Hierarchies of Intensity Preference Aggregations

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ABSTRACT

This paper deals with aggregation of fuzzy individual opinions into a single group opinion, based upon hierarchical intensity aggregation rules. Characterization theorems are given, and it is also shown that Montero's rationality and standard ethical conditions propagate under hierarchical aggregations.

KEYWORDS: *Aggregation rules, fuzzy preferences, group decision-making*

1. INTRODUCTION

In this paper we investigate properties of hierarchical aggregation of intensity preferences, i.e., aggregation of intensity preferences that in turn represent aggregate opinions of group of individuals.

Our results relate to the ongoing research work on axiomatic approaches to group decision-making in a fuzzy environment (see [4, 7, 8, 11]). In particular, this paper is related to the model started in [9, 10] and subsequently characterized in [6]. Arrow's paradox in group decision-making (cf. [1]) has been translated in these papers into a fuzzy context, showing that his negative result can be avoided in several ways. Aggregation of preferences will be obtained here through intensity aggregation rules that will allow the successive aggregation of alternatives. In particular we will show how this intensity aggregation rules can be combined in an hierarchical fashion and we will study what functional properties are preserved by the hierarchical combinations.

This approach reflects common events in real life. Consider for instance the following two extreme circumstances:

- Each member of a deciding committee expresses her/his opinion on the basis of a personal search of the whole society (usually democratic political parties in nonfederal states claim that they are expressing the best consensus laws for the whole country).
- The society has been divided into disjoint groups, in such a way that each member of the deciding committee represents the aggregated opinion of one of these groups (as in democratic federal states where each state is assumed to define its own aggregated opinion about any issue, to be aggregated with the other states opinions).

In between these two extreme cases, global aggregated preference can be obtained by allowing individuals to influence any a priori fixed partial aggregations (social opinion is an aggregation of nondisjoint subsets, in such a way that some individuals have influence in more than one of these smaller groups). For instance,

- agreements between owners and workers in a firm are aggregations of two different groups that may not be disjoint. Some workers may own a portion of the firm and therefore their opinions have positive weight in both sides of any labor conflict.

In all of the above cases, the goal is to produce a social opinion.

This goal may be reached in many ways. In [2, 3], several measures of individual and group consensus are proposed, analyzed, and then used to generate associated measures of distance to consensus, by considering several common goals of group discussion. In the next section, we will introduce a degree of rationality (of the aggregation method) that in some way can be viewed as a particular goal in consensus procedures, allowing a measure of distance from consistency.

2. PRELIMINARIES

We will analyze hierarchical aggregation of individual fuzzy preferences in the context of the model proposed in [6]. At the basis of such a model are non-absolutely irrational (in the sense of [9, 10]) complete fuzzy preference relations. That is each individual is assumed to be able to express her/his opinion about any possible set of alternatives through some complete fuzzy binary preference relation, allowing an aggregated group opinion in terms of another complete fuzzy preference relation. The above concepts are formalized as follows.

Let $\mu: X \times X \to [0, 1]$ be a fuzzy preference relation over an arbitrary finite set of alternatives X. $\mu(x, y)$ represents the degree to which the

relation *x not worse than y* holds. The completeness hypothesis is expressed by

$$\mu(x, y) + \mu(y, x) \geq 1 \qquad \forall x, y \in X. \tag{2.1}$$

Following [5], completeness is required in order to assure that all individuals consider the set of alternatives on which they are expressing their opinions, feasible and comprehensive.

The values

$$\mu_I(x, y) = \mu(x, y) + \mu(y, x) - 1$$

$$\mu_B(x, y) = \mu(x, y) - \mu_I(x, y)$$

$$\mu_W(x, y) = \mu(y, x) - \mu_I(x, y) \tag{2.2}$$

can be understood, respectively, as the degree to which the two alternatives are indifferent (xIy), the degree of strict preference of x over y, (xBy, *x is better than y*) and the degree of strict preference of y over x (xWy, *x is worse than y*). A cycle of preferences will be defined over chains $G = (x_1 - x_2 - \cdots x_k - x_1)$ of k distinct alternatives as

$$x_1 P_1 x_2 P_2 \cdots x_k P_k x_1$$

where $P_h \in \{W, I, B\}$ for all $h = 1, 2, \ldots, k$. A cycle $x_1 P_1 x_2 P_2 \cdots x_k P_k x_1$ is irrational if either

- $P_h \in \{B, I\}$ for all $h = 1, 2, \ldots, k$ and $B \in \{P_h: h = 1, 2, \ldots, k\}$; or
- $p_h \in \{W, I\}$ for all $h = 1, 2, \ldots, k$ and $W \in \{P_h: h = 1, 2, \ldots, k\}$.

We say that a cycle is rational if it is not irrational. Then, given any fuzzy preference μ over a fixed set of alternatives and a chain of alternatives, we can look for all possible rational cycles of preferences, weigh them in a natural way and assign to the chain a degree of rationality (see [6, 9]). Specifically, this is done as follows. Given a cycle $C \equiv x_1 P_2 \cdots x_k P_k x_1$ where $P_h \in \{B, I, W\}$ for all $h = 1, 2, \ldots, k$, the natural weight associated to C and denoted by $\Delta(C)$ will be

$$\Delta(C) = \Pi_{h=1}^k \mu_{P_h}(x_h, x_{h+1})$$

where $x_{k+1} = x_1$ for convenience.

Therefore, given a chain $G = (x_1 - x_2 - \cdots -x_k - x_1)$ a natural degree of rationality associated to G and denoted by $A_\mu(G)$ can be defined as

$$A_\mu(G) = \sum_{C \in \text{rat.cycles}} \Delta(C).$$

As proven in [6, 9], $A_\mu(G)$ verifies

$$1 - A_\mu(G) = \Pi_{h=1}^k \mu(x_h, x_{h+1})$$

$$+ \Pi_{h=1}^k \mu(x_{h+1}, x_h) - 2\Pi_{h=1}^k \mu_I(x_h, x_{h+1}). \quad (2.3)$$

In view of (2.3), once a finite set of alternatives X has been fixed, rationality can be defined as a fuzzy property $A: \mathscr{P}(X) \rightarrow [0, 1]$ with

$$A(\mu) = \min_G A_\mu(G) \quad (2.4)$$

and where $\mathscr{P}(X)$ is the set of all complete fuzzy preferences.

Once a group of $n \geq 2$ individuals is fixed, we should be able to aggregate their opinions about any set of alternatives in a coherent way. Therefore, in [6] were defined aggregation operations that can take into account any extra alternative x so to properly extend any previous aggregated opinion relative to a collection of alternatives not containing x. The key properties are the standard conditions

- (IIA) Independence of Irrelevant Alternatives: each aggregated preference relation $\mu(x, y)$ depends solely on the values $\mu^i(x, y)$, i.e. on the individual preference intensities of x over y.
- (UD) Unrestricted Domain: the aggregation rule is defined over all possible profiles of fuzzy preferences. Intensity aggregation rules are defined as follows.

DEFINITION 2.1   *An intensity aggregation rule is any mapping $\phi: [0, 1]^n \rightarrow [0, 1]$ which assigns a fuzzy preference intensity to each profile of individual fuzzy preference intensities.*   □

If we only assume (IIA) and (UD), an intensity aggregation rule $\phi: [0, 1]^n \rightarrow [0, 1]$ may depend on the pair of alternatives x, y. Specifically, we may have

$$\phi(\mu^1(x, y), \cdots, \mu^n(x, y)) \neq \phi(\mu^1(w, z), \cdots, \mu^n(w, z))$$

even though $\mu^i(x, y) = \mu^i(w, z)$ for all $i = 1, 2, \ldots, n$.

We assume then the following condition

- (N) *Neutrality*: given any permutation of the set of alternatives π, if $\nu^i(x, y) = \mu^i(\pi(x), \pi(y))$ for all $i = 1, 2, \ldots, n$ and any pair of alternatives x, y, then

$$\phi(\nu^1(x, y), \cdots, \nu^n(x, y)) = \phi(\mu^1(\pi(x), \pi(y)), \cdots, \mu^n(\pi(x), \pi(y)))$$

As a consequence, it is clear that the same intensity aggregation mapping ϕ will be associated to any pair of alternatives and therefore each possible aggregation procedure is characterized by one of these intensity aggregation mappings.

For the time being, we will suppose that conditions IIA, UD, N hold.

Given ϕ and n individuals expressing their opinion on the set of alternatives X, the aggregated preference μ defined on $X \times X$ associated to ϕ is defined as

$$\mu(x, y) = \phi(\mu^1(x, y), \cdots, \mu^n(x, y)) \qquad \forall x, y \in X.$$

Standard ethical conditions may also be imposed on the intensity aggregation rules, among them:

- (NNR) *Non-negative Responsiveness*:

$$\phi(a_1, a_2, \ldots, a_n) \geq \phi(b_1, b_2, \ldots, b_n)$$

if $a_i \geq b_i$ for all $i = 1, 2, \ldots, n$.
- (PR) *Positive Responsiveness*:

$$\phi(a_1, a_2, \ldots, a_n) > \phi(b_1, b_2, \ldots, b_n)$$

if $a_i \geq b_i$ for all $i = 1, 2, \ldots, n$ and there exist $1 \leq j \leq n$ such that $a_j > b_j$.
- (A) *Anonymity*: given any permutation $\pi: \{1, \ldots, n\} \rightarrow \{1, \ldots, n\}$, we have

$$\phi(a_1, a_2, \ldots, a_n) = \phi(a_{\pi(1)}, \ldots, a_{\pi(n)}).$$

- (U) *Unanimity*: if $a_i = a$ for all $i = 1, 2, \ldots, n$, then

$$\phi(a_1, a_2, \ldots, a_n) = a.$$

- (CS) *Citizen Sovereign*: for any given $a \in [0, 1]$ there exists a profile $(a_1, a_2, \ldots, a_n) \in [0, 1]^n$ such that

$$\phi(a_1, a_2, \ldots, a_n) = a.$$

- (ND) *Non-dictatorship*: there is no individual i such that

$$\phi(a_1, a_2, \ldots, a_n) = a_i$$

for any $(a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n) \in [0, 1]^{n-1}$.

Definitions of completeness and rationality can be naturally extended to intensity aggregation rules.

In particular, an intensity aggregation rule is said to be complete if the associated aggregated fuzzy preference is complete for any profile of complete individual preferences.

The fuzzy property of rationality is extended to intensity aggregation rules in the following way.

DEFINITION 2.2 *Given n individuals, an intensity aggregation rule $\phi: [0,1]^n \to [0,1]$ is non-absolutely irrational (NAI), or simply non-irrational, if for any arbitrary finite set of alternatives X, the associated aggregated preference $\mu: X \times X \to [0,1]$ is complete and non-absolutely irrational, i.e., $A(\mu) > 0$, whenever all individuals are complete and non-absolutely irrational themselves, i.e., $A(\mu^i) > 0$ for all $i = 1, 2, \ldots, n$, with $\mu^i: X \times X \to [0,1]$ for all i.*

It is clear that in this way both individual and social opinions are required to belong to the set of Non-absolutely Irrational (NAI) complete fuzzy preference relations. Therefore, we are in fact modifying the unrestricted domain condition.

One characterization of complete intensity aggregations rules is given by the following lemma proven in [6].

LEMMA 2.1 *Given an intensity aggregation rule $\phi: [0,1]^n \to [0,1]$, ϕ is complete if and only if*

$$\phi(a_1, \ldots, a_n) + \phi(b_1, \ldots, b_n) \geq 1$$

whenever $a_i + b_i \geq 1$ for all $i = 1, 2, \ldots, n$. ∎

The main result proven in [6] is the following.

THEOREM 2.1 *Let $\phi: [0,1]^n \to [0,1]$ be a complete intensity aggregation rule verifying condition A and such that $\phi(1, \ldots, 1) = 1$ and $\phi(0, \ldots, 0) = 0$. Then ϕ is NAI if and only if the following conditions hold:*

$$(i) \ if \ a_i + b_i > 1$$

for all $i = 1, 2, \ldots, n$ then $\phi(a_1, \ldots, a_n) + \phi(b_1, \ldots, b_n) > 1$;

$$(ii) \ \phi(a_1, \ldots, a_n) = 1 \ implies \ a_i = 1 \ for \ all \ i = 1, 2, \ldots, n. \quad ∎$$

More specifically, from the proof of theorem 2.1. it can be concluded that in order for a complete intensity aggregation rule to be NAI, conditions (i) and (ii) are sufficient.

In the next section we will introduce our main definition and prove our main results.

3. HIERARCHICAL AGGREGATION RULES

Let us first introduce some useful notation.

By $[i_1, i_1, \ldots, i_n]$ we will denote the ordered list whose first element is i_1, second element is i_2, and so on. $\mathbb{I}$ will denote the empty list and given a list L, $|L|$ will denote the length of the list. Moreover, let $\cdot$ be the classical list concatenation or composition operator. So, given two lists L_1, L_2 with $n = |L_1|$ and $m = |L_2|$, $L = L_1 \cdot L_2$ is the list of length $n + m$ whose first n elements are the elements of L_1 and whose last m elements are the elements of L_2. Moreover, given a list L the notation $j \in L$ has the obvious intended meaning. Finally, given a list L we define the operator $\star$ that produces the set of elements of the list L, i.e. $\star L = \{j: j \in L\}$.

Given a list L and m lists $L_1, \ldots, L_m$ we say that the list $\mathscr{L} = [L_1, \ldots, L_m]$ is a cover of L if the following conditions are verified:

- $L_k \neq \mathbb{I}$ for all $k = 1, 2, \ldots, m$;
- for all $k = 1, 2, \ldots, m$, no two elements of L_k are equal;
- $\star L = \bigcup_{i=1}^{m} \star L_i$.

Given a list of indices $I = [i_1, \ldots, i_m]$ and an intensity aggregation rule ϕ, we introduce the following notation

$$\phi(a_h | h \in I) \equiv \phi(a_{i_1}, \ldots, a_{i_m}).$$

The following is our main definition.

DEFINITION 3.1 *Let I be a finite list of individuals and let $[I_1, \ldots, I_{c_o}]$ be a fixed cover of I. Let $m = |I|$ and $m_k = |I_k|$ for all $k = 1, 2, \ldots, c_0$. A hierarchical aggregation is characterized by a collection $\phi_0, \phi_1, \phi_2, \ldots, \phi_{c_0}$ of intensity aggregation rules with $c_0 \geq 2$ and with $m_k > 1$ for some $1 \leq k \leq c_0$, such that $\phi_k: [0, 1]^{m_k} \to [0, 1]$ for all $k = 0, 1, 2, \ldots, c_0$, in such a way that the composition*

$$\phi \equiv \phi_0(\phi_1, \phi_2, \ldots, \phi_{c_0}): [0, 1]^m \to [0, 1]$$

defined as follows

$$\phi(a_k | k \in I) = \phi_0\Big(\phi_1(a_h | h \in I_1), \ldots, \phi_{c_0}\big(a_h | h \in I_{c_0}\big)\Big)$$

is an intensity aggregation rule.   □

To clarify definition 3.1. consider the following example. Let $I = [1, 2, 3, 4]$ and for $c_0 = 3$ consider the cover $[I_1, I_2, I_3]$ with $I_1 = [1, 2]$, $I_2 = [2, 3, 1]$, $I_3 = [4, 1, 3]$. So given any input (a_1, a_2, a_3, a_4) we have

$$\phi(a_1, a_2, a_3, a_4) = \phi_0(\phi_1(a_1, a_2), \phi_2(a_2, a_3, a_1), \phi_3(a_4, a_1, a_3)).$$

Assuming an a priori fixed cover $[I_1, \ldots, I_{c_0}]$ of I, each hierarchical aggregation is therefore characterized by the composition $\phi_0(\phi_1, \phi_2, \ldots, \phi_{c_0})$. Our main goal is checking if ethical and rational conditions imposed on the basic aggregation maps $\phi_0, \phi_1, \phi_2, \ldots, \phi_{c_0}$, propagate under these compositions, that is, if hierarchical aggregations lead to ethical and rational intensity aggregation rules whenever each basic aggregation rule is ethical and rational.

If a property P that holds for $\phi_0, \phi_1, \phi_2, \ldots, \phi_{c_0}$ holds for $\phi_0(\phi_1, \ldots, \phi_{c_0})$ as well (for any fixed cover) we will say that P propagates under hierarchical aggregation.

As a simple consequence of functional composition many properties propagate under hierarchical aggregation, among them completeness, non-negative responsiveness (NNR), positive responsiveness (PR), unanimity (U), and conditions (i) and (ii) of theorem 2.1.

Our first result is the following.

THEOREM 3.1 *If $\phi_0, \phi_1, \phi_2, \ldots, \phi_{n_0}$ are NAI intensity aggregation rules then $\phi \equiv \phi_0(\phi_1, \phi_2, \ldots, \phi_{n_0})$ is also NAI, i.e., rationality propagates under hierarchical aggregation.*

Proof Let X be a set of alternatives and let $\mu^1, \ldots, \mu^n$ be n NAI individuals. Let us denote by ν the aggregation function associated to ϕ and by ν_k the aggregation function associated to ϕ_k for all $k = 0, 1, 2, \ldots, n_0$. Since all the individuals $\mu^1, \ldots, \mu^n$ are NAI so are $\nu_1, \ldots, \nu_{n_0}$. On the other hand, since ϕ_0 is NAI and in view of definition 2.2, we have that its aggregation function ν_0 defined as

$$\nu_0(x, y) = \phi_0\big(\nu_1(x, y), \ldots, \nu_{n_0}(x, y)\big)$$

is NAI. Therefore, since

$$\nu(x, y) = \nu_0(x, y)$$

we have that ϕ is NAI. ∎

It is easy to observe that anonymity (A) and citizen sovereign (CS) do not in general propagate. Clearly, anonymity propagates in the extreme case in which the cover $[I_1, \ldots, I_{c_0}]$ verifies the condition $I_k = I_h$ for all $h, k = 1, \ldots, c_0$.

Anonymity propagates also in the special case of balanced aggregations, defined as follows.

A hierarchical aggregation $\phi \equiv \phi_0(\phi_1, \ldots, \phi_{c_0})$ with list of indices I and cover $[I_1, \ldots, I_{c_0}]$ is balanced when:
- (b1) $\phi_h = \phi_j$ for all $h, j = 1, 2, \ldots, c_0$;
- (b2) for every permutation π of I there exists a permutation δ of the set of indices $\{1, \ldots, c_0\}$ such that for all $h = 1, 2, \ldots, c_0$,

$$\bigstar I_{\delta(h)} = \{\pi(i) | i \in I_h\}.$$

For instance, consider the hierarchical aggregation $\phi_0(\phi_1, \phi_1, \phi_1)$ with $\phi_1 : [0, 1]^2 \to [0, 1]$ and with the associated list $I = [1, 2, 3]$ and cover $[I_1, I_2, I_3]$ with $I_1 = [2, 1], I_2 = [1, 3], I_3 = [3, 2]$. Given any permutation $\pi : \{1, 2, 3\} \to \{1, 2, 3\}$ it is clear that condition (b2) holds.

The following simple theorem proves our claim.

THEOREM 3.2   *Anonymity propagates under balanced hierarchical aggregations.*

Proof   Let $\phi : [0, 1]^n \to [0, 1]$ be a balanced hierarchical amalgamation. Specifically, let $\phi \equiv \phi_0(\phi_1, \ldots, \phi_1)$ with $\phi_0 : [0, 1]^{c_0} \to [0, 1]$. Let $[I_1, \ldots, I_{c_0}]$ be the cover for $I = [1, 2, \ldots, n]$.

Let us suppose that ϕ_0 and ϕ_1 verify anonymity and let π be a permutation of the set of indices $\{1, 2, \ldots, n\}$. We want to prove that $\phi(a_1, \ldots, a_n) = \phi(a_{\pi(1)}, \ldots, a_{\pi(n)})$.

From the definition of hierarchical aggregation we have

$$\phi(a_{\pi(1)}, \ldots, a_{\pi(n)}) = \phi_0\Big(\phi_1\big(a_{\pi(h)}|h \in I_1\big), \ldots, \phi_1\big(a_{\pi(h)}|h \in I_{c_0}\big)\Big).$$

In view of condition (b2) there exists a permutation δ of the set of indices $\{1, 2, \ldots, c_0\}$ such that for all $h = 1, 2, \ldots, c_0$ we have $\star I_{\delta(h)} = \{\pi(j)|j \in I_h\}$. Therefore, since ϕ_1 verifies anonymity we have that for all $h = 1, \ldots, c_0$,

$$\phi_1\big(a_j|j \in I_h\big) = \phi_1\big(a_j|j \in I_{\delta(h)}\big) = \phi_1\big(a_{\pi(j)}|j \in I_h\big).$$

Thus,

$$\phi_0\Big(\phi_1\big(a_{\pi(h)}|h \in I_1\big), \ldots, \phi_1\big(a_{\pi(h)}|h \in I_{c_0}\big)\Big)$$
$$= \phi_0\Big(\phi_1\big(a_j|j \in I_{\delta(1)}\big), \ldots, \phi_1\big(a_j|j \in I_{\delta(c_0)}\big)\Big).$$

Finally, since ϕ_0 verifies anonymity we can conclude that the theorem is proven.   ∎

Concerning Citizen Sovereign, we observe that any nondecreasing intensity aggregation rule ϕ satisfying CS must be a continuous mapping such that $\phi(1, \ldots, 1) = 1$ and $\phi(0, \ldots, 0) = 0$. Therefore, if $\phi_0, \phi_1, \ldots, \phi_{n_0}$ are nondecreasing and satisfy CS, ϕ is nondecreasing, continuous, and satisfies CS. Continuity is a very important property from a practical point of view. Indeed, jumps in the aggregation rule would imply that small changes in the input may produce big changes in the output, leading to aggregation rules that are not stable, according to the following definition.

DEFINITION 3.2   *An intensity aggregation rule ϕ is stable if there exists a constant K (called stability constant) such that for all $\epsilon > 0$ and for all $i = 1, \ldots, m$,*

$$|\phi(a_1, \ldots, a_{i-1}, a_i + \epsilon, a_{i+1}, \ldots, a_m) - \phi(a_1, \ldots, a_m)| \leq K\epsilon$$

for all $a_1, \ldots, a_m$.   □

The following theorem shows that stability propagates under hierarchical aggregation.

THEOREM 3.3 Let $\phi_0, \phi_1, \ldots, \phi_{c_0}$ be stable intensity aggregation rules. Then $\phi \equiv \phi_0(\phi_1, \ldots, \phi_{c_0})$ is stable.

Proof Let for $k = 0, \ldots, c_0$, K_k be the stability constant of ϕ_k. Following definition 3.1 let I be the set of individuals and $[I_1, \ldots, I_{c_0}]$ a cover of I with I_k associated to ϕ_k for all $k = 1, \ldots, c_0$. Let us show that

$$|\phi(a_1, \ldots, a_{i-1}, a_i + \epsilon, a_{i+1}, \ldots, a_m) - \phi(a_1, \ldots, a_m)| \leq K\epsilon$$

for a certain constant K.

Let us first suppose for simplicity that there exists a unique j such that $i \in I_j$. Therefore, if we put $\alpha_k = \phi_k(a_h | h \in I_k)$ for $k = 1, 2, \ldots, c_0$, we have

$$\phi(a_1, \ldots, a_{i-1}, a_i + \epsilon, a_{i+1}, \ldots, a_c) = \phi_0(\alpha_1, \ldots, \alpha_{j-1}, \alpha'_j, \alpha_{j+1}, \alpha_{c_0}),$$

where from the hypothesys of stability ϕ_j, $|\alpha'_j - \alpha_j| \leq K_j\epsilon$. Therefore, since ϕ_0 is stable we can conclude that

$$|\phi(a_1, \ldots, a_{i-1} a_i + \epsilon, a_{i+1}, \ldots, a_m) - \phi(a_1, \ldots, a_m)| \leq K_0 K_j\epsilon.$$

If on the other hand there exist $j_1, \ldots, j_l$ such that i belongs to all the lists $I_{j_1}, \ldots, I_{j_l}$, reasoning as above we obtain

$$|\phi(a_1, \ldots, a_{i-1}, a_i + \epsilon, a_{i+1}, \ldots, a_m) - \phi(a_1, \ldots, a_m)|$$

$$\leq K_0(K_{j_l} + K_{j_2} + \cdots + K_{j_l})\epsilon,$$

which proves the theorem.   ∎

FINAL COMMENTS

In this paper we have introduced hierarchical aggregation on intensity preference. The practical importance of such aggregation rules is quite clear, since many complex group decision-making procedures are defined as hierarchical aggregations. Therefore, the fact that standard ethical conditions as well as rationality propagate under general conditions is a very significant result. Anonymity, in general, does not propagate, however we must notice that such an ethical condition is related to very specific problems and commonly assumed. Indeed, in many decision-making processes, individuals are not considered all equal but they are weighted according to various factors like for instance, the individual experience and knowledge of the specific subject. As a final remark about the usefulness of

hierarchical aggregations, we recall the propagation of stability. Such a result, apart from its theoretical interest, legitimizes hierarchical aggregations from an applicative point of view, since it guarantees the stability of the whole process once it has been put into practice.

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References

1. Arrow, K. J., *Social Choice and Individual Values*, John Wiley & Sons, Inc., New York, 1964.

2. Bezdek, J. C., Spillman, B., and Spillman, R., Fuzzy relation spaces for group decision theory. *Fuzzy Sets and Systems* 1:225–268, 1978.

3. Bezdek, J. C., Spillman, B., and Spillman, R., Fuzzy relation spaces for group decision theory: an application. *Fuzzy Sets and Systems* 2:5–14, 1978.

4. Cholewa, W., Aggregation of fuzzy opinions: an axiomatic approach. *Fuzzy Sets and Systems* 17:249–258, 1985.

5. Cutello, V., and Montero, J., A model for amalgamation in group decision making. In *NAFIPS 1992*, Puerto Vallarta, Mexico, 1992. Proceedings of the North America Fuzzy Information Processing Society International Conference on Fuzzy Set Theory and Applications.

6. Cutello, V., and Montero, J., A characterization of rational amalgamation operations. *International Journal of Approximate Reasoning* 8(4):325–344, 1993.

7. Fung, L. W., and Fu, K. S., An axiomatic approach to rational decision making in fuzzy environment, in *Fuzzy Sets and their Applications to Cognitive and Decision Processes* (L. A. Zadeh, K. S. Fu, K. Tanaka, and M. Shimura, eds.), Academic Press, 227–256, 1975.

8. Montero, J., A note on Fung-fu's theorem. *Fuzzy Sets and Systems* 17:259–269, 1985.

9. Montero, J., Arrow's theorem under fuzzy rationality. *Behavioral Science*, 267–273, 1987.

10. Montero, J., Social welfare functions in a fuzzy environment. *Kybernetes* 16:241–245, 1987.

11. Ovchinnikov, S., Means and social welfare function in fuzzy binary relation spaces, in *Multiperson Decision Making Using Fuzzy Sets and Possibility Theory* (J. Kacprzyk and M. Fedrizzi, eds.), Kluwer, 143–154, 1990.