QUASICOINCIDENCE FOR INTUITIONISTIC FUZZY POINTS

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The introduction of intuitionistic fuzzy sets is due to K. T. Atanassov, who also proposed some problems about this subject. D. Çoker defined the intuitionistic fuzzy topological spaces and, with some coworkers, studied these spaces. In this paper, we define and study the notion of quasicoincidence for intuitionistic fuzzy points and obtain a characterization of continuity for maps between intuitionistic fuzzy topological spaces

The introduction of "intuitionistic fuzzy sets" is due to Atanassov [1], and this theory has been developed in many papers [2, 3, 4]. This author proposed as an open problem "to investigate the topological and geometric properties of the IFSs" remarking that "some first steps in this direction are made" [4].

In this paper, we define for intuitionistic fuzzy sets the notion of quasicoincidence and the corresponding neighborhood structure (see [9]). These concepts allow us to obtain a characterization of continuity for maps between two intuitionistic fuzzy topological spaces.

(For notions on ordinary fuzzy topology used in this paper, see [7, 8].)

First, we list some previous definitions.

Definition 1 [1]. Let X be a nonempty set. An intuitionistic fuzzy set (IFS) A of X is an object having the form

$$A = \{\langle x, \mu_A, \gamma_A \rangle \mid x \in X\}, \tag{1}$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership of each element $x \in X$ to an ordinary subset of X, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Notation 2. $0_\sim = \langle x, 0, 1 \rangle$ and $1_\sim = \langle x, 1, 0 \rangle$.

Definition 3 [2]. Let X be a nonempty set, and let A and B be two IFSs of X. Then,

(a) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$;

(b) $A = B$ if $A \subseteq B$ and $B \subseteq A$;

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(c) $A \cup B = \{\langle x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B \rangle \mid x \in X\}$;
(d) $A \cap B = \{\langle x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B \rangle \mid x \in X\}$.

Definition 4 [5]. Let $\{A_j \mid j \in J\}$ be an arbitrary family of IFSs of X. Then,
(a) $\bigcap A_j = \{\langle x, \wedge\mu_{Aj}, \vee\gamma_{Aj} \rangle \mid x \in X\}$;
(b) $\bigcup A_j = \{\langle x, \vee\mu_{Aj}, \wedge\gamma_{Aj} \rangle \mid x \in X\}$.
(For other definitions concerning IFSs used in this paper, see [5, 6].)

Definition 5. Let $A = \{\langle x, \mu_A, \gamma_A \rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B, \gamma_B \rangle \mid x \in X\}$ be two IFSs. Say that A quasicoincides with B, denoted by AqB if μ_A quasicoincides with μ_B and γ'_A quasicoincides with γ'_B.

Remark 6. If AqB, we have that $A \cap B \neq 0_\sim$. ($\mu_A q \mu_B$ implies that $\mu_A \wedge \mu_B \neq 0$, then $A \cap B \neq 0_\sim$).

Remark 7. If A and B verify that $\gamma_A = \mu'_A$ and $\gamma_B = \mu'_B$, then AqB if, and only if, $\mu_A q \mu_B$.

Proposition 8. *Let A and B be two IFSs of X, let $f : X \to Y$ be a map between two nonempty sets X and Y, then if AqB, $f(A)qf(B)$.*

Proof. AqB if and only if $\mu_A q \mu_B$ and $\gamma'_A q \gamma'_B$. Then, we have that $f(\mu_A)qf(\mu_B)$ and $f(\gamma'_A)qf(\gamma'_B)$, that is, $(1 - f(\gamma'_A))'q(1 - f(\gamma'_B))'$. Thus $f(A)qf(B)$. □

Proposition 9. *Let X and Y be two nonempty sets, let $f : X \to Y$ be a map, let A be an IFS of X, and let C be an IFS of Y. If $f(A)qC$, $Aqf^{-1}(C)$.*

Proof. $f(A)qC$ if and only if $f(\mu_A)q\mu_C$ and $f(\gamma'_A)q\gamma'_C$. Then, $\mu_A q f^{-1}(\mu_C)$ and $\gamma'_A q f^{-1}(\gamma'_C)$, that is, $\gamma'_A q(f^{-1}(\gamma_C))'$ (because $f^{-1}(\gamma'_C) = f^{-1}(\gamma_C)'$). □

Remark 10. If A, B, and C are IFSs of X, such that AqB, and $B \subseteq C$, then AqC.

Definition 11. Let (X, τ) be an IFTS, and let p be an IFP of X. Say that an IFS N of X is a Q-neighborhood of p if there exists an IFOS A of (X, τ) such that pqA and $A \subseteq N$.

Theorem 12. *Let (X, τ) be an* IFTS, *let p be an* IFP *of X, and let $\mathcal{U}_Q(p)$ be the family of all the Q-neighborhoods of p in (X, τ), then,*
(1) *$N \in \mathcal{U}_Q(p)$ implies that pqN,*
(2) *$N_1, N_2 \in \mathcal{U}_Q(p)$ imply that $N_1 \cap N_2 \in \mathcal{U}_Q(p)$,*
(3) *if $N \in \mathcal{U}_Q(p)$ and $N \subseteq M$, then $M \in \mathcal{U}_Q(p)$,*
(4) *if $N \in \mathcal{U}_Q(p)$, there exists $M \in \mathcal{U}_Q(p)$, $M \subseteq N$, such that, for every* IFP *e which quasicoincides with M, $M \in \mathcal{U}_Q(e)$.*

Proof. (1) $N \in \mathcal{U}_Q(p)$ if and only if there exists an IFOS A such that pqA and $A \subseteq N$, then pqN (by Remark 10).

(2) N_1, N_2 are Q-neighborhoods of p if and only if there exist two IFOSs A_i such that pqA_i, $A_i \subseteq N_i$ $(i = 1,2)$, then, if $p = c(\alpha, \beta)$, we have that $c_\alpha q \mu_{A_i}$, $\mu_{A_i} \leq \mu_{N_i}$, $c_{1-\beta} q \gamma'_{A_i}$, $\gamma_{A_i} \geq \gamma_{N_i}$ $(i = 1,2)$, then $c_\alpha q \mu_{A_1 \cap A_2}$, $\mu_{A_1 \cap A_2} \leq \mu_{N_1 \cap N_2}$, $c_{1-\beta} q \gamma'_{A_1 \cap A_2}$, $\gamma_{A_1 \cap A_2} \geq \gamma_{N_1 \cap N_2}$, and $pq(A_1 \cap A_2)$, $A_1 \cap A_2 \subseteq N_1 \cap N_2$, with $A_1 \cap A_2$ an IFOS.

(3) It is obvious.

(4) $N \in \mathcal{U}_Q(p)$ if and only if there exists an IFOS A such that pqA and $A \subseteq N$, then A is also a Q-neighborhood of p, and for each IFP e such that eqA, A is a Q-neighborhood of e. □

Proposition 13. *Let X be a nonempty set, for each* IFP *p of X, let $\mathcal{U}_Q(p)$ be a family of* IFSs *verifying (1), (2), and (3) of the theorem, then $\tau = \{U\,\text{IFS} \mid U \in \mathcal{U}_Q(p)$ if $pqU\}$ is an* IFT *in X. If also the family verifies (4), then $\mathcal{U}_Q(p)$ is the system of Q-neighborhoods of p in (X,τ).*

Proof. $1_\sim \in \tau$ by (3).

$U_i \in \tau$ $(i = 1,2)$, and $pq(U_1 \cap U_2)$, then $U_i \in \mathcal{U}_Q(p)$ and $U_1 \cap U_2 \in \mathcal{U}_Q(p)$ by (2).

$\{U_j\}_{j\in J} \in \tau$, $pq \cup U_j$ with $p = c(\alpha,\beta)$ if and only if $c_\alpha q\mu_{\cup U_j}$ and $c_{1-\beta}q\gamma'_{\cup U_j}$ and it is equivalent to $c_\alpha q\mu_{\cup U_{j_0}}$ (for some j_0 of J) and $c_{1-\beta}q\gamma'_{\cup U_j}$ (for all j of J). Then pqU_{j_0} for some $j_0 \in J$, $U_{j_0} \in \mathcal{U}_Q(p)$ for some $j_0 \in J$, and $\cup U_j \in \mathcal{U}_Q(p)$ by (3).

Finally, if $N \in \mathcal{U}_Q(p)$, there exists $M \in \mathcal{U}_Q(p)$, $M \subseteq N$, such that for every IFP e which quasicoincides with M, we have that $M \in \mathcal{U}_Q(e)$, then $M \in \tau$, pqM, $M \subseteq N$, and N is a Q-neighborhood of p in (X,τ). Conversely, for every Q-neighborhood N of p in (X,τ), there is an $A \in \tau$ such that pqA, $A \subseteq N$, then for every IFP e which quasicoincides with A, we have that $A \in \mathcal{U}_Q(e)$, thus $N \in \mathcal{U}_Q(p)$. □

Proposition 14. *Let X, Y be two nonempty sets, let $f : X \to Y$ be a map, let τ be an* IFT *in X, and let s be an* IFT *in Y. Then, $f : (X,\tau) \to (Y,s)$ is continuous if, and only if, for each* IFP *p of X, and for each Q-neighborhood V of $f(p)$, there exists a Q-neighborhood U of p such that $f(U) \subseteq V$.*

Proof. If V is a Q-neighborhood of $f(p)$, there exists an IFOS G such that $f(p)qG$ and $G \subseteq V$, then $pqf^{-1}(G)$ (by Proposition 9), and $f^{-1}(G)$ is an IFOS such that $f^{-1}(G) \subseteq f^{-1}(V)$. Thus, $f^{-1}(V)$ is a Q-neighborhood of p and $f(f^{-1}(V)) \subseteq V$.

Conversely, for each $G \in s$, we have that, for every IFP p such that $pqf^{-1}(G)$ is $f(p)qf(f^{-1}(G))$ (by Proposition 8), then $f(p)qG$, and G is a Q-neighborhood of $f(p)$. By the hypothesis, there exists a Q-neighborhood U of p such that $f(U) \subseteq G$, then $U \subseteq f^{-1}(G)$ and $f^{-1}(G) \in \mathcal{U}_Q(p)$. From Proposition 13, it follows that $f^{-1}(G) \in \tau$. □

References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets*, VII ITKR's Session, Sofia, 1983 (Central Sci. Tech. Library, Bulg. Acad. Sci, 1984).

[2] ______, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems **20** (1986), no. 1, 87–96.

[3] ______, *More on intuitionistic fuzzy sets*, Fuzzy Sets and Systems **33** (1989), no. 1, 37–45.

[4] ______, *Intuitionistic Fuzzy Sets. Theory and Applications*, Studies in Fuzziness and Soft Computing, vol. 35, Physica-Verlag, Heidelberg, 1999.

[5] D. Çoker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems **88** (1997), no. 1, 81–89.

[6] D. Çoker and M. Demirci, *On intuitionistic fuzzy points*, Notes IFS **1** (1995), no. 2, 79–84.

[7] Y.-M. Liu and M.-K. Luo, *Fuzzy Topology*, Advances in Fuzzy Systems—Applications and Theory, vol. 9, World Scientific, New Jersey, 1997.

[8] N. Palaniappan, *Fuzzy Topology*, CRC Press, Florida, 2002.

[9] P. M. Pu and Y.-M. Liu, *Fuzzy topology. I. Neighborhood structure of a fuzzy point and Moore-Smith convergence*, J. Math. Anal. Appl. **76** (1980), no. 2, 571–599.

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