Study of the sensibility variations of the earth tide records

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ABSTRACT

The idea is to use the observed time variations of the tidal amplitudes in order to estimate the variations of the calibration coefficients of the tidal records. The amplitude variations are revealed through the so called normalized tidal parameters determined in very short intervals of the records. Then the calibration variations are approximated through a regression analysis. The analysis takes into account the artificial changes of the sensibility.

1. INTRODUCTION

It is a well known and evident fact that the variations of the sensibility can be manifested as variations of the tidal amplitudes. Here we shall use this and develop a technique for the determination of the sensibility/amplitude variations before the tidal analysis. We shall namely use the filtered numbers computed for the analysis and an amplitude factor δ determined through them.

In another paper we intend to develop a method for analysis where the sensibility/amplitude variations can be incorporated in the observational equations.

Let us note that the pure amplitude variations are interesting as possible earthquake and volcano precursors. Our study is orientated towards this heavy problem. First of all we have to be able to establish the total sensibility/amplitude variations. Only after that, through a comparison with the calibration data, it could be possible to separate the amplitude variations.

2. NORMALIZED TIDAL PARAMETERS

The basic equation for the Earth tide data is

$$g(t) = \sum_{j=1}^{m} \delta_j h_j \cos(\phi_j + \omega_j t + \varkappa_j) + D(t) + \varepsilon(t),\ t \in (a,b) \qquad (1)$$

Here $g(t)$ is the ordinate at time t of the observed tidal phenomena, h_j, ϕ_j and ω_j are the theoretical elements of the j-th tide, δ_j and $\varkappa_j$ are the Earth tidal parameters, $D(t)$ is the drift and $\varepsilon(t)$ is the noise, i.e. the observational error at time t. The expression $t\in(a,b)$ means that we dispose by a record of $g(t)$ in a time interval (a,b).

The equation (1) can be written for a set of values $t\in(a,b)$, for example every hour, so that (1) is in fact a system of equations. The object of the tidal analysis is to solve this system and find the estimates $\tilde{\delta}_j$ and $\tilde{\varkappa}_j$ of δ_j and $\varkappa_j$ respectively.

We shall suppose that this is done, i.e. $\tilde{\delta}_j$ and $\varkappa_j$ are obtained by using one or another method of analysis.

Let us see what will happen if we replace

$$h_j \text{ by } \tilde{h}_j = \tilde{\delta}_j h_j \text{ and } \phi_j \text{ by } \tilde{\phi}_j = \phi_j + \tilde{\varkappa}_j. \tag{2}$$

The equations (1) then become

$$g(t) = \sum_{j=1}^{m} \delta_{Nj}\, \tilde{h}_j \cos(\tilde{\phi}_j + \omega_j t + \varkappa_{Nj}) + D(t) + \varepsilon(t),\ t\in(a,b) \tag{3}$$

with new unknowns denoted here by δ_{Nj} and $\varkappa_{Nj}$. We shall call them «normalized parameters» because they have the following properties.

If we solve the system (3) in the same way as (1), i.e. by applying the same method of analysis on the same data for $t\in(a,b)$, we shall get identically the estimates

$$\tilde{\delta}_{N1} = \tilde{\delta}_{N2} = \ldots = \tilde{\delta}_{Nm} = 1 \text{ and } \tilde{\varkappa}_{N1} = \tilde{\varkappa}_{N2} = \ldots = \tilde{\varkappa}_{Nm} = 0. \tag{4}$$

On the basis of this the equations (3) can be replaced by

$$g(t) = \delta_N \sum_{j=1}^{m} \tilde{h}_j \cos(\tilde{\phi}_j + \omega_j t + \varkappa_N) + D(t) + \varepsilon(t),\ t\in(a,b). \tag{5}$$

If we apply once again the same method for analysis for $t\in(a,b)$ we shall get as estimates of δ_N and $\varkappa_n$

$$\tilde{\delta}_N = 1 \text{ and } \tilde{\varkappa}_N = 0. \tag{6}$$

The properties (4) and (6) of the normalized parameters will be accomplished independently on the properties of the noise and the drift, even if there are some systematic errors. However, if we analyze another time interval, for example a subinterval (a_i,b_i) with a central epoch t_i, we shall get an estimate $\tilde{\delta}_N = \tilde{\delta}_N(t_i)$ generally depending on the time due to the noise and all possible errors. If (a_i,b_i) are n subintervals of (a,b) with central epochs t_i, $i = 1,2,\ldots n$, the empirically obtained function $\tilde{\delta}_N(t_i)$ can be used for studying some systematic errors in particular an error coming from a sensibility/amplitude variation.

If there is a constant error in the calibration it will not affect the values of the normalized parameter $\bar{\delta}_N(t_i)$. But, if the calibration coefficient is variable and the variations are not introduced correctly, these kind of systematic errors can be studied through $\bar{\delta}_N(t_i)$.

For example let $C(t_1) < C(t_2)$ are the true values of the calibration coefficients at time t_1 and t_2 respectively. If the difference between $C(t_1)$ and $C(t_2)$ is not taken into account, the expected values of δ_N will be $\delta_N(t_2) < \delta_N(t_1)$ instead of $\delta_N(t_2) = \delta_N(t_1) = 1$.

This could be used for studying the sensibility/amplitude variations through a processing of calibrated data. However, as it will be discussed in the next paragraph, more information about the sensibility can be obtained if we process the data before the calibration.

3. ANALYSIS OF RAW DATA

Let $y(t)$, $t \in (a,b)$, are the raw (non-calibrated) data of a tidal record, i.e. $y(t)$ are ordinates in some conventional units, like *mm* or *mV*, while $g(t)$ are expressed in tidal units, e.g. μgal. The relation between $y(t)$ and $g(t)$ is

$$g(t) = C(t) \cdot y(t) \text{ or } y(t) = R(t) \cdot g(t),\ R(t) = 1/C(t), \tag{7}$$

where $C(t)$ is the calibration coefficient at time t. The coefficient $R(t)$ can be called a response coefficient. It describes how the initial input geophysical signal $g(t)$ is transformed by the instrumentation into the output $y(t)$, namely

input signal $g(t)$	⇒	recording instrumentation e.g. gravimeter, tiltmeter, etc.	⇒	output on a record $y(t)=R(t).g(t)$

The coefficients C and R are considered as functions of t as there can be variations in the sensibility or the amplitudes. They can be represented as

$$R(t) = R_0 + \Delta R(t) \text{ and } C(t) = C_0 + \Delta C(t) \tag{8}$$

where R_0 and C_0 are some constants.

By using (1) we get the equation

$$y(t) = \sum_{j=1}^{m} d_j(t) h_j \cos(\phi_j + \omega_j t + k_j) + D(t) + \varepsilon(t),\ t \in (a,b) \tag{9}$$

where $D(t)$ and $\varepsilon(t)$ are now the drift and the noise in the units of $y(t)$. The amplitude factor δ_j in (1) is here replaced by

$$d_j(t) = \delta_j R(t) \tag{10}$$

while the phase shift remains the same.

We can apply the analysis on the raw data $y(t)$ in the same way as on $g(t)$ for $t \in (a,b)$, by using the equations (9) instead of (1). Then we shall get some estimates $\tilde{d}_j$ as amplitude factors which are a kind of a mean value of $d_j(t)$. It seems very natural to define the constants R_0 and C_0 in relation with $\tilde{d}_j$ as it follows

$$\tilde{d}_j = \delta_j R_0 = \delta_j / C_0 \tag{11}$$

This is not perfectly correct because R_0 and C_0 thus defined are not just the same for $j = 1,2,...m$. However, it can be shown that (11) is justified when $R(t)$ is a linear function, i.e. the error in this expression is of a second order. Then, as we do not pretend for a very high precision, we can afford us to use it.

If we replace similarly to (2)

$$h_j \text{ by } \tilde{h}_j = \tilde{d}_j h_j \text{ and } \phi_j \text{ by } \tilde{\phi}_j = \phi_j + \tilde{k}_j \tag{12}$$

the equations (9) become, like (3),

$$y(t) = \sum_{j=1}^{m} d_{Nj}(t) \tilde{h}_j \cos(\phi_j + \omega_j t + k_{Nj}) + D(t) + \varepsilon(t), \; t \in (a,b) \tag{13}$$

where

$$d_{Nj}(t) = d_j(t) / \tilde{d}_j. \tag{14}$$

Now, if we accept (11) and apply here (10), we get

$$d_{Nj}(t) = \frac{\delta_j R(t)}{\delta_j R_0} = \frac{R(t)}{R_0} = \frac{C_0}{C(t)} = \delta_N(t) \tag{15}$$

where $\delta_N(t)$ is a normalized amplitude factor which depends on t but which does not depend on the index j. That is why we can replace (13) through

$$y(t) = \delta_N(t) \sum_{j=1}^{m} \tilde{h}_j \cos(\tilde{\phi}_j + \omega_j t + k_N) + D(t) + \varepsilon(t). \tag{16}$$

If we analyze, again in the same way, the data $y(t)$ for $t \in (a,b)$ we shall get an estimate of the normalized parameter

$$\tilde{\delta}_N(t_0) = 1, \tag{17}$$

related to the central epoch t_0 of (a,b). If we process the intervals (a_i,b_i) we shall get estimates $\tilde{\delta}_N(t_i)$ depending on t.

According to (15) $\delta_N(t_i)$ will be related with the calibration coefficients. Something more, from here we get a rather strong result that we can compute the calibration coefficient $C(t)$, as well as the response coefficient $R(t)$ at time t through

$$C(t) = C_0 / \tilde{\delta}_N(t), \; R(t) = R_0 \tilde{\delta}_N(t) = \tilde{\delta}_N(t) / C_0 \tag{18}$$

provided we know C_0. Actually, C_0 is not known but we can use, as an approximation, a mean value of the calibration coefficients. Then (18) can be used, with much precaution, to check up some values of $C(t)$ and their relative variations.

4. DETERMINATION OF THE NORMALIZED TIDAL PARAMETERS FROM VERY SHORT TIME INTERVALS

The replacement of the equations (1) by (5) as well as (9) by (16) has the following important advantage. Through this we take into account the differences within the tidal parameters for the different tidal waves. The consequences are: (i) we have only one couple of unknowns, namely δ_N and $\varkappa_N$ and (ii) we can estimate such a limited number of unknowns from very short tidal records.

If the method for analysis (Venedikov, 1966, Melchior and Venedikov, 1968, Venedikov, 1984, see also Melchior, 1978) is used, the original record is first subdivided into intervals of 48 hours or another length of this order, then these intervals are filtered. As the intervals (a_i, b_i) used above we can take just these filtered intervals.

For such an interval, for a given tidal group, for example for SD, we obtain one couple of filtered numbers, $U(t_i)$ and $V(t_i)$, by applying respectively an even and an odd filter. The equations (1) are transformed in the following way:

$$U(t) = \sum_{j=1}^{m} \delta_j c_j h_j \cos(\phi_j + \omega_j t + \varkappa_j) + \varepsilon_1(t),$$
$$V(t) = \sum_{j=1}^{m} \delta_j s_j h_j \sin(\phi_j + \omega_j t + \varkappa_j) + \varepsilon_2(t) \qquad (19)$$

The effect of the filters is: the drift is eliminated, the noise is transformed into $\varepsilon_1(t)$ and $\varepsilon_2(t)$ and the tides are multiplied by the amplifying factors of the filters c_j and s_j.

Without repeating the details, by using the normalization already described, we can get the following equations

$$U(t) = \delta_N(t) \sum_{j=1}^{m} c_j \bar{h}_j \cos(\bar{\phi}_j + \omega_j t + \varkappa_N) + \varepsilon_1(t) \text{ and}$$
$$V(t) = \delta_N(t) \sum_{j=1}^{m} s_j \bar{h}_j \sin(\bar{\phi}_j + \omega_j t + \varkappa_N) + \varepsilon_2(t). \qquad (20)$$

For a given fixed $t = t_i$, i.e. for a given interval (a_i, b_i), the system (20) of two equations can be easily solved and we can get the value $\tilde{\delta}_N(t_i)$, of course with a corresponding influence of the noise.

If $U(t)$ and $V(t)$ are filtered numbers obtained from the raw data $y(t)$ we can relate $\tilde{\delta}_N(t_i)$ with $C(t_i)$ or $R(t_i)$ through (18).

5. A REGRESSION ANALYSIS OF THE NORMALIZED PARAMETERS WITH THE USE OF CALIBRATION DATA *(LINEAR INTERPOLATION)*

Let the calibrations are made at the epochs $\tau = \tau_1, \tau_2, \ldots \tau_\nu$ and the observed values of the calibration coefficients are (τ_1), $c(\tau_2) \ldots c(\tau_\nu)$. We shall consider the widely spread practice to make a linear interpolation between every two consecutive calibrations. According to it, the calibration coefficient at time t, say $c(t)$, is computed after

$$c(t) = c(\tau_k) + a_{k+1}\,(t-\tau_k), \text{ if } \tau_k \leqslant t \leqslant \tau_{k+1}. \tag{21}$$

where the coefficients a_k are computed after

$$\begin{aligned} a_1 &= c(\tau_1) \\ a_k &= (c(\tau_k)-c(\tau_{k-1}))\;/\;(\tau_k-\tau_{k-1}),\; k = 2,\ldots\nu \end{aligned} \tag{22}$$

When this expression is used this has the meaning, that for the observations $c(\tau_k)$, $k = 1,2,\ldots\nu$, we accept the equations

$$\begin{aligned} c(\tau_1) &= a_1 \\ c(\tau_2) &= a_1 + a_2(\tau_2-\tau_1) \\ c(\tau_3) &= a_1 + a_2(\tau_2-\tau_1) + a_3(\tau_3-\tau_2) \\ &\ldots\ldots\ldots\ldots \\ c(\tau_\nu) &= a_1 + a_2(\tau_2-\tau_1) + a_3(\tau_3-\tau_2) + \ldots + a_\nu\,(\tau_\nu-\tau_{\nu-1}) \end{aligned} \tag{23}$$

This is a system of ν equations with the same number ν of unknowns, the coefficients a_k. The solution of the system is namely (22).

There is no room for any statistical processing of (23), for example for the Method of the Least Squares, because the number of the equations (23) is just equal of the number of the unknowns. For this reason we cannot estimate the precision of a_k. Actually, as we have zero degrees of freedom, the estimates a_k have indefinite intervals of confidence. Something more, any error in a_k is directly transferred as a systematic error onto the data.

These weaknesses of the scheme can be partly reduced if we use our normalized parameters and the equations (18). This can be done in the following way.

We shall introduce a variable

$$x(t_i) = C_0\;/\;\delta_N(t_i),\; i = 1,2,\ldots n. \tag{24}$$

Then, according to (18) and (21), we shall have the observational equations for χ

$$\begin{aligned} x(t) &= a_1 + a_2\,(t-\tau_1), \text{ for } \tau_1 \leqslant t \leqslant \tau_2 \\ x(t) &= a_1 + a_2\,(\tau_2-\tau_1) + a_3\,(t-\tau_2), \text{ for } \tau_2 \leqslant t \leqslant \tau_3 \\ &\ldots\ldots\ldots\ldots \\ x(t) &= a_1 + a_2\,(\tau_2-\tau_2) + \ldots + a_\nu\,(t-\tau_{\nu-1}), \\ &\text{for } \tau_{\nu-1} \leqslant t \leqslant \tau_\nu,\; t = t_i,\; i = 1,2,\ldots n \end{aligned} \tag{25}$$

If there are discontinues changes in $C(t)$, there are further complications in the scheme which is without this not very comfortable for computations. Let us suppose that such a change we have after τ_ν, i.e. we have new sensibility and new calibrations made at time $\tau_{\nu+1}$, $\tau_{\nu+2}, \ldots \tau_{\nu+\mu}$. Then for $\tau_{\nu+1}$ we must introduce a new calibration constant at the place of a_1 and write new equations, independent from the equations (25), like

$$x(t) = a_{\nu+1} + a_{\nu+2}(\tau_{\nu+2}-\tau_{\nu+1}) + \ldots + a_{\nu+k}(t-\tau_{\nu+k-1}), \text{ for } \tau_{\nu+k-1} \leq t\ \tau_{\nu+k}. \quad (26)$$

If there is another change of the sensibility we must create another new system of equations, like (26), and so on. What is interesting, through (25) or (25), (26) and all other equations like (26), we can determine the coefficients a_k, $k = 1,\ldots\nu$ or $k = 1,\ldots\nu,\ldots\nu + \mu$ etc. Then, on the basis of the expressions (21) through (23) we can compute the calibration coefficient $C(t)$ at any time t. And all this without using the observed calibration coefficients $c(\tau_k)$, i.e. the calibrations themselves. The only application of the calibrations is to determine the mean value C_o for the computation of $x(t)$ after (24).

Evidently, this kind of equations are to be solved by the Method of the least squares.

6. SOME RESULTS

This technique was applied on the gravity observations by a LCR gravimeter in the Geodynamic station Cueva de los Verdes in Lanzarote, the Canary Islands. An interesting moment in these data is that there are several artificial changes of the sensibility. There were slow inclination of the gravimeter which produced slow linear variations of the calibration coefficient.

When there was a new adjustment of the position of the gravimeter, it provoked a jump in the sensibility. Evidently, before and after these jumps there were made corresponding calibrations.

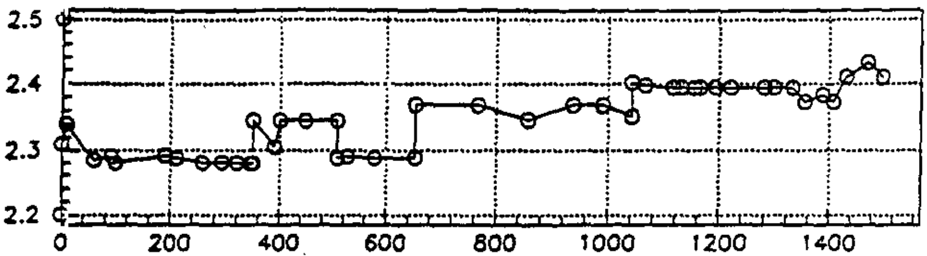

Figure 1. Observed calibration coefficients

The jumps are clearly seen on Figure 1 on which is plotted the empirically obtained calibration curve. There are indicated all determinations of the calibration coefficients.

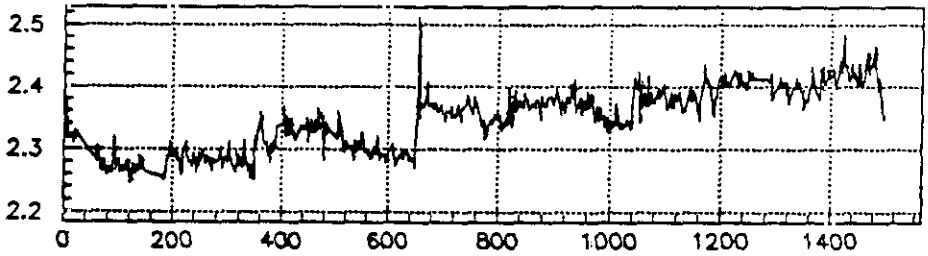

Figure 2. The curve of $x(t_i) = C_o / \delta_N(t_i)$ where δ_N are normalized δ factors obtained from *non-calibrated data.*

In Figure 2 the curve of $x(t_i) = C_o / \delta_N(t_i)$ is plotted where $\delta_N(t_i)$ is obtained from the raw data $y(t)$, i.e. the data before the calibration are used. According to our consideration above, this curve should describe the variations of the sensibility. There is possible a systematic shift due to the more or less arbitrary choice of C_0 in (24). One can see that there is indeed a close relation with the curve on Figure 1.

The data from Fig. 2 were adjusted as it was suggested in paragraph 5. The adjustment allows jumps at the points where are made changes in the sensibility. The curve obtained is plotted on Figure 3. It is close to Figure 1 but the two curves are not identical. In particular, there are some differences between the estimated and the experimental jumps.

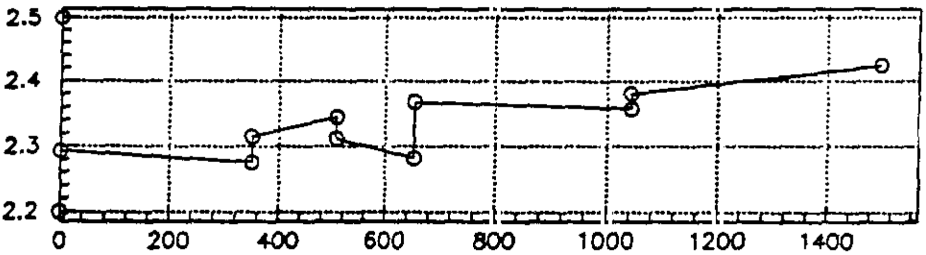

Figure 3. Adjusted calibration curve.

On Figure 4 is plotted the quantity (24), $x(t_i) = C_o / \delta_N(t_i)$, where δ_N is obtained through the processing of the calibrated data. The applied calibration

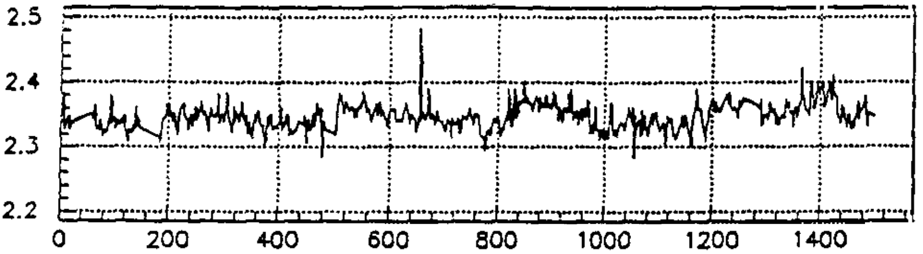

Figure 4. The curve of $x(t_i) = C_o / \delta_N(t_i)$ where δ_N are normalized δ factors obtained from *calibrated data* by using the *observed calibration coefficients* (Figure 1).

coefficients are those on Figure 1. In the ideal case of a perfect calibration $x(t_i)$ should be a constant. One can see that there are some systematic deviations of $x(t_i)$ from a constant.

Through the adjusted curve on Figure 3 the data were again calibrated. On the last Figure 5 the quantity $x(t_i)$ is again plotted. Now the normalized δ_N used is taken from the newly calibrated data. Compared to Fig. 4, here we are somewhat closer to the ideal case $x(t)$ = const.

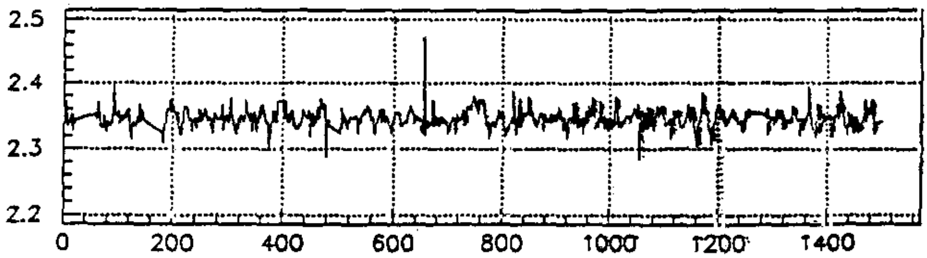

Figure 5. The curve of $x(t_i) = C_0 d / \delta_N(t_i)$ where δ_N are normalized δ factor obtained from *calibrated data* by using the *adjusted calibration curve* (Figure 3).

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