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Black holes in modified gravity theories

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Abstract. In the context of $f(R)$ gravity theories, the issue of finding static and spherically symmetric black hole solutions is addressed. Two approaches to study the existence of such solutions are considered: first, constant curvature solutions, and second, the general case (without imposing constant curvature) is also studied. Performing a perturbative expansion around the Einstein-Hilbert action, it is found that only solutions of the Schwarzschild-(Anti-) de Sitter type are present (up to second order in perturbations) and the explicit expressions for these solutions are provided in terms of the $f(R)$ function. Finally we consider the thermodynamics of black holes in Anti-de Sitter space-time and study their local and global stability.

1. Introduction

In the last years, increasing attention has been paid to modified theories of gravity in order to understand several open cosmological issues such as the present accelerated expansion of the universe [1]. Some of those theories modify general relativity by adding higher powers of the scalar curvature R , i.e. $f(R)$ theories, the Riemann and Ricci tensors or their derivatives [2]. It is therefore quite natural to ask about black hole (BH) features in those gravitational theories since, on the one hand, some BH signatures may be peculiar to Einstein's gravity and others may be robust features of all generally covariant theories of gravity. On the other hand, the obtained results may lead to rule out some models which will be in disagreement with expected physical results. For those purposes, research on thermodynamical quantities of BH is of particular interest.

In this work we will restrict ourselves to the so called $f(R)$ gravity theories in metric formalism in Jordan's frame where the gravitational Lagrangian is given by $R + f(R)$ and modified Einstein's equations are usually fourth order in the metric. Using the euclidean action method [3, 4] in order to determine different thermodynamical quantities, Anti-de Sitter (AdS) BHs in $f(R)$ models and their stability will be studied [5].

2. Black Holes in $f(R)$ gravities

The most general static and spherically symmetric $D \geq 4$ dimensional external metric for the gravitational field produced by a nonrotating object in $f(R)$ gravity theories can be written as:

$$ds^2 = e^{-2\Phi(r)} A(r) dt^2 - A^{-1}(r) dr^2 - r^2 d\Omega_{D-2}^2 \equiv \lambda(r) dt^2 - \mu^{-1}(r) dr^2 - r^2 d\Omega_{D-2}^2 \quad (1)$$

where $d\Omega_{D-2}^2$ is the metric on the S^{D-2} sphere. In this case the scalar curvature R in D dimensions depends only on r and for the first parametrization in equation (1) is given by

$$R = \frac{1}{r^2} \left\{ (D-2)[(D-3)(1-A) + 2r(-A' + A\Phi')] + r^2[3A'\Phi' - A'' - 2A(\Phi'^2 - \Phi'')] \right\}. \quad (2)$$

where the prime denotes derivative with respect to r from now on. Therefore the most general static and spherically symmetric metrics with constant scalar curvature R_0 can be found solving the equation $R = R_0$. For a constant $\Phi(r) = \Phi_0$ it is straightforward to see that the solution is:

$$A(r) = 1 + a_1 r^{3-D} + a_2 r^{2-D} - \frac{R_0}{D(D-1)} r^2 \quad (3)$$

with $a_{1,2}$ being arbitrary integration constants. For the particular case $D = 4$, $R_0 = 0$ and $\Phi_0 = 0$, the metric can be written exclusively in terms of the function $A(r) = 1 + a_1 r^{-1} + a_2 r^{-2}$ where by establishing the identifications $a_1 = -2G_N M$ and $a_2 = Q^2$, this solution corresponds to a charged massive BH solution with mass M and charge Q (Reissner-Nordström solution).

On the other hand, if the constant curvature case $R = R_0$ is considered, the modified Einstein's equation from $f(R)$ theories for the function $A(r)$ reduces to:

$$R + f(R) + (1 + f_R) \left[A'' + \frac{(D-2)}{r} (A' - 2A\Phi') - 3A'\Phi' + 2A(\Phi'^2 - \Phi'') \right] = 0 \quad (4)$$

where f_R holds for $df(R)/dR$. As it is explained in [5], the vacuum constant curvature solutions of $f(R)$ gravities are given by:

$$R_0 = \frac{D f(R_0)}{2(1 + f_{R_0}) - D} \quad (5)$$

If the hypothesis $\Phi'(r) = 0$ is made, then equation (4), by using (5), reduces in this case to

$$A'' + (D-2) \frac{A'}{r} = -\frac{2}{D} R_0 \rightarrow A(r) = c_1 + c_2 r^{3-D} - \frac{R_0}{D(D-1)} r^2 \quad (6)$$

where $c_{1,2}$ are two arbitrary constants. However this solution has no constant curvature in the general case since, as it can be seen in expression (3), the constant curvature requirement demands $c_1 = 1$. Then, for negative R_0 , this solution is basically the D dimensional generalization obtained by Witten [4] of the BH in AdS space-time solution considered by Hawking and Page [3]. With the natural choice $\Phi_0 = 0$ the solution can be written as:

$$A(r) = 1 - (R_S/r)^{D-3} + r^2/l^2 \quad (7)$$

where R_S and l have the usual interpretations found in the literature.

Thus we have concluded that the only static and spherically symmetric vacuum solutions with constant (negative) curvature of any $f(R)$ gravity is just the Hawking-Page BH in AdS space. However this kind of solution is not the most general static and spherically symmetric metric with constant curvature as can be seen by comparison with the solutions given by expression (3).

3. Perturbative results

In Einstein-Hilbert (EH) theory the most general static solution with spherical symmetry is the one with constant curvature. However, this is not guaranteed to be the case in $f(R)$ theories. The problem of finding the general static spherically symmetric solution in arbitrary $f(R)$ theories without imposing the constant curvature condition is in principle too complicated. For that reason a perturbative analysis of the problem is performed here, assuming that the modified action is a small perturbation around EH theory of the form $f(R) = -(D-2)\Lambda_D + \alpha g(R)$ where $\alpha \ll 1$ is a dimensionless parameter and $g(R)$ is assumed to be analytic. Assuming that

the $\lambda(r)$ and $\mu(r)$ functions appearing in the second parametrization of the metric (1) are also analytical in α , they can be written as follows

$$\lambda(r) = \lambda_0(r) + \sum_{i=1}^{\infty} \alpha^i \lambda_i(r) \quad ; \quad \mu(r) = \mu_0(r) + \sum_{i=1}^{\infty} \alpha^i \mu_i(r) \quad (8)$$

where $\{\lambda_0(r), \mu_0(r)\}$ are the unperturbed solutions for the EH action with cosmological constant Λ_D given by $\mu_0(r) = 1 + \frac{C_1}{r^{D-3}} - \frac{\Lambda_D}{(D-1)} r^2$ and $\lambda_0(r) = -C_2(D-2)(D-1)\mu_0(r)$ which are the standard BH solutions in a D dimensional AdS spacetime with constant curvature $R_0 \equiv D\Lambda_D$.

By inserting the proposed $f(R)$ expansion and equation (8) in modified Einstein's equations the first and second order equations in α are obtained, whose solutions are

$$\begin{aligned} \lambda_1(r) &= (D-1)(D-2) \left\{ C_4 + \frac{C_1 C_4 - C_2 C_3}{r^{D-3}} - \left[\frac{C_4 \Lambda_D}{D-1} + \frac{C_2 A(g; D, \Lambda_D)}{(D-1)(D-2)} \right] r^2 \right\} \\ \mu_1(r) &= \frac{C_3}{r^{D-3}} + \frac{A(g; D, \Lambda_D)}{(D-2)(D-1)} r^2 \\ \lambda_2(r) &= C_6 + \frac{C_5 + C_6 C_1}{r^{D-3}} - \left[\frac{C_6 \Lambda_D}{D-1} - A(g; D, \Lambda_D) \left(C_4 + C_2 g_{R_0} - \frac{2C_2 R_0 g_{R_0 R_0}}{D-2} \right) \right] r^2 \\ \mu_2(r) &= \left(C_3 C_4 + \frac{-C_5}{(D-2)(D-1)} \right) \frac{C_2^{-1}}{r^{D-3}} + \frac{A(g; D, \Lambda_D) (2R_0 g_{R_0 R_0} - (D-2)g_{R_0})}{(D-2)^2(D-1)} r^2 \end{aligned} \quad (9)$$

where $g_{R_0} \equiv dg(R)/dR|_{R=R_0}$, $g_{R_0 R_0} \equiv d^2g(R)/dR^2|_{R=R_0}$, and $A(g; D, \Lambda_D) \equiv g(R_0) - 2\Lambda_D g_{R_0}$. Higher order solutions $\{\lambda_{3,4,\dots}(r), \mu_{3,4,\dots}(r)\}$ can be obtained by inserting the previous results in the corresponding equations order $\alpha^{3,4,\dots}$ which become increasingly complicated.

Notice that from the obtained results up to second order in α , the corresponding metric has constant scalar curvature for any value of the parameters $C_{1,\dots,6}$. As a matter of fact, this metric is nothing but the standard Schwarzschild- AdS geometry, and can be easily rewritten in the usual form by making a trivial time reparametrization. Therefore, at least up to second order, the only static, spherically symmetric solutions which are analytical in α are the standard Schwarzschild- AdS space-times.

4. Black-hole thermodynamics

For simplicity we will concentrate only on constant curvature AdS BH solutions given by expression (7) with $\Phi(r) \equiv 0$ as a natural choice. Then, usual definitions of temperature lead to $\beta \equiv 1/T = 4\pi l^2 r_H / [(D-1)r_H^2 + (D-3)l^2]$ where r_H satisfies $A(r_H) = 0$. Notice that the temperature is a function of r_H only, i.e. it depends only on the BH size. It is easy to prove that T has a minimum T_0 at r_{H0} and whose existence was established in [3] for $D = 4$. Notice that for any $T > T_0$, there are two possible BH sizes: one corresponding to the small BH phase with $r_H < r_{H0}$ and the other corresponding to the large BH phase with $r_H > r_{H0}$. The remaining thermodynamic quantities are determined by using the Euclidean method so that the free energy F is given by

$$F = -\frac{(R_0 + f(R_0))\mu_{D-2}}{36\pi(D-1)G_D} (l^2 r_H^{D-3} - r_H^{D-1}) \quad (10)$$

where provided $-(R_0 + f(R_0)) > 0$, which is the usual case in EH gravity, $F > 0$ for $r_H < l$ and $F < 0$ for $r_H > l$. On the other hand, the heat capacity C can be written as:

$$C = \frac{\partial E}{\partial T} = \frac{\partial E}{\partial r_H} \frac{\partial r_H}{\partial T} = \frac{-(R_0 + f(R_0))(D-2)\mu_{D-2} r_H^{D-2} l^2}{8G_D(D-1)} \frac{(D-1)r_H^2 + (D-3)l^2}{(D-1)r_H^2 - (D-3)l^2}. \quad (11)$$

Let us assume from now on that, like in the EH case, the condition $(R_0 + f(R_0)) < 0$ holds. This is the required condition, as explicitly shown in [5] to get both positive energy and entropy in these theories and according to (5) it implies $1 + f_{R_0} > 0$ in agreement with cosmological requirements [6]. With this assumption, we find $C > 0$ for $r_H > r_{H0}$ (the large BH region) and $C < 0$ for $r_H < r_{H0}$ (the small BH region). For $r_H \sim r_{H0}$ (T close to T_0) C is divergent. In any case, for $f(R)$ theories with $R_0 + f(R_0) < 0$, we have found a similar scenario to the one described in full detail in [3] for the EH case: for $T > T_0$ two possible BH solutions are present, the small BH and the large BH: The small one has $C < 0$ and $F > 0$ as the standard Schwarzschild BH. For the large BH two possibilities appear; if $T_0 < T < T(r_H = l)$ then $C > 0$ and $F > 0$ and the BH will decay by tunnelling into radiation, but if $T > T(r_H = l)$ then $C > 0$ and $F < 0$. In this case F of the large BH will be less than that of pure radiation.

To illustrate the previous calculations, a model $f(R) = \alpha(-R)^\beta$ is considered where the heat capacity C and the free energy F signs will be studied to know the local and global stability of BHs. This model has a nonvanishing constant curvature solution $R_0 = -[(2 - D)/\alpha(2\beta - D)]^{1/(\beta-1)}$ calculated by using (5) and therefore two separated regions, where $R_0 < 0$ holds, have to be studied: Region 1 $\{\alpha < 0, \beta > D/2\}$ and Region 2 $\{\alpha > 0, \beta < D/2\}$. Further models following an analogous procedure were presented in [5].

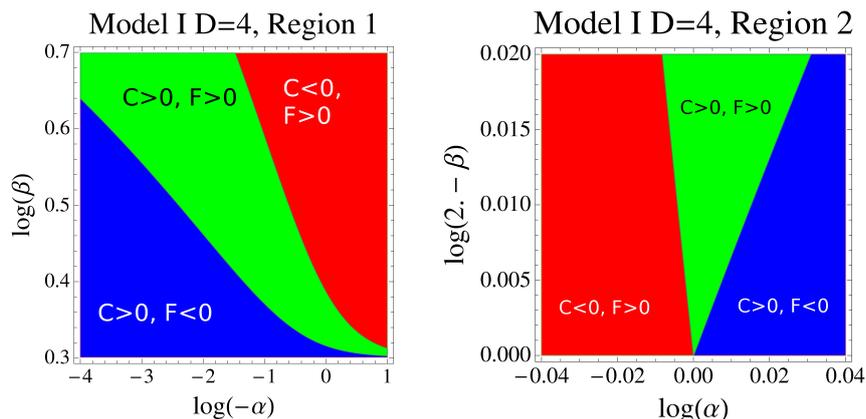


Figure 1. Regions for C and F signs have been compared in the (α, β) plane for Model I in $D = 4$. Region 1 (left), Region 2 (right). Schwarzschild radius $R_S = 2$ was assumed.

Acknowledgments

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