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SHORT PAPER
On some fuzzy maps

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Abstract

Purpose – To define some new kinds of fuzzy maps related with c-zero dimensionality fuzzy, and various notions of fuzzy connectedness (due to Fatteh and Bassan, and Ajmal and Kohli, respectively), and obtain several properties about them.

Design/methodology/approach – The paper shows several properties of some new fuzzy maps.

Findings – The paper shows new results on fuzzy maps and various notios of fuzzy connectedness.

Research limitations/implications – Clearly, this paper is devoted to fuzzy topological spaces.

Practical implications – The main applications are in the mathematical field.

Originality/value – The paper shows original results on fuzzy topology.

Keywords Mathematics, Topology, Fuzzy logic

Paper type Research paper

1. Introduction

We define in this paper the zero dimensional fuzzy maps, related with the notion of c-zero dimensional fuzzy set (due to Ajmal and Kohli), and the concepts of monotone fuzzy and c_i-monotone fuzzy maps ($i = 1, 3$), related with the notions of fuzzy connected set (due to Fatteh and Bassan), and of c_i-connected set (due to Ajmal and Kohli), respectively.

We obtain some results on these new fuzzy maps and the old notions of zero dimensional and monotone continuos maps. Nevertheless, a theorem on composition of fuzzy perfect maps using these new fuzzy maps does not seem possible (see Remark 2).

The previous definitions on fuzzy topology are in the Liu and Luo's book (Liu and Luo, 1997), and the reference for *General Topology* is the Engelking's book (Engelking, 1989).

2. Main results

Definition 1. Let (X, τ) and (Y, s) be two fts. A F- continuous map $\tilde{f}: (X, \tau) \to (Y, s)$ will be called zero dimensional fuzzy if each fiber of $\tilde{f}$ is c-zero dimensional fuzzy (i.e.: if every fuzzy point x_1 in the fiber and every fuzzy open set μ containing x_1, there exists a crisp clopen fuzzy set σ in it such that $x_1 \leq \sigma \leq \mu$ (Ajmal and Kohli, 1994)).

Proposition 1. Let (X, τ) and (Y, s) be two induced fts, and $f : X \to Y$ an onto map. If $\tilde{f}$ is zero dimensional fuzzy, then f is zero dimensional.

Proof. For each $y \in Y$, each $x \in f^{-1}(y)$, and each open set U of X, such that $x \in f^{-1}(y) \cap U$, let $y_1 : Y \to I$ defined by:

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$$\begin{cases} y_1(y) = 1 \\ y_1(y') = 0 \text{ for all } y' \neq y \end{cases}$$

Then, $\tilde{f}^{-1}(y_1)(x) = y_1(f(x)) = y_1(y) = 1$ and $x_1 \leq \tilde{f}^{-1}(y_1)$ (and also, $x_1 \leq \chi_U$).

From the hypothesis, we have that, there exists a crisp clopen fuzzy set σ such that $x_1 \leq \sigma \leq \chi_U$. Then $\sigma = \chi_S$ for some clopen set S in X, and finally $x \in S \subset U$. □

Remark 1. $\tilde{f}^{-1}(y_\alpha) = \alpha\chi_{f^{-1}(y)}$ for every fuzzy point y_α.

$$\left(\text{Because } \tilde{f}^{-1}(y_\alpha)(x) = y_\alpha(f(x)) = \begin{cases} \alpha, & \text{if } f(x) = y (\Leftrightarrow x \in f^{-1}(y)) \\ 0, & \text{if } f(x) \neq y (\Leftrightarrow x \notin f^{-1}(y)) \end{cases}\right)$$

Remark 2. The converse of the above proposition is false.

(For each fuzzy point y_α, each $x_1 \leq \tilde{f}^{-1}(y_\alpha)$ and each open fuzzy set μ such that $x_1 \leq \mu$, we have, by Remark 1, that $1 = \alpha\chi_{f^{-1}(y)}(x)$, then $x \in f^{-1}(y)$ and $\alpha = 1$).

Definition 2. Let (X, τ) and (Y, s) be two fts. A F- continuous map $\tilde{f}: (X, \tau) \rightarrow (Y, s)$ will be called monotone fuzzy if the fibers of $\tilde{f}$ are fuzzy connected (i.e.: there is not a clopen proper fuzzy set contained in it (Fatteh and Bassan, 1985)).

Proposition 2. Let (X, τ) and (Y, s) be two induced fts, and $f: X \rightarrow Y$ an onto map. If $\tilde{f}$ is monotone fuzzy, then f is monotone.

Proof. Let S be a clopen set such that $\varnothing \neq Sf^{-1}(y)$ where $y \in Y$, and $y_1 : Y \rightarrow I$ is defined by:

$$\begin{cases} y_1(y) = 1 \\ y_1(y') = 0 \text{ for all } y' \neq y \end{cases}$$

Then $\chi_S \leq \tilde{f}^{-1}(y_1)$ (because $\tilde{f}^{-1}(y_1) = \chi_{f^{-1}(y)}$).

And χ_s is clopen fuzzy if S is a clopen set. □

Definition 3. Let (X, τ) and (Y, s) be two fts. A F- continuous map $\tilde{f}: (X, \tau) \rightarrow (Y, s)$ will be called c_1-monotone fuzzy (resp. c_3-monotone fuzzy) if the fibers of $\tilde{f}$ are c_1-connected (resp. c_3-connected (Ajmal and Kohli, 1989)).

Proposition 3. Let (X, τ) and (Y, s) be two induced fts, and $f: X \rightarrow Y$ an onto map. If $\tilde{f}$ is c_1-monotone fuzzy (resp. c_3-monotone fuzzy), then f is monotone.

Proof. If $f^{-1}(y) = A_1 \cup A_2$, with A_i non-empty open in $f^{-1}(y)$ and $A_1 \cap A_2 = \varnothing$.

Let $y_1 : Y \rightarrow I$ be the fuzzy point of support y and value 1. Then:

$$\tilde{f}^{-1}(y_1) = \chi_{f^{-1}(y)} = \chi_{A_1 \cup A_2} = \chi_{A_1} \vee \chi_{A_2}.$$

Since, $Ai = G_i \cap f^{-1}(y)$ for some open G_i of X ($i = 1, 2$), then χ_{G_i} is open fuzzy and $\tilde{f}^{-1}(y_1) \leq \chi_{G_1} \vee \chi_{G_2}$:

$$(\chi_{G_1} \wedge \chi_{G_2})(x) = \chi_{G_1 \cap G_2}(x) \leq (1 - \tilde{f}^{-1}(y_1))(x) = \begin{cases} 0, & \text{if } x \in f^{-1}(y) \\ 1, & \text{if } x \notin f^{-1}(y) \end{cases}$$

for all $x \in X$,

because $x \in G_1 \cap G_2$ implies that $x \notin f^{-1}(y)$ (from $A_1 \cap A_2 = \varnothing$), and:

$$(\chi_{G_i} \wedge \tilde{f}^{-1}(y_1))(x) = \begin{cases} 0, & \text{if } x \notin G_i \\ \tilde{f}^{-1}(y_1), & \text{if } x \in G_i \end{cases},$$

and then $\chi_{G_i} \wedge \tilde{f}^{-1}(y_1) \neq 0$, and $\tilde{f}^{-1}(y_1)$ is not c_1-connected.

On the other hand, $G_i \cap f^{-1}(y) \neq \varnothing$ $(i = 1,2)$, then $\chi_{G_i}(\chi_{f^{-1}(y)})' = 1 - \tilde{f}^{-1}(y_1)$ and $\tilde{f}^{-1}(y_1)$ is not c_3-connected. □

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