Characterisation of Marginally Turbulent Square Duct Flow

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1 Introduction

We have performed direct numerical simulation of fully developed turbulent flow in a straight duct with square cross-section. The main objective of the present study is to determine the minimal requirements for maintaining turbulence in duct flows [1]. A detailed analysis of this limit regime allows to elucidate the dominant mechanisms governing the marginally turbulent state, where the self-sustaining coherent structures, i.e. streamwise vortices and streaks, have a cross-streamwise length scale comparable with the duct width. It is therefore expected that the coherent structures are of direct relevance to the appearance of secondary flow of Prandtl's second kind.

2 Methodology and Results

For the numerical simulations we have used a pseudo-spectral method applied to the primitive variable formulation of the incompressible Navier-Stokes equations. The time advancement is based on a three-step Runge-Kutta scheme with implicit viscous terms, and continuity is imposed by means of a pressure-correction method. A dealiased Fourier expansion is employed in the streamwise (x) direction, while Chebyshev-polynomial expansions are used in the cross-streamwise (y, z) directions. Turbulence statistics resulting from the present simulations are in good agreement with those obtained in former finite-difference simulations [2] as well as experimental measurements [3].

In order to determine the critical Reynolds number and the minimum streamwise period allowing for self-sustained turbulence, the parameter values have been gradually reduced from their initial values $Re_b \equiv u_b h/\nu = 2205$ (based on bulk velocity u_b and duct half-width h) and $L_x/h = 4\pi$, respectively. It is found that turbulence can be maintained above $Re_b \simeq 1080$ (cf. Fig. 1). If we use the mean friction velocity u_τ as a velocity scale, the corresponding

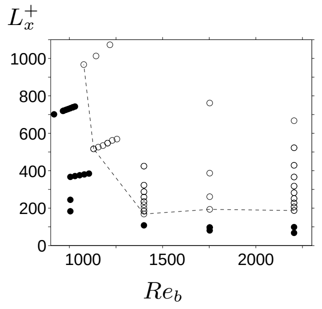

Fig. 1. Map of the critical values for the bulk Reynolds number Re_b and the streamwise box length in wall units L_x^+. (●) laminar flow; (○) turbulent.

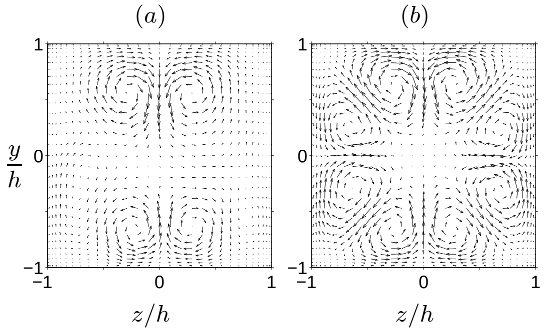

Fig. 2. Mean velocity in the cross-sectional plane of a marginally turbulent square duct at $Re_b = 1180$. (a) Data averaged along the streamwise direction and over a time period of $100\,h/u_b$; (b) average over a much longer temporal interval (approximately $3300\,h/U_b$).

lowest Reynolds number is $Re_\tau \equiv u_\tau h/\nu \simeq 75$. In viscous length units the minimal streamwise period has a value of $L_x^+ \equiv L_x u_\tau/\nu \simeq 200$, approximately independent of the Reynolds number. These critical values are comparable to their counterparts in plane channel flow [1, 4].

Focusing upon one of the marginal cases ($Re_b = 1180$, $L_x/h = 2\pi$), we observe persistent periods of time characterized by two almost quiescent walls (opposite to each other) while turbulence activity is concentrated on the other two walls. During those intervals, a single low-velocity streak is located around the bisector of each one of the active walls whereas no streak is on the other pair of parallel walls. Each streak is flanked by staggered streamwise vortices of alternating signs. Therefore, the mean flow exhibits a pair of counter-rotating streamwise vortices, when the flow field is averaged over intervals of $\mathcal{O}(100)$ bulk flow time units, as shown in Fig. 2*a* (active walls being located at $y/h = \pm 1$ in this example). It is observed that the pairs of active walls alternate in time, thus leading to a long-time average secondary flow consisting of a superposition of the two possible states, as shown in Fig. 2*b*. The latter image shows an 8-vortex pattern similar to the well-known secondary flow observed at higher Reynolds number. The tendency to exhibit the 4-vortex state can be quantified by evaluating the distribution of the streamwise vorticity in the cross-stream plane, i.e. by integrating (the square of) its streamwise average separately over the four sectors delimited by the diagonals and comparing the sums of the values pertaining to pairs of opposite walls. We have found that this measure of inequality is largest for the lowest Reynolds numbers and for the shortest domains. Similarly, the mean interval between successive reorientations of the instantaneous streamwise-averaged vortex pattern strongly decreases with the Reynolds number (figures omitted).

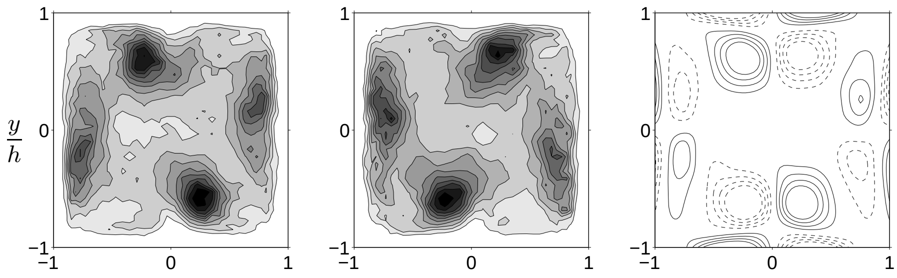

Fig. 3. Statistical data for a case with $Re_b = 1143$ and $L_x/h = 4\pi$, accumulated from 960 instantaneous flow fields over a time interval of $915h/u_b$. (*a*) Contours indicating .1(.1).9 times the maximum probability of occurrence of vortex centers with positive streamwise vorticity (increasing from white to black) ; (*b*) the probability for vortices with negative streamwise vorticity; (*c*) the average streamwise vorticity over the same interval (negative values dashed).

Our conjecture is that the secondary flow in this marginal state is a footprint of the quasi-streamwise vortices associated with the near-wall turbulence regeneration cycle. In order to confirm this scenario, we have identified the central axes of the streamwise vortices by means of the criterion of Kida and Miura [5]. The accumulated data has been used to determine the p.d.f. of the position of vortex centers in the cross-plane. Fig. 3 shows the result for one low-Reynolds-number case, averaged over an interval during which turbulence activity is primarily found near the walls at $y/h = \pm 1$. Comparing the most probable positions for the coherent vortex centers associated with positive/negative streamwise vorticity on the one hand (cf. Fig. 3*a*-*b*) to the mean secondary vorticity on the other hand (Fig. 3*c*) a striking correspondence can be noted. In this case, where the coherent structures are highly constrained by the geometry, the instantaneous streamwise vortices are practically locked into their positions. For higher Reynolds numbers, the p.d.f.s associated with positive/negative streamwise vorticity are expected to overlap near the wall-bisector, restricting the selectivity to the corner region.

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