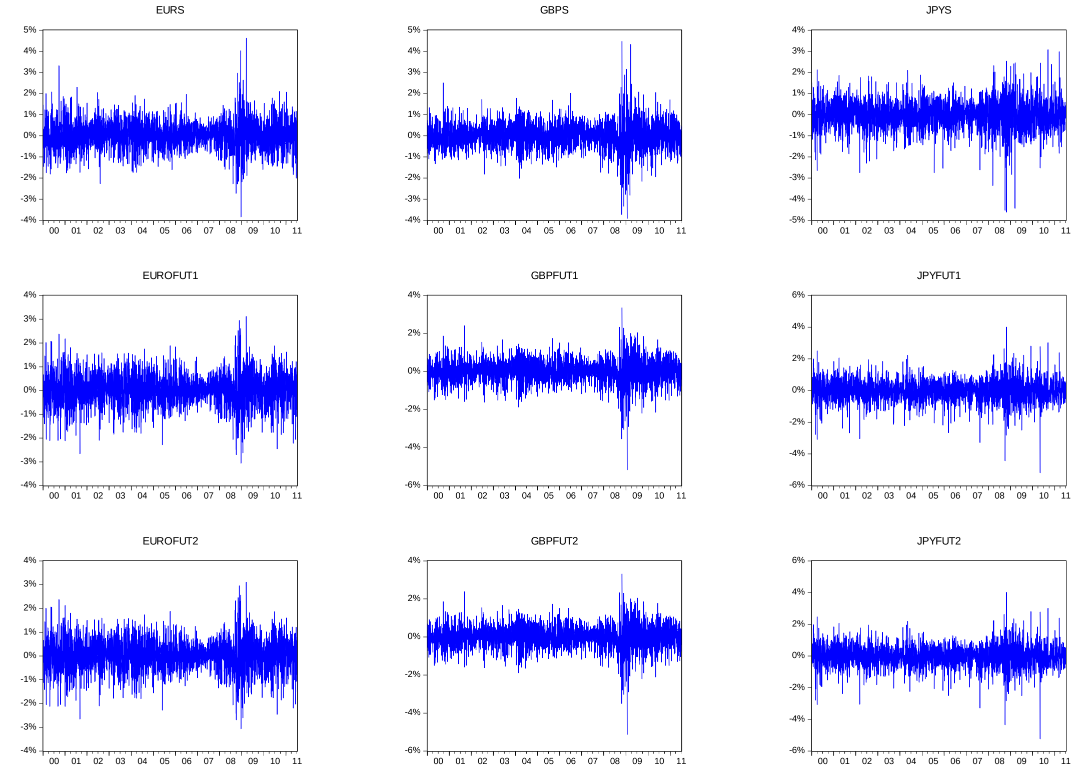

Figure 2. Estimated Conditional Volatilities of Returns

USD/EUR	USD/GBP	USD/JPY

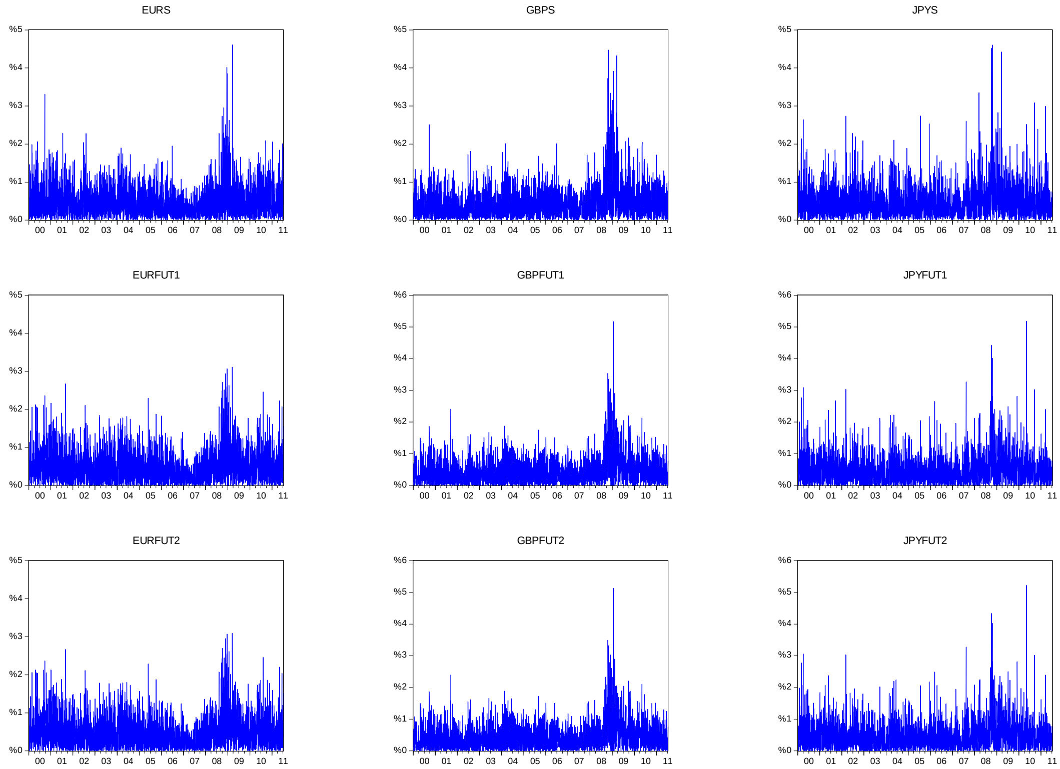

Figure 3. DCC Estimates

USD/EUR	USD/GBP	USD/JPY

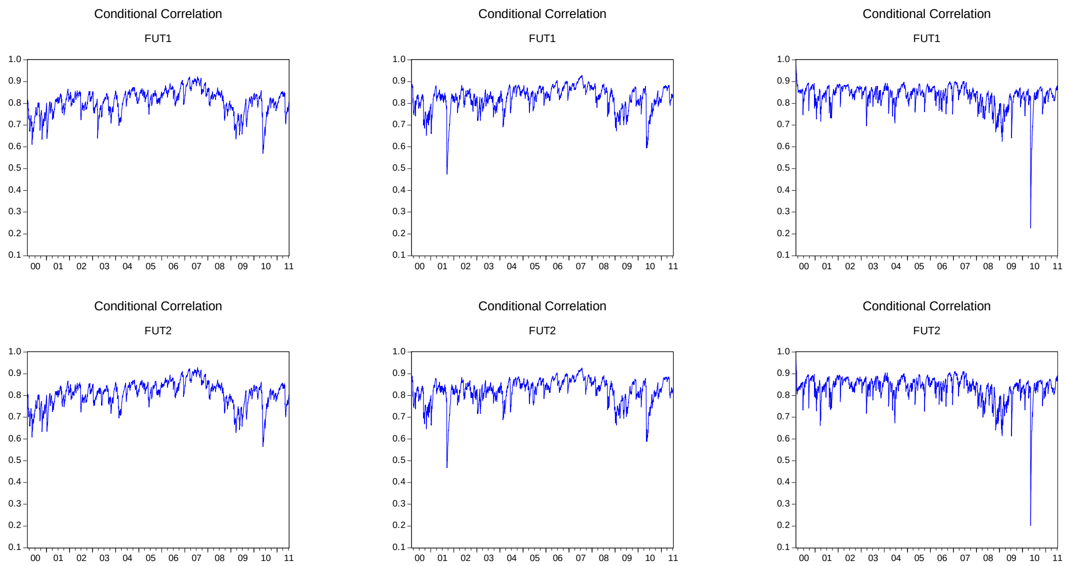

Figure 4. Optimal Hedge Ratios

USD/EUR	USD/GBP	USD/JPY

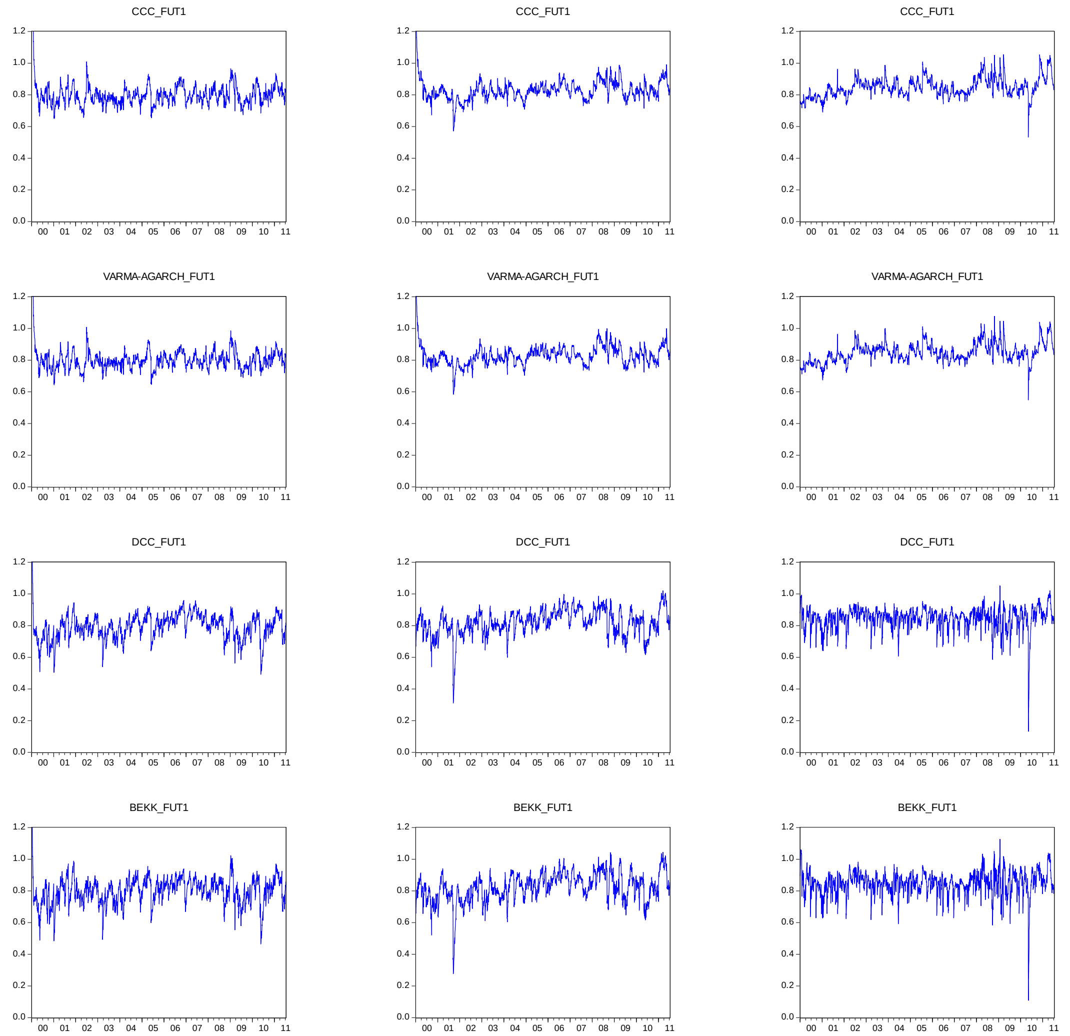

Table 1. EUR Descriptive Statistics

Returns	EUROS	EUROFUT1	EUROFUT2
Mean	0.0107	0.0101	0.0098
Maximum	4.6174	3.1184	3.1007
Minimum	-3.8445	-3.0568	-3.0620
Std. Dev.	0.6485	0.6552	0.6532
Skewness	0.1477	-0.0939	-0.0962
Kurtosis	5.6389	4.3878	4.3556
Jarque-Bera	882.87	245.58	234.73

Table 2. GPB Descriptive Statistics

Returns	GBPS	GBPFUT1	GBPFUT2
Mean	-0.0007	-0.0010	-0.0011
Maximum	4.4745	3.3542	3.3147
Minimum	-3.9182	-5.1703	-5.1326
Std. Dev.	0.6103	0.6010	0.6021
Skewness	-0.0552	-0.3793	-0.3560
Kurtosis	7.3609	6.7628	6.5791
Jarque-Bera	2382.7	1844.9	1667.4

Table 3. JPY Descriptive Statistics

Returns	JPYS	JPYFUT1	JPYFUT2
Mean	-0.0077	-0.0075	-0.0070
Maximum	3.0770	4.0082	4.0187
Minimum	-4.6098	-5.1906	-5.2289
Std. Dev.	0.6594	0.6642	0.6594
Skewness	-0.4246	-0.3285	-0.3020
Kurtosis	6.5503	6.9307	6.9282
Jarque-Bera	1668.5	1988.5	1977.8

Table 4. CCC Estimates

Panel a: EURS_EURFUT1								
Returns	C	ω	α	β	α + β	Constant Conditional Correlation	Log-likelihood	AIC
EURS	0.025532 (0.0165)	0.001324 (0.1455)	0.036873 (0.0000)	0.960795 (0.0000)	0.997668	0.798867 (0.0000)	-4105.754	2.738605
EURFUT1	0.022376 (0.0454)	0.001886 (0.0329)	0.034411 0.0000	0.962024 0.0000	0.996435			

Panel b: GBPS_GBPFUT1								
Returns	C	ω	α	β	α + β	Constant Conditional Correlation	Log-likelihood	AIC
GBPS	0.014299 (0.1479)	0.002229 (0.0086)	0.030472 (0.0000)	0.962650 (0.0000)	0.993122	0.812791 (0.0000)	-3367.432	2.247209
GBPFUT1	0.012066 (0.2202)	0.002608 (0.0055)	0.028671 (0.0000)	0.963302 (0.0000)	0.991973			

Panel c: JPYS_JPYFUT1								
Returns	C	ω	α	β	α + β	Constant Conditional Correlation	Log-likelihood	AIC
JPYS	0.003894 (0.7283)	0.006035 (0.0269)	0.037076 (0.0000)	0.948975 (0.0000)	0.986051	0.811211 (0.0000)	-4239.943	2.827915
JPYFUT1	0.005502 (0.6230)	0.009677 (0.0205)	0.033193 (0.0055)	0.944285 (0.0000)	0.977478			

Note: p_values in parentheses.

Table 5. VARMA-AGARCH Estimates

Panel a: EURS_EURFUT1									
Returns	C	ω	α	β	γ	α+β+γ	Constant Conditional Correlation	Log-likelihood	AIC
EURS	0.016007 (0.1350)	0.001446 (0.1021)	0.025002 (0.0027)	0.962974 (0.0000)	0.018713 (0.0489)	1.006689	0.799869 (0.0000)	-4098.227	2.734926
EURFUT1	0.011699 (0.2921)	0.002113 (0.0135)	0.020336 (0.0105)	0.964626 (0.0000)	0.021366 (0.0483)	1.006328			

Panel b: GBPS_GBPFUT1									
Returns	C	ω	α	β	γ	α+β+γ	Constant Conditional Correlation	Log-likelihood	AIC
GBPS	0.004260 (0.6588)	0.002587 (0.0021)	0.014752 (0.0446)	0.963269 (0.0000)	0.027465 (0.0117)	1.005486	0.813575 (0.0000)	-3356.194	2.241061
GBPFUT1	0.001990 (0.8368)	0.002791 (0.0021)	0.016439 (0.0301)	0.964716 (0.0000)	0.020035 (0.0644)	1.00119			

Panel c: JPYS_JPYFUT1									
Returns	C	ω	α	β	γ	α+β+γ	Constant Conditional Correlation	Log-likelihood	AIC
JPYS	0.004648 (0.6808)	0.006036 (0.0315)	0.038539 (0.0006)	0.949630 (0.0000)	-0.004651 (0.7528)	0.983518	0.811191 (0.0000)	-4239.519	2.828964
JPYFUT1	0.005544 (0.6287)	0.009533 (0.0208)	0.032053 (0.0331)	0.944463 (0.0000)	0.002810 (0.8728)	0.979326			

Note: p_values in parentheses.

Table 6. DCC Estimates

Panel a: EURS_EURFUT1									
Returns	C	ω	α	β	α + β	θ_1	θ_2	Log-likelihood	AIC
EURS	0.020377 (0.0532)	0.004386 (0.0022)	0.027374 (0.0000)	0.961108 (0.0000)	0.988482	0.023709 (0.0000)	0.961618 (0.0000)	-4077.809	2.721337
EURFUT1	0.018240 (0.0910)	0.006484 (0.0002)	0.034551 (0.0000)	0.949876 (0.0000)	0.984427				

Panel b: GBPS_GBPFUT1									
Returns	C	ω	α	β	α + β	θ_1	θ_2	Log-likelihood	AIC
GBPS	0.010971 (0.2487)	0.004841 (0.0001)	0.038697 (0.0000)	0.946627 (0.0000)	0.985324	0.041639 (0.0000)	0.936892 (0.0000)	-3303.009	2.205663
GBPFUT1	0.009160 (0.3353)	0.006684 (0.0001)	0.049814 (0.0000)	0.930739 (0.0000)	0.980553				

Panel c: JPYS_JPYFUT1									
Returns	C	ω	α	β	α + β	θ_1	θ_2	Log-likelihood	AIC
JPYS	0.003990 (0.7237)	0.019263 (0.0000)	0.043614 (0.0000)	0.911911 (0.0000)	0.955525	0.050768 (0.0000)	0.862159 (0.0000)	-4173.424	2.784974
JPYFUT1	0.005513 (0.6364)	0.048472 (0.0000)	0.069140 (0.0001)	0.824529 (0.0000)	0.893669				

Note: p_values in parentheses.

Table 7. BEKK Estimates

Panel a: EURS_EURFUT1									
Returns	C	**C**		**A**		**B**		Log-likelihood	AIC
EURS	0.020559 (0.0488)	0.003112 (0.0001)		0.163463 (0.0000)	0.000000	0.982880 (0.0000)	0.000000	-4101.786	2.735964
EURFUT1	0.019446 (0.0664)	0.004368 (0.0001)	0.006801 (0.0003)	0.000000	0.213424 (0.0000)	0.000000	0.969448 (0.0000)		

Panel a: GBPS_GBPFUT1									
Returns	C	**C**		**A**		**B**		Log-likelihood	AIC
GBPS	0.009175 (0.3332)	0.003862 (0.0000)		0.195416 (0.0000)	0.000000	0.975276 (0.0000)	0.000000	-3315.017	2.212324
GBPFUT1	0.007995 (0.3992)	0.004889 (0.0000)	0.007320 (0.0001)	0.000000	0.238545 (0.0000)	0.000000	0.960766 (0.0000)		

Panel a: JPYS_JPYFUT1									
Returns	C	**C**		**A**		**B**		Log-likelihood	AIC
JPYS	0.001488 (0.8959)	0.022401 (0.0000)		0.215503 (0.0000)	0.000000	0.950569 (0.0000)	0.000000	-4181.067	2.788730
JPYFUT1	0.001963 (0.8719)	0.046133 (0.0000)	0.083282 (0.0000)	0.000000	0.298570 (0.0000)	0.000000	0.855350 (0.0000)		

Note: p_values in parentheses.

Table 8A. Alternative hedging strategies (USD/EUR)

	MODEL	OHR	Var. PF	HE	Var. UnHed	OPT. W
	Student-t					
FUT1	**CCC**	0.805	0.158	62.5%	0.420	0.536
FUT1	**VARMA-AGARCH**	0.805	0.157	62.7%	0.420	0.536
FUT1	**DCC**	0.794	0.157	62.7%	0.420	0.542
FUT1	**BEKK**	0.802	0.157	62.6%	0.420	0.542
FUT2	**CCC**	0.808	0.157	62.7%	0.420	0.532
FUT2	**VARMA-AGARCH**	0.808	0.156	62.9%	0.420	0.532
FUT2	**DCC**	0.797	0.156	62.9%	0.420	0.535
FUT2	**BEKK**	0.804	0.156	62.8%	0.420	0.537
	Gaussian					
FUT1	**CCC**	0.792	0.158	62.5%	0.420	0.544
FUT1	**VARMA-AGARCH**	0.792	0.157	62.7%	0.420	0.545
FUT1	**DCC**	0.784	0.157	62.7%	0.420	0.554
FUT1	**BEKK**	0.792	0.157	62.6%	0.420	0.550
FUT2	**CCC**	0.799	0.157	62.7%	0.420	0.532
FUT2	**VARMA-AGARCH**	0.799	0.156	62.9%	0.420	0.533
FUT2	**DCC**	0.791	0.156	62.9%	0.420	0.538
FUT2	**BEKK**	0.798	0.156	62.8%	0.420	0.537

Notes: Optimal Hedging Ratio (**OHR**), Variance of Portfolios (**Var. PF**), Hedging Effective Index (**HE**), Variance of ungedged portfolio (**Var. UnHed)** and Optimal Portfolio Weights (**OPT. W**).

Table 8B. Alternative hedging strategies (USD/GBP)

	MODEL	OHR	Var. PF	HE	Var. UnHed	OPT. W
	Student-t					
FUT1	**CCC**	0.829	0.126	66.2%	0.372	0.496
	VARMA-AGARCH	0.830	0.125	66.3%	0.372	0.498
	DCC	0.822	0.126	66.2%	0.372	0.497
	BEKK	0.826	0.126	66.2%	0.372	0.490
FUT2	**CCC**	0.826	0.127	65.9%	0.372	0.510
	VARMA-AGARCH	0.826	0.126	66.1%	0.372	0.512
	DCC	0.817	0.127	65.9%	0.372	0.511
	BEKK	0.821	0.127	65.9%	0.372	0.505
	Gaussian					
FUT1	**CCC**	0.816	0.126	66.2%	0.372	0.499
	VARMA-AGARCH	0.815	0.125	66.3%	0.372	0.503
	DCC	0.818	0.126	66.1%	0.372	0.500
	BEKK	0.822	0.127	66.0%	0.372	0.495
FUT2	**CCC**	0.812	0.127	65.9%	0.372	0.510
	VARMA-AGARCH	0.812	0.126	66.1%	0.372	0.513
	DCC	0.813	0.127	65.8%	0.372	0.513
	BEKK	0.817	0.128	65.7%	0.372	0.508

Notes: Optimal Hedging Ratio (**OHR**), Variance of Portfolios (**Var. PF**), Hedging Effective Index (**HE**), Variance of ungedged portfolio (**Var. UnHed)** and Optimal Portfolio Weights (**OPT. W**).

Table 8C. Alternative hedging strategies (USD/JPY)

	MODEL	OHR	Var. PF	HE	Var. UnHed	OPT. W
	Student-t					
FUT1	**CCC**	0.849	0.153	64.8%	0.435	0.463
FUT1	**VARMA-AGARCH**	0.849	0.153	64.8%	0.435	0.464
FUT1	**DCC**	0.845	0.153	64.8%	0.435	0.475
FUT1	**BEKK**	0.849	0.154	64.7%	0.435	0.474
FUT2	**CCC**	0.853	0.154	64.6%	0.435	0.450
FUT2	**VARMA-AGARCH**	0.853	0.154	64.6%	0.435	0.450
FUT2	**DCC**	0.850	0.154	64.7%	0.435	0.464
FUT2	**BEKK**	0.854	0.154	64.6%	0.435	0.468
	Gaussian					
FUT1	**CCC**	0.803	0.152	65.0%	0.435	0.535
FUT1	**VARMA-AGARCH**	0.802	0.152	65.0%	0.435	0.537
FUT1	**DCC**	0.812	0.153	64.8%	0.435	0.566
FUT1	**BEKK**	0.817	0.153	64.7%	0.435	0.570
FUT2	**CCC**	0.810	0.153	64.8%	0.435	0.514
FUT2	**VARMA-AGARCH**	0.809	0.153	64.8%	0.435	0.515
FUT2	**DCC**	0.818	0.154	64.6%	0.435	0.549
FUT2	**BEKK**	0.823	0.154	64.5%	0.435	0.555

Notes: Optimal Hedging Ratio (**OHR**), Variance of Portfolios (**Var. PF**), Hedging Effective Index (**HE**), Variance of ungedged portfolio (**Var. UnHed)** and Optimal Portfolio Weights (**OPT. W**).