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"LIES AND LAYOFFS.
ASSYMMETRIC INFORMATION
AND UNEMPLOYEMENT EQUILIBRIUM"

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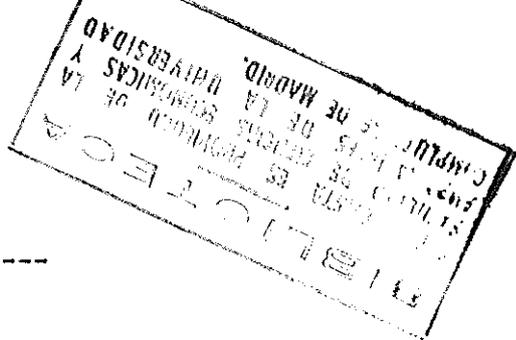
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LIES AND LAYOFFS



Implicit contracts were initially seen as a form of insurance. Workers bought a less variable wage at the expense of a slightly lower average wage. But as long as this idea of implicit contracts persisted, they could do little to explain unemployment. A risk-averse worker is unlikely to buy a steadier wage if it is at the increased risk of losing his job. He might do so if the disutility of labour or the level of unemployment benefit were high enough, but in such cases we are effectively back with the classical version of unemployment. Indeed, as Akerlof and Miyazaki (1980) have shown, for a given amount of unemployment to be generated, unemployment benefit must be higher in the implicit-contract case than it need be with spot auction markets.

All this changed when asymmetric information was added to the mixture. Hart (1983) and Azariadis (1983) both take up the case where workers do not know the true state of demand for the firm's product. They can, however, work out whether a particular contract will induce the firm to tell the truth about the state of demand, and such contracts are the only ones they will accept.

This rules out the contract which might otherwise be optimal -- one where wages adjust to maintain full employment. For if wages are flexible but employment fixed, rational employers will forever claim that demand is depressed, thus cutting their costs without affecting their revenues. If this is to be avoided, the firm too must suffer if it tells an untrue tale of woe. It will only do so if its revenue is reduced, and its revenue can only fall if it lays workers off. A contract with flexible wages, then, must involve flexible employment.

Despite the advanced mathematical dress in which this idea usually appears (see especially Hart (1983)), it appeals immediately to anyone who has ever had to listen to a sorrowful employer without being able to send the tears for chemical analysis. It asks, however, to be embedded in a complete macro-economic model and Frank (1986) has done the embedding.

In Frank's model there are two possible states of real demand, Q^+ and Q^- . Q^+ , Frank shows, always yields full employment. Truth-telling requires that at Q^+ it is at least as profitable to admit demand is at Q^+ as to pretend that it is at Q^- . Although workers are risk-neutral, firms are risk-averse in that they insist on a given minimum profit even in the event of Q^- . They must also pay (and, as profit-maximisers, will only pay) their workers enough to stop them deserting to rival firms. Firms are monopolistic competitors and thus face downward-sloping demand curves.

These propositions suffice for Frank to derive a determinate "reaction curve" showing the level of output which will be generated at each level of demand. Provided firms are sufficiently risk-averse, there will be a level of demand (besides the full-employment level) which generates a level of output equal to itself -- an unemployment equilibrium. This unemployment equilibrium is influenced by, but is not necessarily equal to, Q^- .

The limitations of Frank's model, however, are severe. Not everyone will care for his assumption of risk-neutral workers and risk-averse firms. Unemployment equilibrium may -- and in Frank's only illustrative example does -- require negative wages. But above all there is no reason why the "unemployment equilibrium" level of demand should correspond to Q^- except through knife-edge luck; yet Q^- is supposed to be the only level of demand (except the full-employment-inducing Q^+) that anyone can countenance. In fact a double miracle is required: first, firms have to expect the unique

level of Q^* which generates an unemployment equilibrium equal to itself. Secondly this unique level must occur. If it does not do so, there is no adjustment process: the expected level of demand, the actual level of demand, and the level of output which it generates are all different to one another, and no clue is given as to how, or whether, they are brought into line. It is thus beside the point to complain that Frank's unemployment equilibrium is "unstable" (as it appears to be from his reaction curve diagram): if no adjustment process (or even non-adjustment process) is specified, stability and instability cease to have any meaning.

It is the either/or nature of anticipated demand that is the source of the trouble. In the model presented below, firms have a continuous probability distribution of expected demand, from zero to the full-employment level of output. An adjustment process is specified and the possibility of both stable and unstable flex-price unemployment equilibria demonstrated. The unsatisfactory assumption of risk-neutral workers but risk-averse firms is no longer required. Nor shall we be issuing any unfortunates with a negative pay packet although, like Frank, we shall foster both analytical simplicity and sturdy independence by abolishing unemployment benefit.

II

Assume that each firm starts with a team of \bar{N} workers, any of whom can be N employed or laid off as circumstances dictate. Layoffs, when they occur, are random. The utility of labour is zero and there is no unemployment benefit.

Firms are risk-averse to the extent that they always prefer a contract which guarantees breaking even to a contract which does not. Workers may be anything from risk-neutral to infinitely risk-averse.

Firms are monopolistic competitors. Each firm faces an identical demand curve. Aggregate demand equals QF , where F is the number of firms. Hence aggregate demand per firm = Q . Let output per firm = q .

Price is a non-linear function of Q/q :

$$\text{i.e. } p = (Q/q)^c \quad \text{where } 0 < c < 1$$

Output is a not necessarily linear function of employment:

$$\text{i.e. } q = N^{d/1-c} \quad \text{where } d < 1 \text{ (or competitive equilibrium would not exist).}$$

and where $\begin{pmatrix} < \\ d \\ > \end{pmatrix} = 1-c$ means $\begin{pmatrix} \text{diminishing} \\ \text{constant} \\ \text{increasing} \end{pmatrix}$ returns.

$$\text{Hence revenue (R)} = pq = Q^c q^{1-c} = Q^c N^d$$

The wage and employment levels offered are a function of what the firm declares to be the demand for its goods (Q')

$$\text{Thus: } W = W(Q')$$

$$N = N(Q')$$

$$\text{cost (C)} = WN(Q')$$

Now, for the firm to tell the truth, the level of declared demand which maximises profits must equal actual demand at all levels of actual demand:

$$\text{i.e. } d\Pi/dQ' = 0 ; \quad d^2\Pi/dQ'^2 < 0 \quad \text{at all } Q=Q'$$

So what forms must $W(Q')$ and $N(Q')$ take? Because $d\Pi/dQ' = 0$ must hold at all $Q=Q'$, it is an identity. The upshot of this is that neither $W(Q')$ nor $N(Q')$ can contain more than a single term (see Appendix (1) for proof.)

Let us therefore write them as:

$$W = kQ'^a \quad (1)$$

$$N = lQ'^b \quad k, l > 0 \quad (2)$$

$$\text{Hence } C = klQ'^{a+b}$$

$$\text{and } R = \frac{1-c}{1} Q^c Q'^{bd}$$

$$\text{Hence profit } (\Pi) = l^d Q^c Q^{bd} - k l Q^{a+b} \quad (3)$$

The (first order) truth-telling condition thus requires that

$$b d l^d Q^c Q^{bd-1} = k l (a+b) Q^{a+b-1} \quad \text{at all } Q=Q'$$

i.e. that $b d l^d Q^{c+bd-1} = k l (a+b) Q^{a+b-1}$

This must mean either that $a = b = 0$

$$\text{or that (i) } b d l^d = k l (a+b) \quad (4)$$

$$\text{and (ii) } c + b d = a + b \quad (b)$$

Let us, without undue originality, call the first possibility the zero option. Here wages and employment are fixed, the latter at full employment.¹ A risk-neutral firm will always choose the zero option. Suppose, first, that workers are risk-neutral too. Then the expected wage-bill that the firm must pay is invariant to the contract chosen. But of all possible contracts the zero option maximises revenue. (All layoffs reduce revenue). Hence it maximises expected profits. If workers are risk-averse this becomes true a fortiori. In issuing the zero option, the firm is selling the workers insurance and can offer a lower expected wage-bill.

But the zero option is obviously very risky to the firm, and in fact we have ruled it out with our assumption that the firm always prefers a contract which guarantees breaking even to a contract which does not. Our firm, therefore, must choose the second type of contract. Using (4) and (5) to derive k and b in terms of a and l , and substituting into (1) and (2), we get a contract of the form:

¹Because if both wages and employment are fixed, there can be no "truth-telling" problem and hence no need for layoffs.

$$W(Q') = \frac{c-a}{c-ad} dl^{d-1} Q'^a \quad (6)$$

$$N(Q') = aQ' \frac{c-a}{1-d} \quad (7)$$

The fact that the first-order condition is an identity has cost us two of the original four degrees of freedom (a,b,k,l). The fact that maximum possible demand must involve full employment (or no one is optimising) will lose us a third. The fact that firms will pay their workers the minimum expected wage required to retain them removes the last degree of freedom and gives a uniquely determined contract.

III

Let the maximum level of demand anticipated by firms be \bar{Q} . As just said, \bar{Q} must imply full employment. The only purpose of layoffs is to penalise a firm for declaring a level of demand below the true one. But a declared demand of \bar{Q}' , by definition, cannot be below the true one. Unemployment at \bar{Q}' would thus simply be a pointless waste. Hence $N(\bar{Q}') = \bar{N}$. But the truth-telling condition also ensures that $\bar{Q} \rightarrow \bar{Q}'$.

$$\text{Hence } N(\bar{Q}) = N(\bar{Q}') = \bar{N}$$

Now let us assume that the maximum demand which firms expect to have to face corresponds to the full employment level of national income.

$$\text{i.e. } \bar{Q} = q(\bar{N})$$

$$\text{But: } \bar{N} = N(\bar{Q}) \quad \text{Hence: } \bar{Q} = q(N(\bar{Q}))$$

$$\text{i.e. } \bar{Q} = \frac{d}{1-c} \frac{cd-ad}{(1-d)(1-c)}$$

$$\text{which yields: } 1 = \bar{Q} \frac{1-c-d+ad}{d(1-d)}$$

Hence, from (6) and (7):

$$N = \bar{Q} \frac{1-c-d+ad}{d(1-d)} Q^{\frac{c-a}{1-d}} \quad (8)$$

$$W = \frac{c}{c/d - a} \bar{Q} \frac{c+d-ad-1}{d} Q^a \quad (9)$$

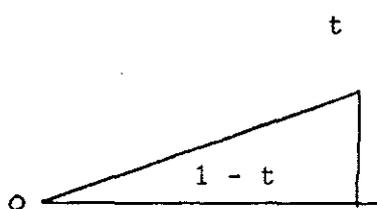
Only one degree of freedom is left. We begin on its elimination by noting that the second-order condition for truth-telling requires that $a < c$. (See Appendix (2).) Next we use the fact that, if firms are to optimise, any contract must equate expected wage to expected marginal product of labour, the latter being given by the marginal value product of labour at full employment times the probability that full employment will occur (call this t).

$$\text{Hence } E(W) = t \frac{dR}{dN} (Q=\bar{Q}) = t d \bar{Q}^{\frac{c+d-1}{d}} \quad ** \quad (10)$$

The above equation gives the required expected wage. The actual expected wage will be

$$E(W) = \int_0^{\bar{Q}} \frac{W(Q)N(Q)}{N(Q)} \text{prob}(Q) dQ$$

Further assumptions must be made as to the subjective probabilities of different states of demand. If $\text{prob}(Q=\bar{Q})$ is t , then $\text{prob}(Q < \bar{Q}) = 1-t$. Let us suppose that, within this constraint, the probability of a particular Q is proportional to the size of that Q i.e. that the overall probability distribution is as below:



 **This represents both what the worker's existing firm will pay to retain him and what any other firm would be prepared to pay to attract him. It is thus the equilibrium expected wage across the economy.

This yields the result that

$$\text{prob}Q(Q < \bar{Q}) = \frac{2(1-t)Q}{\bar{Q}^2}$$

Consequently

$$\begin{aligned} E(W) &= tW(\bar{Q}) + \int_0^{\bar{Q}} \frac{W(Q)N(Q)}{N(\bar{Q})} \frac{2(1-t)Q}{\bar{Q}^2} dQ \quad (11) \\ &= t \frac{c-a}{c/d-a} \bar{Q} \frac{c+d-1}{d} + \int_0^{\bar{Q}} \left(\frac{c-a}{c/d-a} \right) \bar{Q} \frac{c+d-ad-1}{d} Q^a \frac{Q^{1-d}}{\bar{Q}^{1-d}} \frac{2(1-t)Q}{\bar{Q}^2} dQ \\ &= t \left(\frac{c-a}{c/d-a} \right) \bar{Q} \frac{c+d-1}{d} + 2(1-t) \left(\frac{c-a}{c/d-a} \right)^{c+d-ad-1} = \frac{c-a}{1-d} - 2 \int_0^{\bar{Q}} Q^{\frac{c-a}{1-d} + a+1} dQ \\ &= \left[t \left(\frac{c-a}{c/d-a} \right) + 2(1-t) \left(\frac{c-a}{c/d-a} \right) \left(\frac{1-d}{c-ad+2-2d} \right) \right] \bar{Q} \frac{c+d-1}{d} \quad (12) \end{aligned}$$

Hence, putting (9') and (10') together:

$$td = t \left(\frac{c-a}{c/d-a} \right) + 2(1-t) \left(\frac{c-a}{c/d-a} \right) \left(\frac{1-d}{c-ad+2-2d} \right)$$

$$\text{i.e. } t \left(\frac{a-ad}{c/d-a} \right) = 2(1-t) \left(\frac{c-a}{c/d-a} \right) \left(\frac{1-d}{c-ad+2-2d} \right)$$

$$\text{i.e. } \frac{t}{1-t} = \frac{2(c-a)}{(c-ad+2-2d)a} \quad (13)$$

Hence a is determined by c , d and t . The solution is messy in the extreme, but the important points are that $a \rightarrow c$ as $t \rightarrow 0$, as can be seen by inspection, and that $a \rightarrow 0$ as $t \rightarrow 1$, which is less obvious:

As $t \rightarrow 1$, $t/1-t \rightarrow \infty$, hence $a(c-ad+2-2d) \rightarrow 0$. This means either that $a \rightarrow 0$ or that $a \rightarrow \frac{c+2}{d} - 2$. But $a = \frac{c+2}{d} - 2$ violates the second-order truth-telling condition. The firm would be minimising its profits by telling the truth

about demand. Hence this class of contracts must be ruled out and we are left with the case where $a \rightarrow 0$ as $t \rightarrow 1$.

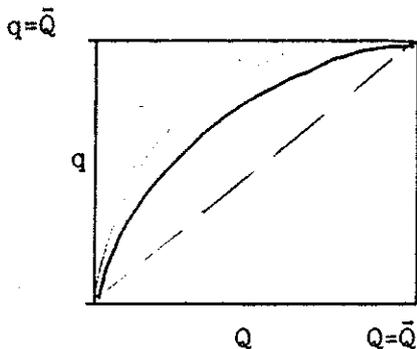
Now we can start looking at the prospects for an unemployment equilibrium.

Recollect equation (8):

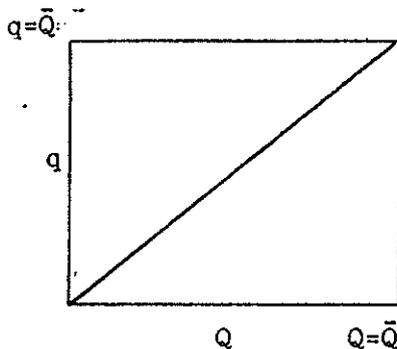
$$N = \bar{Q} \frac{1-c-d+ad}{d(1-d)} \frac{c-a}{Q^{1-d}} \quad (Q=Q')$$

This yields $q = \bar{Q} \frac{cd-ad}{(1-d)(1-c)}$ (14)

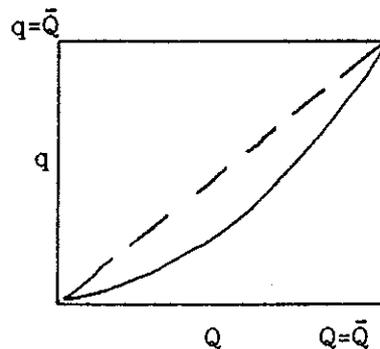
We have our equation telling us what output (per firm) will be at each level of demand (per firm). The curve will be concave, straight or convex according to the value of $\frac{cd-ad}{(1-d)(1-c)}$. The adjustment process is taken to be a simple "Keynesian" one: last week's output equals this week's demand.



$cd-ad < (1-d)(1-c)$
 stable full-employment equilibrium
 unstable zero-employment equilibrium



$cd-ad = (1-d)(1-c)$
 every level of unemployment is an equilibrium



$cd-ad > (1-d)(1-c)$
 unstable full-employment equilibrium
 stable zero-employment equilibrium

But as $t \rightarrow 1$ and $a \rightarrow 0$, the condition for unemployment equilibrium approximates increasingly to

$$cd \geq (1-d)(1-c)$$

i.e. that $c + d \geq 1$

i.e. that returns are constant or increasing.

Conversely, as $t \rightarrow 0$ and $a \rightarrow c$, the condition for unemployment equilibrium approximates to

$$(1-d)(1-c) \leq 0$$

i.e. that either $c \geq 1$ (profit-maximising output zero) or that $d \geq 1$ (profit-maximising output infinite.)

The overall position is that, for given c , the higher the value that t is, the lower the value of a will be, and hence the lower the value of d compatible with unemployment equilibrium. As $t \rightarrow 0$ we approach the paradoxical position where unemployment equilibrium would require increasing returns too strong for any kind of equilibrium. But as $t \rightarrow 1$, we approach the position where any increasing of returns, however mild, will yield an unemployment equilibrium.

To conclude this section, let us look more closely at the variable a . It will, as stated, be chosen by the firm so as to be just sufficient to prevent workers looking elsewhere. But does just sufficient mean just high enough or just low enough? Is a positively or inversely related to the expected wage? The answer is unambiguous. A rise in a will lower the expected wage and raise expected profits.

Effect on wages

$$E(W) = \bar{Q} \frac{c+d-1}{d} \left(\frac{c-a}{c/d-a} \right) \left(t+2(1-t) \frac{1-d}{c-ad+2-2d} \right)$$

$$\text{Let } \frac{c-a}{d-a} = u, \quad \frac{1-d}{c-ad+2-2d} = w$$

$$\therefore \frac{d}{da} \left(\frac{E(W)}{\bar{Q} \frac{c+d-1}{d}} \right) = u't + 2(1-t)(uw' + u'w)$$

$$\therefore \frac{d}{da} \left(\frac{E(W)}{(1-t)\bar{Q} \frac{c+d-1}{d}} \right) = \frac{u't}{1-t} + 2uw' + 2u'w$$

$$\text{But } \frac{t}{1-t} = \frac{2(c-a)}{a(1-d)} w \quad (11) \quad \text{and } w' = \frac{-d}{1-d} w^2$$

$$\therefore \frac{d}{da} \left(\frac{E(W)}{(1-t)\bar{Q} \frac{c+d-1}{d}} \right) = wu' \left(\frac{2c-2ad}{a(1-d)} \right) - \frac{2d}{1-d} w^2$$

The second term of this expression must be negative ($d < 1$). As for the first term, the facts that $c > a$ (second-order condition for truth-telling) and $d < 1$ ensure that w , $a(1-d)$ and $2c-ad$ are all positive and that u' is negative. Hence the first term is also negative.

$$\therefore \frac{d}{da} \left(\frac{E(W)}{(1-t) \left(\bar{Q} \left(\frac{c+d-1}{d} \right) \right)} \right) < 0 \quad \therefore \frac{dE(W)}{da} < 0.$$

Effect on profits

At level of demand Q , profits are:

$$\begin{aligned} R(Q) - C(Q) &= Q^c (N(Q))^d - W(Q)N(Q) \\ &= \frac{ad-c-d+1}{Q} \frac{c-ad}{1-d} \frac{c-ad}{1-d} - \frac{c-a}{c/d-a} \frac{ad-c-d+1}{Q} \frac{c-ad}{1-d} \\ &= \frac{c/d-c}{c/d-a} \bar{Q} \left(\frac{Q}{\bar{Q}} \right)^{1-d} \end{aligned}$$

which can be seen by inspection to be increasing in a . Higher a , therefore, does not just raise expected profits: it raises profits at all levels of output.

Hence the unique determination of a by t , c and d . Were it to be any higher, workers would not stay with the firm. Were it lower, the firm would be throwing away profits.

IV

However, the implicit assumption in the above, and in equation (13) in particular, is that workers are risk-neutral. For only if they are risk-neutral can we specify a unique minimum expected wage sufficient to keep them with their firm. Once risk-aversion is posited, we must replace this with the minimum sufficient expected utility of income.

Suppose, then, that the worker's utility function takes the form

$$U = W^v, \text{ where } v < 1.$$

Then, by analogy with equation (11):

$$E(U) = t(W(\bar{Q}))^v + \int_0^{\bar{Q}} \frac{(W(Q))^v N(Q)}{N(\bar{Q})} \frac{2(1-t)Q}{\bar{Q}^2} dQ$$

which yields

$$E(U) = \left(\frac{cd-ad}{c-a} \right)^v \bar{Q}^{\frac{c+d-1}{d}v} \left[t + \frac{2(1-t)(1-d)}{2-2d+av-avd+c-a} \right] \quad (15)$$

Equations (11) and (12) now become special cases with $v=1$.

It would seem plausible that firms, provided their own risk-aversion is not too great, can capitalise on workers' risk-aversion ($v < 1$) by "selling them insurance." That is, that they can pay them a lower overall expected wage provided that they lower the variance of (wage x probability of being employed) across the different states of demand.

We have assumed that the firm's own risk-aversion takes the form of always preferring a contract which guarantees breaking even to a contract which does not. This simply rules out the zero option while leaving all other "truth-telling" contracts available. Workers' risk-aversion does not alter the fact that, if firms are to optimise and tell the truth, any contract must take the form

$$N = \bar{Q}^{\frac{1-c-d+ad}{d(1-d)}} Q^{\frac{c-a}{1-d}}$$

$$W = \frac{c}{c/d} - \frac{a}{a} \bar{Q}^{\frac{c+d-ad-1}{d}} Q^a$$

This limits the range of insurance policies the firm can sell. All it can do is alter the value of a . The question is whether, in a risk-averse case, there exists a higher value of a than obtains in the risk-neutral case,

which would both increase the firm's profits and, by reducing the variance of the wage, raise the worker's expected utility despite his lower expected wage.

Consider what happens to equation (15) as risk-aversion approaches infinity.

As $v \rightarrow 0$,

$$E(U) \longrightarrow t + \frac{2(1-t)(1-d)}{2-2d+c-a}$$

This expression is monotonically increasing in a . Thus if risk-aversion were infinite, workers and employers alike would prefer a contract where a took on its maximum value of c . Equation (14) then simply collapses into:

$$q = \bar{Q}$$

Full employment output is produced whatever the level of demand.

However, equation (9) also collapses, into $W = 0$. Workers have insured themselves into a zero wage! How can this possibly maximise anyone's utility? The answer is that $v=0$ will never actually be attained, but that as it is approached, the worker is approaching the position where his only concern is to maximise the probability of receiving some income, i.e. minimise the probability of being unemployed, however low the wage when in work.

But now consider what happens to (15) as the perceived probability of a slump approaches zero. As $t \rightarrow 1$,

$$E(U) \longrightarrow \frac{cd - ad^v}{c - ad} \bar{Q} \frac{c+d-1}{d} v$$

which (given $c > a$, $d < 1$, $v > 0$) is monotonically decreasing in a . Workers, as in the risk-neutral case, prefer a to be as low as possible.

v and t are thus in a symmetrical relationship. Fix t below 1 and, however close to 1 you make it, there is always some value of v low enough to render $dE(U)/da$ positive. But fix v above zero and, however small you make it, there is always some value of t large enough to render $dE(U)/da$ negative. It is this last fact which concerns us here. Risk-aversion, by threatening to raise the value of a above its "risk-neutral" level, threatens the condition for unemployment equilibrium that

$$cd - ad > (1 - d)(1 - c)$$

But however great the degree of risk-aversion, there will be some subjective probability distribution which will cause workers to choose the lowest available value of a, i.e. to choose, from among the contracts the firm is prepared to let them have, the one they would have chosen had they been risk-neutral.

This does not sound like conventional insurance. Discounting transactions costs, a risk-averse customer and a risk-neutral insurer can always do business. It is true that, in the case we have just looked at, the firm was not entirely risk-neutral: we made it rule out the zero option. But it was modelled as risk-neutral where the possibility of a slightly lower expected wage in return for a slightly steadier wage was concerned. And yet there is a whole area in (t,v) space where no transaction of this kind will be feasible. The reasons lie in the restrictive nature of the truth-telling requirement.

This at first seems counter-intuitive. If the worker insures himself against the results of a slump, surely that would deter the firm from inventing an imaginary slump. Far from particular insurance policies falling foul of the truth-telling condition, one might expect any kind of insurance to reinforce it. No driver, after all, need worry that his insurers will invent

a cracked chassis when he has really only scratched a wing! Yet a worker who "buys insurance" from his employer does have to make sure that he is not inducing the employer to exaggerate hard times when they occur. The paradox vanishes when we realise that some contracts may hold wages up so effectively in a mild slump that it pays the firm to get wages right down by pretending that 1931 has just occurred. Complete insurance of both wages and employment (the zero option) can never produce this hazard : and it is reiterated that a wholly risk-neutral firm will always offer the zero option. But anything short of the zero option is liable to.

To summarise, then : the more risk-averse the workforce, the more restricted the range of values of c , d and t which yield an unemployment equilibrium. As risk-aversion approaches infinite strength, unemployment equilibrium approaches impossibility. But as long as risk-aversion remains finite, there will be some states of expectations (t) at which workers and employers will forge exactly the same contract as would have obtained with risk-neutral workers. Furthermore, such a state of expectations (high t) will be of precisely the kind which only requires very mildly increasing returns for unemployment equilibrium to occur.

V

The more implicit the contract, the more general the analysis. In this final section, therefore, we emphasise that employers need communicate with workers only by making current wage and employment offers. Workers communicate with employers only by accepting these offers or threatening to leave. No one promises anything for the future. No one reveals their expectations of future boom or slump. And the "declared" level of demand need not be declared.

Let us start with the last proposition. Assuming that \bar{Q} , c and d are known, the worker can use his first contract to solve simultaneously for Q' and

a. The following week he uses a and the new wage offer to work out the current Q' , which he then combines with a to "predict" the new employment offer. If his prediction is accurate, a truth-telling contract is in force.

Note that the value of t is irrelevant to this operation. But whether the worker finds his contract acceptable will depend on his, and the firm's, respective estimates of t . Once again we keep things as Trappist as possible by assuming that t is not discussed. There are, of course, a large number of ways that subjective expectations can differ between worker and firm, between firm and firm, and between worker and worker. Suppose for simplicity that there are only two firms (the analysis is not thereby materially altered) f_1 and f_2 . Their teams of workers are w_1 and w_2 respectively. We will concentrate on two particular types of case.

$$(i) \quad t(f_1) = t(f_2) \neq t(w_1) = t(w_2)$$

Here there is nothing the workers can do about the fact that they see the future differently from their employers. W and N are determined by a , which depends on the expected wage the firm thinks it must offer. This in turn depends on its estimate of the worker's expected marginal value product, both to itself and to competitors, which is a function of its estimate of t (probability that the marginal worker will actually be used.) Thus if $t(f_1) = t(f_2)$ then $a_1 = a_2$, $N(Q)_1 = N(Q)_2$ and $W(Q)_1 = W(Q)_2$ for all Q . Workers' own expectations have no influence on a , N or W , and (obviously) they have no incentive to transfer from one firm to another.

$$(ii) \quad t(f_1) < t(f_2) = t(w_1) = t(w_2)$$

Here firm 2 will offer a better contract than firm 1. Workers will leave or threaten to leave f_1 until it comes into line. The only qualification to this is that, as we have seen, contracts can only be varied along the single path of varying a . The better contract (higher expected wage) has

a lower a than the inferior one. But although the overall expected wage is inversely related to a , there is always some level of Q low enough for the expected wage at that level of demand to be positively related to a . (See Appendix (3) for proof). Suppose that demand is initially in this depressed state. Then firm 1 will initially pay out a larger wage bill than firm 2. In this case there will be no immediate worker migration either way. The w_1 's realise that "overpayment" now signifies "underpayment" in the next boom, but, rationally, will not complain until then. The w_2 's realise that their current "underpayment" will be compensated if they stay.

APPENDIX

- (1) Because $d\Pi/dQ' = 0$ must hold at all $Q=Q'$, it is an identity. Therefore neither $N(Q')$ can contain more than a single term.

Proof Suppose that $N(Q')$ at least were more complicated than this:

$$N = \sum_{i=1}^n l_i Q'^{b_i} \quad \text{where } n > 1$$

$$W = \sum_{i=1}^m k_i Q'^{a_i} \quad \text{where } m > 1$$

$$\text{Then } dR/dQ' = Q'^c d \left[\left(\sum_{i=1}^n l_i Q'^{b_i} \right)^{d-1} \right] \sum_{i=1}^n b_i l_i Q'^{b_i-1}$$

which, because of the term in square brackets (d is not an integer), cannot be articulated into an additive series of terms, either when $Q=Q'$ or at any other time. But dC/dQ' will always be articulable in this fashion, whatever the values of m and n . Hence $dR/dQ' = dC/dQ'$ at $Q=Q'$ cannot be the identity we require. Hence N cannot have more than one term.

But then dR/dQ' will only have one term. Then dC/dQ' must have but one term (or, again, the required identity will not exist.) This in turn requires

that C takes the form $\alpha_1 Q'^{\beta} + \alpha_2$.

Since N has only one term this must mean:

$$N = 1Q'^b \quad ; \quad W = k_1 Q'^a + k_2 Q'^{-b}$$

Consider the three possibilities for b:

(1) b negative. This makes employment inversely related to demand, which violates truth-telling. (If $b < 0$, then $a > c$ (equation (5)), which violates truth-telling. (See Appendix (2)).

(2) b zero. Here $N = 1$; $W = k_1 Q'^a + k_2$. But here negative or zero a violates our assumption that the firm always prefers a contract that guarantees breaking even.¹ Positive a violates the truth-telling condition (the firm will always do best to declare demand as zero.)

(3) So b must be positive. But in this case, unless $k_2=0$, wages will approach either plus or minus infinity as demand approaches zero. Plus infinity violates the firm's preference for breaking even. The second possibility will always be suboptimal but the proof is tedious. We therefore simply (and reasonably) assume that no one will work for negative wages. Hence $k_2=0$. Hence W cannot have more than one term.

(2) The second-order truth-telling condition is that $a < c$

Proof Let $x = \frac{ad-d-c+1}{1-d}$

Then, from (3), and the fact that $Q=Q'$,

$$\Pi = \bar{Q}^x \left(Q^c Q' \frac{cd-ad}{1-d} - \frac{c-a}{c/d-a} Q' \frac{c-ad}{1-d} \right)$$

$$\dots \frac{d\Pi}{dQ'} = \bar{Q}^x \left(Q^c \frac{cd-ad}{1-d} Q' \frac{cd-ad-1+d}{1-d} - \frac{cd-ad}{1-d} Q' \frac{c-ad-1+d}{1-d} \right)$$

¹Unless $k_1 + k_2 \leq 0$, $a = 0$. But this means negative or zero wages at all Q. The firm will lose all its workers to rival firms.

$$\therefore \frac{d^2 \Pi}{dQ^2} = \bar{Q}^x \frac{cd-ad}{1-d} \left(\frac{cd-ad-1+d}{1-d} Q^c Q' \frac{cd-ad-2+2d}{1-d} - \frac{c-ad-1+d}{1-d} Q' \frac{c-ad-2+2d}{1-d} \right)$$

$$\therefore \text{At } Q' = Q, \frac{d^2 \Pi}{dQ^2} = \bar{Q} \frac{ad-d-c+1}{1-d} \frac{c(ad-cd)}{1-d} Q \frac{c-ad-2+2d}{1-d}$$

$$\therefore \frac{d^2 \Pi}{dQ^2} \text{ (at } Q' = Q) < 0 \text{ iff } a < c.$$

(3) There is always a positive level of Q below which $dE(W)/da > 0$, where $E(W)$ refers to expected wage at a given level of Q.

Proof $E(W)$ (given Q) = $\frac{w(Q)N(Q)}{N(\bar{Q})}$

$$= \frac{c-a}{c/d-a} Q \frac{c-ad}{1-d} \bar{Q} \frac{c-2cd+2d-d^2+ad^2}{d(1-d)}$$

Let $\frac{c-2cd+2d-d^2+ad^2}{d(1-d)} = y$

Let $\log\left(\frac{E(W)}{\bar{Q}^y}\right) = x$

The general rule is that, if $x = \log\left(\frac{f(a)}{K}\right)$, where $f(a)$ and K are both positive and K is independent of a , then:

$$\frac{dX}{da} \text{ has the same sign as } f'(a) \text{ for all } f(a)$$

$$\frac{d^2 X}{da^2} \text{ has the same sign as } f''(a) \text{ at } f'(a) = 0.$$

In this case, $X = \log(c-a) - \log(c/d-a) + \frac{c-ad}{1-d} \log Q + \frac{ad}{1-d} \log \bar{Q}$

$$\therefore \frac{dX}{da} = \frac{-1}{c-a} + \frac{1}{c/d-a} - \log Q + \frac{d}{1-d} \log \bar{Q}.$$

Hence $\log Q$ has a real value when $\frac{dX}{da} = 0$.

Hence $Q > 0$ when $\frac{dX}{da} = 0$

Hence $Q > 0$ when $\frac{dE(W)}{da} = 0$

$$\frac{d^2X}{da^2} = \frac{-1}{(c-a)^2} + \frac{1}{(c/d-a)^2} < 0 \text{ at all } Q$$

$$\therefore \frac{d^2X}{da^2} < 0 \text{ when } \frac{dE(W)}{da} = 0$$

$$\therefore \frac{d^2E(W)}{da^2} < 0 \text{ when } \frac{dE(W)}{da} = 0$$

Hence when Q is below its unique positive value at which $\frac{dE(W)}{da} = 0$, $\frac{dE(W)}{da} > 0$.

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