Branon radiative corrections to collider physics and precision observables

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(Received 18 November 2005; published 17 February 2006)

In the context of brane-world scenarios, we study the effects produced by the exchange of virtual massive branons. A one-loop calculation is performed which generates higher-dimensional operators involving SM fields suppressed by powers of the brane tension scale. We discuss constraints on this scenario from colliders such as HERA, LEP and Tevatron and prospects for future detections at LHC or ILC. The most interesting phenomenology comes from new four-particles vertices induced by branon radiative corrections, mainly from four-fermion interactions. The presence of flexible branes modifies also the muon anomalous magnetic moment and the electroweak precision observables.

DOI: 10.1103/PhysRevD.73.035008 PACS numbers: 11.25.Mj, 11.10.Lm, 11.15.Ex

I. INTRODUCTION

In recent years, it has been shown that a generic property of brane-world models [1] with low tension ($\tau \equiv f^4 \ll \Lambda^4$, where τ is the brane tension and Λ is the scale below which the description given by the brane-world scenario is appropriate) is the presence of new modes $\pi^\alpha(x)$ called branons which roughly correspond to excitations of the brane position along the extra compactified dimensions. The relevant tree-level phenomenology of branons has been studied for colliders and also for astrophysics and cosmology in terms of their mass M and brane tension parameter f, and it has been suggested that massive branons could be natural candidates for dark matter in this kind of models [2].

In this work we study the phenomenology of branon radiative corrections. Branon loops are interesting mainly for two reasons. First because precision tests of the standard model (SM) usually enforce strong constraints on physics beyond it and thus make possible to reject many new models, or at least to set bounds on their parameters. The second reason is that branon loops provide new physical effects, such as four-fermion interactions, which can be searched for in present and next generation colliders.

As it is the case for branon tree-level effects, the loop corrections can be obtained from the effective action for branons described in detail in [3]. This effective action can be expanded in powers of $\partial\pi/f^2$ and $M^2\pi^2/f^4$ [4,5]:

$$S_{\rm eff}[\pi] = S_{\rm eff}^{(0)}[\pi] + S_{\rm eff}^{(2)}[\pi] + S_{eff}^{(4)}[\pi] + \ldots. \quad (1)$$

The zeroth order term is just a constant, the $\mathcal{O}((\pi/f)^2)$ contribution contains the branon free action:

$$S_{\rm eff}^{(2)}[\pi] = \frac{1}{2}\int_{M_4} d^4x(\delta_{\alpha\beta}\partial_\mu\pi^\alpha\partial^\mu\pi^\beta - M^2_{\alpha\beta}\pi^\alpha\pi^\beta). \quad (2)$$

where $M^2_{\alpha\beta}$ is the squared branon mass matrix corresponding to the different branon excitations π^α, with α running from one to the number of effective extra dimensions N. The couplings to the SM fields Φ living on the brane (or any suitable extension of it) in the presence of a gravitational background which for simplicity we will assume to be flat ($g_{\mu\nu} = \eta_{\mu\nu}$), can be described at low energies by the action:

$$S_{SM}[\Phi, \pi] = \int_{M_4} d^4x\Big[\mathcal{L}_{SM}(\Phi) + \frac{1}{2}\delta_{\alpha\beta}\partial_\mu\pi^\alpha\partial^\mu\pi^\beta - \frac{1}{2}M^2_{\alpha\beta}\pi^\alpha\pi^\beta + \frac{1}{8f^4}(4\delta_{\alpha\beta}\partial_\mu\pi^\alpha\partial_\nu\pi^\beta - M^2_{\alpha\beta}\pi^\alpha\pi^\beta\eta_{\mu\nu})T^{\mu\nu}_{SM}\Big] + \mathcal{O}(\pi^4), \quad (3)$$

where $T^{\mu\nu}_{SM}$ is the conserved SM energy-momentum tensor evaluated in the background metric:

$$T^{\mu\nu}_{SM} = -\left(g^{\mu\nu}\mathcal{L}_{SM} + 2\frac{\delta\mathcal{L}_{SM}}{\delta g_{\mu\nu}}\right)\bigg|_{g_{\mu\nu}=\eta_{\mu\nu}}. \quad (4)$$

It is interesting to note that there is no single branon interactions due to the parity conservation on the brane by the gravitational action. Thus branons are absolutely stable. This fact is crucial for the branon phenomenology and, in particular, for cosmology since it makes them natural WIMP candidates for dark matter [2]. In addition, the quadratic expression in (3) is valid for any internal extra-dimension space K_N, regardless the particular form of its metric $\gamma_{\alpha\beta}$. Indeed the form of the couplings only depends on the number N of branon fields and the brane tension. Dependence on the geometry of the extra dimensions will appear only at higher orders. Here we are assuming that the bulk D dimensional space-time ($D = 4 + N$) can be split as $\mathcal{M}_D = M_4 \times K_N$, where M_4 is the standard four-dimensional Minkowski space and K_N is some compact and homogeneous space of dimension N with Gaussian coordinates y^α. Then the N branon fields ($\alpha = 1, \ldots, N$) can be chosen so that $\pi^\alpha(x) = f^2 y^\alpha(x)$ where $y^\alpha = y^\alpha(x)$ represents the position of the excited brane in the extra-dimension space K_N. The brane ground state corresponds to $\pi^\alpha = 0$.

From the action above it is clear that branons always interact by pairs with the SM matter fields. In addition, due to their geometric origin, those interactions are very similar to the gravitational ones since the π^α fields couple to all the matter fields through the energy-momentum tensor and with the same strength suppressed by a f^4 factor. The interaction between bulk gravitons and SM fields is given by

$$S_h = \frac{1}{\bar{M}_P}\sum_p \int_{M_4} d^4x h^{(p)}_{\mu\nu}(x) T^{\mu\nu}_{SM}(x), \tag{5}$$

where $\bar{M}_P^2 \equiv M_P^2/4\pi$ is the squared reduced Planck mass ($M_P = 1.2 \times 10^{19}$ GeV) and $h^{(p)}_{\mu\nu}$ are the Kaluza-Klein (KK) modes of the bulk graviton $h_{\mu\nu}(x, y)$ corresponding to the 4×4 part of the bulk metric:

$$g_{\mu\nu}(x, y) = \eta_{\mu\nu} + \frac{2h_{\mu\nu}(x, y)}{\bar{M}_D^{1+N/2}}, \tag{6}$$

where $\bar{M}_D^{D-2} = M_D^{D-2}/(4\pi)$ and M_D is the D dimensional Planck scale (fundamental scale of gravity) related with the usual four-dimensional Planck scale by $M_P^2 = V(K_N)M_D^{D-2}$ with $V(K_N)$ being the volume of the internal space K_N (notice that $M_4 \equiv M_P$). By introducing a complete orthonormal set of functions $f_p = f_p(y)$ on K_N with normalization:

$$\int_{K_n} dV(K_N) f_p^*(y) f_q(y) = V(K_n)\delta_{pq} \tag{7}$$

and $f_p(0) = 1$, the KK mode decomposition for the graviton field becomes

$$h_{\mu\nu}(x, y) = \frac{1}{\sqrt{V(K_N)}}\sum_p h^{(p)}_{\mu\nu}(x) f_p(y). \tag{8}$$

When computing radiative corrections, divergent integrals appear. As our effective actions are not renormalizable all our results will be given in terms of some energy cutoff Λ, which could be taken as the value where the whole brane-world picture breaks down and a more fundamental approach is needed. Then our results will be given in terms of four parameters, namely, the number of branons or extra dimensions N, the branon mass M (for simplicity we will assume at the end that all of them are degenerate $M_{\alpha\beta} = \delta_{\alpha\beta}M_\beta = \delta_{\alpha\beta}M$), the brane tension scale f ($\tau = f^4$) and the cutoff Λ.

In this work we will assume all the time for simplicity that the brane is infinitely thin, i.e. it is considered as a completely fundamental and structureless object. However in many realistic cases one expects to have some width scale L for the brane. This is for instance the case when the brane appears as a kink or more generally as a soliton coming from a more fundamental theory like M-theory (see for instance [6,7]). In all these cases it is clear that the thin brane approximation is valid only provided the energies considered, and consequently Λ, is smaller than the scale $\Lambda_L \equiv L^{-1}$. On the other hand one also typically expects to have the brane tension scale f of the same order of Λ_L. All these comments should be taken into account when the results given in this work are used in the context of any precise realization of the brane world and the SM fields confinement mechanism.

The plan of the paper goes as follows: In Sec. II we reobtain the result of [8] concerning the suppression of the coupling between SM fields on the brane and bulk fields, by integrating out the branon fields instead of using arguments based on normal ordering. In Sec. III we study the effects of branon loops on the SM particle parameters and find the effective action describing the new induced interactions. The corresponding phenomenological consequences are considered in Sec. IV, where we also set the bounds coming from the branon loops on the parameters f, M, N and the scale Λ. Further constraints can be obtained from two loops effects and their impact on the electroweak precision observables and the muon anomalous magnetic momentum which can be found in Sec. V. In Sec. VI we summarize and comment our results and in Appendixes A, B, and C we define the divergent integrals appearing in our computations, the Feynman rules corresponding to the effective Lagrangian describing the branon-loops effects, and the associated cross sections.

II. GRAVITON COUPLING SUPPRESSION

Probably the most immediate effect of virtual branons is the suppression of the coupling of SM particles and the KK modes bulk fields like the graviton. When branon fluctuations are taken into account this effective coupling is described by the action:

$$S_h = \frac{1}{\bar{M}_P}\sum_p \int_{M_4} d^4x h^{(p)}_{\mu\nu}(x) T^{\mu\nu}_{SM}(x) f_p(\pi) \tag{9}$$

due to the fact that the brane is no more sitting at $\pi = 0$ but is moving around this point. Now the branons fields can be integrated out in the usual way to find:

$$S_h = \frac{1}{\bar{M}_P}\sum_p \int_{M_4} d^4x h^{(p)}_{\mu\nu}(x) T^{\mu\nu}_{SM}(x) \langle f_p(\pi)\rangle, \tag{10}$$

where the f_p expectation value is given by

$$\langle f_p(\pi)\rangle = \int [d\pi] e^{iS^{(2)}_{\rm eff}[\pi]} f_p(\pi). \tag{11}$$

In the limit of massless branons, the branon effective action is just a nonlinear sigma model (NLSM) based on a coset which is isomorphic to K_N. Therefore the invariant path integral measure should include an additional factor proportional to the square root determinant of the coset metric to ensure that quantum corrections do not spoil the Ward identities of the NLSM. The extra term in the measure amounts to an extra term in the effective action proportional to Λ^4. This term is important when dealing with branon loop corrections to the branon self-interactions (for instance branon-branon elastic scattering) [9]. However in this work we are mainly interested in interactions between a couple of branons and SM particles and hence we can safely neglect this measure term.

To compute the path integral above, we need to know the precise form of the $f(\pi)$ functions which depends on the K_N geometry. For example for the case of the torus $K_N = T^N$:

$$f_{\vec{n}}(y) = \exp\left(i\frac{\vec{n}\,\vec{y}}{R}\right), \qquad (12)$$

where $\vec{n} = (n_1, n_2, \ldots, n_N)$ is a N dimensional vector with integer and positive or zero components and R is the torus radius (common for all coordinates). Then we have

$$\langle f_{\vec{n}}(\pi)\rangle = \left\langle \exp\left(i\frac{\vec{n}\,\vec{\pi}}{Rf^2}\right)\right\rangle = \exp\left(-\frac{1}{2R^2f^4}\sum_{\alpha=1}^{N} n_\alpha^2 G_{\alpha\alpha}(0)\right), \qquad (13)$$

where $G_{\alpha\beta}(x)$ is the branon propagator

$$G_{\alpha\beta}(x-y) = \int d\tilde{q}\, e^{-iq(x-y)}\frac{\delta_{\alpha\beta}}{q^2 - M_\alpha^2 + i\epsilon} \qquad (14)$$

and $d\tilde{q} \equiv d^4q/(2\pi)^4$. Using a cutoff Λ to regularize the divergent integral we find

$$G_{\alpha\beta}(0) = \frac{1}{16\pi^2}\left[\Lambda^2 - M_\alpha^2 \log\left(\frac{\Lambda^2}{M_\alpha^2} + 1\right)\right]\delta_{\alpha\beta}. \qquad (15)$$

Then the effective action becomes

$$S_h = \frac{1}{\bar{M}_P}\sum_{\vec{n}}\int_{M_4} d^4x\, g_{\vec{n}} h_{\mu\nu}^{(\vec{n})}(x) T_{SM}^{\mu\nu}(x). \qquad (16)$$

In other words, the effect of branon quantum fluctuations amounts to introducing the KK mode dependent couplings $g_{\vec{n}}$ which are given for toroidal compactification by

$$g_{\vec{n}} = \exp\left(-\frac{\Lambda^2}{32\pi^2R^2f^4}\sum_{\alpha=1}^{N} n_\alpha^2\left[1 - \frac{M_\alpha^2}{\Lambda^2}\log\left(\frac{\Lambda^2}{M_\alpha^2} + 1\right)\right]\right). \qquad (17)$$

Thus the coupling of SM matter to higher KK modes is exponentially suppressed. This result was first obtained in [8] for massless branons by using an argument based on normal ordering. Our derivation here is more natural in the context of the path integral treatment of branon quantum fluctuations, and also it can be applied to massive branons in any extra-dimension space K_N. In any case this coupling suppression has very interesting consequences from the phenomenological point of view. It improves the unitarity behavior of the cross section for producing gravitons from SM particles and, in addition, it solves the problem of the divergences appearing even at the tree level when one considers the KK graviton tower propagators for dimension equal or larger than 2. Moreover whenever we have $v \equiv Rf^2 \ll \Lambda$, KK gravitons decouple from the SM particles, so that at low energies the only brane-world related particles that must be taken into account are branons. In the following we will assume this to be the case and accordingly we will deal only with SM particles and branons.

III. BRANON-LOOPS EFFECTS ON SM PARTICLES

In order to study the effect of virtual branons on the SM particles, it is useful to introduce the SM effective action $\Gamma_{SM}^{\text{eff}}[\Phi]$ obtained after integrating out the branon fields:

$$e^{i\Gamma_{SM}^{\text{eff}}[\Phi]} = \int [d\pi] e^{iS_{SM}[\Phi,\pi]} = e^{i\int d^4x \mathcal{L}_{SM}}(\text{Det}[O])^{-1/2}, \qquad (18)$$

where the O operator is defined as $O = A + B$ with:

$$A_{\alpha\beta}(x,y) = -\delta_{\alpha\beta}[\partial_\mu\partial^\mu + M_\alpha^2]\delta(x-y). \qquad (19)$$

$$B_{\alpha\beta}(x,y) = \frac{-1}{f^4}\delta_{\alpha\beta}\left[T_{SM}^{\mu\nu}\left(\partial_\mu\partial_\nu + \frac{M_\alpha^2}{4}\eta_{\mu\nu}\right)\right]\delta(x-y). \qquad (20)$$

This SM effective action can be computed in a systematic way by using standard procedures (see for instance [10]). Thus we have

$$\Gamma_{SM}^{\text{eff}}[\Phi] = \int d^4x \mathcal{L}_{SM} + \frac{i}{2}\text{Tr}(\ln[\text{O}]), \qquad (21)$$

and

$$\text{Tr}\,(\ln[\text{O}]) = \text{Tr}(\ln[\text{A}]) + \text{Tr}(\ln[1 + \text{BA}^{-1}]). \qquad (22)$$

The first term does not depend on the SM fields and it can only contribute to the renormalization of the cosmological constant. Expanding the logarithm, we obtain the usual expression:

$$\Gamma_{SM}^{\text{eff}}[\Phi] = \int d^4x \mathcal{L}_{SM} - \frac{i}{2}\sum_{k=1}^{\infty}\frac{(-1)^k}{k}\text{Tr}(\text{BA}^{-1})^k = \int d^4x \mathcal{L}_{SM} + \sum_{k=1}^{\infty}\Gamma^{(k)}[\Phi], \qquad (23)$$

where A^{-1} is the branon (π^α) free propagator:

$$A_{\alpha\beta}^{-1}(x,y) = G_{\alpha\beta}(x-y). \qquad (24)$$

Then the first contribution $\Gamma^{(1)}$, reads:

$$\Gamma^{(1)}[\Phi] = \frac{i}{2}\int d^4x d^4y B^{\alpha\beta}(x,y) G_{\alpha\beta}(x-y) = C_1\int d^4x T_{SM\mu}^{\mu}, \qquad (25)$$

where, assuming all the branons to be degenerate ($M_\alpha = M$, $\alpha = 1, \ldots, N$), the C_1 constant is given by

$$C_1 \eta_{\mu\nu} = i\frac{N}{8f^4}\int d\tilde{q}\frac{4q_\mu q_\nu - M^2\eta_{\mu\nu}}{q^2 - M^2 + i\epsilon}. \quad (26)$$

The second contribution to the effective action is

$$\begin{aligned}\Gamma^{(2)}[\Phi] &= -\frac{i}{4}\int d^4x d^4y d^4z d^4t B_{\alpha\beta}(x,y)G_{\beta\gamma}(y-z) \\ &\quad \times B_{\gamma\delta}(z,t)G_{\delta\alpha}(t-x) \\ &= -i\frac{N}{4f^8}\int d^4x d^4y d^4p e^{-ip(y-x)}T^{\mu\nu}(x)T^{\rho\sigma}(y) \\ &\quad \times \Big[J^{(M)}_{\mu\nu\rho\sigma}(p) - \frac{M^2}{4}(\eta_{\mu\nu}J^{(2)}_{\rho\sigma}(p) + \eta_{\rho\sigma}J^{(2)}_{\mu\nu}(p)) \\ &\quad + \frac{M^4}{16}\eta_{\mu\nu}\eta_{\rho\sigma}J^{(0)}(p)\Big], \end{aligned} \quad (27)$$

where from now on $T^{\mu\nu} = T^{\mu\nu}_{SM}$ and the integrals $J^{(I)}$ are defined in Appendix A. It is convenient to split the final expression into a local divergent term $\Gamma^{(2)}_L[\Phi]$ and a nonlocal finite term: $\Gamma^{(2)}_{NL}[\Phi]$.

$$\Gamma^{(2)}[\Phi] = \Gamma^{(2)}_L[\Phi] + \Gamma^{(2)}_{NL}[\Phi]. \quad (28)$$

Now by using the equation of motion at the zero order: $\partial_\mu T^{\mu\nu}_{SM} = 0$, the local piece can be written in terms of six constants W_i, $(i = 1, 2, \ldots, 6)$:

$$\begin{aligned}\Gamma^{(2)}_L[\Phi] = \int dx\{&W_1 T^{\mu\nu}T_{\mu\nu} + W_2 T^\mu_\mu T^\nu_\nu + W_3 T^{\mu\nu}\Box T_{\mu\nu} \\ &+ W_4 T^\mu_\mu \Box T^\nu_\nu + W_5 T^{\mu\nu}\Box^2 T_{\mu\nu} + W_6 T^\mu_\mu \Box^2 T^\nu_\nu\}\end{aligned} \quad (29)$$

and the nonlocal one in terms of two functions $D_j(p)$, $(j = 1, 2)$:

$$\begin{aligned}\Gamma^{(2)}_{NL}[\Phi] = \int dx\,dy\,dp\, e^{-ip(y-x)}\{&D_1(p)T^{\mu\nu}(x)T_{\mu\nu}(y) \\ &+ D_2(p)T^\mu_\mu(x)T^\nu_\nu(y)\}.\end{aligned} \quad (30)$$

These two functions are given by

$$D_1(p) = \frac{-iN}{480f^8}(p^2 - 4M^2)^2 J^F(p,M), \quad (31)$$

$$\begin{aligned}D_2(p) = -i\frac{N}{960f^8}\{&(p^2 + 6M^2)(p^2 - 4M^2) \\ &+ 15M^4\}J^F(p,M),\end{aligned} \quad (32)$$

with

$$J^F(p,M) = \begin{cases} \frac{i}{(4\pi)^2}\Big[2 - \sqrt{1 - \frac{4M^2}{p^2}}\ln\Big(\frac{\sqrt{1-\frac{4M^2}{p^2}}+1}{\sqrt{1-\frac{4M^2}{p^2}}-1}\Big)\Big]; & p^2 \le 0, \\ \frac{i}{(4\pi)^2}\Big[2 + 2\sqrt{\frac{4M^2}{p^2} - 1}\tan^{-1}\Big(\frac{1}{\sqrt{\frac{4M^2}{p^2}-1}}\Big)\Big]; & 0 < p^2 \le 4M^2, \\ \frac{i}{(4\pi)^2}\Big[2 - \sqrt{1 - \frac{4M^2}{p^2}}\ln\Big(\frac{1+\sqrt{1-\frac{4M^2}{p^2}}}{1-\sqrt{1-\frac{4M^2}{p^2}}}\Big) + i\pi\Big]; & 4M^2 < p^2. \end{cases} \quad (33)$$

Thus for the particular case of massless branons we find:

$$\begin{aligned}D_1(p) = 2D_2(p) &= \frac{-iNp^4 J^F(p, M=0)}{480f^8} \\ &= -\frac{Np^4\ln(-p^2)}{480(4\pi)^2 f^8},\end{aligned} \quad (34)$$

which is in agreement with previous results [11,12].

In the general case, the local actions $\Gamma^{(1)}[\phi]$ and $\Gamma^{(2)}_L[\phi]$ are divergent and therefore need to be regularized. By using a cutoff Λ, the C_1 constant appearing in $\Gamma^{(1)}[\phi]$ is given by

$$C_1(\Lambda, f) = -\frac{N\Lambda^4}{16(4\pi)^2 f^4}. \quad (35)$$

The W_i constants can be found in Table I. Λ could represent the width of brane or any other mechanism that modified the short-distance theory to cure the ultraviolet behavior of branons. However, for our purposes, Λ is just a phenomenological parameter. From the point of view of the effective theory, Λ/f parametrizes how strongly (or weakly) coupled quantum brane is, and therefore controls the unknown relative importance of tree-level versus loop

TABLE I. Regularized constants computed by using a cutoff Λ.

Coefficient	Cutoff regularized value
W_1	$\frac{N(\Lambda^4 - 4\Lambda^2M^2 - 2M^4 - 6M^4\ln(\frac{M^2}{\Lambda^2}))}{96(4\pi)^2 f^8}$
W_2	$\frac{N((\Lambda^2 + M^2)^2 + 3M^4\ln(\frac{M^2}{\Lambda^2}))}{192(4\pi)^2 f^8}$
W_3	$\frac{-N(15\Lambda^2 + 2M^2 + 30M^2\ln(\frac{M^2}{\Lambda^2}))}{1440(4\pi)^2 f^8}$
W_4	$\frac{-N(5\Lambda^2 - M^2)}{960(4\pi)^2 f^8}$
W_5	$\frac{N(17 - 60\ln(\frac{M^2}{\Lambda^2}))}{28\,800(4\pi)^2 f^8}$
W_6	$\frac{N(17 - 60\ln(\frac{M^2}{\Lambda^2}))}{57\,600(4\pi)^2 f^8}$

branon effects. From (35) we can see that the perturbative loop analysis only makes sense for approximately $\Lambda \lesssim 4\sqrt{\pi} f N^{-1/4}$.

The first term of the effective action $\Gamma^{(1)}[\phi]$ is proportional to the trace of the energy-momentum tensor so it would vanish if the SM were a scale invariant theory. By using the equation of motion $\partial_\mu T^{\mu\nu}_{SM} = 0$ it is possible to show that the only effect of this term is to renormalize the boson (scalars or gauge fields) masses $m^r_{\Phi,A} = (1 - 2C_1)^{1/2} m_{\Phi,A}$ and the fermion masses as $m^r_\psi = (1 - C_1) m_\psi$. Therefore, this first correction to the SM action does not have any measurable effect. On the other hand, the local action $\Gamma^{(2)}_L[\phi]$ is important from the phenomenological point of view. At low enough energies the dominant terms are the ones proportional to W_1 and W_2. Thus the SM Lagrangian is complemented by the additional effective Lagrangian given by

$$\triangle L_{\text{eff}} = W_1 T_{\mu\nu} T^{\mu\nu} + W_2 T^\mu_\mu T^\nu_\nu, \tag{36}$$

where for $\Lambda \gg M$

$$W_1 = \frac{N\Lambda^4}{96(4\pi)^2 f^8}, \qquad W_2 = \frac{N\Lambda^4}{192(4\pi)^2 f^8}. \tag{37}$$

When this is not the case, one should use the full result in Table I. From this Lagrangian it is possible to obtain the corresponding Feynman rules (see Appendix B). The most relevant contributions of branon loops to the SM particle phenomenology are the four-fermion interactions and fermion pair annihilation into two gauge bosons, whose cross sections can be found in Appendix C.

IV. PHENOMENOLOGICAL CONSEQUENCES: CONSTRAINTS

An effective Lagrangian similar to (36) was obtained in [12,13] by integrating at the tree level the Kaluza-Klein modes of gravitons propagating in the bulk and some of its phenomenological consequences were studied there. Thus it is easy to translate some of the results from these references to the present context.

Concerning the four-fermion interactions $\bar\psi_a(p_1)\psi_a(p_2) \longrightarrow \bar\psi_b(p_3)\psi_b(p_4)$ (see Appendix B 1), the most interesting case is the Bhabha scattering at LEP. From the Lagrangian in (35), it is possible to find a four-fermion interaction, whose amplitude is given by

$$\mathcal{M}^{4\psi} = \bar v_{a'}(p_1)\bar u_b(p_4) V^{4\psi}_{a'abb'}(-p_1, -p_2, p_3, p_4) v_{b'}(p_3) \times u_a(p_2). \tag{38}$$

Neglecting the fermion masses and assuming that all of them are different, the amplitude is just:

$$\begin{aligned}\mathcal{M}^{4\psi}(p_1, p_2, p_3, p_4) = \frac{W_1}{4} &\bar v_{a'}(p_1)\bar u_b(p_4) \\ &\times [\gamma_{a'a\mu}\gamma^\mu_{bb'}(p_2 - p_1)_\nu (p_4 - p_3)^\nu \\ &+ \gamma_{a'a\mu}\gamma_{bb'\nu}(p_2 - p_1)^\nu (p_4 - p_3)^\mu] \\ &\times v_{b'}(p_3) u_a(p_2)\end{aligned} \tag{39}$$

which can be compared with the analogous amplitude given in literature for the graviton case [14,15]. On the other hand, the explicit form of the cross section is given in Appendix C, which agrees with [15,16]. These interactions would lead to modifications of the scattering cross sections such as

$$e^+e^- \to \ell\bar\ell, q\bar q \tag{40}$$

$$q\bar q \to \ell\bar\ell, q\bar q, \tag{41}$$

where $\ell = e, \mu, \tau$. The Bhaba scattering, fermion pair production in e^+e^- colliders and Drell-Yan production at hadron colliders have been studied in detail from the point of view of the KK- graviton virtual exchange [15]. Dilepton and dijet channels have been studied at LEP and dielectron production at Tevatron. Also these processes are interesting for $e^\pm p \to e^\pm p$ interactions observed at HERA.

For $\bar\psi(p_1), \psi(p_2) \longrightarrow A^a_\mu(p_3), A^b_\nu(p_4)$ interactions (see Appendix B 2), diphoton production have been studied at LEP and Tevatron, whereas WW and ZZ production have been studied also at LEP. Exchange of virtual branons can also contribute to processes like

$$e^+e^-, q\bar q \to \gamma\gamma, W^+W^-, ZZ \quad \text{and} \quad gg, \tag{42}$$

$$\gamma\gamma, gg \to \ell\bar\ell, q\bar q. \tag{43}$$

The contribution to the gauge boson production is given by

$$\begin{aligned}\mathcal{M}^{2\psi\to 2A} = &\bar v_{a'}(p_1) V^{2\psi 2A}_{a'a\mu\nu}(-p_1, -p_2, p_3, p_4) u_a(p_2) \\ &\times \epsilon^{*\mu}(p_3, \sigma_{b'})\epsilon^{*\nu}(p_4, \sigma_b).\end{aligned} \tag{44}$$

and to fermion-antifermion production:

$$\begin{aligned}\mathcal{M}^{2A\to 2\psi} = &\bar u_{a'}(p_1) V^{2\psi 2A}_{a'a\mu\nu}(p_1, p_2, -p_3, -p_4) v_a(p_2) \\ &\times \epsilon^{\mu}(p_3, \sigma_{b'})\epsilon^{\nu}(p_4, \sigma_b).\end{aligned} \tag{45}$$

In fact, for the graviton case, the $e^+e^- \to \gamma\gamma, W^+W^-, ZZ$ processes have been studied in detail in [17], as well as the $gg \to l^+l^-$ [15]. Moreover, explicit expressions for the fermion-antifermion and diphoton production cross sections are given in Appendix C, or alternatively in [16].

Using the processes described above, we have obtained limits on the parameter combination $f^2/(N^{1/4}\Lambda)$ from different experiments at LEP, HERA and Tevatron, which are summarized in Table II. Also, with the same analogy with the Kaluza-Klein gravitons, we can estimate the con-

TABLE II. Lower limits on $f^2/(N^{1/4}\Lambda)$ (in GeV) from virtual branon searches at colliders (results at the 95% C.L.): HERA [18], LEP-II [19] and Tevatron-I [20]. $\sqrt{s}$ is the center-of-mass energy of the total process, and $\mathcal{L}$ is the total integrated luminosity.

Experiment	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (pb^{-1})	$f^2/(N^{1/4}\Lambda)$ (GeV)
HERA[c]	0.3	117	52
Tevatron-I[a,b]	1.8	127	69
LEP-II[a]	0.2	700	59
LEP-II[b]	0.2	700	75
Combined			81

[a]The two-photon channels.
[b]The e^+e^- channels.
[c]The $e^+p(e^-p)$ channels.

TABLE III. Estimated sensitivities on the parameter $f^2/(N^{1/4}\Lambda)$ (results in GeV at the 95% C.L.) for various colliders with center-of-mass energies and integrated luminosities as indicated.

Experiment	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (pb^{-1})	$f^2/(N^{1/4}\Lambda)$ (GeV)
Tevatron-II[a,b]	2.0	2×10^3	83
Tevatron-II[a,b]	2.0	3×10^4	108
ILC[b]	0.5	5×10^5	261
ILC[b]	1.0	2×10^5	421
LHC[b]	14	1×10^4	332
LHC[b]	14	1×10^5	383

[a]The two-photon channels.
[b]The e^+e^- channels.
[c]The $e^+p(e^-p)$ channels.

straints from future colliders. With that purpose, we have taken into account the estimations calculated by Hewett [15] for future linear colliders, Tevatron and LHC (see Table III).

V. TWO LOOPS EFFECTS: ELECTROWEAK PRECISION OBSERVABLES AND MUON ANOMALOUS MAGNETIC MOMENT

Electroweak precision measurements are very useful to find constraints on new physics models. The so-called oblique corrections (those corresponding to the W, Z, and γ two point functions) use to be described in terms of the S, T, U [21] or the ϵ_1, ϵ_2 and ϵ_3 parameters [22]. The first order correction coming from the Kaluza-Klein gravitons in the ADD models for rigid branes to the parameter: $\bar{\epsilon} \equiv \delta M_W^2/M_W^2 - \delta M_Z^2/M_Z^2$ was computed in [23]. This result can be written as

$$\delta\bar{\epsilon} \simeq \frac{20\Lambda^2(M_Z^2 - M_W^2)}{3(4\pi)^2}(5W_1 + 2W_2). \tag{46}$$

Translating this result to our context as in the previous section we find:

$$\delta\bar{\epsilon} \simeq \frac{5(M_Z^2 - M_W^2)}{12(4\pi)^4}\frac{N\Lambda^6}{f^8} \tag{47}$$

Notice that this is in fact a two-loop result since it is obtained from a one-loop computation by using an effective Lagrangian which is coming from another one-loop computation.

The experimental value of $\bar{\epsilon}$ obtained from LEP [24] is $\bar{\epsilon} = (1.27 \pm 0.16)\times 10^{-2}$. This value is consistent with the SM prediction for a light Higgs $m_H \leq 237$ GeV at 95% C.L. On the other hand, the theoretical uncertainties are 1 order of magnitude smaller [22] and therefore, we can estimate the constraints for the branon contribution at 95% C.L. as $|\delta\bar{\epsilon}| \leq 3.2\times 10^{-3}$. Thus it is possible to set the bound:

$$\frac{f^4}{N^{1/2}\Lambda^3} \geq 3.1\mathrm{GeV}(95\%\ \mathrm{C.L.}) \tag{48}$$

This result has a stronger dependence on Λ (Λ^6) than the interference cross section between the branon and SM interactions (Λ^4). Therefore, the constraints coming from this analysis are complementary to the previous ones.

A further constraint to the branon parameters can be obtained from the μ anomalous magnetic moment. The first branon contribution to this parameter can be obtained from a one-loop computation with the Lagrangian given by (35). The result for the KK graviton tower was first calculated by [25] and confirmed by [23] in a different way, and can be written as

$$\delta a_\mu \simeq \frac{2m_\mu^2\Lambda^2}{3(4\pi)^2}(11W_1 - 12W_2), \tag{49}$$

which for the branon case can be translated into:

$$\delta a_\mu \simeq \frac{5m_\mu^2}{114(4\pi)^4}\frac{N\Lambda^6}{f^8}. \tag{50}$$

This expression is qualitatively similar to other $g-2$ contributions obtained in different analyses of extra-dimension models [26]. The result depends on the cutoff Λ in the same way as the electroweak precision parameters. However the experimental situation is slightly different. In a sequence of increasingly more precise measurements, the 821 Collaboration at the Brookhaven Alternating Gradient Syncrotron has reached a fabulous relative precision of 0.5 parts per million in the determination of $a_\mu = (g_\mu - 2)/2$ [27]. These measurements provide a stringent test not only of new physics but also of the SM. Indeed, the present result is only marginally consistent with the SM. Taking into account the e^+e^- collisions to calculate the $\pi^+\pi^-$ spectral functions, the deviation with respect to the SM prediction is at 2.6 standard deviations [28]. In particular: $\delta a_\mu \equiv a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM}) = (23.4 \pm 9.1)\times 10^{-10}$. Using Eq. (50) we can estimate the *preferred* parameter region for branons to provide the observed difference:

$$6.0\ \text{GeV} \geq \frac{f^4}{N^{1/2}\Lambda^3} \geq 2.2\ \text{GeV}\ (95\%\ \text{C.L.}).$$

We observe that the correction to the muon anomalous magnetic moment is in the right direction and that it is possible to avoid the present constraints and improve the observed experimental value obtained by the E821 Collaboration.

If there is really new physics in the muon anomalous magnetic moment, and this new physics is due to branon radiative corrections, the phenomenology of these particles should be observed at the LHC and in a possible future ILC (see Table III). In particular, the LHC should observe an important difference in the channels: $pp \rightarrow e^+e^-$ and $pp \rightarrow \gamma\gamma$ with respect to the SM prediction. The ILC should observe the most important effect in the process: $e^+e^- \rightarrow e^+e^-$. Moreover, in [29] it was shown that the same parameter region in which branons could explain the muon anomalous magnetic moment is also compatible with a cosmological branon relic abundance enough to account for the observed dark matter relative density [30].

VI. CONCLUSIONS

In this work we have studied the phenomenological consequences of branon radiative corrections, by calculating the one-loop effective action for SM particles, obtained after integrating the branon fields out. We have found new interaction vertices of SM particles, in particular, new four-fermion interactions and interactions involving two fermions and two gauge fields. Our results, computed with a cutoff regulator and assuming an infinitely thin brane, have some similarity with those obtained by integrating the graviton KK modes at the tree level in ADD models, and, accordingly, we have been able to translate the different constraints to the branon case. Thus, we have obtained limits for the combination of parameters $f^2/(N^{1/4}\Lambda)$ from present experiments at LEP and Tevatron, and also for future colliders (ILC and LHC).

We have also considered the branon two-loop effect on electroweak precision observables and on the muon anomalous magnetic moment. We have evaluated the corresponding corrections and obtained the preferred parameter range for branons in order to fit the Brookhaven results, and at the same time, to be consistent with LEP precision measurements. In Fig. 1 we include all those limits and also the parameter region in which the theory can be considered as strongly interacting, i.e. ($\Lambda \gtrsim 4\sqrt{\pi} f N^{-1/4}$) and for which the loop expansion is no longer valid.

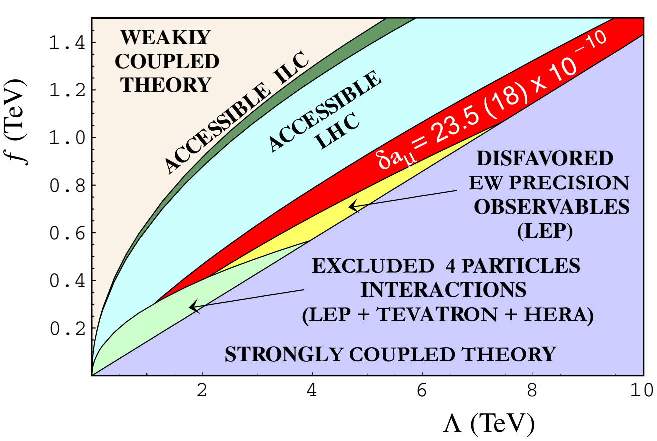


FIG. 1 (color online). Main limits from branon radiative corrections in the $f - \Lambda$ plane for a model with $N = 1$. The (red) central area shows the region in which the branons account for the muon magnetic moment deficit observed by the E821 Collaboration [27,28], and at the same time, are consistent with present collider experiments (whose main constraint comes from the Bhabha scattering at LEP) and electroweak precision observables. Prospects for future colliders are also plotted.

ACKNOWLEDGMENTS

J. A. R. C. thanks C. P. Martin for important comments on the renormalization procedure. This work is supported in part by DGICYT (Spain) under Projects No. FPA 2004-02602 and No. FPA 2005-02327, by NSF Grant No. PHY–0239817 and by the Fulbright-MEC (Spain) program.

APPENDIX A: DIVERGENT INTEGRALS

Definitions of various divergent integrals used in the text:

$$J^{(0)}(p) = \int d\tilde{q} \frac{1}{(q^2 - M^2 + i\epsilon)((p+q)^2 - M^2 + i\epsilon)}. \tag{A1}$$

$$J^{(2)}_{\mu\nu}(p) = \int d\tilde{q} \frac{q_\mu q_\nu}{(q^2 - M^2 + i\epsilon)((p+q)^2 - M^2 + i\epsilon)}. \tag{A2}$$

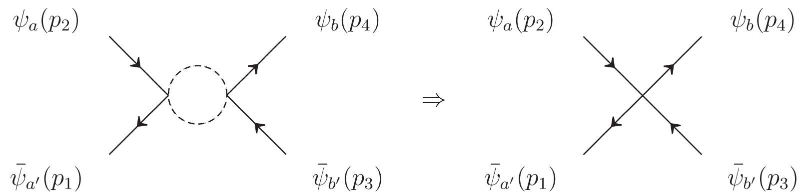
$$J^{(M)}_{\mu\nu\rho\sigma}(p) = \int d\tilde{q} \frac{q_\mu q_\nu (p+q)_\rho (p+q)_\sigma}{(q^2 - M^2 + i\epsilon)((p+q)^2 - M^2 + i\epsilon)}. \tag{A3}$$

APPENDIX B: EFFECTIVE FEYNMAN RULES

In this section we give the most important effective Feynman rules, obtained by the integration of the branons. We are going to present the fundamental new vertices with outgoing momenta from (36).

1. Effective 4-fermion vertex

One of the most relevant contribution of virtual branons to the phenomenology of the SM particles is the effect on four-fermion interactions. For a generic four-fermion process, the branons induce a new effective vertex of the form:



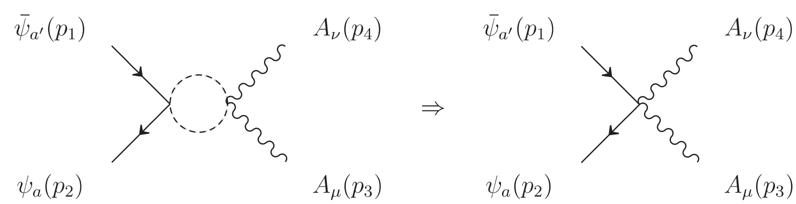
$$\begin{aligned} V^{4\psi D}_{a'abb'}(p_1, p_2, p_3, p_4) = \frac{W_1}{4}[&32m_{\psi_a}\delta_{a'a}m_{\psi_b}\delta_{bb'} + \gamma_{a'a\mu}\gamma^{\mu}_{bb'}(p_2 - p_1)_\nu(p_4 - p_3)^\nu + 12m_{\psi_b}\delta_{bb'}\gamma_{a'a\mu}(p_2 - p_1)^\mu \\ &+ 12m_{\psi_a}\delta_{a'a}\gamma_{bb'\mu}(p_4 - p_3)^\mu + (\gamma_{a'a\mu}\gamma_{bb'\nu} + 4\gamma_{a'a\nu}\gamma_{bb'\mu})(p_2 - p_1)^\nu(p_4 - p_3)^\mu] \\ &+ \frac{W_2}{2}[8m_{\psi_a}\delta_{a'a} + 3\gamma_{a'a\mu}(p_2 - p_1)^\mu][8m_{\psi_b}\delta_{bb'} + 3\gamma_{bb'\nu}(p_4 - p_3)^\nu]. \end{aligned} \tag{B1}$$

In the case in which the fermion fields a and b are the same, one has to take into account two different effects in order to obtain the vertex from the above expression. On one hand, a factor of 2 due to the quadratic term in the SM energy-momentum tensor is not present, and on the other hand, the symmetrization with respect to the change: $\{p_1, p_2\} \leftrightarrow \{p_3, p_4\}$ should be performed. Therefore, the general form of the four-fermion vertex is given by

$$V^{4\psi}_{a'abb'}(p_1, p_2, p_3, p_4) = V^{4\psi D}_{a'abb'}(p_1, p_2, p_3, p_4) + \tfrac{1}{2}\delta_{ab}\delta_{a'b'}[V^{4\psi D}_{bb'a'a}(p_3, p_4, p_1, p_2) - V^{4\psi D}_{a'abb'}(p_1, p_2, p_3, p_4)]. \tag{B2}$$

2. Effective $\bar{\psi}$, ψ, A_μ, A_ν vertex

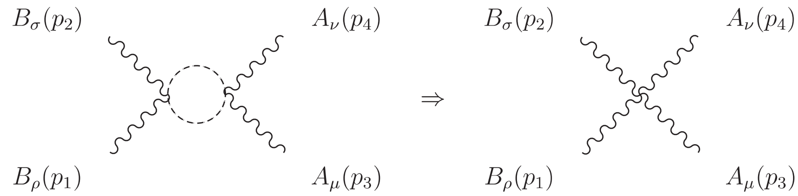
The exchange of virtual branons can also contribute to processes involving both a fermion pair and a gauge field pair As shown in the diagram, branons induce a new effective vertex, which, with the same momenta assignment as before, takes the form:



$$\begin{aligned} V^{2\psi 2A}_{a'a\mu\nu}(p_1, p_2, p_3, p_4) = W_1[&[(p_2 - p_1)^\lambda(p_{3\lambda}p_{4\mu}) - (p_4, p_3)(p_2 - p_1)_\mu]\gamma_{a'a\nu} \\ &+ [(p_2 - p_1)^\lambda(p_{4\lambda}p_{3\nu}) - (p_4, p_3)(p_2 - p_1)_\nu]\gamma_{a'a\mu} \\ &- [(p_2 - p_1)^\lambda\gamma^\sigma_{a'a}(p_{4\lambda}p_{3\sigma} - p_{3\lambda}p_{4\sigma} - \eta_{\lambda\sigma}(p_4, p_3))]\eta_{\mu\nu} \\ &- [(p_2 - p_1)_\lambda p_{4\mu}p_{3\nu} - (p_2 - p_1)_\mu p_{4\lambda}p_{3\nu} - (p_2 - p_1)_\nu p_{3\lambda}p_{4\mu}]\gamma^\lambda_{a'a} \\ &- m_A^2[(4m_\psi\delta_{a'a} + \gamma^\lambda_{a'a}(p_2 - p_1)_\lambda)\eta_{\mu\nu} + (p_2 - p_1)_\mu\gamma_{a'a\nu} + (p_2 - p_1)_\nu\gamma_{a'a\mu}]] \\ &- 2W_2m_A^2[8m_\psi\delta_{a'a} + 3\gamma^\lambda_{a'a}(p_2 - p_1)_\lambda]\eta_{\mu\nu}. \end{aligned} \tag{B3}$$

3. Effective B_ρ, B_σ, A_μ, A_ν vertex

Another effective interaction produced by virtual branon exchange can also contribute to processes involving two different gauge field pairs. Indeed, branons induce the following effective vertex:



$$\begin{aligned}
V^{2B2A}_{\mu\nu\rho\sigma}(p_1, p_2, p_3, p_4) = {} & 4W_1[p_{2\mu}p_{3\sigma}(p_{1\rho}p_{4\nu} - p_{1\nu}p_{4\rho}) + p_{1\mu}p_{3\rho}(p_{2\sigma}p_{4\nu} - p_{2\nu}p_{4\sigma}) + p_{3\mu}(p_{2\sigma}p_{4\rho}p_{1\nu} - p_{2\nu}p_{4\sigma}p_{1\rho}) \\
& - (p_3, p_4)[p_{2\mu}(-p_{1\nu}\eta_{\sigma\rho} + p_{1\rho}\eta_{\nu\sigma}) + p_{1\mu}(-p_{2\nu}\eta_{\sigma\rho} + p_{2\rho}\eta_{\nu\sigma}) + p_{2\nu}p_{1\rho}\eta_{\sigma\mu} + p_{1\nu}p_{2\sigma}\eta_{\mu\rho}) \\
& - 2p_{1\rho}p_{2\sigma}\eta_{\mu\nu}] - (p_1, p_2)[p_{4\nu}(p_{3\sigma}\eta_{\mu\rho} + p_{3\rho}\eta_{\mu\sigma}) + p_{3\mu}(p_{4\rho}\eta_{\sigma\nu} + p_{4\sigma}\eta_{\nu\rho}) - 2p_{3\mu}p_{4\nu}\eta_{\sigma\rho} \\
& - (p_{4\rho}p_{3\sigma} + p_{3\rho}p_{4\sigma})\eta_{\mu\nu}] - (p_2, p_4)[p_{3\sigma}(p_{1\rho}\eta_{\mu\nu} - p_{1\nu}\eta_{\mu\rho}) + p_{3\mu}p_{1\nu}\eta_{\rho\sigma}] \\
& - (p_2, p_3)[p_{4\sigma}(p_{1\rho}\eta_{\mu\nu} - p_{1\mu}\eta_{\nu\rho}) + p_{4\nu}p_{1\mu}\eta_{\rho\sigma}] - (p_1, p_4)[p_{3\rho}(p_{2\sigma}\eta_{\mu\nu} - p_{2\nu}\eta_{\mu\sigma}) \\
& + p_{3\mu}p_{2\nu}\eta_{\rho\sigma}] - (p_1, p_3)[p_{2\mu}(p_{4\nu}\eta_{\rho\sigma} - p_{4\rho}\eta_{\nu\sigma}) + p_{4\rho}p_{2\sigma}\eta_{\mu\nu}] \\
& - (p_1, p_3)[p_{2\mu}(p_{4\nu}\eta_{\rho\sigma} - p_{4\rho}\eta_{\nu\sigma}) + p_{4\rho}p_{2\sigma}\eta_{\mu\nu}] \\
& + (p_1, p_4)(p_2, p_3)[\eta_{\rho\nu}\eta_{\mu\sigma} + \eta_{\rho\mu}\eta_{\nu\sigma} - 2\eta_{\rho\sigma}\eta_{\mu\nu}] \\
& + \eta_{\rho\sigma}\eta_{\mu\nu}[(p_1, p_4)(p_2, p_3) + (p_1, p_3)(p_2, p_4)] + m_A^2[-p_{4\mu}(-2p_{3\nu}\eta_{\rho\sigma} + p_{3\rho}\eta_{\nu\sigma} + p_{3\sigma}\eta_{\nu\rho}) \\
& - p_{3\nu}(p_{4\rho}\eta_{\mu\sigma} + p_{4\sigma}\eta_{\mu\rho}) + \tfrac{1}{2}\eta_{\mu\nu}(p_{4\rho}p_{3\sigma} + p_{3\rho}p_{4\sigma}) \\
& + (p_3, p_4)(\eta_{\nu\sigma}\eta_{\mu\rho} + \eta_{\mu\sigma}\eta_{\rho\nu} - \eta_{\rho\sigma}\eta_{\mu\nu})] + m_B^2[-p_{2\rho}(-2p_{1\sigma}\eta_{\mu\nu} + p_{1\mu}\eta_{\sigma\nu} + p_{1\nu}\eta_{\sigma\mu}) \\
& - p_{1\sigma}(p_{2\mu}\eta_{\rho\nu} + p_{2\nu}\eta_{\mu\rho}) + \tfrac{1}{2}\eta_{\rho\sigma}(p_{2\mu}p_{1\nu} + p_{1\mu}p_{2\nu}) \\
& + (p_1, p_2)(\eta_{\nu\sigma}\eta_{\mu\rho} + \eta_{\mu\sigma}\eta_{\rho\nu} - \eta_{\rho\sigma}\eta_{\mu\nu})] + m_A^2 m_B^2[\eta_{\nu\sigma}\eta_{\mu\rho} + \eta_{\mu\sigma}\eta_{\rho\nu}]] \\
& + 2W_2 m_A^2 m_B^2 \eta_{\mu\nu}\eta_{\rho\sigma}.
\end{aligned} \tag{B4}$$

In the case in which the gauge bosons A and B are the same, one should again take into account the points commented above. In particular the symmetrization with respect to the change: $\{p_1, p_2, \mu, \nu\} \leftrightarrow \{p_3, p_4, \rho, \sigma\}$ should be performed. Therefore, the vertex with four identical gauge boson can be written as

$$\begin{aligned}
V^{4A}_{\mu\nu\rho\sigma}(p_1, p_2, p_3, p_4) = {} & \tfrac{1}{2}[V^{2B2A}_{\mu\nu\rho\sigma}(p_1, p_2, p_3, p_4) \\
& + V^{2B2A}_{\rho\sigma\mu\nu}(p_3, p_4, p_1, p_2)].
\end{aligned} \tag{B5}$$

APPENDIX C: CROSS SECTIONS

In this section we show the modifications in the cross section of four-particles processes derived from virtual branon exchange in terms of $s \equiv (p_1 + p_2)^2$, $t \equiv (p_1 - p_3)^2$ and $u \equiv (p_2 - p_3)^2$. We are neglecting the masses of these particles, which means $s + t + u = 0$.

1. σ_1: $f(p_1)\bar{f}(p_2) \to \gamma(p_3)\gamma(p_4)$

The diphoton production by fermion-antifermion annihilation with electric charge Q_f and number of colors N_f is given by

$$\frac{d\sigma_1}{dt} = \frac{s^2 - 2tu}{N_f s^2}\left[\frac{2\pi\alpha Q_f^2}{\sqrt{tu}} + \frac{W_1}{2}\sqrt{tu}\right]^2. \tag{C1}$$

2. σ_2: $g(p_1)g(p_2) \to l^+(p_3)l^-(p_4)$

On the contrary, the dilepton production by gluon annihilation does not present interference term:

$$\frac{d\sigma_2}{dt} = \frac{W_1^2 tu}{64\pi s^2}[s^2 - 2tu]. \tag{C2}$$

3. σ_3: $g(p_1)g(p_2) \to \gamma(p_3)\gamma(p_4)$

The situation is similar for the diphoton production by gluon annihilation, since there is no SM contribution at tree level:

$$\frac{d\sigma_3}{dt} = \frac{W_1^2}{64\pi s^2}[s^4 - 2tu(2s^2 - tu)]. \tag{C3}$$

4. σ_4: $e^-(p_1)e^+(p_2) \to f(p_3)\bar{f}(p_4)(f \neq \nu_e, e^-)$

To illustrate the four-fermion interaction contribution, we can write the cross section for the fermion-antifermion production (except ν_e and e^-) in e^+e^- collisions in terms of the vector $v_f = T_f - 2Q_f \sin^2\theta_W$ and axial $a_f = T_f$ couplings of the particular fermion field:

$$\frac{d\sigma_4}{dt} = \frac{d\sigma_4}{dt}\bigg|_{\rm SM} + \frac{N_f W_1^2}{128\pi s^2}[s^4 - 2tu(5s^2 - 16tu)] - \frac{N_f \alpha W_1}{4s^3}\left\{Q_e Q_f (t-u)^3 + \frac{1}{\sin^2 2\theta_W}\frac{s}{s - M_Z^2}[v_e v_f (t-u)^3 + a_e a_f s(s^2 - 6tu)]\right\}. \quad \text{(C4)}$$

5. σ_5 : $e^-(p_1)e^+(p_2) \rightarrow e^-(p_3)e^+(p_4)$

For the Bhabha scattering, the cross section presents more terms since one has to take into account the t-channel contributions:

$$\frac{d\sigma_5}{dt} = \frac{d\sigma_4}{dt} + \frac{W_1^2}{128\pi s^2}[40s^4 + 6t(31st^2 - 21s^3 - 40s^2u) + 9t^4] - \frac{\alpha W_1}{4s^3}\left\{\frac{Q_e^2}{t}[9s^4 + 22ts^3 + 24t^2s^2 - 11t^3u - 10t^4] + \frac{s}{\sin^2 2\theta_W}\frac{v_e^2 + a_e^2}{s - M_Z^2}[u(4t^2 - 4s^2 + 5tu)] + \frac{1}{\sin^2 2\theta_W}\frac{s}{t - M_Z^2}[v_e^2(8s^3 + 6ts^2) + (v_e^2 + a_e^2) \times [s^3 + 12s^2t - 5t^2(3u + 2t)]]\right\}. \quad \text{(C5)}$$

All these results are in agreement with the expressions calculated for other kind of models which predict the same Lagrangian (35). In particular, for KK gravitons in the ADD model: $W_2 = -W_1/(N+2)$ and W_1 is related to a new energy scale: M_S [15–17]. For example, $W_1 = 4\lambda/M_S^4$ in [15] (where typically $\lambda = \pm 1$, takes into account the unknownness of the exact theory). So we can use the effective vertices and cross sections given in the Appendixes B and C for branons or gravitons using the corresponding definitions of the parameters W_1 and W_2. In fact, we can estimate directly the bounds over $f^2/(\Lambda N^{1/4})$ using the bounds over M_S. In the most interesting cases, the contribution of the term proportional to W_2 is zero or negligible and it is a good estimation to take, for $\lambda = 1$:

$$\frac{f^2}{\Lambda N^{1/4}} = \frac{M_S}{4(24\pi^2)^{1/4}} \simeq 0.064 M_S.$$

Typically, when $f \ll M_D$ the most important signal of brane worlds comes from branons. In such a case, we can estimate $f^2/(\Lambda N^{1/4})$ as it is shown in Table II.

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