International Journal of Modern Physics D
Vol. 13, No. 10 (2004) 2275–2279
© World Scientific Publishing Company

World Scientific
www.worldscientific.com

DARK GEOMETRY*

J. A. R. CEMBRANOS,† A. DOBADO‡ and A. L. MAROTO§

Departamento de Física Teórica,
Universidad Complutense de Madrid, 28040 Madrid, Spain
†*jruizcem@uci.edu*
‡*dobado@fis.ucm.es*
§*maroto@fis.ucm.es*

Received 18 May 2004
Communicated by D. V. Ahluwalia-Khalilova

Extra-dimensional theories contain additional degrees of freedom related to the geometry of the extra space which can be interpreted as new particles. Such theories allow to reformulate most of the fundamental problems of physics from a completely different point of view. In this essay, we concentrate on the brane fluctuations which are present in brane-worlds, and how such oscillations of the own space–time geometry along curved extra dimensions can help to resolve the Universe missing mass problem. The energy scales involved in these models are low compared to the Planck scale, and this means that some of the brane fluctuations distinctive signals could be detected in future colliders and in direct or indirect dark matter searches.

Keywords: Extra dimensions; brane-world; dark matter.

The elegant geometric description of the gravitational interaction which is the heart of General Relativity (GR) has inspired several attempts to find higher-dimensional generalizations which could unify all the fundamental interactions in a single picture. Based on the early proposal by Kaluza and Klein (KK),[1] the space–time was assumed to have more than four dimensions, and the extra spatial dimensions were compactified with a tiny radius of the order of the Planck length. Thus, the momentum component along the extra dimensions was seen from the four-dimensional viewpoint as the mass of the KK tower states associated to the ordinary particles. In this sense, a new infinite set of particles arose whose mass spectrum was determined by the geometry of the extra space. This would be the first example of a theory in which not only the geometry of space–time is fixed by its matter content as in GR, but also the properties of the own matter fields are determined by the

---

*This essay received an “honorable mention” in the 2004 Essay Competition of the Gravity Research Foundation.

space–time geometry. This appealing possibility was also present somehow in the old string theories, whose consistency required the introduction of six additional dimensions, which typically were also compactified at the Planck scale. Here again the topology and the geometry of the extra dimension space determined the particle content.

More recently, the construction of extra-dimensional models has been revived within the so called brane-world scenario.[2] These models are inspired in the conjectured M-theory which pretends to be an unification of the old known consistent string theories through a web of different dualities. M-theory includes also non-perturbative effects leading to the emergency of a new extra dimension and a complete set of new higher-dimensional states generically called branes.

Unlike the old Kaluza–Klein theories, in the new brane-world models the size of the extra dimensions could be as large as a fraction of a millimeter. The main assumption of this scenario is that by some (unknown) mechanism, matter fields are constrained to live in a three-dimensional hypersurface (brane) embedded in the higher dimensional (bulk) space. Only gravity is able to propagate in the bulk space, but the fundamental scale of gravity in D dimensions M_D can be much lower than the Planck scale, the volume of the extra dimensions being responsible for the actual value of the Newton constant in four dimensions.

Since in these models gravity can live in the bulk, a tower of KK modes would be present in the theory, but in addition, new fields which are characteristic (distinctive) of these models and also have a strong geometric origin (branons) appear in our four-dimensional space–time. In this essay, we will concentrate on some interesting properties of these fields. Indeed, since the energy scales involved in the brane-world scenario can be much lower than the Planck scale, the phenomenological consequences of branon physics could be found in current collider experiments or in astrophysical and cosmological observations.

The fact that rigid objects are incompatible with GR implies that the brane-world must be dynamical and can move and fluctuate along the extra dimensions. Branons are precisely the fields parametrizing the position of the brane in the extra coordinates. Thus, in four dimensions branons could be detected through their contribution to the induced space–time metric. From a more fundamental point of view, branons can be understood as the Goldstone bosons (GB) corresponding to the spontaneous breaking of the translational invariance along the extra dimensions which is produced by the presence of the brane.[3]

Let us consider our four-dimensional space–time M_4 to be embedded in a D-dimensional bulk space whose coordinates will be denoted by (x^μ, y^m), where x^μ, with $\mu = 0, 1, 2, 3$, correspond to the ordinary four-dimensional space–time and y^m, with $m = 4, 5, \ldots, D-1$, are coordinates of the compact extra space. For simplicity, we will assume that the bulk metric tensor takes the following form:

$$ds^2 = \tilde{g}_{\mu\nu}(x) W(y) dx^\mu dx^\nu - g'_{mn}(y) dy^m dy^n \qquad (1)$$

where the warp factor is normalized as $W(0) = 1$.

Working in the probe-brane approximation, our 3-brane universe is moving in the background metric given by Eq. (1) which is not perturbed by its presence. The position of the brane in the bulk can be parametrized as $Y^M = (x^\mu, Y^m(x))$, and we assume for simplicity that the ground state of the brane corresponds to $Y^m(x) = 0$.

In the simplest case in which the metric is not warped along the extra dimensions, i.e. $W(y) = 1$, the transverse brane fluctuations are massless and they can be parametrized by the GB fields $\pi^\alpha(x)$, $\alpha = 4, 5, \ldots, D-1$. In that case, we can choose the y coordinates so that the branon fields are proportional to the extra-space coordinates: $\pi^\alpha(x) = f^2 \delta^\alpha_m Y^m(x)$, where the proportionality constant is related to the brane tension $\tau = f^4$.

In the general case, the curvature generated by the warp factor explicitly breaks the translational invariance in the extra space. Therefore, branons acquire a mass matrix which is given precisely by the bulk Riemann tensor evaluated at the brane position:

$$M^2_{\alpha\beta} = \tilde{g}^{\mu\nu} R_{\mu\alpha\nu\beta}|_{y=0} \,. \tag{2}$$

The fact that the brane can fluctuate implies that the actual metric on the brane is no longer given by $\tilde{g}_{\mu\nu}$, but by the induced metric which includes the effect of warping through the mass matrix:

$$g_{\mu\nu}(x,\pi) = \tilde{g}_{\mu\nu}(x)\left(1 + \frac{M^2_{\alpha\beta}\pi^\alpha\pi^\beta}{4f^4}\right) - \frac{1}{f^4}\partial_\mu\pi^\alpha\partial_\nu\pi^\alpha + \mathcal{O}(\pi^4)\,, \tag{3}$$

The dynamics of branons can be obtained from the Nambu–Goto action by introducing the above expansion. In addition, it is also possible to get their couplings to the ordinary particles just by replacing the space–time by the induced metric in the Standard Model (SM) action. Thus, we get up to quadratic terms in the branon fields:

$$\begin{aligned} S_{Br} = \int_{M_4} d^4x\sqrt{\tilde{g}}\biggl[&\frac{1}{2}\left(\tilde{g}^{\mu\nu}\partial_\mu\pi^\alpha\partial_\nu\pi^\alpha - M^2_{\alpha\beta}\pi^\alpha\pi^\beta\right) \\ &+ \frac{1}{8f^4}\left(4\partial_\mu\pi^\alpha\partial_\nu\pi^\alpha - M^2_{\alpha\beta}\pi^\alpha\pi^\beta\tilde{g}_{\mu\nu}\right)T^{\mu\nu}_{\mathrm{SM}}\biggr]\,. \end{aligned} \tag{4}$$

We can see that branons interact with the SM particles through their energy–momentum tensor. The couplings are controlled by the brane tension scale f and they are universal very much like those of gravitons. For large f, branons are therefore weakly interacting particles.

The sign of the branon fields is determined by the orientation of the brane submanifold in the bulk space. Under a parity transformation on the brane ($x^i \to -x^i$), the orientation of the brane changes sign, provided the ordinary space has an odd number of dimensions, whereas it remains unchanged for even spatial dimensions. In the case in which we are interested with three ordinary spatial dimensions, branons are therefore pseudoscalar particles. Parity on the brane then requires that branons always couple to SM particles by pairs, which ensures that they are stable particles. This fact can have important consequences in cosmology as we show below.

Recent observations[4] seem to confirm that the universe contains an important fraction of non-luminous matter $\Omega_{\rm DM} \sim 0.23$. Moreover, it is known that most of that matter cannot be made of any known particle. Thus, the existence of dark matter candidates beyond the SM or appropriate modifications of GR appear as inescapable consequences of the problem.

In an expanding universe, the number density of a massive, stable particle species follows the thermal equilibrium abundance and declines exponentially with the temperature, provided it is in equilibrium with radiation. However, if the species decouples, its abundance remains frozen with respect to the entropy density. This means that if decoupling occurs early enough, i.e. the species is weakly interacting, its relic number density could be cosmologically relevant and explain the missing mass problem. Accordingly, we find that, in the brane-world scenario, the fluctuations of the own space–time geometry along curved extra dimensions are natural dark matter candidates, since as seen before, they satisfy all these requirements.[5]

An explicit computation shows that for sufficiently large f and M, branons can constitute the total dark matter of the universe (see Fig. 1). In particular, they would contribute as cold dark matter in the parameter range: $M \sim f \gtrsim 200$ GeV. The large value of f would explain why they have not been detected yet in collider experiments. Another interesting possibility is the case with very light branons:

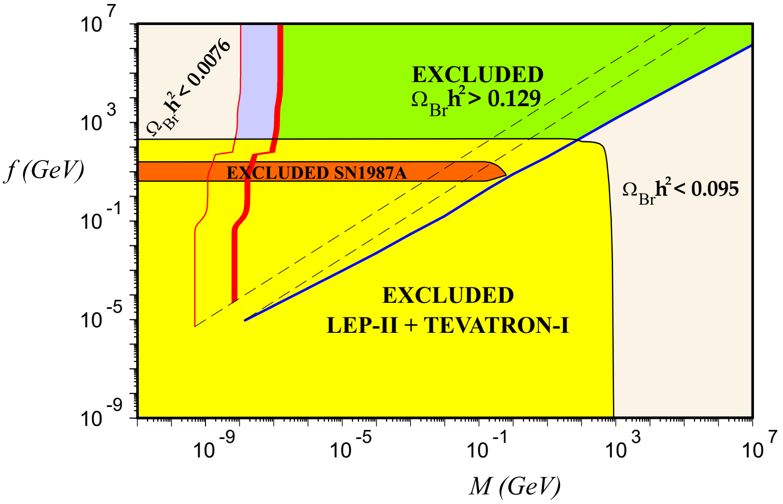

Fig. 1. Relic abundance in the $f - M$ plane for a model with one branon of mass: M. The two continuous lines on the left correspond to the WMAP limits[4] $\Omega_{Br}h^2 = 0.0076$ and $\Omega_{Br}h^2 = 0.129 - 0.095$ for hot-warm relics, whereas the continuous right line corresponds to the latter limit for cold relics. The lower area is excluded by single-photon processes at LEP-II together with monojet signal at Tevatron-I. The upper area is also excluded by cosmological branon overproduction. The astrophysical constraints are less restrictive and they mainly come from supernova cooling by branon emission.

$M \sim 100$ eV and $f \gtrsim 200$ GeV, where they would play the role of hot dark matter (Fig. 1).

Branons could not only be natural cosmological dark matter candidates, but they could also make up the galactic halo and explain the local dynamics. In such a case, they could be detected in future direct search experiments such as CDMS, CRESST II or GENIUS through their interactions with target nucleons. They could also be found in indirect searches by the MAGIC or GLAST telescopes due to their annihilation into photons in the galactic halo; or by antimatter detectors such as AMS because of their annihilation into charged particles. These searches complement those in future high-energy colliders, such as LHC or Linear Colliders which could find branon signals, provided $f \lesssim 1$ TeV and $M \lesssim 6$ TeV. If this possibility were realized in Nature, it would provide a fascinating link between the geometry of our space–time and an important fraction of the matter content of the universe, providing a solution to the dark matter problem.

Acknowledgments

This work has been partially supported by the DGICYT (Spain) under the project numbers FPA 2000-0956 and BFM2002-01003.

References

1. T. Kaluza, *Sitzungsberichte of the Prussian Acad. of Sci.* (1921), p. 966; O. Klein, *Z. Phys.* **37**, 895 (1926).
2. N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B429**, 263 (1998); N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Rev.* **D59**, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B436**, 257 (1998).
3. R. Sundrum, *Phys. Rev.* **D59**, 085009 (1999); A. Dobado and A. L. Maroto, *Nucl. Phys.* **B592**, 203 (2001).
4. D. N. Spergel *et al.*, *Astrophys. J. Suppl.* **148**, 175 (2003).
5. J. A. R. Cembranos, A. Dobado and A. L. Maroto, *Phys. Rev. Lett.* **90**, 241301 (2003).

This article has been cited by:

1. F.D Albareti, J.A.R Cembranos, A. de la Cruz-Dombriz. 2012. Focusing of geodesic congruences in an accelerated expanding Universe. *Journal of Cosmology and Astroparticle Physics* **2012**:12, 020-020. [CrossRef]
2. J. Cembranos, V. Gammaldi, A. Maroto. 2012. Possible dark matter origin of the gamma ray emission from the Galactic Center observed by HESS. *Physical Review D* **86**:10. . [CrossRef]
3. J.A.R Cembranos, A. de la Cruz-Dombriz, B. Montes Núñez. 2012. Gravitational collapse in f (R) theories. *Journal of Cosmology and Astroparticle Physics* **2012**:04, 021-021. [CrossRef]
4. J. Cembranos, A. de la Cruz-Dombriz, V. Gammaldi, A. Maroto. 2012. Detection of branon dark matter with gamma ray telescopes. *Physical Review D* **85**:4. . [CrossRef]
5. Jose Cembranos, J. Díaz-Cruz, Lilian Prado. 2011. Impact of dark matter direct searches and the LHC analyses on branon phenomenology. *Physical Review D* **84**:8. . [CrossRef]
6. Jose A R Cembranos, Jose H Montes de Oca Y, Lilian Prado. 2011. Dark Matter and Higgs Sector. *Journal of Physics: Conference Series* **315**, 012012. [CrossRef]
7. Jose A R Cembranos. 2011. R 2 Dark Matter. *Journal of Physics: Conference Series* **315**, 012004. [CrossRef]
8. J. Cembranos, A. de la Cruz-Dombriz, A. Dobado, R. Lineros, A. Maroto. 2011. Photon spectra from WIMP annihilation. *Physical Review D* **83**:8. . [CrossRef]
9. Jose Cembranos, Louis Strigari. 2008. Diffuse MeV gamma rays and galactic 511 keV line from decaying WIMP dark matter. *Physical Review D* **77**:12. . [CrossRef]
10. J A R Cembranos, A Dobado, A L Maroto. 2007. Some model-independent phenomenological consequences of flexible braneworlds. *Journal of Physics A: Mathematical and Theoretical* **40**:25, 6631-6640. [CrossRef]
11. J. Cembranos, A. Dobado, A. Maroto. 2006. Dark matter clues in the muon anomalous magnetic moment. *Physical Review D* **73**:5. . [CrossRef]
12. J. Cembranos, A. Dobado, A. Maroto.. 2006. Branon radiative corrections to collider physics and precision observables. *Physical Review D* **73**:3. . [CrossRef]