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Corrections and new developments in rigid earth nutation theory

III. Final tables "REN-2000" including crossed-nutation and spin-orbit coupling effects

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Abstract. We present here the new tables REN-2000 of the nutation for a rigid Earth model, starting from Hamiltonian theory, with a level of truncature at 0.1 μas for individual coefficients instead of 5 μas (Kinoshita & Souchay 1990). For this presentation to be achieved we first carry out the calculations of the second-order effects due to crossed-nutations and spin-orbit coupling, at the same level of truncation as above. This paper is the third and last one in the frame of the complete reconstruction of the theory of the rigid Earth nutation. It is the complementary part to previous studies concerning the luni-solar nutation involving indirect planetary effects (Souchay & Kinoshita 1996), and the influence of the second-order geopotential (J_3, J_4) and of the direct planetary effect (Souchay & Kinoshita 1997). Quasi-diurnal and sub-diurnal nutations coming from the harmonics of degree 2, 3 and 4 of the geopotential are also included in REN-2000, their values being taken from Folgueira et al. (1998a,b). A presentation of the series REN-2000 is done at the end of the paper, with separated informations for each contribution.

Key words: reference systems — earth

1. Introduction

Considering the increasing accuracy of the determination of the coefficients of nutation by modern techniques such as VLBI, and the large correction to the conventional IAU1976 value of the general precession in longitude p_a as given by Lieske et al. (1977), Souchay & Kinoshita (1996) made important corrections for the largest coefficients due to the leading luni-solar effect coming from the J_2 Earth's geopotential, in particular for the leading terms of period 18.6 y, 9.3 y, 1 y, 182 d, 13.66 d, with respect to their corresponding values in Kinoshita & Souchay (1990). They also confirmed the presence and the value of an out-of-phase component for the 18.6y and 9.3y terms both in longitude and obliquity, already pointed out by Williams (1994). At last they made some corrections to the tables listed in Kinoshita & Souchay (1990) according to some remarks made by Williams (private communication).

In a second paper, Souchay & Kinoshita (1997) calculated again the coefficients of nutation due to the second-order J_3, J_4, $C_{2,2}$, and $S_{2,2}$ coefficients of the Earth's geopotential, and also the direct action of the planets on nutation, with a truncation limit of 0.1 μas for the coefficients of $\Delta\ \psi$ cos ε and $\Delta\varepsilon$, that is to say 50 times smaller than the truncation limit of the series in Kinoshita & Souchay (1990).

The results were listed by Souchay & Kinoshita (1997) and compared with Hartmann & Soffel (1995) and Williams (1995) respectively for each of the two kinds of effects mentioned above. In the two comparisons the agreement is remarkable, for the absolute difference in the amplitude of the coefficients does not exceed 1 μas except for a few ones, although the total number of these coefficients is much larger than in the Kinoshita & Souchay series. Notice that the three ways of determination of the coefficients are quite different: Hartmann & Soffel (1995) compute them from tidal waves, Williams (1995) is using the torque approach and Souchay & Kinoshita (1997) use Hamiltonian equations.

The fact that the results are very close together is a very probing confirmation of the validity of the terms found. The present paper is the third and final one in the scope of a global check of the coefficients of nutation for a rigid Earth model (Souchay & Kinoshita 1996, 1997). It is devoted to the last and more delicate part concerning the computation at the 0.1 μas level of second-order coupling effects which can be divided into two categories: the first one which can be quoted as the *spin-orbit coupling effect* is the interaction between the orbital motion of the Moon and the J_2 component of the Earth geopotential characterizing the ellipticity of the Earth. The second one, which can be called the *crossed-nutation effect*, is the influence of the nutation itself on the torque exerted by the Moon and the Sun: in a few words, when calculating this torque, we must take into account the small contribution due to the displacement of the figure axis coming from the nutation itself.

The first coupling effect above has been pointed out for the first time by Kubo (1982) who made a rough calculation of the perturbations on the longitude and latitude coordinates λ and β of the Moon, caused by the Earth's flattening, then simultaneously of the perturbations on the nutation. In the frame of a global reconstruction of the theory of the nutation for a rigid Earth model, Kinoshita & Souchay (1990) found a few terms down to their level of truncation of their series, that is to say 0.005 μas (milliarcsecond).

The second coupling effect was already partially computed by Kinoshita (1977) when elaborating a new theory of nutation starting from Hamiltonian formalism. It was more accurately re-calculated by Kinoshita & Souchay (1990) down to 0.005 μas.

In the following we will compute the two kinds of second-order coupling effects which have just been explained, with a double objective: one is to catch all the coupling terms down to 0.1 μas instead of 5 μas (Kinoshita & Souchay 1990). The other one is to carry out all calculations with a computer instead of manually as it was the case in this last paper. The advantage, in addition of avoiding miscalculations, is to push farer the development of the luni-solar potential instead of keeping only its leading terms, and thus to take into account some possible coupling interactions previously neglected.

Moreover one of the best way to check the validity of the series of nutation determined analytically is to carry out a numerical integration of the nutation, and to study the residuals between the results given by these two methods. This was already done by Souchay & Kinoshita (1991) who showed that the residuals were about 20 times smaller, both in longitude and in obliquity, than those found by Kubo & Fukushima (1988), as well as by Schastock et al. (1989) before the reconstruction of the analytical theory by Kinoshita & Souchay (1990). This proved that the relatively important second-order analytical corrections due to the coupling effects described above and calculated in this last paper were justified, and was a probing confirmation of the theory. Moreover a new comparison between our new series and numerical integration using the numerical ephemeris DE403 of the JPL, is on the way (Souchay 1998).

In the following we describe the methods from which we computed the coefficients of the nutation related to the crossed-nutation and to the spin-orbit coupling effect. Then, we present our final tables of nutation REN-2000 for a rigid Earth model, including all the improvements done previously (Souchay & Kinoshita 1996, 1997) and in the present paper. Notice that in order to be complete at the level of 0.1 μas our tables REN-2000 include also the diurnal and sub-diurnal components of the nutation related to the $C_{3,i}$ and $S_{3,i}$ of the geopotential, as calculated by Folgueira et al. (1998a) and those related to the $C_{4,i}$ and $S_{4,i}$ coefficients, as calculated by Folgueira et al. (1998b). These new contributions not included in previous tables (Kinoshita & Souchay 1990) will be presented in the end of the present paper.

2. The crossed-nutation coupling effect

The second-order potential characterizing this effect is involving the Andoyer variables h and H of the rotation of the Earth (Kinoshita 1977), and we can use the same formulation as in Kinoshita & Souchay (1990) for the expression of the second-order determining function involved, that is to say:

$$W_2^{\mathrm{cr.}} = \frac{1}{2}\int \Big[\frac{\partial(U_1^{\mathrm{per}})}{\partial h} \times \frac{\partial(W_1)}{\partial H} - \frac{\partial(U_1^{\mathrm{per}})}{\partial H} \times \frac{\partial(W_1)}{\partial h}\Big]\mathrm{d}t \qquad (1)$$

where U_1^{per} is the periodic part of the first-order potential U_1 due to the combined action of the Moon and of the Sun, and W_1 is the first-order determining function which is determined from a simple integration:

$$W_1 = \int U_1^{\mathrm{per}}\mathrm{d}t \qquad (2)$$

U_1 can be easily expressed as in the following way:

$$\begin{aligned} U_1 = {} & \frac{\kappa^2 M_{\mathrm{M}}}{a_{\mathrm{M}}^3} \\ & \times\Big[\frac{2C-A-B}{2}\Big] \times \left(\frac{a_{\mathrm{M}}}{r_{\mathrm{M}}}\right)^3 \times \Big[\frac{1}{2}(3\cos^2 I - 1)P_2^0(\sin\beta_{\mathrm{M}}) \\ & -\frac{1}{2}\sin 2I P_2^1(\sin\beta_{\mathrm{M}})\sin(\lambda_{\mathrm{M}} - h) \\ & -\frac{1}{4}\sin^2 I P_2^2(\sin\beta_{\mathrm{M}})\cos 2(\lambda_{\mathrm{M}} - h)\Big] \\ & +\frac{\kappa^2 M_{\mathrm{S}}}{a_{\mathrm{S}}^3} \times \Big[\frac{2C-A-B}{2}\Big] \times \left(\frac{a_{\mathrm{S}}}{r_{\mathrm{S}}}\right)^3 \\ & \times\Big[\frac{1}{2}(3\cos^2 I - 1)P_2^0(\sin\beta_{\mathrm{S}}) \\ & -\frac{1}{4}\sin^2 I P_2^2(\sin\beta_{\mathrm{S}})\cos 2(\lambda_{\mathrm{S}} - h)\Big] \end{aligned} \qquad (3)$$

h is the canonical variable representing the general precession in longitude ($h = -p_{\mathrm{A}}$).

M_M and M_S are respectively the masses of the Moon and the Sun,(λ_M, β_M, r_M) and (λ_S, β_S, r_S) are their respective set of spherical coordinates with respect to the mean ecliptic and the mean equinox of the date. Notice that in a first approximation β_S which represents the latitude of the Sun with respect to the mean ecliptic of the date, can be set to $\beta_\mathrm{S} = 0$, and that the terms of nutation due to the small displacements of the Sun with respect to the mean ecliptic have been calculated by Souchay & Kinoshita (1996). a_M and a_S are the basic constant values for the semi-major axes of the Moon and of the Earth, considering the keplerian motion. I is the obliquity angle ($I = -\varepsilon$) associating the two canonical variables H and G, by the trivial equation (Kinoshita 1977):

$$H = G\cos I \tag{4}$$

where G is the amplitude of the angular momentum of the Earth. G being constant, any partial derivative with respect to H is such as (Kinoshita 1977):

$$\frac{\partial[...]}{\partial H} = -\frac{1}{G\sin I}\frac{\partial[...]}{\partial I}. \tag{5}$$

The combination of Eqs. (1) to (5) leads to the following expression for W_2:

$$W_2^{\mathrm{cr.}} = \frac{1}{2}\int[\mathbf{A}]\mathrm{d}t \tag{6}$$

with:

$$[\mathbf{A}] = (\sin I\cos^2 I)A_1 + \left(\frac{\sin^2 I\cos I}{2}\right)A_2 + (\cos I\cos 2I)A_3 + \left(\frac{\sin I\cos 2I}{2}\right)A_4 + \left(\frac{\cos^2 I\sin I}{2}\right)A_5 + \left(\frac{\sin^2 I\cos I}{4}\right)A_6 \tag{7}$$

and:

$$A_1 = B_1^\mathrm{M}\times(C_0^\mathrm{M}+C_0^\mathrm{S}) - C_1^\mathrm{M}\times(B_0^\mathrm{M}+B_0^\mathrm{S}) \tag{8.1}$$

$$A_2 = (C_2^\mathrm{M}+C_2^\mathrm{S})\times(B_0^\mathrm{M}+B_0^\mathrm{S}) - (B_2^\mathrm{M}+B_2^\mathrm{S})\times(C_0^\mathrm{M}+C_0^\mathrm{S}) \tag{8.2}$$

$$A_3 = B_1^\mathrm{M}\times C_3^\mathrm{M} - B_3^\mathrm{M}\times C_1^\mathrm{M} \tag{8.3}$$

$$A_4 = B_3^\mathrm{M}\times(C_2^\mathrm{M}+C_2^\mathrm{S}) - C_3^\mathrm{M}\times(B_2^\mathrm{m}+B_2^\mathrm{S}) \tag{8.4}$$

$$A_5 = (B_4^\mathrm{M}+B_4^\mathrm{S})\times(C_2^\mathrm{M}+C_2^\mathrm{S}) - (C_4^\mathrm{M}+C_4^\mathrm{S})\times(B_2^\mathrm{M}+B_2^\mathrm{S}) \tag{8.5}$$

$$A_6 = (C_4^\mathrm{M}+C_4^\mathrm{S})\times B_1^\mathrm{M} - (B_4^\mathrm{M}+B_4^\mathrm{S})\times C_1^\mathrm{M}. \tag{8.6}$$

The functions B_i^M and C_i^M are parts of the potential due to the Moon. They are expressed as follows:

$$B_0^\mathrm{M} = \left(\frac{k_\mathrm{M}}{2}\right)\left(\frac{a_\mathrm{M}}{r_\mathrm{M}}\right)^3(-1+3\sin^2\beta_\mathrm{M}) \tag{9.1}$$

$$B_1^\mathrm{M} = k_\mathrm{M}\left(\frac{a_\mathrm{M}}{r_\mathrm{M}}\right)^3\sin\beta_\mathrm{M}\cos\beta_\mathrm{M}\cos(\lambda_\mathrm{M}-h) \tag{9.2}$$

$$B_2^\mathrm{M} = k_\mathrm{M}\left(\frac{a_\mathrm{M}}{r_\mathrm{M}}\right)^3\cos^2\beta_\mathrm{M}\sin 2(\lambda_\mathrm{M}-h) \tag{9.3}$$

$$B_3^\mathrm{M} = k_\mathrm{M}\left(\frac{a_\mathrm{M}}{r_\mathrm{M}}\right)^3\sin\beta_\mathrm{M}\cos\beta_\mathrm{M}\sin(\lambda_\mathrm{M}-h) \tag{9.4}$$

$$B_4^\mathrm{M} = k_\mathrm{M}\left(\frac{a_\mathrm{M}}{r_\mathrm{M}}\right)^3\cos^2\beta_\mathrm{M}\cos 2(\lambda_\mathrm{M}-h) \tag{9.5}$$

$$C_0^\mathrm{M} = \int B_0^\mathrm{M}\mathrm{d}t \tag{9.6}$$

$$C_1^\mathrm{M} = \int B_1^\mathrm{M}\mathrm{d}t \tag{9.7}$$

$$C_2^\mathrm{M} = \int B_2^\mathrm{M}\mathrm{d}t \tag{9.8}$$

$$C_3^\mathrm{M} = \int B_3^\mathrm{M}\mathrm{d}t \tag{9.9}$$

$$C_4^\mathrm{M} = \int B_4^\mathrm{M}\mathrm{d}t. \tag{9.10}$$

The functions B_i^S and B_i^S are the corresponding parts of the potential due to the Sun, by taking into account: $\sin\beta_\mathrm{S} = 0$

$$B_0^\mathrm{S} = -\left(\frac{k_\mathrm{S}}{2}\right)\left(\frac{a_\mathrm{S}}{r_\mathrm{S}}\right)^3 \tag{10.1}$$

$$B_2^\mathrm{S} = k_\mathrm{S}\left(\frac{a_\mathrm{S}}{r_\mathrm{S}}\right)^3\sin 2(\lambda_\mathrm{S}-h) \tag{10.2}$$

$$B_4^\mathrm{S} = k_\mathrm{S}\left(\frac{a_\mathrm{S}}{r_\mathrm{S}}\right)^3\cos 2(\lambda_\mathrm{S}-h) \tag{10.3}$$

$$C_0^\mathrm{S} = \int B_0^\mathrm{S}\mathrm{d}t \tag{10.4}$$

$$C_2^{\rm S} = \int B_2^{\rm S} {\rm d}t \quad (10.5)$$

$$C_4^{\rm S} = \int B_4^{\rm S} {\rm d}t \quad (10.6)$$

$k_{\rm M}$ and $k_{\rm S}$ are the scaling factors used to compute the nutation, their expression is (Kinoshita 1977):

$$k_{\rm M} = 3 \times H_{\rm d} \times \Big(\frac{M_{\rm M}}{M_{\rm M}+M_{\rm E}}\Big) \times \Big(\frac{n_{\rm M}^2}{\omega_{\rm E}}\Big) \quad (11.1)$$

$$k_{\rm S} = 3 \times H_{\rm d} \times \Big(\frac{M_{\rm S}}{M_{\rm M}+M_{\rm S}+M_{\rm E}}\Big) \times \Big(\frac{n_{\rm S}^2}{\omega_{\rm E}}\Big) \quad (11.2)$$

where $M_{\rm S}$, $M_{\rm E}$ and $M_{\rm M}$ are respectively the mass of the Sun, the Earth and the Moon and $\omega_{\rm E}$ the angular speed of rotation of the Earth. $n_{\rm M}$ and $n_{\rm S}$ are respectively the relative mean motions of the Moon and of the Sun. Souchay & Kinoshita (1996) calculated the values of $k_{\rm M}$ and $k_{\rm S}$ by choosing an up-to-date value of the general precession in longitude from which they depend directly, by a relationship explained in detail by Kinoshita & Souchay (1990). We keep these values, that is to say: $k_{\rm M} = 7546.71733''$/J cy and: $k_{\rm S} = 3475.41352''$/J cy. The coefficients of nutation coming from W_2 as given by Eqs. (6) to (10) are determined by the following formula:

$$\begin{aligned}
\Delta\psi_{\rm cr}^{W2} &= -\Delta h = -\frac{\partial W_2^{\rm cr}}{\partial H} = \Big[\frac{1}{G\sin I}\Big]\frac{\partial W_2^{\rm cr}}{\partial I} \\
&= \Big[\frac{\cos^3 I - 2\cos I \sin^2 I}{\sin I}\Big] \times \int A_1 {\rm d}t \\
&+ \Big[\frac{2\cos^2 I - \sin^2 I}{2}\Big] \times \int A_2 {\rm d}t \\
&- \Big[\frac{\sin I \cos 2I + 2\cos I \sin 2I}{\sin I}\Big] \times \int A_3 {\rm d}t \\
&+ \Big[\frac{\cos I \cos 2I - 2\sin I \sin 2I}{2\sin I}\Big] \times \int A_4 {\rm d}t \\
&+ \Big[\frac{2\cos^2 I - \sin^2 I}{4}\Big] \times \int A_5 {\rm d}t \\
&+ \Big[\frac{\cos^3 I - 2\cos I \sin^2 I}{2\sin I}\Big] \times \int A_6 {\rm d}t \quad (12)
\end{aligned}$$

$$\begin{aligned}
\Delta\varepsilon_{\rm cr}^{W2} &= -\Delta I = -\Big[\frac{1}{G\sin I}\Big]\frac{\partial W_2}{\partial h} \\
&= -\frac{\cos^2 I}{2}\Big[\frac{\partial A_1}{\partial h}\Big] - \frac{\sin 2I}{8}\Big[\frac{\partial A_2}{\partial h}\Big] - \frac{\cos I \cos 2I}{2\sin I}\Big[\frac{\partial A_3}{\partial h}\Big] \\
&- \frac{\cos 2I}{2}\Big[\frac{\partial A_4}{\partial h}\Big] - \frac{\cos^2 I}{2}\Big[\frac{\partial A_5}{\partial h}\Big] - \frac{\sin 2I}{16}\Big[\frac{\partial A_6}{\partial h}\Big]. \quad (13)
\end{aligned}$$

The expressions $\Delta\psi_{\rm cr.}$ and $\Delta\varepsilon_{\rm cr.}$ which characterize the crossed-nutation effect are given by (Kinoshita & Souchay 1990):

$$\Delta\psi_{\rm cr} = \Delta\psi_{\rm cr}^{W2} - \frac{1}{2}\Big\{\frac{\partial W_1}{\partial H}, W_1\Big\} \quad (14.1)$$

$$\Delta\varepsilon_{\rm cr.} = \Delta\varepsilon_{\rm cr}^{W2} - \frac{1}{2G\sin I}\Big\{W_1, \frac{\partial W_1}{\partial h}\Big\} \quad (14.2)$$

which gives, after development:

$$\Delta\psi_{\rm cr} = \Delta\psi_{\rm cr}^{W2} - \frac{1}{2}\Big[\Big(\Delta\varepsilon \times \frac{\partial(\Delta\psi)}{\partial I}\Big) + \Big(\Delta\psi \times \frac{\partial(\Delta\psi)}{\partial h}\Big)\Big] \quad (15.1)$$

$$\Delta\varepsilon_{\rm cr} = \Delta\varepsilon_{\rm cr}^{W2} + \frac{1}{2}\Big[\Big(\Delta\varepsilon \times \frac{\partial(\Delta\varepsilon)}{\partial I}\Big) + \Big(\Delta\psi \times \frac{\partial(\Delta\varepsilon)}{\partial h}\Big) - \cot\varepsilon(\Delta\varepsilon)^2\Big] \quad (15.2)$$

$\Delta\psi$ and $\Delta\varepsilon$ are the nutations at the first order given by the basic relationships:

$$\Delta\psi = -\Delta h = \frac{1}{G\sin I}\frac{\partial W_1}{\partial I} \quad (16.1)$$

$$\Delta\varepsilon = -\Delta I = -\frac{1}{G\sin I}\frac{\partial W_1}{\partial h} \quad (16.2)$$

where W_1 is calculated by the intermediary of Eqs. (2) and (3).

For the computations relative to the present effect, we catch all the coefficients up to a relative 10^{-8} with respect to the largest term, in the expressions $B_i^{\rm M}$, $B_i^{\rm S}$, $C_i^{\rm M}$ and $C_i^{\rm S}$ which are used in Eqs. (8.1) to (8.6) and (9.1) to (9.10). In a similar way, we take all the coefficients of nutation in $\Delta\psi$ and $\Delta\varepsilon$ larger than 0.1 μas for the combinations inside (15.1) and (15.2), which is quite enough if we want to reach the 0.1 μas level for the resulting coefficients. These computations are made with the Broucke (1980) subroutines for manipulation of Fourier series, which are the same we used previously for the reconstruction of the theory of nutation for a rigid Earth model (Kinoshita & Souchay 1990).

The results are listed in Table 1. This table constitutes a big improvement with respect to previous computations carried out manually (Zhu & Groten 1989; Kinoshita & Souchay 1990) by picking up only the largest coefficients in the potential. It seems that it is much more difficult to select these terms with a theory of nutation based on the classical equation for the angular momentum (Bretagnon et al. 1997; Roosbeek & Dehant 1997), because there is no clear method to separate them from the spin-orbit effect described in the next chapter.

We can remark also that no less than 68 coefficients are present above the 0.1 μas level, which demonstrates its rather big influence. The number of coefficients is still 24 for $\Delta\psi$, and 20 for $\Delta\varepsilon$ up to 1 μas. The by far largest

Table 1. List of the coefficients of rigid Earth nutation coming from the crossed-nutation effects

l_M	l_S	F	D	Ω	Period day	$\Delta\psi$ sin μas	$\Delta\varepsilon$ cos μas
0	−2	2	−2	3	−6810.493	−0.4	
0	0	0	0	1	−6798.384	28.8	−27.7
0	2	−2	2	−1	−6786.317	−0.2	−1.2
0	0	0	0	2	−3399.192	1220.6	−238.1
0	0	0	0	3	−2266.128	−21.6	4.2
0	0	0	0	4	−1699.596	0.1	
−2	0	2	0	2	1615.748	−0.3	0.1
−2	0	2	0	0	1095.175	−0.2	
0	1	0	0	2	409.234	0.2	
0	1	0	0	1	385.998	−1.0	−0.4
0	−1	2	−2	3	385.959	−1.9	0.5
0	1	0	0	0	365.260	−1.0	0.2
0	0	1	−1	1	365.242	−0.2	
0	−1	2	−2	2	365.225	1.5	−1.7
0	1	0	0	−1	346.636	−1.4	−0.2
0	0	1	−1	0	346.620	−0.4	
0	−1	2	−2	1	346.604	1.5	−0.8
2	0	0	−2	−1	199.840	−0.1	
0	0	2	−2	4	192.989	−1.4	0.3
0	0	2	−2	3	187.662	117.7	−17.3
0	0	2	−2	2	182.621	−1.7	0.3
0	0	2	−2	1	177.844	−92.8	73.1
0	0	2	−2	0	173.310	0.4	−1.0
0	1	2	−2	3	123.969	4.6	−1.0
0	1	2	−2	2	121.749	−0.6	0.2
0	−1	4	−4	4	121.745	−0.1	
0	1	2	−2	1	119.607	−3.6	3.8
0	0	4	−4	4	91.311	4.3	−0.8
0	1	4	−4	4	73.049	0.3	
1	0	−2	2	−2	32.451	0.1	
−1	0	0	2	2	32.112	−0.1	
−1	0	0	2	1	31.961	−0.2	0.2
−1	0	0	2	0	31.812	−0.3	
−1	0	0	2	−1	31.664	−0.2	−0.2
−1	0	0	2	−2	31.517	−0.1	
1	0	0	0	2	27.780	−0.8	0.2
1	0	0	0	1	27.667	−0.6	
1	0	0	0	0	27.555	−0.8	
1	0	0	0	−1	27.443	−0.7	
1	0	0	0	−2	27.333	−0.8	−0.2
−1	0	2	0	3	27.201	−1.1	0.3
−1	0	2	0	2	27.093	−0.3	
−1	0	2	0	1	26.985	0.8	−0.4
1	0	2	−2	3	24.027	−0.3	
1	0	2	−2	2	23.942	−0.1	
1	0	2	−2	1	23.858	0.1	
0	0	0	2	1	14.797	−0.2	
0	0	0	2	0	14.765	−1.3	0.8
0	0	2	0	4	13.716	−0.2	
1	0	1	0	0	13.691	0.3	
0	0	2	0	3	13.688	19.0	−2.8
0	0	2	0	2	13.661	4.8	−0.9
0	0	2	0	1	13.633	−15.2	12.0
0	0	2	0	0	13.606	2.1	0.5
0	0	4	−2	4	12.710	1.4	−0.3
0	0	4	−2	3	12.686	0.3	
1	0	0	2	0	9.614	0.2	0.1
−1	0	2	2	3	9.570	0.6	−0.1
−1	0	2	2	1	9.543	−0.3	0.2
1	0	2	0	3	9.145	2.4	−0.6
1	0	2	0	2	9.133	0.6	−0.1
1	0	2	0	1	9.121	−1.9	1.5
1	0	4	−2	4	8.698	0.2	
0	0	2	2	3	7.103	0.3	
0	0	2	2	1	7.088	−0.2	0.1
2	0	2	0	3	6.866	0.3	
2	0	2	0	1	6.852	−0.2	0.1
0	0	4	0	4	6.830	0.1	

component with argument 2Ω and amplitude 1.2206 μas results naturally from the interactions between the nutations of the leading component at the first order with argument Ω. It was already calculated for the first time by Kinoshita & Souchay (1990). As it was not taken into account before, this explained the big 9.3 y signature when comparing the previous analytical nutation (Kinoshita 1977) with numerical integration (Schastok et al. 1989), which disappeared after the reconstruction of the theory, as was shown by Souchay & Kinoshita (1991). Notice also the 2 coefficients with amplitude 117.7 μas and -92.8 μas in longitude (respectively -17.3 μas and 73.1 μas in obliquity) around the semi-annual period, and the 2 coefficients with amplitude 19.0 μas and -15.2 μas in longitude (respectively -2.8 μas and 12.0 μas in obliquity) around the fortnightly period. At last we can also remark the clustering of coefficients around these two fundamental periods.

3. The spin-orbit coupling effect

In this chapter our aim is to calculate the coefficients of the nutation related to the spin-orbit effect with a better accuracy than previously (Kinoshita & Souchay 1990), and by picking up all the coefficients larger than 0.1 μas. Kubo (1982) showed that the Earth flattening is perturbing the orbital motion of the Moon, and this perturbation itself is modifying the motion of nutation of the Earth. The determination of the perturbation due to this reciprocical influence can be tackled when considering the global Earth-Moon system, not the system formed by the Earth itself, as it is the case in classical theories not involving the Hamiltonian (Woolard 1953). Kinoshita & Souchay (1990) included this effect in their second-order calculations involving the Delaunay canonical angular variables l', g' and h', and action variables L', G' and H'. l' is the mean anomaly of the Moon, g' is the argument of the perigee and h' is the longitude of the node, with respect to the ecliptic. The action variables have the following expressions:

$$L' = \left[\frac{M_E M_M}{M_E + M_M}\right] \times \sqrt{\mu a_M} \tag{17.1}$$

$$G' = L' \times \sqrt{1 - e_M^2} \tag{17.2}$$

$$H' = G' \cos I_M. \tag{17.3}$$

For the calculations to be achieved properly, the spherical coordinates r_M, λ_M and β_M must be replaced by their expressions in function of the canonical variables in the Eq. (3) giving the expression of the potential. β_M is related to the canonical variables H' and G' by the intermediary of the $I_{\rm M}$ variable which represents the inclination of the Moon'sorbit on the ecliptic. We have:

$$\sin\beta_{\rm M} = \sin I_{\rm M}\sin(f'+g'). \quad (18)$$

Moreover $\lambda_{\rm M}$ is the sum of the three angular variables:

$$\lambda_{\rm M} = f'+g'+h' \quad (19)$$

f' being the true anomaly of the Moon. For reasons of commodity, the indices M will be omitted, in the following, concerning the variables $a_{\rm M}$, $r_{\rm M}$ and $e_{\rm M}$. By substituing the values of $\sin\beta_{\rm M}$ and $\lambda_{\rm M}$ in equation (3) we find the following development for $U_{1,{\rm M}}$, which is the expression of the lunar potential at the first order:

$$U_{1,M} = k_{\rm M}G\Big(\frac{a}{r}\Big)^3 \times \Bigg[\Bigg(\frac{1-3\cos^2 I}{16}\Bigg)\times\Bigg[-\cos 2I_{\rm M}-\frac{1}{3}$$

$$+(1-\cos 2I_{\rm M})\cos 2(f'+g')\Bigg] - \Bigg(\frac{\sin 2I}{2}\Bigg)$$

$$\times\Bigg[\Bigg(\frac{\sin 2I_{\rm M}}{4}\Bigg)\cos(h'-h)$$

$$+\Big(\frac{\sin I_{\rm M}}{4}-\frac{\sin 2I_{\rm M}}{8}\Big)\cos(2f'+2g'-h'+h)$$

$$-\Big(\frac{\sin I_{\rm M}}{4}+\frac{\sin 2I_{\rm M}}{8}\Big)\cos(2f'+2g'+h'-h)\Bigg]$$

$$-\Bigg(\frac{\sin^2 I}{4}\Bigg)\times\Bigg[\Bigg(\frac{\sin^2 I_{\rm M}}{2}\Bigg)\cos(2h'-2h)$$

$$+\Bigg(\frac{(1+\cos I_{\rm M})^2}{4}\Bigg)\cos(2f'+2g'+2h'-2h)$$

$$+\Bigg(\frac{(1-\cos I_{\rm M})^2}{4}\Bigg)\cos(2f'+2g'-2h'+2h)\Bigg]. \quad (20)$$

Where $\left(\frac{a}{r}\right)^3$ and f' are themselves a function of the mean anomaly of the Moon l' and of the eccentricity $e_{\rm M}$:

$$\Big(\frac{a}{r}\Big)^3 = 1+\frac{3e^2}{2}+3e\cos l'+\Big(\frac{9e^2}{2}\Big)\cos 2l'+... \quad (21)$$

$$f' = l'+2e\sin l'+\big(\frac{5e^2}{4}\big)\sin 2l'+... \quad (22)$$

The second-order potential $W_2^{\rm cp}$ characterizing the spin-orbit coupling effect has the same expression as in (1), but by substituting the Delaunay's variables h' and H' to the Andoyer's variables h and H:

$$W_2^{\rm cpl.}=\Bigg(\frac{1}{2}\int\Big[\frac{\partial(U_{1,\rm per})}{\partial h'}\times\frac{\partial(W_1)}{\partial H'}-\frac{\partial(U_{1,\rm per})}{\partial H'}\times\frac{\partial(W_1)}{\partial h'}\Big]{\rm d}t\Bigg)_{\rm per}$$

$$+\Bigg(\frac{1}{2}\int\Big[\frac{\partial(U_{1,\rm per})}{\partial g'}\times\frac{\partial(W_1)}{\partial G'}-\frac{\partial(U_{1,per})}{\partial G'}\times\frac{\partial(W_1)}{\partial g'}\Big]{\rm d}t\Bigg)_{\rm per}$$

$$+\Bigg(\frac{1}{2}\int\Big[\frac{\partial(U_{1,per})}{\partial l'}\times\frac{\partial(W_1)}{\partial L'}-\frac{\partial(U_{1,\rm per})}{\partial L'}\times\frac{\partial(W_1)}{\partial l'}\Big]{\rm d}t\Bigg)_{\rm per}. \quad (23)$$

Using the Eqs. (17.1-3) we obtain the derivative of a given function with respect to L', G' and H' starting from its derivatives with respect to a, e and $I_{\rm M}$:

$$\frac{\partial[...]}{\partial L'} = \frac{2L'}{\mu}\times\frac{\partial[...]}{\partial a}+\frac{(1-e^2)}{eL'}\times\frac{\partial[...]}{\partial e} \quad (24)$$

$$\frac{\partial[...]}{\partial G'} = \frac{-\sqrt{1-e^2}}{L'e}\times\frac{\partial[...]}{\partial e}+\frac{\cot I_{\rm M}}{G'}\frac{\partial[...]}{\partial I_{\rm M}} \quad (25)$$

$$\frac{\partial[...]}{\partial H'} = -\frac{1}{G'\sin I_{\rm M}}\times\frac{\partial[...]}{\partial I_{\rm M}}. \quad (26)$$

Because of the expected relative smallness of the nutation coefficients coming from the spin-orbit coupling effect, we can initially restrict ourselves to the leading terms of the potential as given by (20). Practically we can keep the components which remain large enough after integration, that is to say those whose the product of the amplitude and of the inverse of the frequency are the largest ones. As a result of the procedure, we retain in fact 6 terms with the argument $l_{\rm M}$, Ω, 2Ω, $2F+\Omega$, $2F+2\Omega$ and $l_{\rm M}+2F+2\Omega$ ($l_{\rm M}$ is the mean anomaly of the Moon, Ω the mean longitude of the node, and F is given by: $F = L_{\rm M}-\Omega$, where $L_{\rm m}$ is the mean longitude of the Moon).

To have an idea of their respective values, we can refer to the tables of the potential listed in Kinoshita (1977). Each of these components can be represented as a product $H_i(I, I_{\rm M}, e_{\rm M})\times U_i(l', g', h', h)$. This makes the calculations easier for $H_i (i = 1, 6)$ depends only on the canonical action variables, whereas $U_i(l', g', h', h) (i = 1, 6)$ only depends on the angle canonical variables. We can thus adopt for the potential the following development:

$$U = k_{\rm M}G\sum_{i=1}^{6} H_i(I, I_{\rm M}, e)\times u_i(l', g', h', h). \quad (27)$$

With:

$$\begin{aligned}
H_1 &= \Big(\frac{3\cos^2 I - 1}{16}\Big)(1+3\cos 2I_{\rm m})e \\
H_2 &= -\frac{\sin 2I \sin 2I_{\rm M}}{8} \\
H_3 &= \frac{\sin 2I(\sin 2I_{\rm M} + 2\sin I_{\rm M})}{16} \\
H_4 &= -\frac{\sin^2 I \sin^2 I_{\rm M}}{8} \\
H_5 &= -\frac{\sin^2 I(1+\cos I_{\rm M})^2}{16} \\
H_6 &= -\Big(\frac{7\sin^2 I}{32}\Big)(1+\cos I_{\rm M})^2 e
\end{aligned} \tag{28}$$

$$\begin{aligned}
u_1 &= \cos l' \\
u_2 &= \cos(h'-h) \\
u_3 &= \cos(2l'+2g'+h'-h) \\
u_4 &= \cos(2h'-2h) \\
u_5 &= \cos(2l'+2g'+2h'-2h) \\
u_6 &= \cos(3l'+2g'+2h'-2h).
\end{aligned} \tag{29}$$

By combining the Eqs. (23), (24), (25) and (26) with the help of the form given by (27), (28) and (29), then we can get a rather straightforward final expression for the second-order determing function $W_2^{\rm cp}$ related to the coupling effect that we are dealing with here, that is to say:

$$\begin{aligned}
W_2^{\rm cp} = &-\Big(\frac{k_{\rm M}^2 G^2}{2G'\sin I_{\rm M}}\Big)\times A_1 + \Big(\frac{k_{\rm M}^2 G^2 \cot I_{\rm M}}{2G'}\Big)\times A_2 \\
&+\Big(\frac{k_{\rm M}^2 G^2}{32L'}\Big)\times(3\cos^2 I-1)\times\left[\frac{\sqrt{1-e^2}}{e_{\rm M}}\right]\times[1+3\cos 2I_{\rm M}] \\
&\times A_3 \\
&+\Big(\frac{k_{\rm M}^2 G^2}{32L'}\Big)\times(3\cos^2 I-1)\times\left[\frac{1-e^2}{e_{\rm M}}\right]\times[1+3\cos 2I_{\rm M}] \\
&\times A_4 \\
&+\Big(\frac{3k_{\rm M}^2 G^2}{\sqrt{\mu a}}\Big)\times A_5.
\end{aligned} \tag{30}$$

With the following developments for A_i:

$$\begin{aligned}
A_1 &= \sum_{i=1}^{6}\sum_{j=1}^{6} H_i \frac{\partial H_j}{\partial I_{\rm M}} \times \Big[\Big(\frac{\partial u_i}{\partial h'}\Big) w_j - u_j\Big(\frac{\partial W_i}{\partial h'}\Big)\Big] \\
A_2 &= \sum_{i=1}^{6}\sum_{j=1}^{6} H_i \Big(\frac{\partial H_j}{\partial I_{\rm M}}\Big) \times \Big[\Big(\frac{\partial u_i}{\partial g'}\Big) w_j - u_j\Big(\frac{\partial w_i}{\partial g'}\Big)\Big] \\
A_3 &= \sum_{i=1}^{6} H_i \Big[\Big(\frac{\partial w_i}{\partial g'}\Big)\cos l' - \Big(\frac{\partial u_i}{\partial g'}\Big)\frac{\sin l'}{\dot{l}'}\Big] \\
A_4 &= \sum_{i=1}^{6} H_i \Big[\Big(\frac{\partial u_i}{\partial l'}\Big)\frac{\sin l'}{\dot{l}'} - \Big(\frac{\partial w_i}{\partial l'}\Big)\cos l'\Big] \\
A_5 &= \sum_{i=1,6}^{j=1,6} H_i H_j \times \Big[u_i\Big(\frac{\partial w_j}{\partial l'}\Big) - w_j\Big(\frac{\partial u_i}{\partial l'}\Big)\Big]
\end{aligned} \tag{31}$$

where w_i $(i = 1, 6)$ is obtained with a simple integration of u_i:

$$\begin{aligned}
w_1 &= \frac{\sin l'}{\dot{l}'} \\
w_2 &= \frac{\sin(h'-h)}{(\dot{h}'-\dot{h})} \\
w_3 &= \frac{\sin(2l'+2g'+h'-h)}{(2\dot{l}'+2\dot{g}'+2\dot{h}'-2\dot{h})} \\
w_4 &= \frac{\sin(2h'-2h)}{(2\dot{h}'-2\dot{h})} \\
w_5 &= \frac{\sin(2l'+2g'+2h'-2h)}{(2\dot{l}'+2\dot{g}'+2\dot{h}'-2\dot{h})} \\
w_6 &= \frac{\sin(3l'+2g'+2h'-2h)}{(3\dot{l}'+2\dot{g}'+2\dot{h}'-2\dot{h})}\cdot
\end{aligned} \tag{32}$$

Then the nutations in longitude $\Delta\psi_{\rm cp}^{W_2}$ and $\Delta\varepsilon_{\rm cp}^{W_2}$ coming from $W_2^{\rm cp}$ are given by:

$$\Delta\psi_{\rm cp}^{W_2} = -\Delta h = \Big(\frac{1}{G\sin I}\Big)\frac{\partial W_2^{\rm cp}}{\partial I} \tag{33}$$

and:

$$\Delta\varepsilon_{\rm cp}^{W_2} = -\Delta I = -\Big[\frac{1}{G\sin I}\Big]\frac{\partial W_2^{\rm cp}}{\partial h}\cdot \tag{34}$$

The expressions $\Delta\psi_{\rm cp}$ and $\Delta\varepsilon_{\rm cp}$ which characterize the total spin-orbit coupling effect are then given by (Kinoshita & Souchay 1990):

$$\Delta\psi_{\rm cp} = \Delta\psi_{\rm cp}^{W_2} - \frac{1}{2}\left\{\frac{\partial W_1}{\partial H}, W_1\right\}_{\rm cp} \tag{35.1}$$

$$\Delta\varepsilon_{\rm cp} = \Delta\varepsilon_{\rm cp}^{W2} - \frac{1}{2G\sin I}\left\{W_1, \frac{\partial W_1}{\partial h}\right\}_{\rm cp} \tag{35.2}$$

we insist on the fact that as long as we dealt with crossed-nutation, for instance in (14.1) and (14.2) the Poisson brackets $\{...\}_{\rm cr}$ were calculated with respect to the Andoyer canonical variables l, g, and h. In this section which concerns the coupling effect, the Poisson brackets $\{...\}_{\rm cp}$ are calculated with respect to the Delaunay canonical variables l', g' and h'. It is also important to keep in mind that the derivatives with respect to a in the u_i's and the w_i's (where a is the semi-major axis for the keplerian motion) is not 0, for the coefficient $k_{\rm M}$ in the expression of the potential $U_{1,{\rm M}}$ in Eq. (20) contains $(a^3)^{-1}$ at the denominator. Then these derivatives have to be taken into account when calculating the derivatives with respect to L', according to (24). This explains the presence of the coefficient A_5 in (30) and (31).

Let us now introduce the following quantities:

$$K_i = -\Big(\frac{1}{\sin I}\Big)\frac{\partial H_i}{\partial I},$$

and

$$z_i = \left(\frac{\partial w_i}{\partial h}\right)$$

that is to say:

$$K_1 = \left(\frac{3e\cos I}{8}\right) \times (1+3\cos 2I_{\rm M}) \quad (36.1)$$

$$K_2 = -\left(\frac{\cos 2I}{4\sin I}\right) \times \sin 2I_{\rm M} \quad (36.2)$$

$$K_3 = \left(\frac{\cos 2I}{8\sin I}\right) \times (\sin 2I_{\rm M} + 2\sin I_{\rm M}) \quad (36.3)$$

$$K_4 = \left(\frac{\cos I}{4}\right) \times \sin^2 I_{\rm M} \quad (36.4)$$

$$K_5 = \left(\frac{\cos I}{8}\right) \times (1+\cos I_{\rm M})^2 \quad (36.5)$$

$$K_6 = \left(\frac{7e\cos I}{16}\right) \times (1+\cos I_{\rm M})^2 \quad (36.6)$$

and:

$$z_1 = 0 \quad (37.1)$$

$$z_2 = -\frac{\cos(h'-h)}{(h'-h)} \quad (37.2)$$

$$z_3 = -\frac{\cos(2l'+2g'+h'-h)}{(2l'+2g'+h'-h)} \quad (37.3)$$

$$z_4 = -\frac{\cos(2h'-2h)}{(h'-h)} \quad (37.4)$$

$$z_5 = -\frac{\cos(2l'+2g'+2h'-2h)}{(2l'+2g'+2h'-2h)} \quad (37.5)$$

$$z_6 = -2\frac{\cos(3l'+2g'+2h'-2h)}{(3l'+2g'+2h'-2h)}. \quad (37.6)$$

Then the complementary term of the nutation in longitude, which corresponds to the part inside the Poisson brackets in (35.1), is given by:

$$\begin{aligned}
\Delta\psi_{\rm cp}^{\rm comp} &= -\frac{1}{2}\left\{\frac{\partial W_1}{\partial H}, W_1\right\}_{\rm cp} \\
&= \left(\frac{k_{\rm M}^2 G}{4G'\sin I_{\rm M}}\right) \times \sum_{i=1}^{6}\sum_{j=1}^{6}\left[K_i\frac{\partial H_j}{\partial I_{\rm M}} - H_i\frac{\partial K_j}{\partial I_{\rm M}}\right] \times \left(\frac{\partial w_i}{\partial h'}\times w_j\right) \\
&-\left(\frac{k_{\rm M}^2 \cot I_{\rm M}}{2G'}\right)\sum_{i=1}^{6}\sum_{j=1}^{6}\left[K_i\frac{\partial H_j}{\partial I_{\rm M}} - H_i\frac{\partial K_j}{\partial I_{\rm M}}\right] \times \left(\frac{\partial w_i}{\partial g'}\times w_j\right) \\
&+\left(\frac{k_{\rm M}^2 G}{2L'}\right)\times\left(\frac{\sqrt{1-e_{\rm M}^2}}{e_{\rm M}^2}\right)\sum_{i=1}^{6}(K_iH_1 - K_1H_i)\times\frac{\sin l'}{l'}\times\left(\frac{\partial w_i}{\partial g'}\right) \\
&-\frac{k_{\rm M}^2 G}{2L'}\times\left(\frac{1-e_{\rm M}^2}{e_{\rm M}^2}\right)\sum_{i=1}^{6}(K_iH_1 - K_1H_i)\times\frac{\sin l'}{l'}\times\left(\frac{\partial w_i}{\partial l'}\right) \\
&-\frac{3k_{\rm M}^2 G}{L'}\times\sum_{i=1}^{6}\sum_{j=1}^{6}\left[H_iK_j - H_jK_i\right]\times\left(\frac{\partial w_i}{\partial l'}\right)w_j. \quad (38)
\end{aligned}$$

And the complementary term of the nutation in obliquity in (28.2) can be expressed in a similar way by:

$$\begin{aligned}
\Delta\varepsilon_{\rm cp}^{\rm comp} &= \left[\frac{1}{2G\sin I}\right]\left\{W_1, \frac{\partial W_1}{\partial h}\right\}_{\rm cp} \\
&= \left(\frac{k_{\rm M}^2 G}{2G'\sin I_{\rm M}\sin I}\right)\times\sum_{i=1}^{6}\sum_{j=1}^{6}H_j\frac{\partial H_i}{\partial I_{\rm M}} \\
&\times\left[w_i\left(\frac{\partial z_j}{\partial h'}\right) - z_i\left(\frac{\partial w_j}{\partial h'}\right)\right] + \left(\frac{k_{\rm M}^2 G\cot I_{\rm M}}{2G'\sin I}\right) \\
&\times\sum_{i=1}^{6}\sum_{j=1}^{6}H_i\frac{\partial H_j}{\partial I_{\rm M}}\times\left[z_j\left(\frac{\partial w_i}{\partial g'}\right) - w_j\left(\frac{\partial z_i}{\partial g'}\right)\right] \\
&+\left(\frac{k_{\rm M}^2 G}{2L'\sin I}\right)\times\left(\frac{\sqrt{1-e_{\rm M}^2}}{e_{\rm M}}\right)\times\left(\frac{3\cos^2 I - 1}{16}\right) \\
&(1+3\cos 2I_{\rm M})\times\sum_{i=1}^{6}H_i\left(\frac{\sin l'}{l'}\right)\left(\frac{\partial z_i}{\partial g'}\right) \\
&-\left(\frac{k_{\rm M}^2 G}{2L'\sin I}\right)\times\left(\frac{1-e_{\rm M}^2}{e_{\rm M}}\right)\times\left(\frac{3\cos^2 I - 1}{16}\right)\times \\
&(1+3\cos 2I_{\rm M})\times\sum_{i=1}^{6}H_i\left(\frac{\sin l'}{l'}\right)\left(\frac{\partial z_i}{\partial l'}\right) \\
&+\left(\frac{3k_{\rm M}^2 G}{2L'\sin I}\right)\times\sum_{i=1}^{6}\sum_{j=1}^{6}\left[w_i\frac{\partial z_j}{\partial l'} - z_j\frac{\partial w_i}{\partial l'}\right]. \quad (39)
\end{aligned}$$

For our present computations, the parameter $\frac{G}{G'}$ is necessary. It represents the ratio of the spin angular-momentum of the Earth to the orbital angular momentum of the Moon, and can be expressed as follows:

$$\frac{G}{G'} = \frac{J_2}{H_{\rm d}}\times\left(\frac{M_{\rm E}+M_{\rm M}}{M_{\rm M}}\right)\times\left(\frac{\omega}{n}\right)\left(\frac{a_{\rm E}^2}{a_{\rm M}^2}\right)\times\frac{1}{\sqrt{1-e_{\rm M}^2}}. \quad (40)$$

Its value is: $\frac{G}{G'} = 0.206971306$.

Table 2. List of the coefficients of rigid Earth nutation coming from the spin-orbit coupling effects

$l_{\rm M}$	$l_{\rm S}$	F	D	Ω	Period day	$\Delta\psi$ sin μas	$\Delta\varepsilon$ cos μas
0	0	0	0	1	−6798.384	−463.8	123.2
0	0	0	0	3	−2266.128	−0.4	0.2
1	0	0	0	1	27.667	1.2	−0.3
1	0	0	0	−1	27.443	−0.7	0.1
0	0	2	0	3	13.688	−8.0	2.6
0	0	2	0	2	13.661	11.8	−7.5
0	0	2	0	1	13.633	4.5	−2.0
0	0	2	0	0	13.606	0.8	−4.6
0	0	2	0	0	13.579	−0.2	0.0
1	0	2	0	3	9.145	−1.0	−0.3
1	0	2	0	1	9.121	0.7	−0.1

The results related to the spin-orbit effect as studied here are listed in Table 2. We can remark that the number of coefficients down to 0.1 μas is much smaller than what was found in the previous section for the crossed-nutation contribution. Also we can remark that the leading coefficient is by far the 18.6y Ω component, both in longitude and in obliquity, with respective in-phase values of -463.8 μas and 133.9 μas. The analytical expressions for these two leading terms are given by the rather cumbersome following formulas:

$$
\begin{aligned}
(\Delta\psi)^{\rm cp}_{\Omega} &= \left(\frac{k^2_{\rm M} G \sin\Omega}{4G' \sin I_{\rm M}}\right)\\
&\times \left[\left(K_2\frac{\partial H_4}{\partial I_{\rm M}} - H_2\frac{\partial K_4}{\partial I_{\rm M}}\right) - 2\left(K_4\frac{\partial H_2}{\partial I_{\rm M}} - H_4\frac{\partial K_2}{\partial I_{\rm M}}\right)\right]\\
&\times \left[\frac{1}{2(\dot h' - \dot h)^2}\right]\\
&+ \left[\left(K_3\frac{\partial H_5}{I_{\rm M}} - H_3\frac{\partial K_5}{\partial I_{\rm M}}\right) - 2\left(K_5\frac{\partial H_3}{I_{\rm M}} - H_5\frac{\partial K_3}{\partial I_{\rm M}}\right)\right]\\
&\times \left[\frac{1}{2(\dot l' + \dot g' + \dot h' - \dot h)(2\dot l' + 2\dot g' + \dot h' - \dot h)}\right]\Bigg)\\
&- \left(\frac{k^2_{\rm M} \cot I_{\rm M} G \sin\Omega}{4G'}\right) \times \left(K_3\frac{\partial H_5}{\partial I_{\rm M}} - H_3\frac{\partial K_5}{\partial I_{\rm M}}\right)\\
&\times \left[\frac{1}{(\dot l' + \dot g' + \dot h' - \dot h)(2\dot l' + 2\dot g' + \dot h' - \dot h)}\right]\\
&- \left(K_5\frac{\partial H_3}{\partial I_{\rm M}} - H_5\frac{\partial K_3}{\partial I_{\rm M}}\right)\\
&\times \left[\frac{1}{(\dot l' + \dot g' + \dot h' - \dot h)(2\dot l' + 2\dot g' + \dot h' - \dot h)}\right]\Bigg)\\
&- \left(\frac{3k^2_{\rm M} G \sin\Omega}{L'}\right) \times \left[\frac{H_3K_5 - H_5K_3}{(\dot l' + \dot g' + \dot h' - \dot h)(2\dot l' + 2\dot g' + \dot h' - \dot h)}\right]\\
&+ \frac{1}{G\sin I}\left(\frac{\partial W^{\rm cp}_{\Omega}}{\partial I}\right)
\end{aligned}
\tag{41}
$$

and:

$$
\begin{aligned}
(\Delta\varepsilon)^{\rm cp}_{\Omega} &= \left(\frac{k^2_{\rm M} G \cos\Omega}{2G' \sin I_{\rm M} \sin I}\right)\\
&\times \left(\left[\frac{3}{4}\left(H_2\frac{\partial H_4}{\partial I_{\rm M}} + 2H_4\frac{\partial H_2}{\partial I_{\rm M}}\right) \times \frac{1}{(\dot h' - \dot h)^2}\right]\right.\\
&+ \left[\frac{3}{4}\left(H_3\frac{\partial H_5}{\partial I_{\rm M}} + 2H_5\frac{\partial H_3}{\partial I_{\rm M}}\right)\right.\\
&\times \left[\frac{1}{(\dot l' + \dot g' + \dot h' - \dot h)(2\dot l' + 2\dot g' + \dot h' - \dot h)}\right]\Bigg]\Bigg)\\
&- \left(\frac{k^2_{\rm M} \cot I_{\rm M} G \cos\Omega}{2G' \sin I}\right) \times \left[\frac{3}{2}\left(H_3\frac{\partial H_5}{\partial I_{\rm M}} + H_5\frac{\partial H_3}{\partial I_{\rm M}}\right)\right.\\
&\times \left[\frac{1}{(\dot l' + \dot g' + \dot h' - \dot h)(2\dot l' + 2\dot g' + \dot h' - \dot h)}\right]\Bigg]\\
&+ \left(\frac{9k^2_{\rm M} G \cos\Omega}{L' \sin I}\right) \times \left[\frac{H_3H_5}{(\dot l' + \dot g' + \dot h' - \dot h)(2\dot l' + 2\dot g' + \dot h' - \dot h)}\right]\\
&- \frac{1}{G\sin I}\left(\frac{\partial W^{\rm cp}_{\Omega}}{h}\right)
\end{aligned}
\tag{42}
$$

where $W^{\rm cp}_{\Omega}$ itself is expressed as a function of the H_i:

$$
\begin{aligned}
W^{\rm cp}_{\Omega} &= \left(\frac{k^2_{\rm M} G^2 \sin\Omega}{2G' \sin I_{\rm M}}\right)\\
&\times \left(\left[\left(H_2\frac{\partial H_4}{\partial I_{\rm M}} + 2H_4\frac{\partial H_2}{\partial I_{\rm M}}\right) \times \left[\frac{3}{4(\dot h' - \dot h)^2}\right]\right]\right.\\
&+ \left[\left(H_3\frac{\partial H_5}{\partial I_{\rm M}} + 2H_5\frac{\partial H_3}{\partial I_{\rm M}}\right)\right.\\
&\times \left[\frac{(4\dot l' + 4\dot g' + 3\dot h' - 3\dot h)}{4(\dot l' + \dot g' + \dot h' - \dot h)(2\dot l' + 2\dot g' + \dot h' - \dot h)(\dot h' - \dot h)}\right]\Bigg]\Bigg)\\
&- \left(\frac{k^2_{\rm M} \cot I_{\rm M} G^2 \sin\Omega}{2G'}\right) \times \left(\left(H_3\frac{\partial H_5}{\partial I_{\rm M}} + H_5\frac{\partial H_3}{\partial I_{\rm M}}\right)\right.\\
&\times \left[\frac{4\dot l' + 4\dot g' + 3\dot h' - 3\dot h)}{2(\dot l' + \dot g' + \dot h' - \dot h)(2\dot l' + 2\dot g' + \dot h' - \dot h)(\dot h' - \dot h)}\right]\Bigg)\\
&+ \left(\frac{3k^2_{\rm M} G^2 \sin\Omega}{L'}\right) \times (H_3H_5)\left(\left[\frac{1}{(\dot l' + \dot g' + \dot h' - \dot h)(\dot h' - \dot h)}\right]\right.\\
&+ \left.\left[\frac{2}{(2\dot l' + 2\dot g' + \dot h' - \dot h)(\dot h' - \dot h)}\right]\right),
\end{aligned}
\tag{43}
$$

with: $\Omega = h' - h$. In Eqs. (40), (41), and (42), we use the following substitutions, with the help of (27.1-6) and (36.1-6):

$$
\left(K_2\frac{\partial H_4}{\partial I_{\rm M}} - H_2\frac{\partial K_4}{\partial I_{\rm M}}\right) - 2\left(K_4\frac{\partial H_2}{\partial I_{\rm M}} - H_4\frac{\partial K_2}{\partial I_{\rm M}}\right)
$$

$$
= \frac{(1 + 3\cos 2I_{\rm M})\sin^2 I_{\rm M} \sin I}{16} \tag{44.1}
$$

$$
\left(K_3\frac{\partial H_5}{\partial I_{\rm M}} - H_5\frac{\partial K_3}{\partial I_{\rm M}}\right) - 2\left(K_5\frac{\partial H_3}{\partial I_{\rm M}} - H_5\frac{\partial K_3}{\partial I_{\rm M}}\right)
$$

$$
= \left(\cos\frac{I_{\rm M}}{2}\right)^6 \times \frac{(2 - 3\cos I_{\rm M})\sin I}{4} \tag{44.2}
$$

$$
H_3K_5 - H_5K_3 = \frac{1}{4}\left(\cos\frac{I_{\rm M}}{2}\right)^7 \sin\left(\frac{I_{\rm M}}{2}\right)\sin I \tag{44.3}
$$

$$
H_2\frac{\partial H_4}{\partial I_{\rm M}} + 2H_4\frac{\partial H_2}{\partial I_{\rm M}} = \frac{1}{16}(1 + 3\cos 2I_{\rm M})\sin^2 I_{\rm M}\sin^3 I\cos I \tag{44.4}
$$

$$
H_3\frac{\partial H_5}{\partial I_{\rm M}} + 2H_5\frac{\partial H_3}{\partial I_{\rm M}}
$$

$$
= \frac{1}{128}(1 + \cos I_{\rm M})(2 - 3\cos I_{\rm M})
$$

$$
(-5\cos I_{\rm M} - 8\cos 2I_{\rm M} - 3\cos 3I_{\rm M})\sin^3 I\cos I \tag{44.5}
$$

$$H_3\frac{\partial H_5}{\partial I_{\rm M}}+H_5\frac{\partial H_3}{\partial I_{\rm M}}=\frac{1}{128}(1+\cos I_{\rm M})$$

$$(1-2\cos I_{\rm M}-2\cos 3I_{\rm M}-5\cos 2I_{\rm M})\sin^3 I\cos I \tag{44.6}$$

$$H_3H_5=-\frac{1}{8}(\cos\frac{I_{\rm M}}{2})^6\sin I_{\rm M}\sin^3 I\cos I. \tag{44.7}$$

Remark that the second largest component in Table 2 with argument $2F+2\Omega$ and with period 13.66d (fortnightly), was not found in previous calculations (Kinoshita & Souchay 1990), because of their incompleteness. On the opposite the two terms of argument $2F+\Omega$ and $2F+3\Omega$ were already present in these calculations. An important part of the present ones were carried out by the help of Mathematica, a precious tool to compute formal analytical expressions.

The numerical values of the constant terms used in the present section are taken from ELP2000 (Chapront-Touzé & Chapront 1988) except for $k_{\rm M}$ (Souchay & Kinoshita 1996) and the ratio $\frac{G}{G'}$ calculated above. They are given as follows:

$$\begin{aligned}
a_{\rm M} &= 384747.981\ \text{km}\\
e_{\rm M} &= 0.054879905\\
I_{\rm M} &= 5^\circ 7' 47\rlap{.}''40623\\
\dot\Omega &= \dot h'-\dot h=-337.57045\ \text{rd/1000 yrs}\\
\dot l' &= 83286.914\ \text{rd/1000 yrs}\\
\dot g' &= 1047.747\ \text{rd/1000 yrs}\\
\tfrac{G}{G'} &= 0.2069714\\
\tfrac{G}{L'} &= \tfrac{G}{G'}\times\tfrac{G'}{L'}=\tfrac{G}{G'}\times\sqrt{1-e_{\rm M}^2}=0.2066593\\
k_{\rm M} &= 7546.7173289''/\text{J cy.}
\end{aligned}$$

4. Final values for the 18.6 Y. Leading nutation term

The computation of the 18.6 y leading coefficients of nutation, both in longitude and in obliquity, is rather delicate, so that this chapter is devoted to it. As explained in detail by Williams (1994) and Souchay & Kinoshita (1996), these coefficients are the result of the combination of various contributions, which are, for the in-phase coefficient: the first-order component related to the main problem of the Moon, the crossed-nutation effect, the spin-orbit coupling effect, the planetary- tilt effect.

For the out-of-phase components, contributions come from the planetary-tilt effect (Williams 1994) and from the secular variation of the mean obliquity with respect to the moving ecliptic of the date. This last contribution, clearly pointed out and accurately calulated recently by Bretagnon et al. (1997), was not considered in previous works (Kinoshita & Souchay 1990) although already Kinoshita (1977) mentioned that strictly speaking, in the integration of the potential which serves to give the expression of the determining function in Hamiltonian theory, we shoud have to take into account the secular change of the obliquity $I^\star$ (which is the value of the obliquity when not considering its periodical variations, that is to say the mean obliquity itself including its secular variation). By taking into account this effect, we find a correction for the out-of-phase coefficient in longitude of which matches very well the difference found by Bretagnon et al. (1997) when comparing his value with Souchay & Kinoshita (1997), that is to say 0.250 μas.

The calculation related to the correction above consists in replacing the constant value of $I_0=-\varepsilon_0$ at J2000.0 in the expression of the potential by $I_0+\dot I$ where $\dot I$ is the secular variation of the mean obliquity of the date ($I=-\varepsilon$) with respect to the mean ecliptic of the date. The conventional value of $\dot I$ can be found in Lieske et al. (1977), that is to say: $\dot I=468.150''/1000$ yrs. $=0.0022696$ rd /1000 yrs. Then after integrating the potential and applying the canonical equations which serve to the determination of the coefficients of nutation at the first order (Kinoshita 1977), we find that the ratio of the out-of-phase Ω component with respect to the in-phase one is $\rho_\psi^\Omega=-\left(\frac{\dot I}{\dot\Omega}\right)(2\tan 2I_0+\cot I_0)$ in longitude, and: $\rho_\varepsilon^\Omega=\left(\frac{\dot I}{\dot\Omega}\right)\tan I_0$ in obliquity.

Thus the values in milliarcsecond obtained for the out-of-phase component are $0.5161\cos\Omega$ for $\Delta\psi$ and $0.0267\sin\Omega$ for $\Delta\varepsilon$. Notice that these values are the same as those found Roosbeek & Dehant (1997).

Moreover, in longitude only, the secular variation of the obliquity produces an additional out-of-phase component coming from the expression of the complementary part of the Hamiltonian E (Kinoshita 1977) which is related to the change of canonical variables from the fixed ecliptic of the epoch to the moving ecliptic of the date. E can be written, by using the same notations as in Kinoshita (1977):

$$\begin{aligned}
E &= 2\sin^2\frac{\pi_{\rm A}}{2}\left[H\times\frac{{\rm d}\Pi_{\rm A}}{{\rm d}t}+G\sin I'\sin(-h'+\Pi_{\rm A})\frac{{\rm d}\pi_{\rm A}}{{\rm d}t}\right]\\
&+G\sin I'\left[-\sin h'\frac{\rm d}{{\rm d}t}(\sin\pi_{\rm A}\sin\Pi_{\rm A})\right]\\
&= G\sin I'\left[-\sin h'\frac{\rm d}{{\rm d}t}(\sin\pi_{\rm A}\cos\Pi_{\rm A})+\cos h'\frac{\rm d}{{\rm d}t}(\sin\pi_{\rm A}\sin\Pi_{\rm A})\right]\\
&+o(\pi_{\rm A}^2)\\
&= G\sin I'(-q\sin h'+p\cos h')+o(t).
\end{aligned} \tag{45}$$

Where h' is the combination of the general precession in longitude ($-p_{\rm A}$) and of the nutation in longitude: $h'=-p_{\rm A}-\Delta\psi$. By applying the canonical equations:

$$\frac{{\rm d}\Delta h'}{{\rm d}t}=-\frac{1}{G\sin I'}\frac{\partial E}{\partial I'} \tag{46.1}$$

$$\frac{{\rm d}\Delta I'}{{\rm d}t}=\frac{1}{G\sin I'}\frac{\partial E}{\partial h'}, \tag{46.2}$$

we find the following expressions for the derivatives:

$$\frac{\mathrm{d}\Delta h'}{\mathrm{d}t} = q \cot I_0 \Delta h' \tag{47.1}$$

$$\frac{\mathrm{d}\Delta I'}{\mathrm{d}t} = -p\Delta h' \tag{47.2}$$

where p and q are respectively the linear trends of the functions $\sin \pi_{\mathrm{A}} \sin \Pi_{\mathrm{A}}$ and $\sin \pi_{\mathrm{A}} \cos \Pi_{\mathrm{A}}$, and I_0 is the value of the obliquity ($I_0 = -\varepsilon_0$) at the epoch (J2000.0).

By taking: $q = -46''\!.82$/J cy. and $p = 5.341''$/J cy. integration of equations (47.1) and (47.2) leads to the following values:

$\Delta\psi = -268.1\ \mu\mathrm{as} \cos \Omega\ \Delta\varepsilon = 13.3 \cos \Omega$.

Notice that the results are an out-of-phase component in longitude and an in-phase component in obliquity. The ratio of the out-of-phase term to the in-phase leading term in longitude can be approximated as follows:

$$\rho^{\Omega}_{\psi,\mathrm{b}} \approx \left(\frac{\dot{I}}{\dot{\Omega}}\right) \cot I_0. \tag{48}$$

Notice that the ratio ρ^{Ω}_{ψ} found previously can be split into two parts and one of them is exactly the opposite of the ratio $\rho^{\Omega}_{\psi,\mathrm{b}}$, so that the two terms nearly annulate when mixed together. Then the final ratio in longitude of the out-of-phase component with respect to the in-phase one, due to the secular variation of the obliquity, can be written:

$$\rho^{\Omega}_{\psi} + \rho^{\Omega}_{\psi,\mathrm{b}} \approx -2\left(\frac{\dot{I}}{\dot{\Omega}}\right) \times \tan 2I_0. \tag{49}$$

And its value is: $\Delta\psi = 0.5161 - 0.2681 = 0.2480\ \mu$as. This value matches quite well the difference of 0.250 μas already noticed by Bretagnon et al. (1997) when comparing the value of the out-of-phase component with Souchay & Kinoshita (1996), for the contribution above was not included.

Notice that the second-order tilt-effect (Souchay & Kinoshita 1996) gives 0.1351 μas and -0.0298 μas for the out-of-phase part respectively for $\Delta\psi$ and $\Delta\varepsilon$, so that the final values are respectively 0.3831 μas and -0.0031 μas.

The same remark which leads to the calculation of out-of-phase terms depending on $\dot{I}$ can be done for the other coefficients, but these out-of-phase terms are respectively much smaller, so that even the semi-annual term gives a component smaller than 1 μas, that is to say $-0.4\ \mu\mathrm{as} \cos 2\ L_{\mathrm{S}}$ for $\Delta\psi$ and $-0.5\ \mu\mathrm{as} \sin 2\ L_{\mathrm{S}}$ for $\Delta\varepsilon$, where L_{S} is the mean longitude of the Sun.

In Tables 3.1 and 3.2 we present our final values for the rigid Earth nutation leading 18.6y component, respectively in longitude and in obliquity, with the detailed account of all the effects. The difference with Souchay & Kinoshita (1997) is, in milliarcsecond: $\delta(\Delta\psi) = -0.018 \sin \Omega + 0.248 \cos \Omega$ and: $\delta(\Delta\varepsilon) = 0.015 \cos \Omega + 0.027 \sin \Omega$. The big differences for the out-of-phase components come from the new contribution related to the secular variation of the obliquity, as detailed above.

5. Final tables of nutation REN-2000

The results obtained in the preceding chapters constitute the final step of a reconstruction of the theory of the nutation for a rigid Earth model already begun in two previous papers (Souchay & Kinoshita 1996; Souchay & Kinoshita 1997). This reconstruction, in order to match the remarkable accuracy of recent VLBI and LLR observations is at the level of truncation of 0.1 μas for individual coefficients, both for $\Delta\psi \cos \varepsilon_0$ and $\Delta\varepsilon$. This is 1000 times smaller than the level of truncation of the rigid Earth nutation series which served as a reference for the the UAI1980 conventional nutation (Kinoshita 1977; Seidelmann 1982).

Our series, called REN2000 (REN for Rigid Earth Nutation) are ready and available at this level of truncation. They give the nutation for the axis of angular-momentum, the axis of figure and the axis of rotation, in separate files. The basic plane of reference and the basic point of reference are respectively the mean dynamical ecliptic and the mean equinox of the date. Hamiltonian theory is very well suited to give the nutations with respect to a moving point (equinox of the date) and a moving plane (the mean dynamical ecliptic), for the change of canonical variables caused by their motions with respect to the fixed ecliptic and the fix equinox of the epoch is quite straightforward (Kinoshita 1977).

The corrected value for the general precession in longitude as pointed out by various authors and using different techniques (Herring et al. 1991; Miyamoto & Soma 1993; Steppe et al. 1994; Williams et al. 1993; Charlot et al. 1995) has been taken into account as well as more accurate recent values for the indirect and direct planetary effects (Souchay & Kinoshita 1996, 1997). Notice that for this last effect, a comparaison was done in this last paper with respect to previous results by Williams (1995), which showed a particularly remarkable agreement.

In Table 4, we present the largest Oppolzer terms which characterize the difference between the nutations for the figure axis and for the angular momentum axis, and in Tables 5.1 and 5.2 we give the values of the leading coefficients of rigid Earth nutation for the angular momentum axis and the figure axis, both in longitude and in obliquity. Little changes can be remarked with respect to the values listed by Souchay & Kinoshita (1996). They are due to the improved second-order computations related to the crossed-nutation effect and the coupling effect as described in the precedent chapters and illustrated by Tables 1 and 2.

Our tables REN-2000 include also the semi-diurnal coefficients related to the triaxiality of the Earth (Souchay &

Table 3.1. Summary of the various contributions to the 18.6 y leading coefficient of the rigid Earth nutation in longitude, figure axis

Contribution	in-phase sin (μas) Souchay & Kinoshita (1996)	in-phase sin (μas) this paper	out-of-phase cos (μas) Souchay & Kinoshita (1996)	out-of-phase cos (μas) this paper
Main Problem Moon	−17285.279	−17285.284		
Planetary tilt−effect	−0.032	−0.032	0.135	0.135
Sun	0.008	0.008		
First-order compl.	1.759	1.759		
Coupling effects	−0.433	−0.435		
J_4		0.001		
Sec. var. of obliquity				0.248
Oppolzer	3.403	3.392		
Total	−17280.574	−17280.592	0.135	0.383

Table 3.2. Summary of the various contributions to the 18.6 y leading coefficient of the rigid Earth nutation in obliquity, figure axis

Contribution	in-phase cos μas Souchay & Kinoshita (1996)	in-phase cos μas this paper	out-of-phase sin μas Souchay & Kinoshita (1996)	out-of-phase sin μas this paper
Main Problem Moon	9228.809	9228.809		
Planetary tilt-effect	0.003	0.003	−0.030	−0.030
Sun	−0.004	−0.004		
Coupling effects	0.102	0.095		
J_4	0.007	0.007		
Sec. var. of obliquity		0.013		0.027
Oppolzer	−1.008	−1.005		
Total	9227.902	9227.917	−0.030	−0.003

Kinoshita 1997), as well as the diurnal, semi-diurnal and 1/3 d period nutations related to the tesseral harmonics $C_{3,n}$ and $S_{3,n}$, $(n = 1, 2, 3)$. These last ones were studied in a complementary paper by Folgueira et al. (1998a), by taking the same Hamiltonian method as in the present paper. Notice that the amplitude of these waves reaches 40 μas as was already noticed by Bretagnon et al. (1997). The waves related to the tesseral harmonics $C^4_{m,n}$ and $S^4_{m,n}$ (Folgueira et al. 1998b) are also taken into account, as well as the two waves up to our level of truncation (0.1 μas) for the influence of J_4 (Souchay & Kinoshita 1996).

In order to have a clear insight of various contributions to the rigid Earth nutation, we list in Table 6 the number of coefficients related to each specific effect, up to 0.1 μas. The symbol used in the tables in order to reckon the effect involved, is indicated here. At last, the bibliographic reference from which the coefficients of our tables REN-2000 have been picked up is also given for each specific effect. For a better understanding of the presentation of our tables REN-2000, we give in the following some comments and remarks that we would like to present, with respect to each contribution.

Main problem of the Moon

The major contribution to the nutation, as considering the importance of the effect and of the number of coefficients, comes from the Main Problem of the Moon, that is to say from its orbital motion when being

Table 4. The largest Oppolzer terms (figure axis - ang. momentum axis) in rigid Earth nutation; the symbol "C" indicates that the coefficients is the combination of two contributions (lunar+solar)

l_M	l_S	F	D	Ω	Period day	$\Delta\psi$ sin μas	$\Delta\varepsilon$ cos μas	
0	0	0	0	1	−6798.384	3.3919	−1.0051	
0	0	2	−2	2	182.621	−7.5716	2.7658	C
0	1	2	−2	2	121.749	−0.4449	0.1653	C
1	0	0	0	1	27.667	0.2887	−0.0873	
−1	0	0	0	1	−27.443	0.2702	−0.0783	
−1	0	2	0	2	27.093	0.4730	−0.1740	
1	0	2	−2	2	23.942	0.1242	−0.0456	
0	0	2	0	2	13.661	−17.3648	6.4119	
0	0	2	0	1	13.633	−3.5565	1.1181	
−1	0	2	2	2	9.557	−0.6521	0.2419	
−1	0	2	2	1	9.543	−0.1329	0.0417	
1	0	2	0	2	9.133	−3.4506	1.2808	
1	0	2	0	1	9.121	−0.7028	0.2208	
0	0	2	2	2	7.096	−0.5699	0.2124	
0	0	2	2	1	7.088	−0.1156	0.0368	
2	0	2	0	2	6.859	−0.4747	0.1771	
1	0	2	2	2	5.643	−0.1434	0.0537	

Table 5.1. The largest components of rigid Earth nutation, longitude part, angular momentum axis and figure axis, for epoch J2000.0. When some coefficients are the combination of a lunar and solar contribution, they are mixed together

						Ang. mom.	axis	Figure	axis
l_M	l_S	F	D	Ω	Period day	$\Delta\psi$(sin) μas	$\Delta\psi$(cos) μas	$\Delta\psi$(sin) μas	$\Delta\psi$(cos) μas
0	0	0	0	1	−6798.384	−17283.9972	0.3826	−17280.5921	0.3826
0	0	0	0	2	−3399.192	209.0958	0.0047	209.0296	0.0047
0	1	0	0	0	365.260	125.5027		125.5036	
0	−1	2	−2	2	365.225	21.2480		21.3112	
0	0	2	−2	2	182.621	−1269.9036		−1277.4752	
0	0	2	−2	1	177.844	12.4161		12.5103	
0	1	2	−2	2	121.749	−49.4928		−49.9377	
−1	0	0	2	0	31.812	14.9442		14.9588	
1	0	0	0	1	27.667	5.8206		6.1093	
1	0	0	0	0	27.555	67.6795		67.7677	
1	0	0	0	−1	27.443	5.7020		5.4318	
−1	0	2	0	2	27.093	11.4450		11.9188	
0	0	0	2	0	14.765	6.0158		6.0432	
0	0	2	0	2	13.661	−204.1468		−221.5116	
0	0	2	0	1	13.633	−34.2965		−37.8530	
−1	0	2	2	2	9.557	−5.1941		−5.8462	
1	0	2	0	2	9.133	−26.1325		−29.5831	

Table 5.2. The largest components of rigid Earth nutation, obliquity part, angular momentum axis and figure axis, for epoch J2000.0. when some coefficients are the combination of a lunar and a solar contribution, they are mixed together

						Ang. mom.	axis	Figure	axis
l_M	l_S	F	D	Ω	Period day	$\Delta\varepsilon$(sin) μas	$\Delta\varepsilon$(cos) μas	$\Delta\varepsilon$(sin) μas	$\Delta\varepsilon$(cos) μas
0	0	0	0	1	−6798.384	−0.0031	9228.9220	−0.0031	9227.9170
0	0	0	0	2	−3399.192	0.0032	−90.3611	0.0032	−90.3369
0	1	0	0	0	365.260		0.0013		−0.1349
0	−1	2	−2	2	365.225		−9.1875		−9.2106
0	0	2	−2	2	182.621		550.5690		553.3348
0	0	2	−2	1	177.844		−6.6042		−6.6322
0	1	2	−2	2	121.749		21.4577		21.6200
−1	0	0	2	0	31.812		0.0000		−0.1859
1	0	0	0	1	27.667		−3.1073		−3.1946
1	0	0	0	0	27.555		0.0000		−0.9725
1	0	0	0	−1	27.443		3.0458		2.9675
−1	0	2	0	2	27.093		−4.9625		−5.1365
0	0	0	2	0	14.765		0.0008		−0.1611
0	0	2	0	2	13.661		88.5092		94.9211
0	0	2	0	1	13.633		18.3148		19.4127
−1	0	2	2	2	9.557		2.2519		2.4938
1	0	2	0	2	9.133		11.3290		12.6098

disturbed only by the Sun. It is worthy to notice that the arguments of the terms coming from this effect are expressed in function of the only Delaunay arguments F, D, l_M, l_S and Ω (Chapront-Touzé & Chapront 1988). The related nutation has been calculated with up-to-date analytical Fourier series of Moon coordinates taken in the recent version of ELP-2000 (Chapront-Touzé & Chapront 1988), with a truncature at a relative 10^{-11} for all the Fourier series concerned in the calculations in order to avoid cut-off problems (Souchay & Kinoshita 1996). The Moon's coordinates in ELP2000 are given with respect to the moving ecliptic and equinox of the date, which constitute a precious advantage for our theory is itself established starting from the moving ecliptic and equinox. Therefore we have not further coordinates transformations to carry out, as it would be the case if our basic equations were given in an inertial frame. Concerning the nutation due to the main problem of the Moon at the first-order, we find no significant difference with older calculations (Kinoshita & Souchay 1990), except for the contribution coming from the planetary tilt effect described in detail and calculated by Williams (1994) and recalculated by Souchay & Kinoshita (1996), and which only influences the coefficient with argument Ω and 2Ω. Notice that the number of terms is the same for the three axe in $\Delta\psi$. On the other hand, the number of terms for $\Delta\varepsilon$ is noticeably larger for the figure axis than for the angular momentum axis (486 instead of 432). This is due to the fact that the terms in the potential with argument not containing Ω can give birth to a nutation component in obliquity only for the figure axis, and not the axis of angular momentum.

Indirect planetary effects, Moon's part

These terms are coming from the perturbations exerted by the planets on the orbital motion of the Moon, and consequently on the nutation as computed from the lunar potential. Their argument is systematically expressed in function of the Delaunay's ones (F, D, l_M, l_S and Ω) and a combination of the mean longitudes of the planets expressed with respect to the fixed equinox and ecliptic of the epoch J2000.0. Notice that the coefficients larger than 5 μas resulting from our recalculations have been listed by Souchay & Kinoshita (1996), with some corrections with respect to previous tables (Kinoshita & Souchay 1990). In order to have a qualitative idea of the effect studied here, we show in Figs. 1.1 and 1.2, respectively for a 2 years time span and a 180 years time span, the curve of the indirect planetary effects on the nutations due to the Moon. At high frequencies this curve is dominated by two terms with periods 13.659 d and 13.663 d respective arguments $-l_M + 2F + 2\Omega + 18\lambda_{Ve} - 16\lambda_{Ea}$ and $l_M + 2F + 2\Omega - 18\lambda_{Ve} + 16\lambda_{Ea}$ and the same amplitude 14.1 μas. At low frequency the peak-to-peak amplitude

Table 6. Number of coefficients larger than 0.1 μas included in the tables REN-2000, according to their origin

axis of	$\Delta\psi$ ang. mom.	$\Delta\varepsilon$ ang. mom.	$\Delta\psi$ rot.	$\Delta\varepsilon$ rot.	$\Delta\psi$ fig.	$\Delta\varepsilon$ fig.	symbol	reference
Main Moon	583	432	583	433	583	486	M1	Souchay & Kinoshita (1996)
Sun	275	176	275	176	275	176	S	Souchay & Kinoshita (1996)
2nd. order	62	52	62	54	62	55	T2	This paper
Ind. plan. Moon.	313	217	313	217	313	217	PM	Souchay & Kinoshita (1996)
Direct plan.	233	168	233	168	233	168	ME,VE MA, JU SA,UR	Souchay & Kinoshita (1997)
J_3	16	13	16	13	16	13		Souchay & Kinoshita (1997)
$C_{2,2}$, $S_{2,2}$ (1/2 d)	34	27	34	27	34	27	TR	Souchay & Kinoshita (1997)
$C_{3,1}$, $S_{3,1}$ (1 d)	12	7	12	7	57	38	CS31	Folgueira et al. (1997)
$C_{3,2}$, $S_{3,2}$ (1/2 d)	2	2	2	2	6	3	CS32	Folgueira et al. (1997)
$C_{3,3}$, $S_{3,3}$ (1/3 d)	3	2	3	2	3	3	CS33	Folgueira et al. (1997)
$C_{4,1}$, $S_{4,1}$ (1 d)	1	1	1	1	3	3	CS41	Folgueira et al. (1997)
TOTAL	1534	1097	1534	1100	1585	1189		

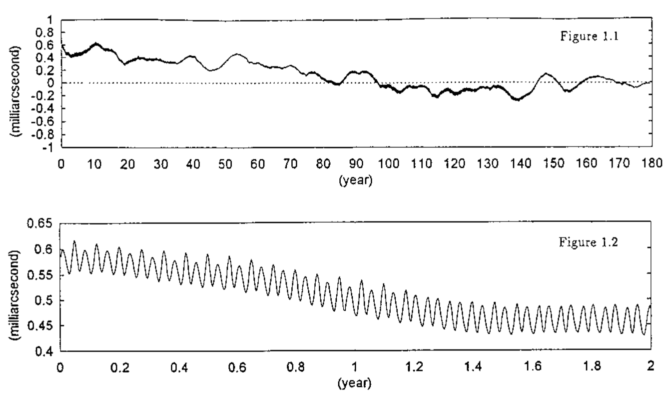

Fig. 1. Indirect planetary effect on the nutation $\Delta\psi$ exerted by the Moon, for a 180 years time span (Fig. 1.1) and a 2 yearts time span (Fig. 1.2)

reaches 1 μas in a little more than 100 years. Similar observations can be done for the nutation in obliquity.

Sun

The contribution of the Sun to the nutation, calculated from VSOP series (Bretagnon & Francou 1988) for the orbital elements of the Earth is characterized by two different kinds of terms: some of them (15 terms for $\Delta\psi$ and 8 for $\Delta\varepsilon$) are expressed as a function of the Delaunay arguments only, as the leading component of argument $2L_S = 2F - 2D + 2\Omega$. The others are expressed as a function of the Delaunay's arguments and of the mean longitudes of the planets. These last terms which are quite numerous (260 terms for $\Delta\psi$ and 168 for $\Delta\varepsilon$) are sometimes quoted as *indirect planetary effect*, but we can observe that this appellation, although being adopted, looks a little wrong, for the planets themselves have some influence in the value of the first category of terms, which are therefore not the product of a simple two-bodies problem.

Only a few significant differences are found for the terms already taken into account previously (Kinoshita & Souchay 1990), despite the new high order of relative truncation of the coefficients (10^{-11}). Souchay & Kinoshita (1996) made the necessary corrections, essentially due to the new value of the general precession in longitude with respect to the IAU conventional value (Lieske et al. 1977), but also to several erroneous values in the tables of Kinoshita & Souchay (1990) as was pointed out by Williams (private communication). Notice that some

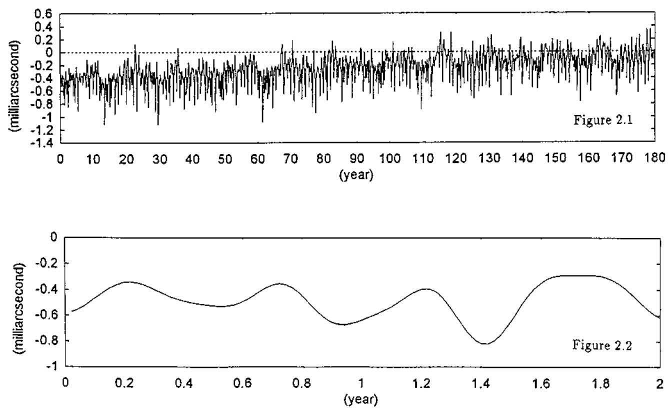

Fig. 2. Indirect planetary effect on the nutation $\Delta\psi$ exerted by the Sun, for a 180 years time span (Fig. 2.1) and a 2 years time span (Fig. 2.2)

of the corrections, reaching roughly the value of 4 μas, originate from the fact that the latitude of the Sun with respect to the moving ecliptic of the date is not rigorously equal to zero (Souchay & Kinoshita 1996). We show for $\Delta\psi$ the indirect planetary effect due to the Sun (following the somewhat improper appellation as explained above), for a short time span (2 years), respectively in Fig. 2.1 and for a long one (100 years) in Fig. 2.2. Notice that the aspect of the curve is quite different than in the case of the Moon (Figs. 1.1 and 1.2) for no significant term is present at less than the half-year period. By contrast the combination of numerous contributing coefficients between 0.5 y and 10 years leads to a very disturbed curve at the scale of one year.

Second-order components

We call second-order components those which involve a second-order effect, but it must be emphasized that in the general case they are the combination of this second-order effect and a first-order contribution, which is often much more larger. The leading 18.6 y coefficient is then for instance ranged into this category. The second-order effects can be divided into two independent parts (orbital coupling and crossed-nutation) which have been studied in detail in the preceding sections. Because of the new truncation limit the number of components undergoing second-order modifications has considerably increased with respect to previous results: 62 terms in $\Delta\psi$ and 52 terms in $\Delta\varepsilon$ compared with only 7 terms both in $\Delta\psi$ and $\Delta\varepsilon$ found by Kinoshita & Souchay (1990). Nevertheless the truncation limit is not the only explanation: some new coefficients are found here but not in this last paper, although their amplitude was up to the old level of truncation, that is to say 5 μas. The reason must be found in the completion of the present calculations. Notice also that crossed-nutation terms can come from the interaction of one nutation component originating in the potential exerted by the Moon and one nutation component originating in the potential exerted by the Sun. In an opposite side, the Sun does not contribute at all to spin-orbit coupling.

Direct planetary effect

This effect has been computed recently at the 0.1 μas level both by Williams (1995) and Souchay & Kinoshita (1997). As it is shown in tables from this last paper, the agreement between the two works is quasi-perfect, despite the fact that the way of calculations is different. For the great majority of coefficients, there is no difference in the amplitude, at the level of truncation above (0.1 μas). Notice that even the influence of Mercury and Uranus has to be taken into account. The major contribution comes from Venus, Mars and Jupiter. Venus itself contributes to 147 coefficients in $\Delta\psi$, that is much more than all the other planets together. The reason must be found both in its proximity to the Earth and therefore in the big number of significant terms in the potential having as argument the combination of the mean longitudes of the two planets (the Earth and Venus). Here also the computations have been done with a relative 10^{-11} truncation of intermediate series. The terms of nutation related to the direct planetary effect in our tables REN-2000 can be easily reckoned by the fact that their arguments is only a fuction of the mean longitudes of the planets. In other words, they are the only terms for which the Delaunay arguments $(l_{\mathrm{M}}, l_{\mathrm{S}}, F, D, \Omega)$ are not present.

Terms due to J_3

The second-order potential J_3 of the Earth gives birth to 17 coefficients in longitude and in obliquity up to 0.1 μas as was listed by Souchay & Kinoshita (1997). It is characterized by a very large set of frequencies, the smallest period (argument $l_{\mathrm{M}} + 3F + 3\Omega$) being 6.8 d, and the largest one (argument $-l_{\mathrm{S}} + F - D + \Omega$) being 20935 y. The leading coefficient is closed to 0.1 μas both in longitude and in obliquity, and has a 8.85 y period, with argument $-l_{\mathrm{M}} + F + \Omega$. Here also the comparison with Hartmann et al. (1995) is quasi-perfect for the biggest difference in amplitude is 0.6 μas, and is less than 0.1 μas for 15 coefficients among the 20 coefficients which have been recognized by these last authors (for their level of truncature is larger).

Terms due to $C_{2,2}$ and $S_{2,2}$.

The coefficients of the geopotential $C_{2,2}$ and $S_{2,2}$ which are characterizing the triaxiality of the Earth, naturally lead to the presence of coefficients of nutation with quasi semi-diurnal period, and their joint effect leads to a beating (Souchay 1993) dominated by 3 coefficients at periods 0.518 d, 0.500 d and 0.499 d, with respective arguments $2\Phi - 2F - 2\Omega$, $2\Phi - 2F + 2D - 2\Omega$ and 2Φ, and respective amplitudes 27.1 μas, 12.5 μas and -37.8 μas in longitude, 11.0 μas, 4.7 μas and 15.0 μas in obliquity. Notice that the largest coefficient originates from both the influence of the Sun and the influence of the Moon. Two points must be emphasized: at first the angle Φ is not the angle of the sidereal rotation of the Earth, for it is shifted by a phase which corresponds to the longitude difference between the prime meridian and the axis of minimum moment of inertia (Bretagnon et al. 1997), which amounts roughly to 15°. This point is detailed in the next chapter. Secondly the coefficients of nutation coming from this effect as they were listed by Kinoshita & Souchay (1990) up to 5 μas and by Souchay & Kinoshita (1997), up to 0.1 μas correspond to the axis of angular momentum. By their semi-diurnal periodic properties, the respective nutation terms for the figure axis have the opposite sign and a very close amplitude (at the present level of truncation), as can be shown from straightforward relationships (Kinoshita 1977).

Thus it can be easily shown that for a given coefficient of the nutation of the angular momentum axis due to the triaxiality, with amplitude ($\Delta\psi_{\rm A.M.}$) and period P in days ($P \approx 0.5$ d), the respective coefficient of nutation will be:

$$(\Delta\psi)_{\rm fig.} = -(\Delta\psi)_{\rm A.M.} + \left[(\Delta\psi)_{\rm A.M.} \times \left(\frac{P_{\rm g} - 2P}{P_{\rm g} - P}\right)\right] \quad (50)$$

where $P_{\rm g}$ is the period of the canonical variable g which represents the angle between the intersection of the plane $\Sigma_{\rm am}$ perpendicular to the angular momentum vector with the ecliptic of the date, and the intersection between $\Sigma_{\rm am}$ and the equator of figure (Kinoshita 1977). The numerical value for $P_{\rm g}$ being: $P_{\rm g} \approx 0.997$ d, we can observe that the second term at the right-hand side of (50) is small in comparison to the first one, and then that the amplitude for $(\Delta\psi)_{\rm fig}$ is close to that of $(\Delta\psi)_{\rm A.M.}$ with the opposite sign. The more P is close to $P_{\rm g}/2$, the more the absolute values of these amplitudes become relatively closer one to each other, for the second term is gradually vanishing.

So it seems necessary to remind of this property, in order to avoid any misunderstanding concerning the tables, where the opposite sign between the values for the angular momentum axis and those for the figure axis, might be interpreted as an error. In Table 7.1, all the coefficients of rigid-Earth nutation in $\Delta\psi$ up to 1 μas due to the triaxiality of the Earth, marked by the symbol $CS22$, have been gathered together with the coefficients coming from the $C_{3,i}$ and $S_{3,i}$ geopotential, which will be studied in the next section. $C_{3,2}$ and $S_{3,2}$ give also birth to quasi semi-diurnal components of nutation, which do not reach the 1 μas level. Table 7.2 is the table corresponding to Table 7.1, in obliquity.

Terms due to $C_{3,i}$ and $S_{3,i}$, i = 1,3.

The terms of nutation related to the geopotential harmonics $C_{3,i}$ and $S_{3,i}$ have been computed for the first time recently, by Bretagnon et al. (1997), and can be ranged into three categories: the quasi-diurnal terms, which originate from $C_{3,1}$ and $S_{3,1}$, the quasi semi-diurnal terms, which originate from $C_{3,2}$ and $S_{3,2}$, and at last the quasi 1/3 d periodic terms, which originate from $C_{3,3}$ and $S_{3,3}$. One of the specific properties of the diurnal terms is that the amplitudes of the angular momentum components is much smaller than the amplitude of the component which is the figure axis counterpart. In other words, the Oppolzer terms are themselves much bigger than the amplitude of the nutation of the angular momentum, which is not the case of all the other nutations, for which it is by one or two orders smaller, when non negligible (except the contribution of $C_{2,2}$ and $S_{2,2}$, for which the order is the same). Folgueira et al. (1998a) calculated the contribution of the $C_{3,i}$'s and $S_{3,i}$'s with the help of Hamiltonian canonical equations, following the same way of calculation as Kinoshita (1977). It can be seen from these calculations that the relatively large amplitude of Oppolzer terms comes from the fact that some components in the determining function W do not contain the variable g at the denominator (g has a nearly diurnal period), so that they keep on remaining big after integration. Here also the agreement for the three kinds of contribution is very good between the results found and Bretagnon et al. (1997), as is shown in a comparative table (Souchay 1997). Notice that the largest diurnal component has an amplitude of 38.5 μas in longitude and 15.2 μas in obliquity, with respective arguments $\Phi - F - \Omega - \tau_{3,1}$ where: $\tau_{3,1} = -S_{3,1}/C_{3,1}$. The characterization of the angle Φ will be given in the chapter 6. Concerning the contribution studied here, the tables in REN-2000 are the same as in Folgueira et al. (1998a) when the phases $\tau_{3,i}, (i = 1, 3)$ have been replaced in each argument by their numerical value so that the nutation terms are split into *sine* and *cosine* components. In Tables 7.1 and 7.2, we show the largest of the nutations from $C_{3,1}$ and $S_{3,1}$, respectively in longitude and in obliquity, indicated by the symbol $CS31$.

Terms due to $C_{4,i}$ and $S_{4,i}$, i = 1,4.

They have been computed by Bretagnon et al. (1997) and by Folgueira et al. (1998b). The coefficients retained in the tables REN-2000 are coming from this last paper. Only 5 components are larger than 0.1 μas (3 for $\Delta\psi$ and 2 for $\Delta\varepsilon$). The leading one can be written, in μas: $\Delta\psi = 1.4 \sin\Phi - 1.2 \cos\Phi$, and: $\Delta\varepsilon = 0.4 \cos\Phi + 0.5 \sin\Phi$ For these terms also the amplitude of the coefficients is much bigger for the figure axis than for the axis of angular momentum.

6. Arguments and constant terms used in REN-2000 series

It seems useful to gather in Table 8 the values of all the constant terms and parameters which have been

Table 7.1. The largest terms of rigid Earth nutation with period $P \leq 2$ d, longitude part, both for the angular momentum axis and the figure axis

							ang. mom.	axis	fig.	axis	origin
ϕ	l_M	l_S	F	D	Ω	Period (day)	$\Delta\psi$ (sin) μas	$\Delta\psi$ (cos) μas	$\Delta\psi$ (sin) μas	$\Delta\psi$ (cos) μas	
1	−1	0	−1	0	−1	1.07545	−0.4	−0.1	−3.3	−0.4	$C_{3,1}$, $S_{3,1}$
1	0	0	−1	0	0	1.03521	−0.1	0.0	−5.3	−0.7	$C_{3,1}$, $S_{3,1}$
1	0	0	−1	0	−1	1.03505	−2.5	−0.3	−35.4	−4.4	$C_{3,1}$, $S_{3,1}$
1	0	0	−1	0	−2	1.03489	0.1	0.1	1.8	0.2	$C_{3,1}$, $S_{3,1}$
1	−1	0	−1	2	−1	1.00243	0.0	0.0	−1.6	−0.2	$C_{3,1}$, $S_{3,1}$
1	1	0	−1	0	0	0.99772	0.0	0.0	−3.1	−0.4	$C_{3,1}$, $S_{3,1}$
1	1	0	−1	0	−1	0.99758	−0.1	0.0	−20.0	−2.5	$C_{3,1}$, $S_{3,1}$
1	1	0	−1	0	−2	0.99743	0.0	0.0	1.1	0.1	$C_{3,1}$, $S_{3,1}$
1	0	0	0	0	0	0.99726	0.0	0.0	−1.2	1.3	$C_{4,1}$, $S_{4,1}$
1	−1	0	1	0	2	0.99711	0.0	0.0	−1.2	−0.2	$C_{3,1}$, $S_{3,1}$
1	−1	0	1	0	1	0.99696	0.1	0.0	24.1	3.0	$C_{3,1}$, $S_{3,1}$
1	−1	0	1	0	0	0.99682	0.0	0.0	4.0	0.5	$C_{3,1}$, $S_{3,1}$
1	−1	1	1	0	1	0.99425	0.0	0.0	1.3	0.1	$C_{3,1}$, $S_{3,1}$
1	1	0	1	−2	1	0.99216	0.0	0.0	−7.1	−0.8	$C_{3,1}$, $S_{3,1}$
1	0	0	1	0	1	0.96215	1.2	0.1	−38.2	−4.7	$C_{3,1}$, $S_{3,1}$
1	0	0	1	0	0	0.96201	0.1	0.0	−6.0	−0.7	$C_{3,1}$, $S_{3,1}$
1	1	0	1	0	1	0.92969	0.2	0.0	−2.9	−0.4	$C_{3,1}$, $S_{3,1}$
1	0	0	3	0	3	0.89884	0.1	0.0	1.1	0.1	$C_{3,1}$, $S_{3,1}$
2	0	0	−3	0	−3	0.52752	4.3	−2.5	−4.9	2.8	$C_{2,2}$, $S_{2,2}$
2	−1	0	−2	0	−2	0.52743	4.5	−2.6	−5.1	2.9	$C_{2,2}$, $S_{2,2}$
2	0	0	−2	0	−1	0.51756	4.3	−2.5	−4.7	2.7	$C_{2,2}$, $S_{2,2}$
2	0	0	−2	0	−2	0.51753	23.5	−13.5	−25.5	14.7	$C_{2,2}$, $S_{2,2}$
2	−1	0	0	0	0	0.50782	−1.7	1.0	1.8	−1.0	$C_{2,2}$, $S_{2,2}$
2	0	0	−2	2	−2	0.50000	10.3	−5.9	−10.5	6.0	$C_{2,2}$, $S_{2,2}$
2	0	0	0	0	0	0.49863	−31.1	17.8	31.3	−18.0	$C_{2,2}$, $S_{2,2}$
2	0	0	0	0	−1	0.49860	−4.3	2.4	4.3	−2.5	$C_{2,2}$, $S_{2,2}$
2	1	0	0	0	1	0.48981	−1.7	1.0	1.7	−1.0	$C_{2,2}$, $S_{2,2}$

used for the construction of the series REN-2000 through the present study and the two other ones (Souchay & Kinoshita 1996, 1997). In fact all those which are related to the lunar potential are taken from the theory ELP-2000 (Chapront-Touzé & Chapront 1988), as the polynomial expressions of the Delaunay's arguments which have to be used when calculating the nutation for a given date. All those which are related to the Sun and to the planets are taken from the theory VSOP82 (Bretagnon 1982), as the mean longitudes of the planets, but it must be noticed that the mean longitude of the Sun L_S is replaced in some cases by the expression in function of the Delaunay's variables, that is to say: $L_S = F - D + \Omega$ as it is was already mentioned in the paragraph above related to the effects of the Sun. This substitution was already adopted in the rigid Earth nutation tables for the IAU conventional nutation theory (Seidelmann 1982).

The corrections which are about to occur concerning the expressions of the arguments and of the values constant terms in the recent past (Simon et al. 1994) or in the future seem to be too small to affect in a significant manner the value of the global nutation. The general precession in longitude constitutes the only exception to this assertion, for it has been shown by Souchay & Kinoshita (1996) that the correction of $-0.3266''$/cy. with respect to the conventional IAU value (Lieske et al. 1977) lead to individual corrections of the coefficients of nutation proportional to their amplitude, reaching 1 mas for the leading term of 18.6y nutation in longitude. Notice that our value for the general precession in longitude p_A at J2000.0 is then $5028''.7700$/cy instead of $5029''.0966$/cy (Lieske et al. 1977).

In our tables REN-2000, the coefficients having the same argument but with different kinds of contributions are not mixed together. They are kept separately in

Table 7.2. The largest terms of rigid Earth nutation with period $P \leq 2$ d, obliquity part, both for the angular momentum axis and the figure axis

							ang. mom.	axis	fig.	axis	origin
ϕ	l_M	l_S	F	D	Ω	Period (day)	$\Delta\varepsilon$ (sin)	$\Delta\varepsilon$ (cos)	$\Delta\varepsilon$ (sin)	$\Delta\varepsilon$ (cos)	
							μas	μas	μas	μas	
1	−1	0	−1	0	−1	1.07545	0.0	0.0	−0.1	1.1	$C_{3,1},\ S_{3,1}$
1	0	0	−1	0	0	1.03521	0.0	0.1	−0.3	2.2	$C_{3,1},\ S_{3,1}$
1	0	0	−1	0	−1	1.03505	0.0	0.1	−1.6	12.9	$C_{3,1},\ S_{3,1}$
1	0	0	−1	0	−2	1.03489	0.0	0.0	0.1	−0.7	$C_{3,1},\ S_{3,1}$
1	−1	0	−1	2	−1	1.00243	0.0	0.0	−0.1	0.6	$C_{3,1},\ S_{3,1}$
1	1	0	−1	0	0	0.99772	0.0	0.0	−0.2	1.2	$C_{3,1},\ S_{3,1}$
1	1	0	−1	0	−1	0.99758	0.0	0.0	−1.0	7.9	$C_{3,1},\ S_{3,1}$
1	1	0	−1	0	−2	0.99743	0.0	0.0	0.1	−0.4	$C_{3,1},\ S_{3,1}$
1	−1	0	1	0	2	0.99711	0.0	0.0	−0.1	0.5	$C_{3,1},\ S_{3,1}$
1	−1	0	1	0	1	0.99696	0.0	0.1	1.2	−9.5	$C_{3,1},\ S_{3,1}$
1	−1	0	1	0	0	0.99682	0.0	0.0	0.2	−1.6	$C_{3,1},\ S_{3,1}$
1	−1	1	1	0	1	0.99425	0.0	0.0	0.1	−0.5	$C_{3,1},\ S_{3,1}$
1	1	0	1	−2	1	0.99216	0.0	0.0	−0.4	2.8	$C_{3,1},\ S_{3,1}$
1	0	0	1	0	1	0.96215	0.1	−0.5	−1.9	15.1	$C_{3,1},\ S_{3,1}$
1	0	0	1	0	0	0.96201	0.0	−0.1	−0.3	2.4	$C_{3,1},\ S_{3,1}$
1	1	0	1	0	1	0.92969	0.0	−0.1	−0.1	1.1	$C_{3,1},\ S_{3,1}$
1	0	0	3	0	3	0.89884	0.0	0.0	0.1	−0.4	$C_{3,1},\ S_{3,1}$
2	−1	0	−2	0	−2	0.52743	−1.1	−1.8	1.2	2.1	$C_{2,2},\ S_{2,2}$
2	0	0	−2	0	−1	0.51756	−1.0	−1.7	1.1	1.9	$C_{2,2},\ S_{2,2}$
2	0	0	−2	0	−2	0.51753	−5.2	−9.1	5.7	9.9	$C_{2,2},\ S_{2,2}$
2	−1	0	0	0	0	0.50782	0.4	0.7	−0.4	−0.7	$C_{2,2},\ S_{2,2}$
2	0	0	−2	2	−2	0.50000	−2.3	−4.1	2.4	4.1	$C_{2,2},\ S_{2,2}$
2	0	0	0	0	0	0.49863	7.1	12.3	−7.1	−12.4	$C_{2,2},\ S_{2,2}$
2	0	0	0	0	−1	0.49860	1.0	1.7	−1.0	−1.7	$C_{2,2},\ S_{2,2}$
2	1	0	0	0	1	0.48981	0.4	0.7	−0.4	−0.7	$C_{2,2},\ S_{2,2}$

order to have a clear insight of the weigh of each contribution. The general presentation is quite similar to the presentation of the tables established from precedent rigid Earth nutation theories (Kinoshita 1977; Kinoshita & Souchay 1990) and of the present conventional tables of the IAU1980 nutation theory. Only the argument Φ has been added, in order to include the quasi-diurnal and subdiurnal nutations.

The definition of the angle Φ is a little subtle, so that we can refer to the Fig. 3 in order to understand it: Φ corresponds to the angle $(l + g + \Delta\Phi_0)$ where (Kinoshita 1977): g is the angle between the equinox of date and the node N between the equator of figure (E_q) and the plane (Σ_am) perpendicular to the angular momentum vector. l is the angle between N and A where A is along the principal axis of the Earth corresponding to the minimum moment of inertia. Note that the the angle J between (E_q) and (Σ_am) is very small (of the order of 10^{-6} rd) so that $(l+g)$ can be considered along (E_q). The phase shift $\Delta\Phi_0$ corresponds to the angle between the Greenwich prime meridian and A, along the equator of figure (E_q). This phase shift explains the presence of out-of-phase components both in longitude and in obliquity. Then we can remark that Φ is the angle of sidereal rotation of the Earth.

Then Φ can be noted as follows, with the same kind of calculations as Bretagnon et al. (1997): $\Phi = 4.89496121282 + 2301216.7526278t$, where t is expressed in rd/1000 y.

Notice that Φ is reckoned from the moving equinox of the date instead of the fixed one so that the secular component is a little different as for these last authors (the difference correponds in fact to the planetary precession χ).

As for the value for the phase $\Delta\Phi_0$, it can be taken from the components $C_{2,2}$ and $S_{2,2}$ of the geopotential: $\Delta\Phi_0 = \frac{1}{2}\ \arctan\left(\frac{S_{2,2}}{C_{2,2}}\right)$. By taking the values of $C_{2,2}$ and $S_{2,2}$ from the the IERS standards (Mc. Carthy 1992), we thus find: $\Delta\Phi_0 = -14.^\circ 928537 = -0.2605521$ rd.

Table 8. Parameters and constant terms used for the construction and the use of the series REN-2000

Argument or Constant term	Value	Units	Origin
$L_{\rm Me}$	$4.402608842 + 26087.903141574 \times t$	rd, 1000 y	VSOP82
$L_{\rm Ve}$	$3.176146697 + 10213.285546211 \times t$	rd, 1000y	VSOP82
$L_{\rm Ea}$	$1.753470314 + 6283.075849991 \times t$	rd, 1000 y	VSOP82
$L_{\rm Ma}$	$6.203480913 + 3340.612426700 \times t$	rd, 1000 y	VSOP82
$L_{\rm Ju}$	$0.599546497 + 529.690962641 \times t$	rd, 1000 y	VSOP82
$L_{\rm Sa}$	$0.874016757 + 213.299104960 \times t$	rd, 1000 y	VSOP82
$L_{\rm Ur}$	$5.481293871 + 74.781598567 \times t$	rd, 1000 y	VSOP82
$l_{\rm M}$	$2.355555898 + 83286.914269554 \times t$	rd, 1000 y	ELP2000
$l_{\rm S}$	$6.24006013 + 6283.01955 \times t$	rd, 1000 y	ELP2000
F	$1.627905234 + 84334.66158131 \times t$	rd, 1000 y	ELP2000
D	$5.198466741 + 77713.771468121 \times t$	rd, 1000 y	ELP2000
Ω	$2.18243920 - 337.57045 \times t$	rd, 1000 y	ELP2000
Φ	$4.894961212 + 2301216.7526278 \times t$	rd, 1000 y	Aoki et al. (1982)
ε	$0.4090928041 - 0.002269655 \times t$	rd, 1000 y	Lieske et al. (1977)
J_2	$-1082.626075\ 10^{-6}$		IERS Standards
J_3	$2.532516\ 10^{-6}$		IERS Standards
J_4	$-1.618563\ 10^{-6}$		IERS Standards
$C_{2,2}$	$1.574410\ 10^{-6}$		IERS Standards
$S_{2,2}$	$-0.903757\ 10^{-6}$		IERS Standards
$C_{3,1}$	$2.190182\ 10^{-6}$		IERS Standards
$S_{3,1}$	$0.269185\ 10^{-6}$		IERS Standards
$C_{3,2}$	$0.308936\ 10^{-6}$		IERS Standards
$S_{3,2}$	$-0.211581\ 10^{-6}$		IERS Standards
$C_{3,3}$	$0.100447\ 10^{-6}$		IERS Standards
$S_{3,3}$	$0.197157\ 10^{-6}$		IERS Standards
$C_{4,1}$	$-0.508638\ 10^{-6}$		IERS Standards
$S_{4,1}$	$-0.449141\ 10^{-6}$		IERS Standards
$M_{\rm Me}/M_{\rm S}$	$0.1660136\ 10^{-6}$		IAU 1976
$M_{\rm Ve}/M_{\rm S}$	$2.4478396\ 10^{-6}$		IAU 1976
$(M_{\rm E} + M_{M})/M_{\rm S}$	$3.0404326\ 10^{-6}$		DE403/LE403
$M_{\rm Ma}/M_{\rm S}$	$3.2271494\ 10^{-6}$		IAU 1976
$M_{\rm Ju}/M_{\rm S}$	$9.5478610\ 10^{-6}$		IAU 1976
$M_{\rm Sa}/M_{\rm S}$	$2.8583679\ 10^{-6}$		IAU 1976
$M_{\rm Ur}/M_{\rm S}$	$4.3727316\ 10^{-6}$		IAU 1976
$M_{\rm M}/(M_{\rm M} + M_{\rm E})$	1.215058210^{-6}		DE403/LE403
$H_{\rm d}$ (dynam. ellipt.)	0.0032737548		Souchay & Kinoshita (1996)
$k_{\rm M}$	$7546.717329''$/cy		Souchay & Kinoshita (1996)
$k_{\rm S}$	$3475.413512''$/cy		Souchay & Kinoshita (1996)

7. Remarks and conclusion

In the present study we have carried out the final step of a complete reconstruction of the tables of nutation for a rigid Earth model REN-2000 (Rigid Earth Nutation), whose one of the main purpose was to catch all the coefficients up to 0.1 μas. We have detailed the calculations and given the coefficients of rigid Earth nutation related to two different kinds of second-order effects, which are the crossed-nutation effects and the spin-orbit coupling effect. We can observe that the Hamiltonian approach used here enabled to separate very easily these two kinds of contributions. Moreover we have given a complete overview of the contributions to the leading 18.6 y nutation with argument Ω, with precise explanations for their origin.

As our results here can be considered as definitive ones following the four other specific studies which lead to the establishment of the tables REN-2000 (Souchay

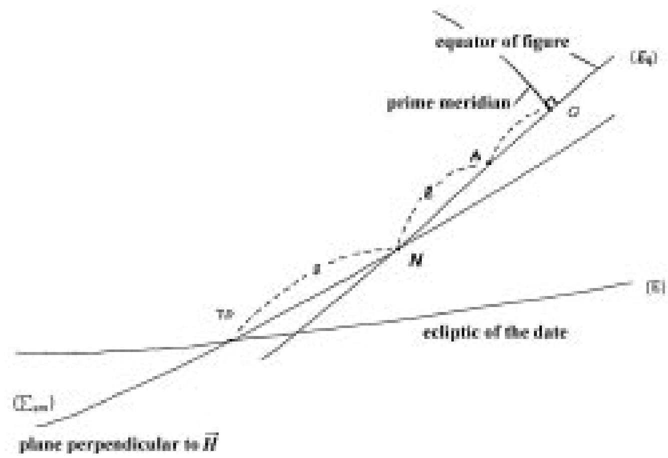

Fig. 3. Variables used to characterize the diurnal and sub-diurnal nutations

& Kinoshita 1996, 1997; Folgueira et al. 1998a; Folgueira et al. 1998b), we have listed here the leading coefficients of rigid Earth nutation for the axis of angular momentum and the axis of figure, with the related Oppolzer terms. Also we have made a final review of all the different contributions to the tables REN-2000, with some explanations on their origin, their way of calculations, their amplitudes and other remarks.

A numerical integration (Souchay 1998) has been done in order to check our analytical results, which is accompanied by a comparison with recent works which have also lead to the establishment of other series of nutation for a rigid Earth model (Bretagnon et al. 1997; Roosbeek & Dehant 1997).

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