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The Higgs sector of the MSSM in the decoupling limit

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Abstract. We study the heavy Higgs sector of the MSSM composed of the $H^{\pm}$, H^0 and A^0 particles in the so-called decoupling limit where $m_{A^0} \gg m_Z$. By integrating out these heavy Higgs particles to one-loop, we compute the effective action for the electroweak gauge bosons and find out that, in the decoupling limit, all the heavy Higgs effects can be absorbed into redefinitions of the Standard Model electroweak parameters. This demonstrates explicitely that the decoupling theorem works for the heavy MSSM Higgs particles. This is also compared with the paradigmatic and different case of the Standard Model heavy Higgs particle. Finally, this work together with our two previous works, complete the demonstration that all the non-standard particles in the MSSM, namely, squarks, sleptons, charginos, neutralinos and the heavy Higgs particles, decouple to one-loop from the low energy electroweak gauge boson physics.

1 Introduction

The absence of any signal from Supersymmetric (SUSY) particles in the existing data indicates that either SUSY theories are not the proper ones for low energy physics beyond the Standard Model (SM) or the SUSY spectrum is above the available energies at present experiments. In the simplest SUSY theory, the Minimal Supersymmetric Standard Model (MSSM), the predicted spectrum is composed of squarks $\tilde{q}$ and sleptons $\tilde{l}$, $\tilde{\nu}$ for the three generations, charginos $\tilde{\chi}^{\pm}_{1,2}$, neutralinos $\tilde{\chi}^{o}_{1,2,3,4}$, gluinos $\tilde{g}$, and the Higgs sector with five Higgs particles, two CP-even Higgs bosons h^o and H^o, a CP-odd or pseudoscalar Higgs boson A^o, and two charged Higgs particles $H^{\pm}$. Although the precise mass bound varies for each particle, it is clear that, at present time, there is little room for light MSSM particles, say lighter than the W gauge boson mass m_W. Particularly stringent are the bounds for the strongly interacting particles, the squarks and gluinos with a lower mass limit already above $200\,GeV$ [1]. Under these circumstances it is a reasonable hypothesis to think of a mass gap between the SM particles and the genuine MSSM particles. In case this energy separation occurs, its size should not be larger than about $1\,TeV$, if the MSSM is required to repair the hierarchy problem. We will assume here the extreme but plausible situation where all the MSSM spectrum lay well above the electroweak scale M_{EW}. For the purpose of this paper we just need to assume the existence of this sizeable gap, but the particular value of the gap width is not relevant. There is just one exception in this large SUSY mass assumption, the lightest CP-even h^o particle which stays close to the SM spectrum. It is well known that when the pseudoscalar mass m_{A^o} is very large, that is much larger than the Z boson mass $m_{A^o} \gg m_Z$, the heavy CP-even, CP-odd and charged Higgs bosons are nearly degenerate, $m_{H^o} \simeq m_{H^{\pm}} \simeq m_{A^o}$, while the h^o particle reaches its maximal mass value which, at tree level, is bounded from above by m_Z, and when radiative corrections are included, this upper bound is shifted towards $\sim 130\,GeV$ [2–14]. In this so-called decoupling limit [15], the lightest SUSY Higgs boson h^o and the SM Higgs boson H_{SM} have very similar properties, since both have similar couplings to fermions and vector bosons and therefore the task of discriminating between these two particles will be quite hard. This equality of couplings is exact at tree level when the decoupling limit is reached asymptotically and both their production rates and decay branching rations are identical. However, it is not known with complete generality if this equality remains beyond tree level. It is a very interesting subject, since in case it does not happen it will provide the clue for discriminating between the SM and MSSM, even in the extreme situation mentioned above where all the rest of the MSSM spectrum is well above the electroweak scale and hence not reachable at present experiments. This topic has been studied by several authors [15–31] by looking to particular observables of interest in phenomenology, as for instance, the parameters S, T and U[1] that measure the radiative corrections at LEP [15,28,29], the h^o production rates at LEP and LHC and the decay branching rations of h^o to $\gamma\gamma$ [24]

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[1] or equivalently Δr, $\Delta\rho$, $\Delta\kappa$ or the ϵ_i parameters

and to $f\bar{f}$ [20,30,31]. Most of these studies analyzed the decoupling of SUSY particles numerically. Although the numerical analysis are complicated since they depend on many MSSM parameters, there are indications from these studies that the SUSY particles indeed tend to decouple in the previous observables when the SUSY masses are taken numerically very large. In particular, the MSSM h^o couplings to $\gamma\gamma$ [24] and to $f\bar{f}$ [30] seem to approach those of the H_{SM} particle in the decoupling limit and in the one loop approximation, confirming therefore the enormous challenge that will be discriminating between these two particles at future high energy colliders as the LHC.

In this paper we study the MSSM Higgs sector in the decoupling limit at a more formal level. Our object of interest is the effective action for the SM particles and the contributions to this action from the loops of the MSSM Higgs sector in the limit where all the Higgs particles, except h^o, are very heavy, namely, when $m_{A^o} \gg m_Z$. We want to demonstrate the decoupling of the MSSM Higgs particles *á la Appelquist Carazzone* [32], meaning that the required proof should show that the decoupling theorem also applies for this particular case. This is the third work belonging to a program that we initiated in [33,34] which aims to demonstrate the decoupling of SUSY particles beyond tree level in each of the MSSM sectors. In generic words, and by following the Appelquist-Carazzone approach, the proof of decoupling of SUSY particles at low energies amounts to first compute the effective action $\Gamma_{\rm eff}[\phi]$ for the SM particles ϕ ($\phi = q, l, \nu, Z, W^\pm, \gamma, g, H_{SM}$) that is generated through functional integration of all the non-standard particles of the MSSM $\tilde{\phi}$ ($\tilde{\phi} = \tilde{q}, \tilde{l}, \tilde{\nu}, \tilde{\chi}^\pm, \tilde{\chi}^o, \tilde{g}, H^\pm, H^o, A^o$)

$$\mathrm{e}^{i\Gamma_{eff}[\phi]} = \int [\mathrm{d}\tilde{\phi}]\, \mathrm{e}^{i\Gamma_{\rm MSSM}[\phi,\tilde{\phi}]}\,, \qquad (1)$$

with

$$\Gamma_{\rm MSSM}[\phi,\tilde{\phi}] \equiv \int dx \mathcal{L}_{\rm MSSM}(\phi,\tilde{\phi})\,;\; \mathrm{d}x \equiv \mathrm{d}^4x\,, \qquad (2)$$

and $\mathcal{L}_{\rm MSSM}$ is the MSSM Lagrangian.

Secondly, one must perform a large SUSY mass expansion of $\Gamma_{\rm eff}[\phi]$ to be valid for low energies, say $M_{EW} \ll M_{\tilde{\phi}}$, and, as a result, one should get finally the following behaviour,

$$\Gamma_{\rm eff}[\phi] = \hat{\Gamma}_{\rm SM}[\phi] + \mathcal{O}\left[\left(\frac{M_{EW}}{M_{\tilde{\phi}}}\right)^n\right]\,, \qquad (3)$$

which means that all the effects of the heavy SUSY particles $\tilde{\phi}$ can be absorbed into redefinitions of the SM couplings and wave functions of the SM fields ϕ, or else they are suppressed by inverse powers of the heavy masses $M_{\tilde{\phi}}$ and therefore vanish in the asymptotic limit $M_{\tilde{\phi}} \to \infty$. We believe that only an explicit computation as the one just outlined can be considered as a formal and general proof of decoupling of non-standard particles from the low energy SM physics.

We have started this program with the computation of the part of the effective action for the electroweak gauge bosons, but, of course, a complete proof of decoupling will require to obtain the total effective action for the other SM particles as well, namely, the fermions, the gluon and the SM Higgs particle itself. In particular, the study of the $h^0 b\bar{b}$ vertex is one of the most interesting observables in the Higgs phenomenology [35]. The reason to start with the electroweak gauge boson sector is, first, for simplicity and, second, because we were interested in studing the implications for some of the precision observables at LEP with external gauge bosons as the S, T and U or related parameters. We have proved that, to one loop level, the functional integration of the various MSSM sparticle sectors factorize in the effective action for electroweak bosons and, therefore, this integration can be performed sector by sector separately. In [33,34] we have completed the integration of squarks, sleptons, charginos and neutralinos in the MSSM to one loop, and have demonstrated their decoupling in the large SUSY masses limit. Since the asymptotic behaviour of the Feynman loop integrals appearing in the computation depend on the relative sizes of the various sparticle masses in the loop propagators, one must perform the computation by assuming a particular hypothesis for these masses. We assumed in [33,34] that the large SUSY masses limit is taken for each sector such that $M^2_{EW} \ll M^2_{\tilde{\phi}_i}$ $\forall i$, but with $|M^2_{\tilde{\phi}_i} - M^2_{\tilde{\phi}_j}| \ll |M^2_{\tilde{\phi}_i} + M^2_{\tilde{\phi}_j}|$ if $i \neq j$. That is, all the SUSY masses are large as compared to the electroweak scale but they are close to each other. This is a plausible hypothesis in the MSSM but is not the most general one for all the sectors. In particular for the squarks of the third generation where, even assuming a common soft-SUSY-breaking mass, one has $(\tilde{m}^2_{t_1} - \tilde{m}^2_{t_2}) \simeq m_t(A_t - \mu\cot\beta)$ and $(\tilde{m}^2_{b_1} - \tilde{m}^2_{b_2}) \simeq m_b(A_b - \mu\tan\beta)$ and, therefore, for large enough values of A_t, A_b, μ and/or $\tan\beta$ the previous hypothesis may not hold. In consequence, for these particular cases where $|M^2_{\tilde{\phi}_i} - M^2_{\tilde{\phi}_j}| \simeq \mathcal{O}|M^2_{\tilde{\phi}_i} + M^2_{\tilde{\phi}_j}|$ for $i \neq j$ an independent demonstration of decoupling should be done.

In the present work we complete the computation of the effective action for electroweak gauge bosons to one loop by integrating out the heavy MSSM Higgs particles, namely the charged $H^\pm$, the pseudoscalar A^o and the heaviest CP-even Higgs boson H^o. We then perform the large mass expansion which in the Higgs sector case corresponds to work in the above mentioned decoupling limit. Notice that for the Higgs sector the previous assumption for the relative Higgs mass values, $|m^2_{H_i} - m^2_{H_j}| \ll |m^2_{H_i} + m^2_{H_j}|$ if $i \neq j$ holds trivially, since when $m_{A^o} \gg m_Z$ the four heavy Higgs bosons, $H^\pm$, A^o and H^o tend to be degenerate with a mass close to m_{A^o}.

The paper is organized as follows. In the second section we define the effective action for the electroweak gauge bosons and summarize the relevant part of the MSSM lagrangian for the purpose of integration of the MSSM Higgs sector to one loop level. The exact results to one loop of the contributions to the effective action from the 2, 3, and 4 point electroweak gauge bosons functions are presented in Sect. 3. We also analyze in that section the behaviour of these functions in the decoupling limit, $m_{A^o} \gg m_Z$, and present the corresponding asymptotic results in terms of the large Higgs masses $m_{H^\pm}, m_{A^o}, m_{H^o}$. In Sect. 4 the previous asymptotic expressions are rewritten in a form that will allow us to conclude on the decoupling of the Higgs sector *á la Appelquist Carazzone* as announced. In particular, by using the common language of renormalization, the required redefinitions of the SM couplings and wave functions for the electroweak bosons are presented in the form of specific contributions to the SM counterterms. Section 5 is devoted to a comparison with the paradigmatic and dramatically different case of the SM with a very heavy Higgs particle, $M_{EW} \ll M_{H_{SM}}$, which is well known not to decouple from low energy electroweak physics [36–43]. We find illustrative to perform this comparison in the language of the effective action. This non-decoupling of the SM Higgs particle has been shown to manifest at one loop level in several observables, as for instance $\Delta\rho$ [38,39,44], and it is being very relevant in the indirect Higgs searches at the present colliders. In Sect. 5 we reobtain this non-decoupling behavior by computing the effective action for electroweak gauge bosons after integration to one loop of the SM Higgs particle and by studying its large $M_{H_{SM}}$ expansion. We will see that the non-decoupling of the Higgs particle manifests in this context as a violation of the decoupling theorem in the four point electroweak gauge functions. Finally, the conclusions of this work are summarized in Sect. 6.

2 Integration of the MSSM Higgs sector to one loop

The effective action for the electroweak gauge bosons, $\Gamma_{\rm eff}[V]$ $(V = A, Z, W^\pm)$ gets contributions to one loop from all the MSSM sectors, except from gluinos which will start contributing at and beyond two loops. This effective action is defined through functional integration of all the sfermions $\tilde f$ $(\tilde q, \tilde l, \tilde\nu)$, neutralinos $\tilde\chi^o$ $(\tilde\chi^o_{1\ldots4})$, charginos $\tilde\chi^\pm$ $(\tilde\chi^\pm_{1,2})$, and the Higgs bosons H $(H^\pm, H^o, A^o)$ by:

$$e^{i\Gamma_{eff}[V]} = \int \left[d\tilde f\right]\left[d\tilde f^*\right]\left[d\tilde\chi^+\right]\left[d\bar{\tilde\chi}^+\right]\left[d\tilde\chi^o\right][dH] \times e^{i\Gamma_{\rm MSSM}\left[V,\tilde f,\tilde\chi^+,\tilde\chi^o,H\right]}, \quad (4)$$

where the relevant part of the MSSM classical action can be written as,

$$\begin{aligned}&\Gamma_{\rm MSSM}[V,\tilde f,\tilde\chi^+,\tilde\chi^o,H]\\ &\equiv \int dx \mathcal{L}_{\rm MSSM}(V,\tilde f,\tilde\chi^+,\tilde\chi^o,H)\\ &= \int dx \mathcal{L}^{(0)}(V) + \int dx \mathcal{L}_{\tilde f}(V,\tilde f) + \int dx \mathcal{L}_{\tilde\chi}(V,\tilde\chi)\\ &\quad + \int dx \mathcal{L}_H(V,H)\\ &\equiv \Gamma_0[V] + \Gamma_{\tilde f}[V,\tilde f] + \Gamma_{\tilde\chi}[V,\tilde\chi] + \Gamma_H[V,H]\,. \end{aligned} \quad (5)$$

Here, $\mathcal{L}^{(0)}(V)$ is the free gauge boson lagrangian at tree level, and $\mathcal{L}_{\tilde f}$, $\mathcal{L}_{\tilde\chi}$ and $\mathcal{L}_H$ are the lagrangians of sfermions, *inos* (i.e. charginos and neutralinos) and Higgs bosons respectively. By looking into the particular form of these lagrangians it is inmediate to see that the integration of the various sectors at the one-loop level can be factorized out, and their contributions to the effective action can be computed separately sector by sector.

In [33,34] we have performed the complete integration to one loop of the sfermions and *inos* sectors. Here we present the corresponding integration of the heavy Higgs sector defined as,

$$e^{i\Gamma^H_{eff}[V]} = \int [dH] e^{i\int dx\left(\mathcal{L}^{(0)}(V)+\mathcal{L}_H(V,H)\right)}, \quad (6)$$

where we have introduced a short hand notation for the heavy Higgs particles,

$$H = \begin{pmatrix} H^1\\ H^2\\ H^o\\ A^o\end{pmatrix}, \quad (7)$$

with H^1 and H^2 being related to the physical charged Higgs particles by $H^\pm \equiv \frac{1}{\sqrt2}\left(H^1 \pm iH^2\right)$, and $\mathcal{L}_H(V,H)$ is the relevant MSSM Higgs sector lagrangian that is given by,

$$\mathcal{L}_H(V,H) = \mathcal{L}^{(0)}(H) + \mathcal{L}_{HVV} + \mathcal{L}_{HHV} + \mathcal{L}_{HHVV}\,. \quad (8)$$

Here $\mathcal{L}^{(0)}(H)$ is the free lagrangian for the heavy Higgs particles,

$$\mathcal{L}^{(0)}(H) = \frac12\left(\partial_\mu H^T \partial^\mu H - H^T M_H^2 H\right), \quad (9)$$

the squared mass matrix is given in terms of the physical Higgs boson masses by

$$M_H^2 \equiv {\rm diag}(m^2_{H^+}, m^2_{H^+}, m^2_{H^o}, m^2_{A^o})\,,\ m_{H^+} = m_{H^-}\,. \quad (10)$$

and we have used the superscript T to denote the transpose matrix. The interaction lagrangian pieces can be written as follows [45],

$$\begin{aligned}\mathcal{L}_{HVV} &= \mathcal{B}^T H\,,\\ \mathcal{L}_{HHV} &= H^T \vee^{(1)\mu} \overleftrightarrow{\partial_\mu} H\,,\\ \mathcal{L}_{HHVV} &= H^T \vee^{(2)} H\,.\end{aligned} \quad (11)$$

where

$$\mathcal{B} \equiv \begin{pmatrix} 0\\ 0\\ g c_{\alpha\beta}\left(m_W W^+_\mu W^{\mu-} + \frac{m_Z}{2c_W} Z_\mu Z^\mu\right)\\ 0\end{pmatrix}, \quad (12)$$

and $\mathbb{V}^{(1)\mu}$, $\mathbb{V}^{(2)}$ are the 4×4 Higgs interaction matrices with one and two gauge bosons respectively defined by,

$$\mathbb{V}^{(1)\mu}\begin{cases}[V^{(1)\mu}]_{ij} &= 0 \text{ if } i=j\,,\\ [V^{(1)\mu}]_{ij} &= -[V^{(1)\mu}]_{ji} \text{ if } i\neq j\,,\\ [V^{(1)\mu}]_{12} = eA^\mu + \frac{gc_{2W}}{2c_W}Z^\mu\,,\\ [V^{(1)\mu}]_{13} = \frac{g}{2}\,s_{\alpha\beta}\,W_2^\mu\,,\ [V^{(1)\mu}]_{14} = \frac{g}{2}W_1^\mu\\ [V^{(1)\mu}]_{23} = -\frac{g}{2}\,s_{\alpha\beta}\,W_1^\mu\,,\\ [V^{(1)\mu}]_{24} = \frac{g}{2}W_2^\mu\,,\ [V^{(1)\mu}]_{34} = -\frac{g}{2c_W}s_{\alpha\beta}Z^\mu\end{cases}$$

$$\mathbb{V}^{(2)}\begin{cases}[V^{(2)}]_{ij} &= [V^{(2)}]_{ji}\,\forall i,j\,,\ [V^{(2)}]_{12}=[V^{(2)}]_{34}=0\,,\\ [V^{(2)}]_{11} &= [V^{(2)}]_{22}\\ &= 2\left[\frac{g^2}{4}W_\mu^+W^{\mu-} + \frac{g^2c_{2W}^2}{8c_W^2}Z_\mu Z^\mu\right.\\ &\quad\left.+\frac{e^2}{2}A_\mu A^\mu + \frac{eg\,c_{2W}}{2c_W}A_\mu Z^\mu\right]\,,\\ [V^{(2)}]_{33} &= [V^{(2)}]_{44}\\ &= 2\left[\frac{g^2}{4}W_\mu^+W^{\mu-} + \frac{g^2}{8c_W^2}Z_\mu Z^\mu\right]\,,\\ [V^{(2)}]_{i3} &= s_{\beta\alpha}\left[-\frac{eg}{2}A_\mu W^{i\,\mu}\right.\\ &\quad\left.+\frac{g^2s_W^2}{2c_W}Z_\mu W^{i\,\mu}\right],\ i=1,2\,,\\ [V^{(2)}]_{i4} &= \left[-\frac{eg}{2}A_\mu W^{i\,\mu} + \frac{g^2s_W^2}{2c_W}Z_\mu W^{i\,\mu}\right]\,,\\ i &= 1,2\,.\end{cases} \tag{13}$$

Here, as usual, g and e are the electroweak and electromagnetic couplings respectively, and we have used a shorthand notation for the sines and cosines of the weak angle θ_W and the β angle ($\tan\beta\equiv\frac{v_2}{v_1}$)) given by

$$\begin{aligned}s_{\alpha\beta}&\equiv\sin(\alpha-\beta)\ ,\ c_{\alpha\beta}\equiv\cos(\alpha-\beta)\,,\\ c_{2W}&\equiv\cos 2\theta_W\ ,\ s_{2W}\equiv\sin 2\theta_W\,,\\ c_W&\equiv\cos\theta_W\ ,\ s_W\equiv\sin\theta_W\,.\end{aligned} \tag{14}$$

Correspondingly, we can define the various contributions to the classical action by,

$$\Gamma_H[V,H] = \langle\mathcal{B}^T H\rangle + \frac{1}{2}\,\langle\,H^T\,A_H\,H\,\rangle\,, \tag{15}$$

where,

$$\begin{aligned}A_H &\equiv A_H^{(0)} + A_H^{(1)} + A_H^{(2)}\,,\\ \langle\mathcal{B}^T H\rangle &\equiv \int \mathrm{d}\tilde{k}\,\mathcal{B}_k^T\,H_k\,,\\ \langle\,H^T\,A_H\,H\,\rangle &\equiv \int \mathrm{d}\tilde{k}\,\mathrm{d}\tilde{p}\,H_k^T A_{Hkp}^{(i)} H_p\,,\ i=0,1,2\,.\end{aligned} \tag{16}$$

with,

$$\mathrm{d}\tilde{k} \equiv \frac{d^4k}{(2\pi)^4}\,, \tag{17}$$

and we have chosen the representation in momentum space which is more convenient for functional integration,

$$\begin{aligned}A_{Hkp}^{(0)} &\equiv (2\pi)^4\,\delta(k+p)\,(k^2-M_H^2)\,,\\ A_{Hkp}^{(1)} &\equiv i\,(2\pi)^4\int\mathrm{d}\tilde{q}\,\delta(k+p+q)\,(k-p)_\mu\,\left[\mathbb{V}^{(1)\,\mu}\right]_q\,,\\ A_{Hkp}^{(2)} &\equiv (2\pi)^4\int\mathrm{d}\tilde{q}\,\mathrm{d}\tilde{r}\,\delta(k+p+q+r)\,\left[\mathbb{V}^{(2)}\right]_{q,r}\,,\\ \mathcal{B}_k^T &\equiv (2\pi)^4\int\mathrm{d}\tilde{q}\,\mathrm{d}\tilde{p}\,\delta(k+p+q)\,\mathcal{B}_{q,p}^T\,.\end{aligned} \tag{18}$$

Once the classical action has been written in the proper form (15), we proceed with the functional integration to one loop of the heavy Higgs particles H. By using the standard path integral techniques we get the following result for the effective action,

$$\Gamma_{eff}^H[V] = \Gamma_0[V] + \frac{i}{2}\mathrm{Tr}\log A_H - \frac{1}{2}\langle\mathcal{B}^T\,A_H^{-1}\,\mathcal{B}\rangle\,, \tag{19}$$

where,

$$\langle\mathcal{B}^T\,A_H^{-1}\,\mathcal{B}\rangle \equiv \int\mathrm{d}\tilde{k}\,\mathrm{d}\tilde{p}\,\mathcal{B}_k^T\,A_{Hkp}^{-1}\,\mathcal{B}_p\,.$$

In (19) we have introduced the functional trace which for a generic matrix operator $C^{ij}(k,p)\equiv C_{kp}^{ij}$ is defined by [33]:

$$\mathrm{Tr}C \equiv \sum_i\int\mathrm{d}\tilde{k}C_{kk}^{ii}\,.$$

Next, by expanding the logarithm and the inverse operator in (19), the effective action can be written as,

$$\begin{aligned}&\Gamma_{eff}^H[V]\\ &= \Gamma_0[V] + \frac{i}{2}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\mathrm{Tr}\left[G_H\left(A_H^{(1)}+A_H^{(2)}\right)\right]^k\\ &\quad -\frac{1}{2}\sum_{k=0}^{\infty}(-1)^k\left\langle\mathcal{B}^T\,\left[G_H\left(A_H^{(1)}+A_H^{(2)}\right)\right]^k\,G_H\,\mathcal{B}\right\rangle,\end{aligned} \tag{20}$$

where G_H is the heavy Higgs propagator matrix, defined as $G_H=\left(A_H^{(0)}\right)^{-1}$, and is given in momentum space by,

$$G_{Hkp} = (2\pi)^4\delta(k+p)\left(k^2-M_H^2\right)^{-1}\,, \tag{21}$$

with

$$\left(q^2-M_H^2\right)^{-1} = \mathrm{diag}\left(\frac{1}{q^2-m_{H^1}^2},\frac{1}{q^2-m_{H^2}^2},\frac{1}{q^2-m_{H^o}^2},\frac{1}{q^2-m_{A^o}^2}\right)\,.$$

Finally, if we keep just the terms that contribute to the two, three and four point V Green functions we get,

$$\begin{aligned}\Gamma_{eff}^H[V] = \Gamma_0[V] &- \frac{1}{2}\,\langle\mathcal{B}^T G_H\mathcal{B}\rangle\\ &+\frac{i}{2}\mathrm{Tr}\left(G_HA_H^{(2)}\right) - \frac{i}{4}\mathrm{Tr}\left(G_HA_H^{(1)}\right)^2\\ &-\frac{i}{2}\mathrm{Tr}\left(G_HA_H^{(1)}G_HA_H^{(2)}\right) + \frac{i}{6}\mathrm{Tr}\left(G_HA_H^{(1)}\right)^3\\ &-\frac{i}{4}\mathrm{Tr}\left(G_HA_H^{(2)}\right)^2\\ &+\frac{i}{2}\mathrm{Tr}\left(G_HA_H^{(1)}G_HA_H^{(1)}G_HA_H^{(2)}\right)\\ &-\frac{i}{8}\mathrm{Tr}\left(G_HA_H^{(1)}\right)^4 + O(V^5)\,.\end{aligned} \tag{22}$$

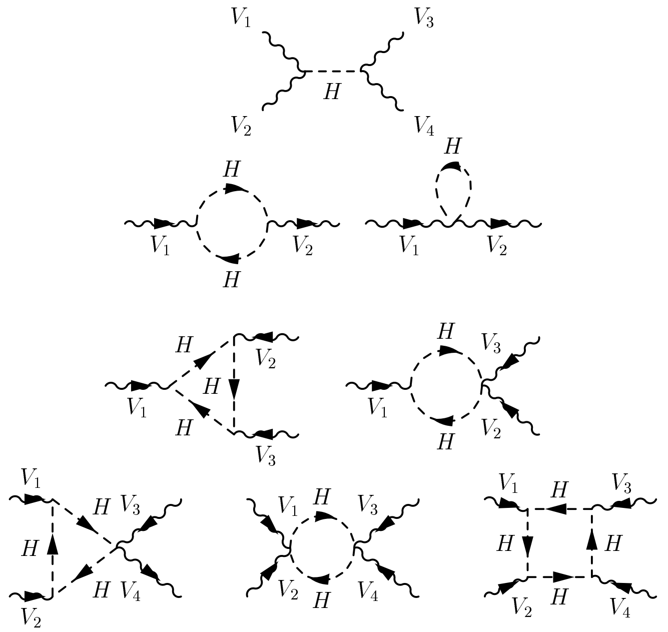

Fig. 1. Generic Feynman diagrams corresponding to the tree-level and one-loop contributions to the two-, three- and four-point functions of electroweak gauge bosons

The various contributions can be clearly identified from this expression. The third and fourth terms give the one-loop contributions to the two-point functions; the two next terms to the three-point functions; and the last three terms correspond to the four-point functions. Notice that there is just one contribution from the Higgs integration at the tree level. This is the second term in (22) and contributes just to the four point functions. Note that it is the unique sector that generates a contribution to the electroweak gauge boson functions at the tree level. As we have seen in [33,34] the integration of sfermions and *inos* in the effective action for electroweak gauge bosons give only contributions starting from one-loop level. In addition, notice also that the resulting effective action in (22) is gauge independent, as expected. This is due to the fact that we only integrate the physical Higgs particles whose interactions with the electroweak gauge bosons are gauge independent.

Finally, and for the purpose of illustration, we have shown in Fig. 1 the Feynman diagrams corresponding to the different terms appearing in the above (22).

3 The n-point functions of electroweak gauge bosons

The effective action can be written in terms of the n-point Green's functions in momentum space, generically as:

$$\begin{aligned}\Gamma_{eff}[V] = &\sum_n \frac{1}{C_{V_1 V_2 \dots V_n}} \int \mathrm{d}\tilde{k}_1 \dots \mathrm{d}\tilde{k}_n (2\pi)^4 \delta(\Sigma_{i=1}^n k_i) \\ &\times \Gamma^{V_1 V_2 \dots V_n}_{\mu\,\nu \dots\,\rho}(k_1\, k_2 \dots\, k_n)\, V_1^{\mu}(-k_1)\, V_2^{\nu}(-k_2) \\ &\dots V_n^{\rho}(-k_n)\,, \end{aligned} \tag{23}$$

where $C_{V_1 V_2 \dots V_n}$ are the proper combinatorial factors accounting for the identical external field, and we have assumed the convention of incoming momenta k_i for the external gauge bosons.

In this section we present the exact results to one-loop for the various contributions to the effective action of (22) coming from the 2, 3 and 4 point functions and write them in terms of the standard one-loop integrals of 't Hooft, Veltman and Passarino [46,47]. We latter analyze the asymptotic behaviour of the electroweak bosons Green's functions in the limit of large Higgs masses. The analysis of the one-loop integrals in the large masses limit have been done by means of the m-Theorem [48].

After working out the functional traces in (22) and by computing the corresponding Feynman integrals in dimensional regularization we get the following contributions, $\Gamma^H_{eff}[V]_{[n]}$, from the $n = 2$, 3 and 4 point functions respectively[2],

$$\begin{aligned}&\Gamma^H_{eff}[V]_{[2]} \\ &= -\pi^2 \int \mathrm{d}\tilde{p}\, \mathrm{d}\tilde{k}\, \delta(p+k) \Bigg\{ \sum_i \left[\vee^{(2)}\right]^{ii}_{p,k} A_0(m_i) \\ &+\frac{1}{4} \sum_{i \neq j} \left[\vee^{(1)\mu}\right]^{ij}_p \left[\vee^{(1)\nu}\right]^{ji}_k I^{i\,j}_{\mu\nu}(k, m_i, m_j) \Bigg\}, \end{aligned} \tag{24}$$

$$\begin{aligned}&\Gamma^H_{eff}[V]_{[3]} \\ &= -i\pi^2 \int \mathrm{d}\tilde{p}\, \mathrm{d}\tilde{k}\, \mathrm{d}\tilde{r}\, \delta(p+k+r) \\ &\times \Bigg\{ \sum_{i \neq j} \left[\vee^{(1)\mu}\right]^{ji}_p \left[\vee^{(2)}\right]^{ij}_{k,r} \frac{1}{2} T^{j\,i}_{\mu}(p, m_i, m_j) \\ &+\frac{1}{6} \sum_{i \neq j \neq k} \left[\vee^{(1)\mu}\right]^{ij}_{-p} \left[\vee^{(1)\nu}\right]^{jk}_{-k} \left[\vee^{(1)\sigma}\right]^{ki}_{-r} \\ &\times T^{i\,j\,k}_{\mu\,\nu\,\sigma}(p, k, m_i, m_j, m_k) \Bigg\}, \end{aligned} \tag{25}$$

$$\begin{aligned}&\Gamma^H_{eff}[V]_{[4]} \\ &= -\frac{1}{2} \sum_i \int \mathrm{d}\tilde{p}\, \mathcal{B}^i_p \frac{1}{p^2 - m_i^2} \mathcal{B}^i_{-p} \\ &+\pi^2 \int \mathrm{d}\tilde{p}\, \mathrm{d}\tilde{k}\, \mathrm{d}\tilde{r}\, \delta(p+k+r) \Bigg\{ \sum_{i,j} \left[\vee^{(2)\mu}\right]^{ij}_{-p,-k} \\ &\times \left[\vee^{(2)}\right]^{ji}_{-r,-t} J^{i\,j}_{p+k}(p+k, m_i, m_j) \\ &+ \sum_{i,j,k} \left[\vee^{(1)\mu}\right]^{ij}_{-p} \left[\vee^{(1)\nu}\right]^{jk}_{-k} \left[\vee^{(2)\sigma}\right]^{ki}_{-r,-t} \\ &\times J^{i\,j\,k}_{\mu\,\nu}(p, k, m_i, m_i, m_k) \\ &+\frac{1}{8} \sum_{i,j,k,l} \left[\vee^{(1)\mu}\right]^{ij}_{-p} \left[\vee^{(1)\nu}\right]^{jk}_{-k} \left[\vee^{(1)\sigma}\right]^{kl}_{-r} \left[\vee^{(1)\lambda}\right]^{li}_{-t} \end{aligned}$$

[2] Notice that in dimensional reduction the results would be the same, since we are not integrating out gauge bosons. This also applies to the results of our two previous papers [33,34]

$$\times J^{i\,j\,k\,l}_{\mu\,\nu\,\sigma\,\lambda}(p,k,r,m_i,m_j,m_k,m_l)\Big\}\,, \qquad (26)$$

In the above expressions the indices i,j,k,l run from 1 to 4 and correspond to the four entries in the heavy Higgs matrix H of (7). In these formulas and in the following, a proper symmetrization over the indices and momenta of the external identical fields, although not explicitely shown, must be assumed. The one loop integrals $T^{j\,i}_{\mu}$, $T^{i\,j\,k}_{\mu\,\nu\,\sigma}$, $J^{i\,j}_{p+k}$, $J^{i\,j\,k}_{\mu\,\nu}$ and $J^{i\,j\,k\,l}_{\mu\,\nu\,\sigma\,\lambda}$ are defined in terms of the standard integrals, A_0, $B_{0,\,\mu,\,\mu\nu}$, $C_{0,\,\mu,\,\mu\nu,\,\mu\nu\sigma}$ and $D_{0,\,\mu\nu,\,\mu\nu\sigma,\,\mu\nu\sigma\lambda}$ [46,47] in appendix A of our previous work [34]. Similarly, the two-point integral $I^{i\,j}_{\mu\nu}$ is defined by,

$$\begin{aligned} & I^{i\,j}_{\mu\nu}(k,m_i,m_j) \qquad (27)\\ & = (4B_{\mu\nu}+2k_\nu B_\mu+2k_\mu B_\nu+k_\mu k_\nu B_0)\,(k,m_i,m_j). \end{aligned}$$

We refer the reader to [34] for these and more details on the Feynman integrals.

Finally, from the previous expressions in (24) and by using the definition in (23) we extract, after a rather tedious computation, the exact results to one loop for the two-point, $\Gamma^{V_1 V_2}_{\mu\nu}$, three-point, $\Gamma^{V_1 V_2 V_3}_{\mu\nu\sigma}$, and four-point, $\Gamma^{V_1 V_2 V_3 V_4}_{\mu\nu\sigma\lambda}$, Green's functions with all the possible choices for the external legs, $V_i = A, Z, W^\pm$ which are collected in appendix A. We would like to mention that we have performed all the one-loop computations of this paper by the standard diagrammatic method as well and we have got the same results.

In the following, and in order to get the n-point Green's functions in the decoupling limit, we use the asymptotic results for the standard one-loop integrals $A_0(m_i)$, $B_{0,\,\mu,\,\mu\nu}(p,m_i,m_j)$, $C_{0,\,\mu,\,\mu\nu,\,\mu\nu\sigma}(p,k,m_i,m_j,m_k)$ and $D_{0,\,\mu,\mu\nu,\,\mu\nu\sigma,\,\mu\nu\sigma\lambda}(p,k,r,m_i,m_j,m_k,m_l)$ that we have computed in dimensional regularization and by using the m-Theorem [48], and were presented in (A.12) of [34]. These expressions are valid if the masses $m_{i,\,j,\,k,\,l}$ in the propagators of the integrals are much larger that the external momenta p, k, r and if the differences of the squared masses involved in the same integral are much smaller than their sums. This last condition is fulfilled in the present case of the heavy MSSM Higgs sector, even after radiative corrections are included in the Higgs mass predictions. In order to illustrate this point we shortly present in the following the approximate MSSM Higgs mass values in the decoupling limit that include the leading radiative corrections. But the conclusions hold even when the full radiative corrections are employed. To be more precise, in the MSSM, using m_{A^o} and $\tan\beta$ as input parameters, and including the leading radiative corrections which can be parametrized in terms of the quantity,

$$\delta \equiv \frac{3\,G_F}{\sqrt{2}\pi^2}\frac{m_t^4}{\sin^2\beta}\log\left(1+\frac{M^2_{\tilde Q}}{m_t^2}\right)\,,$$

the Higgs masses approach the following values, in the decoupling limit, $m_{A^o} \gg m_Z$ [9],

$$\begin{aligned} m_{h^o} &\longrightarrow \sqrt{m_Z^2\cos^2 2\beta+\delta\sin^2\beta}\\ &\quad\times\left[1+\frac{\delta\, m_Z^2\,\cos^2\beta}{2\,m^2_{A^o}\,(m_Z^2\cos^2 2\beta+\delta\sin^2\beta)}\right.\\ &\quad\left.-\frac{m_Z^2\sin^2 2\beta+\delta\cos^2\beta}{2\,m^2_{A^o}}\right]\,,\\ m_{H^o} &\longrightarrow m_{A^o}\left[1+\frac{m_Z^2\sin^2 2\beta+\delta\cos^2\beta}{2\,m^2_{A^o}}\right]\,,\\ m_{H^\pm} &\longrightarrow m_{A^o}\left[1+\frac{m_W^2}{m^2_{A^o}}\right]^{1/2}\,, \qquad (28) \end{aligned}$$

and the mixing angle in the Higgs sector, α, approaches to,

$$\alpha \to \beta-\frac{\pi}{2} \Rightarrow s_{\alpha\beta} \to -1\,.$$

We see, from the previous expressions that indeed, in the decoupling limit, m_{h^o} always stays below a maximum value which can grow up to about $130\,GeV$ depending on the particular value of $\tan\beta$ and the common squark mass $M_{\tilde Q}$[3]. The other Higgs bosons, H^o, $H^\pm$ and A^o become very heavy and approximately degenerate in the decoupling limit, where $m_{H^o} \sim m_{H^\pm} \sim m_{A^o} \gg m_Z$. Therefore the condition that the squared mass differences for the heavy Higgs sector of the MSSM are always smaller than their sums is largely justified in the decoupling limit both to tree level and in the one loop approximation. Notice however that in the present computation of the electroweak Green's functions to one loop level, we use the tree level Higgs masses in the internal propagators. The use of the radiatively corrected Higgs masses would be effectively a two loop effect.

Finally, by considering $s_{\alpha\beta} \to -1$ and inserting the asymptotic expressions of the one loop integrals into (24) and, after some algebra, we get the Green functions in the decoupling limit that are collected in appendix A. These asymptotic results can be summarized by the following generic expressions [34],

$$\Gamma^{V_1\,V_2\ldots V_n}_{\mu\,\nu\ldots\rho} = \Gamma^{V_1\,V_2\ldots V_n}_{0\,\mu\,\nu\ldots\rho} + \Delta\Gamma^{V_1\,V_2\ldots V_n}_{\mu\,\nu\ldots\rho} \qquad (29)$$

where the subscript 0 refers to the tree level functions, and the one-loop contributions to the two, three and four-point functions behave, in the decoupling limit, respectively as follows,

$$\begin{aligned} \Delta\Gamma^{V_1\,V_2}_{\mu\,\nu} &= \left[\Sigma^{V_1\,V_2}_{(0)}+\Sigma^{V_1\,V_2}_{(1)}k^2\right]\,g_{\mu\,\nu}+R^{V_1\,V_2}_{(0)}\,k_\mu k_\nu\\ &\quad+O\left(\frac{k^2}{\Sigma m^2},\frac{\Delta m^2}{\Sigma m^2}\right)\\ \Delta\Gamma^{V_1\,V_2\,V_3}_{\mu\,\nu\,\sigma} &= F^{V_1\,V_2\,V_3}L_{\mu\,\nu\,\sigma}+O\left(\frac{k^2}{\Sigma m^2},\frac{\Delta m^2}{\Sigma m^2}\right)\\ \Delta\Gamma^{V_1\,V_2\,V_3\,V_4}_{\mu\,\nu\,\sigma\,\lambda} &= G^{V_1\,V_2\,V_3\,V_4}\beta_{\mu\,\nu\,\sigma\,\lambda}\\ &\quad+O\left(\frac{k^2}{\Sigma m^2},\frac{\Delta m^2}{\Sigma m^2}\right) \qquad (30) \end{aligned}$$

[3] Similar conclusions are found if the more general hypothesis of non-common squark mass parameter is assumed

where,

$$\Sigma^{V_1\,V_2}_{(0)} \to 0\,,\, R^{V_1\,V_2}_{(0)} = -\Sigma^{V_1\,V_2}_{(1)}\,,\, O\left(\frac{k^2}{\Sigma m^2}, \frac{\Delta m^2}{\Sigma m^2}\right) \to 0, \tag{31}$$

k denotes generically any of the external momenta and Σm^2 and Δm^2 refer generically to sums and differences of Higgs squared masses respectively. The relevant content are in the functions $\Sigma^{V_1\,V_2}_{(1)}$, $F^{V_1\,V_2\,V_3}$ and $G^{V_1\,V_2\,V_3\,V_4}$ which contain a Δ_ϵ proportional term, with Δ_ϵ being defined in (A.1), and a finite contribution that is a logarithmic function of the heavy Higgs masses, m_{H^0}, m_{H^+} and m_{A^0}. These functions are precisely the only remnant of the heavy Higgs particles and, therefore, *a priori*, they summarize all the potential non-decoupling effects of these particles in the low energy electroweak gauge bosons physics. In the next section we will show, however, that these apparent non-decoupling effects are, indeed, nonphysical since they do not manifest in the electroweak observables.

4 Decoupling of the MSSM Higgs particles *á la Appelquist Carazzone*

In the previous section we have presented the asymptotic results of the electroweak gauge boson functions coming from the integration at one loop of the heavy MSSM Higgs particles. We have shown that all the potential non-decoupling effects of these heavy Higgs particles manifest as divergent contributions in $D = 4$ and some finite contributions logarithmically dependent on the heavy Higgs masses. Furthermore, as can be seen in (30), these contributions are both proportional to the tree level functions, so that we expect them to be finally absorbed by some proper redefinition of the low energy SM parameters.

In this section we are going to complete the demonstration of decoupling of the MSSM Higgs particles *á la Appelquist Carazzone* by finding a particular set of counterterms for the SM electroweak parameters which precisely allow to absorb all the mentioned effects. We will also show that these explicit counterterms coincide with the expressions of the corresponding on-shell SM counterterms in the decoupling limit. By using the common language in the renormalization context, it is equivalent to say that the decoupling at the Green functions (or effective action) level manifests if (and only if) the on-shell prescription for the counterterms is fixed. Of course, once the decoupling is shown at the electroweak gauge boson functions level, the decoupling in the observables with external electroweak gauge bosons is automatically ensured, and this latter is obviously independent of the renormalization prescription.

Let us start by stating the condition for decoupling in terms of the renormalized electroweak gauge boson functions. As usual, these functions are obtained as follows,

$$\Gamma^{V_1V_2\ldots V_n}_{R\,\mu\,\nu\ldots\,\rho}(c_{i\,R}) = \Gamma^{V_1V_2\ldots V_n}_{0\,\mu\,\nu\ldots\,\rho}(c_{i\,R}) + \Delta\Gamma^{V_1V_2\ldots V_n}_{\mu\,\nu\ldots\,\rho}(c_{i\,R}) + \delta\Gamma^{V_1V_2\ldots V_n}_{\mu\,\nu\ldots\,\rho}(c_{i\,R})\,, \tag{32}$$

where, once more, Γ_0 denote the tree level contributions, $\Delta\Gamma$ are the one-loop contributions, and $\delta\Gamma$ represent the contributions from the counterterms of the SM parameters and wave functions. All these contributions must be written in terms of the renormalized parameters that we have denoted here generically by $c_{i\,R}$. Now, the decoupling of heavy particles *á la Appelquist Carazzone* is equivalent to the statement that the renormalized Green functions are equal to the corresponding tree level functions, evaluated at the renormalized parameters, plus corrections that go as inverse powers of the heavy masses and vanish in the asymptotic limit. Therefore, it implies the following conditions,

$$\Delta\Gamma^{V_1V_2\ldots V_n}_{\mu\,\nu\ldots\,\rho}(c_{i\,R}) + \delta\Gamma^{V_1V_2\ldots V_n}_{\mu\,\nu\ldots\,\rho}(c_{i\,R}) \approx 0\,;\, k^2 \ll m_i^2, \forall i\,, \tag{33}$$

where, for the present case, m_i are the heavy Higgs masses, k any of the external momenta, and, by ≈ 0 we mean quantities vanishing in the decoupling limit which have been written generically along this paper as being of $O\left(\frac{k^2}{\Sigma m^2}, \frac{\Delta m^2}{\Sigma m^2}\right)$.

In order to find the wanted explicit SM counterterms we need to include in (33) the asymptotic results presented in the previous section for $\Delta\Gamma$, write $\delta\Gamma$ in terms of the SM counterterms and finally solve the complete system of equations with all the two, three and four point functions included.

By using the standard multiplicative renormalization procedure [44,49], the bare SM electroweak fields and parameters, denoted here by a superscript 0, and the renormalized ones are related by,

$$\begin{aligned}
&\mathbf{W}^0_\mu \equiv Z_W^{1/2}\,\mathbf{W}_\mu\;\;,\; B^0_\mu \equiv Z_B^{1/2}\,B_\mu\,,\, \Phi^0 = (Z_\Phi)^{\frac{1}{2}}\Phi,\\
&\xi^0_W \equiv \xi_W(1+\delta\xi_W)\;\;,\; \xi^0_B \equiv \xi_B(1+\delta\xi_B),\\
&g^0 \equiv Z_W^{-1/2}\,(g-\delta g)\;\;,\; g'^0 \equiv Z_B^{-1/2}\,(g'-\delta g')\,,\\
&v^0 = (Z_\Phi)^{\frac{1}{2}}(v-\delta v)\;\;,\; Z_i \equiv 1+\delta Z_i\,,\, i \equiv A,Z,W,B,\Phi\,.
\end{aligned} \tag{34}$$

The counterterms for the physical masses and physical fields are related to the previous ones by,

$$\begin{aligned}
\delta m_W^2 &= m_W^2\left(\delta Z_\Phi - 2\frac{\delta g}{g} - 2\frac{\delta v}{v} - \delta Z_W\right)\\
\delta m_Z^2 &= m_Z^2\left(\delta Z_\Phi - 2c_W^2\frac{\delta g}{g} - 2s_W^2\frac{\delta g'}{g'} - 2\frac{\delta v}{v} - \delta Z_Z\right)\\
\delta Z_A &= s_W^2\delta Z_W + c_W^2\delta Z_B\\
\delta Z_Z &= c_W^2\delta Z_W + s_W^2\delta Z_B\,,
\end{aligned} \tag{35}$$

where, as usual, $s_W^2 = 1 - m_W^2/m_Z^2$ and $e = g s_W$.

The contributions from the various renormalization constants to the two, three and four point functions can be written as [44],

$$\begin{aligned}
\delta\Gamma^{AA}_{\mu\nu} &= \left[-\left(s_W^2\,\delta Z_W + c_W^2\,\delta Z_B\right)\,k^2\right]\,g_{\mu\nu}\\
&\quad+\left[s_W^2\left(\frac{\delta\xi_W}{\xi_W} + \left(1-\frac{1}{\xi_W}\right)\delta Z_W\right)\right.\\
&\quad\left.+c_W^2\left(\frac{\delta\xi_B}{\xi_B} + \left(1-\frac{1}{\xi_B}\right)\delta Z_B\right)\right]\,k_\mu\,k_\nu\,,
\end{aligned}$$

$$
\begin{aligned}
\delta\Gamma^{AZ}_{\mu\nu} &= \Bigg[\frac{s_W}{c_W} m_W^2\left(\frac{\delta g'}{g'}-\frac{\delta g}{g}\right) \\
&\quad -s_W c_W\,(\,\delta Z_W-\delta Z_B\,)\,k^2\Bigg]\,g_{\mu\nu} \\
&\quad +s_W c_W\left[\frac{\delta\xi_W}{\xi_W}+\left(1-\frac{1}{\xi_W}\right)\delta Z_W\right. \\
&\quad \left.-\frac{\delta\xi_B}{\xi_B}-\left(1-\frac{1}{\xi_B}\right)\delta Z_B\right]\,k_\mu\,k_\nu\,, \\
\delta\Gamma^{ZZ}_{\mu\nu} &= \left[\delta m_Z^2+\left(m_Z^2-k^2\right)\right. \\
&\quad \left.\times\left(c_W^2\delta Z_W+s_W^2\delta Z_B\right)\right]\,g_{\mu\nu} \\
&\quad +\left[c_W^2\left(\frac{\delta\xi_W}{\xi_W}+\left(1-\frac{1}{\xi_W}\right)\delta Z_W\right)\right. \\
&\quad \left.+s_W^2\left(\frac{\delta\xi_B}{\xi_B}+\left(1-\frac{1}{\xi_B}\right)\delta Z_B\right)\right]k_\mu k_\nu, \\
\delta\Gamma^{WW}_{\mu\nu} &= \left[\,\delta m_W^2+\left(m_W^2-k^2\right)\,\delta Z_W\right]\,g_{\mu\nu} \\
&\quad +\left[\frac{\delta\xi_W}{\xi_W}+\left(1-\frac{1}{\xi_W}\right)\delta Z_W\right]\,k_\mu\,k_\nu\,, \\
\delta\Gamma^{AW^+W^-}_{\mu\nu\sigma} &= g\,s_W\,\text{Ł}_{\mu\,\nu\,\sigma}\left[\delta Z_W-\frac{\delta g}{g}\right]\,, \\
\delta\Gamma^{ZW^+W^-}_{\mu\nu\sigma} &= g\,c_W\,\text{Ł}_{\mu\,\nu\,\sigma}\left[\delta Z_W-\frac{\delta g}{g}\right]\,, \\
\delta\Gamma^{AAW^+W^-}_{\mu\,\nu\,\sigma\,\lambda} &= -g^2\,s_W^2\,\text{ß}_{\mu\nu\sigma\lambda}\left[\delta Z_W-2\,\frac{\delta g}{g}\right]\,, \\
\delta\Gamma^{AZW^+W^-}_{\mu\,\nu\,\sigma\,\lambda} &= -g^2\,s_W\,c_W\,\text{ß}_{\mu\nu\sigma\lambda}\left[\delta Z_W-2\,\frac{\delta g}{g}\right], \\
\delta\Gamma^{ZZW^+W^-}_{\mu\,\nu\,\sigma\,\lambda} &= -g^2\,c_W^2\,\text{ß}_{\mu\nu\sigma\lambda}\left[\delta Z_W-2\,\frac{\delta g}{g}\right]\,, \\
\delta\Gamma^{W^+W^-W^+W^-}_{\mu\,\nu\,\sigma\,\lambda} &= g^2\,\text{ß}_{\mu\sigma\nu\lambda}\left[\delta Z_W-2\,\frac{\delta g}{g}\right]. \qquad (36)
\end{aligned}
$$

The results for the one-loop contributions to the electroweak gauge boson functions, presented in the previous section and in appendix A, can be rewritten in a more simplified form and in terms of just the heavy m_{A^0} mass as follows,

$$
\begin{aligned}
\Delta\Gamma^{A\,A}_{\mu\,\nu} &= -\frac{e^2}{8\pi^2}\,\mathcal{K}_{\mu\,\nu}\,\Psi_H\,, \\
\Delta\Gamma^{A\,Z}_{\mu\,\nu} &= \frac{eg}{16\pi^2}\frac{(2s_W^2-1)}{c_W}\,\mathcal{K}_{\mu\,\nu}\Psi_H, \\
\Delta\Gamma^{W\,W}_{\mu\,\nu} &= -\frac{g^2}{16\pi^2}\,\mathcal{K}_{\mu\,\nu}\Psi_H\,, \\
\Delta\Gamma^{Z\,Z}_{\mu\,\nu} &= -\frac{g^2}{16\pi^2}\frac{(2s_W^2-1)^2+1}{2c_W^2}\,\mathcal{K}_{\mu\,\nu}\Psi_H, \\
\Delta\Gamma^{AW^+W^-}_{\mu\,\nu\,\sigma} &= \frac{eg^2}{16\pi^2}\,\text{Ł}_{\mu\,\nu\,\sigma}\,\Psi_H\,, \\
\Delta\Gamma^{ZW^+W^-}_{\mu\,\nu\,\sigma} &= \frac{g^3}{16\pi^2}\,c_W\,\text{Ł}_{\mu\,\nu\,\sigma}\,\Psi_H\,, \\
\Delta\Gamma^{AAW^+W^-}_{\mu\,\nu\,\sigma\,\lambda} &= -\frac{e^2g^2}{16\pi^2}\,\text{ß}_{\mu\nu\sigma\lambda}\,\Psi_H\,, \\
\Delta\Gamma^{AZW^+W^-}_{\mu\,\nu\,\sigma\,\lambda} &= -\frac{eg^3}{16\pi^2}\,c_W\,\text{ß}_{\mu\nu\sigma\lambda}\,\Psi_H\,, \\
\Delta\Gamma^{ZZW^+W^-}_{\mu\,\nu\,\sigma\,\lambda} &= -\frac{g^4}{16\pi^2}\,c_W^2\,\text{ß}_{\mu\nu\sigma\lambda}\,\Psi_H\,, \\
\Delta\Gamma^{W^+W^-W^+W^-}_{\mu\,\nu\,\sigma\,\lambda} &= \frac{g^4}{16\pi^2}\,\text{ß}_{\mu\sigma\nu\lambda}\,\Psi_H \qquad (37)
\end{aligned}
$$

where,

$$
\begin{aligned}
\Psi_H &\equiv \frac{1}{6}\left(\,\Delta_\epsilon-\log\frac{m_{A^0}^2}{\mu_o^2}\right)\,, \\
\mathcal{K}_{\mu\,\nu} &\equiv k^2 g_{\mu\,\nu}-k_\mu k_\nu \qquad (38)
\end{aligned}
$$

By plugging the previous results of (35) through (38) into (33) and by solving the system we finally find the following solution for the SM counterterms[4]:

$$
\begin{aligned}
\delta Z_A &= -\frac{e^2}{8\pi^2}\Psi_H \\
\delta m_W^2 &= -m_W^2\,\delta Z_W=\frac{g^2}{16\pi^2}m_W^2\Psi_H \\
\delta m_Z^2 &= -m_Z^2\,\delta Z_Z=\frac{g^2}{16\pi^2}\frac{m_Z^2}{c_W^2}(1-2s_W^2+2s_W^4)\Psi_H, \qquad (39)
\end{aligned}
$$

and,

$$
\begin{aligned}
&\delta\xi_W=\delta Z_W\ \ ,\ \delta\xi_B=\delta Z_B\,, \\
&\frac{\delta g'}{g'}\approx 0\ \ ,\ \frac{\delta g}{g}\approx 0\,. \qquad (40)
\end{aligned}
$$

Notice that, as in our previous formulas, the results for all the counterterms above have corrections, not explicitely shown, that vanish in the asymptotic limit of infinitely heavy m_{A^0}.

To finish this section we find interesting to compare the previous results for the SM counterterms with the corresponding counterterms of the on-shell renormalization prescription which are defined, as usual, by [51]:

$$
\begin{aligned}
\delta m_W^2 &= -\mathrm{Re}\,\Sigma_T^{WW}(m_W^2)\,, \\
\delta Z_W &= \mathrm{Re}\,\frac{\partial\Sigma_T^{WW}(k^2)}{\partial k^2}|_{k^2=m_W^2} \\
\delta m_Z^2 &= -\mathrm{Re}\,\Sigma_T^{ZZ}(m_Z^2)\,, \\
\delta Z_Z &= \mathrm{Re}\,\frac{\partial\Sigma_T^{ZZ}(k^2)}{\partial k^2}|_{k^2=m_Z^2} \\
\delta Z_A &= \mathrm{Re}\,\frac{\partial\Sigma_T^{AA}(k^2)}{\partial k^2}|_{k^2=0}\,, \\
\frac{\delta g}{g} &= \frac{1}{c_W s_W}\,\frac{\Sigma_T^{AZ}(0)}{m_Z} \qquad (41)
\end{aligned}
$$

plus the solution for $\delta g'$ that is a consequence of the $U(1)_Y$ Ward identity,

$$
\frac{\delta g'}{g'}=0, \qquad (42)
$$

[4] Similar results have been found for sfermions, charginos and neutralinos in [50]

Notice that, after plugging our asymptotic expressions for the $\Sigma_T^{V_1\,V_2}$ functions of appendix A into (41), the solutions for the on-shell counterterms coincide with our solutions of (39) and (40) in the decoupling limit.

In summary, we have shown in this section that the heavy MSSM Higgs particles decouple from the low energy electroweak gauge boson physics. We have found as well that the SM counterterms that are needed to absorb all the (non-physical) heavy Higgs effects are precisely the on-shell counterterms, being these consistently evaluated in the decoupling limit.

5 Comparison with the SM Higgs boson case

We present in this section the paradigmatic case of the SM heavy Higgs boson and its comparison with the present case of a MSSM heavy Higgs sector. It is very well known that the SM Higgs particle does not decouple from the low energy electroweak physics. The logarithmic dependent terms on the heavy Higgs mass that appear in various electroweak precision observables to one-loop, as for instance, $\Delta\rho$, Δr.., are clear remnants of the non-decoupling SM Higgs effects. Indeed, it is precisely this non-decoupling phenomenon that is after all being responsible for the present upper Higgs mass limit, $m_H < 230\,GeV$ at 95% CL, which is imposed by the present data not allowing easily to accommodate a heavy Higgs.

We present in the following the results of integrating out the heavy SM Higgs particle at the one-loop level for the electroweak gauge boson part of the SM effective action. The corresponding results for the so-called effective Electroweak Chiral Lagrangian and the chiral parameters were found some years ago in [40,41,**?**,42,52]. We will work here instead in the different context of the effective SM action and the Appelquist Carazzone Theorem that we have chosen in this paper.

By integrating out the physical Higgs boson particle at the one-loop level in the SM, and by following the same procedure as outlined in the previous sections, we have found the following asymptotic results for the two, three and four-point electroweak gauge functions, to be valid in the very large Higgs mass limit, $M_{H_{SM}} \gg M_Z,\, k$,

$$\begin{aligned}
\Delta\Gamma^{AA}_{\mu\nu} &\approx 0\ ,\ \Delta\Gamma^{AZ}_{\mu\nu} \approx 0 \\
\Delta\Gamma^{ZZ}_{\mu\nu} &= \frac{g^2}{16\pi^2}\,\frac{1}{2\,c_W^2}\,M^2_{H_{SM}} \\
&\quad \times\left(\Delta_\epsilon - \log\frac{M^2_{H_{SM}}}{\mu_o^2} + 1\right) g_{\mu\nu}\ , \\
\Delta\Gamma^{WW}_{\mu\nu} &= \frac{g^2}{16\pi^2}\,\frac{1}{2}\,M^2_{H_{SM}}\left(\Delta_\epsilon - \log\frac{M^2_{H_{SM}}}{\mu_o^2} + 1\right) g_{\mu\nu}\ , \\
\Delta\Gamma^{AWW}_{\mu\,\nu\,\sigma} &\approx 0\ ,\ \Delta\Gamma^{ZWW}_{\mu\,\nu\,\sigma} \approx 0 \\
\\
\Delta\Gamma^{AAWW}_{\mu\,\nu\,\sigma\,\lambda} &\approx 0\ ,\ \Delta\Gamma^{AZWW}_{\mu\,\nu\,\sigma\,\lambda} \approx 0 \\
\Delta\Gamma^{ZZWW}_{\mu\,\nu\,\sigma\,\lambda} &= \frac{g^4}{16\pi^2}\,\frac{1}{2\,c_W^2}\left(\Delta_\epsilon - \log\frac{M^2_{H_{SM}}}{\mu_o^2}\right) g_{\mu\nu}\,g_{\sigma\lambda}\ , \\
\Delta\Gamma^{WWWW}_{\mu\,\nu\,\sigma\,\lambda} &= \frac{g^4}{16\pi^2}\,\frac{1}{2}\left(\Delta_\epsilon - \log\frac{M^2_{H_{SM}}}{\mu_o^2}\right) \\
&\quad \times\left(g_{\mu\nu}\,g_{\sigma\lambda} + g_{\mu\lambda}\,g_{\nu\sigma}\right)\ , \\
\Delta\Gamma^{ZZZZ}_{\mu\nu\sigma\lambda} &= \frac{g^4}{16\pi^2}\,\frac{1}{2\,c_W^4}\left(\Delta_\epsilon - \log\frac{M^2_{H_{SM}}}{\mu_o^2}\right) \\
&\quad \times\left(g_{\mu\nu}\,g_{\sigma\lambda} + g_{\mu\lambda}\,g_{\nu\sigma} + g_{\mu\sigma}\,g_{\nu\lambda}\right)\ ,
\end{aligned} \tag{43}$$

where ≈ 0 here means quantities that go with inverse powers of the SM Higgs mass and vanish in the asymptotic limit.

Next, it is inmediate to find out the corresponding SM counterterms given by,

$$\begin{aligned}
\frac{\delta m_W^2}{m_W^2} &= \frac{\delta m_Z^2}{m_Z^2} \\
&= -\frac{g^2}{16\pi^2}\,\frac{1}{2}\,\frac{M^2_{H_{SM}}}{m_W^2}\left(\Delta_\epsilon - \log\frac{M^2_{H_{SM}}}{\mu_o^2} + 1\right)\ , \\
\frac{\delta g}{g} &\approx 0\,,\ \frac{\delta g'}{g'} \approx 0\,, \\
\delta Z_W &= \delta\xi_W \approx 0\,,\ \delta Z_B = \delta\xi_B \approx 0\,.
\end{aligned} \tag{44}$$

By comparing (44) and (39), (40) we already see some differences. While in the MSSM all the Higgs mass dependence, in the decoupling limit, is logarithmic, in the SM case the dominant contribution to the two point functions goes with the square of the Higgs mass. Another relevant difference is in the four point functions. The results in (44) show that the one-loop corrections from the SM Higgs integration are not proportional to the tree level tensor, $ß_{\mu\nu\sigma\lambda}$, and, as a consequence, these can not be absorbed by the SM counterterms. This is a clear indication of the non-decoupling of the Higgs particle.

Finally, by substituting the previous results of (43) and (44) into (32) , we see that the resulting renormalized SM Green functions at low energies, $k \ll M_{H_{SM}}$, are not all equal to the tree level ones evaluated at the renormalized parameters, as in the MSSM case, but there are some extra terms in the four functions given generically by,

$$\begin{aligned}
&\Gamma^{V_1V_2V_3V_4}_{R\,\mu\,\nu\,\sigma\,\lambda}(c_{i\,R}) - \Gamma^{V_1V_2V_3V_4}_{0\,\mu\,\nu\,\sigma\,\lambda}(c_{i\,R}) \\
&\quad = a_5\left(\frac{g^2}{2}W_\mu W^\mu + \frac{g^2}{4c_W^2}Z_\mu Z^\mu\right)^2\ ,
\end{aligned} \tag{45}$$

with,

$$a_5 = \frac{v^2}{8M^2_{H_{SM}}} + \frac{1}{16\pi^2}\,\frac{1}{4}\left(\Delta_\epsilon - \log\frac{M^2_{H_{SM}}}{\mu_o^2}\right)\,. \tag{46}$$

Notice that the value of this effective parameter does not coincide with the so-called electroweak chiral parameter a_5 computed in [40,41,53]. The reason is because this later contains the quantum effects of mixed diagrams with both gauge bosons and the Higgs particle in the loops which are relevant for the computation of the non-decoupling contributions to observables as for instance $\Delta\rho$. In contrast the result presented in (46) does not include these mixed diagrams.

In summary, the previous (45) shows explicitely that the decoupling theorem of Appelquist and Carazzone does not apply in the case of the SM with a very heavy Higgs particle.

6 Conclusions

We have shown in this work that the heavy Higgs Sector of the MSSM composed of the $H^{\pm}$, H^0 and A^0 scalar particles decouple from the electroweak SM gauge boson physics at the one loop level and under the hypothesis that the Higgs masses are well above the electroweak gauge boson masses. The demonstration has consisted of the computation of the effective action for the electroweak gauge bosons that results after the integration to one loop of the $H^{\pm}$, H^0 and A^0 Higgs bosons. We have found that, in the limit of very large m_A^0 as compared to the electroweak scale, all these one-loop effects can be absorbed into redefinitions of the SM parameters, more specifically by the counterterms of (39) and (40).

In this decoupling limit the only remnant to low energies is, therefore, the light MSSM Higgs particle h_0 with a mass below approximately $130\,GeV$. However, it is still an open question if all the interactions of this light Higgs particle with all the SM particles, fermions and gauge bosons, in the decoupling limit and to all orders in perturbation theory, are exactly the same as the SM Higgs particle interactions. In our opinion, it is an interesting subject that is worth to investigate.

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Appendix A

We present here the exact results for the 2, 3 and 4-point Green functions of the electroweak gauge bosons, and their asymptotic results in the decoupling limit, $m_{A^o} \gg m_Z$, and with all the heavy Higgs masses much larger than any of the external momenta.

In order to present these results for the corresponding Green functions, we use the notation introduced in (29). For brevity, we have omitted the arguments of the one-loop integrals and we use the following compact notation:

$$I^{14}_{\mu\nu} \equiv I^{14}_{\mu\nu}(k, m_{H_1}, m_{A^o})\,,\ I^{32}_{\mu\nu} \equiv I^{32}_{\mu\nu}(k, m_{H^o}, m_{H_2})$$
$$T^{14}_{\mu} \equiv T^{14}_{\mu}(p, m_{H^1}, m_{A^o})\,,\ T^{31}_{\nu} \equiv T^{31}_{\nu}(k, m_{H^o}, m_{H^1})\,,$$
$$T^{123}_{\mu\nu\sigma} \equiv T^{123}_{\mu\nu\sigma}(p, k, m_{H^1}, m_{H^2}, m_{H^o})\,,$$
$$T^{231}_{\nu\sigma\mu} \equiv T^{231}_{\nu\sigma\mu}(k, r, m_{H^2}, m_{H^o}, m_{H^1})\,,\ \text{etc.}$$

Let us mention that all the asymptotic expressions below have corrections that are suppressed by inverse powers of the heavy masses, which vanish in the asymptotic large mass limit. They have been evaluated to one loop in dimensional regularization, with:

$$\Delta_\epsilon = \frac{2}{\epsilon} - \gamma_\epsilon + \log(4\pi)\ \ ,\ \ \epsilon = 4 - D\,, \tag{A.1}$$

and μ_o is the scale of dimensional regularization.

Two-point functions

By following the notation given in (29) for the 2-point Green functions, ${\Gamma_0}^{V_1 V_2}_{\mu\nu}$ represent the tree level contributions which are written in a covariant arbitrary gauge R_ξ as,

$$\begin{aligned}
{\Gamma_0}^{V\,V}_{\mu\,\nu}(k) &= (m_V^2 - k^2) g_{\mu\,\nu} + \left(1 - \frac{1}{\xi_V}\right) k_\mu k_\nu \ (V = Z, W)\,,\\
{\Gamma_0}^{A\,A}_{\mu\,\nu} &= -k^2 g_{\mu\,\nu} + \left(1 - \frac{1}{\xi_A}\right) k_\mu k_\nu\,,\\
{\Gamma_0}^{V_1\,V_2}_{\mu\,\nu} &= 0 \ \text{ if } \ V_1 \neq V_2\,,
\end{aligned} \tag{A.2}$$

and $\Delta\Gamma^{V_1\,V_2}_{\mu\,\nu}$ are the one-loop contributions defined in terms of the transverse and longitudinal parts, $\Sigma_T^{V_1\,V_2}$ and $\Sigma_L^{V_1\,V_2}$, by:

$$\begin{aligned}
\Delta\Gamma^{V_1\,V_2}_{\mu\,\nu} &= \Sigma_T^{V_1\,V_2}(k) \left(g_{\mu\,\nu} - \frac{k_\mu k_\nu}{k^2}\right)\\
&\quad + \Sigma_L^{V_1\,V_2}(k)\, \frac{k_\mu k_\nu}{k^2}\,.
\end{aligned} \tag{A.3}$$

The exact results for the one-loop contributions to the two-point Green functions of the electroweak gauge bosons are:

$$\begin{aligned}
\Delta\Gamma^{A\,A}_{\mu\,\nu}(k) = -\frac{e^2}{16\,\pi^2} \Bigg\{ &[A_0(m_{H_1}) + A_0(m_{H_2})]\ g_{\mu\nu}\\
&- \frac{1}{2}\ [\, I^{1\,2}_{\mu\nu} + I^{2\,1}_{\mu\nu} \,] \Bigg\}
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
\Delta\Gamma^{Z\,Z}_{\mu\,\nu}(k) = -\frac{g^2}{16\,\pi^2} \frac{1}{4\,c_W^2} \Bigg\{ &c_{2W}^2 [A_0(m_{H_1}) + A_0(m_{H_2})]\, g_{\mu\nu}\\
&+ [A_0(m_{H^o}) + A_0(m_{A^o})]\, g_{\mu\nu} - \frac{1}{2}\,(2 s_W^2 - 1)^2\\
&\times [\, I^{1\,2}_{\mu\nu} + I^{2\,1}_{\mu\nu} \,] - \frac{1}{2}\, s^2_{\alpha\beta}\ [\, I^{3\,4}_{\mu\nu} + I^{4\,3}_{\mu\nu}] \Bigg\}
\end{aligned} \tag{A.5}$$

$$\begin{aligned}
\Delta\Gamma^{A\,Z}_{\mu\,\nu}(k) = -\frac{e\,g}{16\,\pi^2} \frac{1}{2\,c_W} \Bigg\{ &c_{2W} [A_0(m_{H_1}) + A_0(m_{H_2})]\, g_{\mu\nu}\\
&+ \frac{1}{2}\,(2 s_W^2 - 1)\ [\, I^{1\,2}_{\mu\nu} + I^{2\,1}_{\mu\nu} \,] \Bigg\}
\end{aligned} \tag{A.6}$$

$$\Delta\Gamma^{WW}_{\mu\,\nu}(k) = -\frac{g^2}{16\,\pi^2} \frac{1}{4} \Bigg\{ [A_0(m_{H_1}) + A_0(m_{H_2})$$

$$+A_0(m_{H^o}) + A_0(m_{A^o})]\, g_{\mu\nu} - \frac{1}{4}\left[I^{14}_{\mu\nu} + I^{24}_{\mu\nu} + I^{41}_{\mu\nu} + I^{42}_{\mu\nu} + s^2_{\alpha\beta} \times \left(I^{13}_{\mu\nu} + I^{23}_{\mu\nu} + I^{31}_{\mu\nu} + I^{32}_{\mu\nu}\right)\right]\Bigg\}, \quad (A.7)$$

where $I^{ij}_{\mu\nu}$ has been defined in (27) and A_0 is the scalar one-loop integral, which is defined in [46,47].

By using the asymptotic results of the one-loop integrals that were presented in our previous work [33,34], we obtain the following asymptotic results for the one-loop heavy Higgs contributions to the transverse and longitudinal parts:

- $m^2_{H^\pm} \gg k^2$:

$$\Sigma^{AA}_T(k)_H = -\frac{e^2}{16\,\pi^2}\frac{k^2}{3}\left(\Delta_\epsilon - \log\frac{m^2_{H^+}}{\mu_o^2}\right), \quad (A.8)$$

$$\Sigma^{AZ}_T(k)_H = \frac{e\,g}{16\,\pi^2}\frac{(2s^2_W - 1)}{2c_W}\frac{k^2}{3} \times \left(\Delta_\epsilon - \log\frac{m^2_{H^+}}{\mu_o^2}\right), \quad (A.9)$$

- $m^2_{H^\pm}, m^2_{H^o}, m^2_{A^o} \gg k^2\,; |m^2_{H^o} - m^2_{A^o}| \ll |m^2_{H^o} + m^2_{A^o}|$:

$$\Sigma^{ZZ}_T(k)_H = \frac{g^2}{16\,\pi^2}\frac{1}{4\,c^2_W}\Bigg\{ h(m^2_{H^o}, m^2_{A^o}) - \frac{k^2}{3}\left[(2s^2_W - 1)^2\left(\Delta_\epsilon - \log\frac{m^2_{H^+}}{\mu_o^2}\right) + \left(\Delta_\epsilon - \log\frac{m^2_{H^o} + m^2_{A^o}}{\mu_o^2}\right)\right]\Bigg\}, \quad (A.10)$$

- $m^2_{H^\pm}, m^2_{H^o}, m^2_{A^o} \gg k^2\,; |m^2_{H^o} - m^2_{H^\pm}| \ll |m^2_{H^o} + m^2_{H^\pm}|\,;$ $|m^2_{A^o} - m^2_{H^\pm}| \ll |m^2_{A^o} + m^2_{H^\pm}|$:

$$\Sigma^{WW}_T(k)_H = \frac{g^2}{16\pi^2}\frac{1}{4}\Bigg\{\left[h\left(m^2_{H^+}, m^2_{H^o}\right) + h\left(m^2_{H^+}, m^2_{A^o}\right)\right] - \frac{k^2}{3}\left[\left(\Delta_\epsilon - \log\frac{m^2_{H^+} + m^2_{H^o}}{2\mu_o^2}\right) + \left(\Delta_\epsilon - \log\frac{m^2_{H^+} + m^2_{A^o}}{2\mu_o^2}\right)\right]\Bigg\}, \quad (A.11)$$

where $h(m^2_1, m^2_2)$ is a function defined as:

$$h(m^2_1, m^2_2) \equiv m^2_1 \log\frac{2m^2_1}{m^2_1 + m^2_2} + m^2_2 \log\frac{2m^2_2}{m^2_1 + m^2_2}, \quad (A.12)$$

and whose asymptotic behaviour in the large m_1 and m_2 limit, with $|m^2_1 - m^2_2| \ll |m^2_1 + m^2_2|$ is:

$$h(m^2_1, m^2_2) \to \frac{m^2_1 - m^2_2}{2}\left[\frac{(m^2_1 - m^2_2)}{(m^2_1 + m^2_2)} + O\left(\frac{m^2_1 - m^2_2}{m^2_1 + m^2_2}\right)^2\right]. \quad (A.13)$$

The above results can be written, in a generic form, as: $\Sigma^{V_1\,V_2}_T(k) = \Sigma^{V_1\,V_2}_{T\,(0)} + \Sigma^{V_1\,V_2}_{T\,(1)}\,k^2$, where $\Sigma^{V_1\,V_2}_{T\,(0)}$ and $\Sigma^{V_1\,V_2}_{T\,(1)}$ are k independent functions. The results for the corresponding longitudinal parts can be summarized in short by:

$$\Sigma^{V_1\,V_2}_L(k) = \Sigma^{V_1\,V_2}_{T\,(0)} \;\forall\, V_1\,V_2\,.$$

For example,

$$\Sigma^{WW}_L(k)_H = \frac{g^2}{16\,\pi^2}\frac{1}{4}\left\{h\left(m^2_{H^+}, m^2_{H^o}\right) + h\left(m^2_{H^+}, m^2_{A^o}\right)\right\}. \quad (A.14)$$

Three-point functions

Analogously to the previous case, we define the three-point Green functions by following the notation introduced in (29), with ingoing momenta assignments $V_1^\mu(-p)$, $V_2^\nu(-k)$ and $V_3^\sigma(-r)$. The tree level contributions, $\Gamma_{0\,\mu\,\nu\,\sigma}^{V_1\,V_2\,V_3}$, are given by,

$$\Gamma_{0\,\mu\,\nu\,\sigma}^{AW^+W^-} = e\,Ł_{\mu\nu\sigma}\,,\; \Gamma_{0\,\mu\,\nu\,\sigma}^{ZW^+W^-} = gc_W\,Ł_{\mu\nu\sigma}\,, \quad (A.15)$$

with:

$$Ł_{\mu\,\nu\,\sigma} \equiv [(k-p)_\sigma g_{\mu\,\nu} + (r-k)_\mu g_{\nu\,\sigma} + (p-r)_\nu g_{\mu\,\sigma}]\,, \quad (A.16)$$

and the AW^+W^- and ZW^+W^- exact one-loop contributions are:

$$\Delta\Gamma^{AW^+W^-}_{\mu\,\nu\,\sigma\;H} = -\frac{eg^2}{8}\frac{1}{16\,\pi^2}\Bigg\{ s^2_{\alpha\beta}\left[(T^{13}_\sigma - T^{31}_\sigma)\,g_{\mu\nu} + (T^{31}_\nu - T^{13}_\nu)\,g_{\mu\sigma}\right] + \left[(T^{14}_\sigma - T^{41}_\sigma)\,g_{\mu\nu} + (T^{41}_\nu - T^{14}_\nu)\,g_{\mu\sigma}\right] - \frac{1}{3}s^2_{\alpha\beta}\left[T^{231}_{\nu\sigma\mu} - T^{231}_{\sigma\nu\mu} + T^{321}_{\sigma\mu\nu} - T^{321}_{\nu\mu\sigma} + T^{123}_{\mu\nu\sigma} - T^{123}_{\mu\sigma\nu}\right] - \frac{1}{3}\left[T^{142}_{\nu\sigma\mu} - T^{142}_{\sigma\nu\mu} + T^{412}_{\sigma\mu\nu} - T^{412}_{\nu\mu\sigma} + T^{124}_{\mu\nu\sigma} - T^{124}_{\mu\sigma\nu}\right]\Bigg\}, \quad (A.17)$$

$$\Delta\Gamma^{ZW^+W^-}_{\mu\,\nu\,\sigma\;H} = \frac{g^3}{8\,c_W}\frac{1}{16\,\pi^2}\Bigg\{ s^2_{\alpha\beta}s^2_W\left[(T^{13}_\sigma - T^{31}_\sigma)\,g_{\mu\nu} + (T^{31}_\nu - T^{13}_\nu)\,g_{\mu\sigma}\right] + s^2_W\left[(T^{14}_\sigma - T^{41}_\sigma)\,g_{\mu\nu} + (T^{41}_\nu - T^{14}_\nu)\,g_{\mu\sigma}\right] - \frac{1}{6}s^2_{\alpha\beta}(2s^2_W - 1)\left[T^{231}_{\nu\sigma\mu} - T^{231}_{\sigma\nu\mu} + T^{321}_{\sigma\mu\nu} - T^{321}_{\nu\mu\sigma} + T^{123}_{\mu\nu\sigma} - T^{123}_{\mu\sigma\nu}\right] - \frac{1}{6}(2s^2_W - 1)\left[T^{142}_{\nu\sigma\mu} - T^{142}_{\sigma\nu\mu} + T^{412}_{\sigma\mu\nu} - T^{412}_{\nu\mu\sigma} + T^{124}_{\mu\nu\sigma} - T^{124}_{\mu\sigma\nu}\right] + \frac{1}{6}s^2_{\alpha\beta} \times \left[T^{341}_{\mu\nu\sigma} - T^{341}_{\mu\sigma\nu} + T^{431}_{\mu\nu\sigma} - T^{431}_{\mu\sigma\nu} + T^{143}_{\sigma\mu\nu} - T^{143}_{\nu\mu\sigma} + T^{134}_{\sigma\mu\nu} - T^{134}_{\nu\mu\sigma} + T^{413}_{\nu\sigma\mu} - T^{413}_{\sigma\nu\mu} + T^{314}_{\nu\sigma\mu} - T^{314}_{\sigma\nu\mu}\right]\Bigg\}, \quad (A.18)$$

where T^{ij}_{μ} and $T^{ijk}_{\mu\nu\sigma}$ are the one-loop integrals as defined in [34].

By using the asymptotic results of the above mentioned integrals, we have obtained the following expressions for the three-point functions in the decoupling limit:

$$\Delta\Gamma^{AW^+W^-}_{\mu\,\nu\,\sigma\quad H} = \frac{1}{16\pi^2}\frac{eg^2}{12}Ł_{\mu\,\nu\,\sigma}\left\{2\Delta_\epsilon - \log\frac{2m^2_{H^+}+m^2_{H^o}}{3\mu_o^2} - \log\frac{2m^2_{H^+}+m^2_{A^o}}{3\mu_o^2}\right\}, \tag{A.19}$$

$$\Delta\Gamma^{ZW^+W^-}_{\mu\,\nu\,\sigma\quad H} = \frac{1}{16\pi^2}\frac{g^3}{6c_W}Ł_{\mu\,\nu\,\sigma}\left\{c_W^2\Delta_\epsilon - \frac{1}{2}\times\log\frac{m^2_{H^+}+m^2_{H^o}+m^2_{A^o}}{3\mu_o^2} - \frac{c_{2W}}{4}\times\left(\log\frac{2m^2_{H^+}+m^2_{H^o}}{3\mu_o^2} + \log\frac{2m^2_{H^+}+m^2_{A^o}}{3\mu_o^2}\right)\right\}. \tag{A.20}$$

Four-point functions

Finally, for the 4-point Green functions, $\Gamma^{V_1\,V_2\,V_3\,V_4}_{\mu\,\nu\,\sigma\,\lambda}$, with ingoing momenta assignments $V_1^\mu(-p)$, $V_2^\nu(-k)$, $V_3^\sigma(-r)$ and $V_4^\lambda(-t)$ we have obtained the results presented below. The tree level corresponding contributions different from zero are:

$$\begin{aligned}
\Gamma^{AAW^+W^-}_{0\,\mu\,\nu\,\sigma\,\lambda} &= -e^2ß_{\mu\nu\sigma\lambda},\\
\Gamma^{AZW^+W^-}_{0\,\mu\,\nu\,\sigma\,\lambda} &= -g^2 s_W c_W ß_{\mu\nu\sigma\lambda},\\
\Gamma^{ZZW^+W^-}_{0\,\mu\,\nu\,\sigma\,\lambda} &= -g^2 c_W^2 ß_{\mu\nu\sigma\lambda},\\
\Gamma^{W^+W^-W^+W^-}_{0\,\mu\,\nu\,\sigma\,\lambda} &= g^2 ß_{\mu\sigma\nu\lambda},
\end{aligned} \tag{A.21}$$

where $ß_{\mu\sigma\nu\lambda}$ is defined by,

$$ß_{\mu\,\nu\,\sigma\,\lambda} \equiv [2g_{\mu\,\nu}g_{\sigma\,\lambda} - g_{\mu\,\sigma}g_{\nu\,\lambda} - g_{\mu\,\lambda}g_{\nu\,\sigma}], \tag{A.22}$$

and the exact results for the one-loop contributions of the heavy Higgs sector, $\Delta\Gamma^{V_1\,V_2\,V_3\,V_4}_{\mu\,\nu\,\sigma\,\lambda\quad H}$, are the following:

$$\begin{aligned}
&\Delta\Gamma^{AAW^+W^-}_{\mu\,\nu\,\sigma\,\lambda\quad H}\\
&= \frac{e^2g^2}{16\pi^2}\Big\{g_{\mu\nu}g_{\sigma\lambda}J^{11}_{p+k} + \frac{1}{4}\left(g_{\mu\sigma}g_{\nu\lambda}J^{14}_{p+r} + g_{\nu\sigma}g_{\mu\lambda}J^{14}_{k+r}\right)\\
&+\frac{s^2_{\alpha\beta}}{4}\left(g_{\mu\sigma}g_{\nu\lambda}J^{31}_{p+r} + g_{\nu\sigma}g_{\mu\lambda}J^{31}_{k+r}\right) - \frac{1}{2}g_{\sigma\lambda}\left[J^{111}_{\mu\nu} + J^{111}_{\nu\mu}\right]\\
&-\frac{s^2_{\alpha\beta}}{4}\left[g_{\sigma\nu}J^{113}_{\mu\lambda} + g_{\sigma\mu}J^{113}_{\nu\lambda} + g_{\lambda\nu}J^{113}_{\mu\sigma} + g_{\lambda\mu}J^{113}_{\nu\sigma}\right]\\
&-\frac{1}{4}\left[g_{\sigma\nu}J^{114}_{\mu\lambda} + g_{\sigma\mu}J^{114}_{\nu\lambda} + g_{\lambda\nu}J^{114}_{\mu\sigma} + g_{\lambda\mu}J^{114}_{\nu\sigma}\right]\\
&-\frac{s^2_{\alpha\beta}}{2}g_{\mu\nu}J^{311}_{\lambda\sigma} - \frac{1}{2}g_{\mu\nu}J^{141}_{\lambda\sigma}\\
&+\frac{s^2_{\alpha\beta}}{4}J^{1113}_{\mu\nu\lambda\sigma} + \frac{1}{4}\left[J^{1141}_{\mu\lambda\sigma\nu} + J^{1141}_{\mu\sigma\lambda\nu}\right]\Big\},
\end{aligned} \tag{A.23}$$

$$\begin{aligned}
&\Delta\Gamma^{AZW^+W^-}_{\mu\,\nu\,\sigma\,\lambda\quad H}\\
&= \frac{eg^3}{32\pi^2c_W}\Big\{c_{2W}g_{\mu\nu}g_{\sigma\lambda}J^{11}_{p+k} - \frac{s^2_W}{2}\big(g_{\mu\sigma}g_{\nu\lambda}J^{14}_{p+r}\\
&-g_{\nu\sigma}g_{\mu\lambda}J^{14}_{k+r}\big) - \frac{s^2_{\alpha\beta}s^2_W}{2}\left(g_{\mu\sigma}g_{\nu\lambda}J^{31}_{p+r} - g_{\nu\sigma}g_{\mu\lambda}J^{31}_{k+r}\right)\\
&-\frac{c_{2W}}{2}g_{\sigma\lambda}\left[J^{111}_{\mu\nu} + J^{111}_{\nu\mu}\right] + \frac{s^2_{\alpha\beta}s^2_W}{2}\left[g_{\nu\lambda}J^{113}_{\mu\sigma} + g_{\nu\sigma}J^{113}_{\mu\lambda}\right]\\
&-\frac{s^2_{\alpha\beta}\,c_{2W}}{4}\left[g_{\mu\lambda}J^{113}_{\nu\sigma} + g_{\mu\sigma}J^{113}_{\nu\lambda}\right] + \frac{s^2_W}{2}\left[g_{\nu\lambda}J^{114}_{\mu\sigma} + g_{\nu\sigma}J^{114}_{\mu\lambda}\right]\\
&-\frac{c_{2W}}{4}\left[g_{\mu\lambda}J^{114}_{\nu\sigma} + g_{\mu\sigma}J^{114}_{\nu\lambda}\right] - \frac{s^2_{\alpha\beta}c_{2W}}{2}g_{\mu\nu}J^{311}_{\lambda\sigma}\\
&-\frac{c_{2W}}{2}g_{\mu\nu}J^{141}_{\lambda\sigma} + \frac{s^2_{\alpha\beta}}{4}\big[g_{\mu\lambda}J^{134}_{\nu\sigma} + g_{\mu\sigma}J^{134}_{\nu\lambda} + g_{\mu\lambda}J^{431}_{\nu\sigma}\\
&+g_{\mu\sigma}J^{431}_{\nu\lambda}\big] + \frac{s^2_{\alpha\beta}\,c_{2W}}{4}\left[J^{1113}_{\mu\nu\sigma\lambda} + J^{1113}_{\mu\nu\lambda\sigma}\right] + \frac{c_{2W}}{4}\big[J^{1141}_{\mu\lambda\sigma\nu}\\
&+J^{1141}_{\mu\sigma\lambda\nu}\big] - \frac{s^2_{\alpha\beta}}{4}\left[J^{1134}_{\nu\sigma\mu\lambda} + J^{1134}_{\nu\lambda\mu\sigma}\right]\Big\},
\end{aligned} \tag{A.24}$$

$$\begin{aligned}
&\Delta\Gamma^{ZZW^+W^-}_{\mu\,\nu\,\sigma\,\lambda\quad H}\\
&= \frac{g^4}{64\pi^2c^2_W}\Big\{c^2_{2W}g_{\mu\nu}g_{\sigma\lambda}J^{11}_{p+k} + \frac{1}{2}g_{\mu\nu}g_{\sigma\lambda}\left[J^{33}_{p+k} + J^{44}_{p+k}\right]\\
&+s^4_W\left(g_{\mu\sigma}g_{\lambda\nu}J^{14}_{p+r} + g_{\mu\lambda}g_{\nu\sigma}J^{14}_{k+r}\right) + s^2_{\alpha\beta}s^4_W\big(g_{\mu\sigma}g_{\lambda\nu}J^{31}_{p+r}\\
&+g_{\mu\lambda}g_{\nu\sigma}J^{31}_{k+r}\big) - \frac{c^2_{2W}}{2}g_{\sigma\lambda}\left[J^{111}_{\mu\nu} + J^{111}_{\nu\mu}\right] - \frac{s^2_{\alpha\beta}}{2}g_{\mu\nu}J^{133}_{\sigma\lambda}\\
&-\frac{1}{2}g_{\mu\nu}J^{414}_{\sigma\lambda} + \frac{s^2_{\alpha\beta}\,s^2_W\,c_{2W}}{2}\big[g_{\mu\lambda}J^{113}_{\nu\sigma} + g_{\nu\lambda}J^{113}_{\mu\sigma} + g_{\mu\sigma}J^{113}_{\nu\lambda}\\
&+g_{\nu\sigma}J^{113}_{\mu\lambda}\big] + \frac{s^2_W\,c_{2W}}{4}\big[g_{\mu\lambda}J^{114}_{\nu\sigma} + g_{\nu\lambda}J^{114}_{\mu\sigma} + g_{\mu\sigma}J^{114}_{\nu\lambda}\\
&+g_{\nu\sigma}J^{114}_{\mu\lambda}\big] - \frac{s^2_{\alpha\beta}}{2}\left(g_{\sigma\lambda}J^{434}_{\mu\nu} + g_{\sigma\lambda}J^{343}_{\mu\nu}\right) - \frac{c^2_{2W}}{2}g_{\mu\nu}J^{141}_{\lambda\sigma}\\
&-\frac{s^2_{\alpha\beta}s^2_W}{2}\big[g_{\nu\lambda}J^{431}_{\mu\sigma} + g_{\mu\lambda}J^{431}_{\nu\sigma} + g_{\nu\sigma}J^{431}_{\mu\lambda} + g_{\mu\sigma}J^{431}_{\nu\lambda}\\
&+g_{\mu\lambda}J^{134}_{\nu\sigma} + g_{\nu\lambda}J^{341}_{\mu\sigma} + g_{\mu\sigma}J^{134}_{\nu\lambda} + g_{\nu\sigma}J^{341}_{\mu\lambda}\big]\\
&+\frac{s^2_{\alpha\beta}c^2_{2W}}{4}\left[J^{1113}_{\mu\nu\sigma\lambda} + J^{1113}_{\mu\nu\lambda\sigma}\right] + \frac{c^2_{2W}}{4}\left[J^{1141}_{\mu\lambda\sigma\nu} + J^{1141}_{\mu\sigma\lambda\nu}\right]\\
&+\frac{s^2_{\alpha\beta}}{4}\left[J^{4341}_{\mu\nu\sigma\lambda} + J^{4341}_{\mu\nu\lambda\sigma}\right] + \frac{s^4_{\alpha\beta}}{4}\left[J^{3431}_{\mu\nu\sigma\lambda} + J^{3431}_{\mu\nu\lambda\sigma}\right]\\
&-\frac{s^2_{\alpha\beta}\,c_{2W}}{4}\left[J^{4311}_{\nu\sigma\mu\lambda} + J^{4311}_{\mu\sigma\nu\lambda} + J^{4311}_{\nu\lambda\mu\sigma} + J^{4311}_{\mu\lambda\nu\sigma}\right]\Big\},
\end{aligned} \tag{A.25}$$

$$\begin{aligned}
&\Delta\Gamma^{W^+W^-W^+W^-}_{\mu\,\nu\,\sigma\,\lambda\quad H}\\
&= \frac{g^4}{64\pi^2}\Big\{g_{\mu\nu}g_{\sigma\lambda}\left[J^{11}_{p+k} + \frac{1}{2}J^{33}_{p+k} + \frac{1}{2}J^{44}_{p+k}\right] + g_{\sigma\nu}g_{\mu\lambda}\\
&\times\left[J^{11}_{k+r} + \frac{1}{2}J^{33}_{k+r} + \frac{1}{2}J^{44}_{k+r}\right] + \frac{1}{2}\big[g_{\nu\sigma}J^{414}_{\mu\lambda} - g_{\nu\mu}J^{414}_{\sigma\lambda}\\
&-g_{\lambda\sigma}J^{414}_{\mu\nu} + g_{\lambda\mu}J^{414}_{\sigma\nu}\big] - \frac{s^2_{\alpha\beta}}{2}\big[g_{\nu\sigma}\left(J^{313}_{\mu\lambda} + J^{131}_{\lambda\mu}\right)\\
&+g_{\nu\mu}\left(J^{313}_{\sigma\lambda} + J^{131}_{\lambda\sigma}\right) + g_{\lambda\sigma}\left(J^{313}_{\mu\nu} + J^{131}_{\nu\mu}\right)
\end{aligned}$$

$$+g_{\lambda\mu}\left(J^{313}_{\sigma\nu}+J^{131}_{\nu\sigma}\right)\Big]-\frac{s^2_{\alpha\beta}}{2}\Big[-g_{\nu\sigma}J^{141}_{\lambda\mu}+g_{\nu\mu}J^{141}_{\lambda\sigma}$$
$$+g_{\lambda\sigma}J^{141}_{\nu\mu}-g_{\lambda\mu}J^{141}_{\nu\sigma}\Big]+\frac{s^4_{\alpha\beta}}{4}\Big[J^{3131}_{\mu\nu\sigma\lambda}+J^{1313}_{\mu\lambda\sigma\nu}+J^{1313}_{\mu\sigma\nu\lambda}$$
$$+J^{1313}_{\mu\sigma\lambda\nu}\Big]+\frac{1}{4}\Big[J^{4141}_{\mu\nu\sigma\lambda}+J^{1414}_{\mu\lambda\sigma\nu}+J^{1414}_{\mu\sigma\nu\lambda}+J^{1414}_{\mu\sigma\lambda\nu}\Big]$$
$$+\frac{s^2_{\alpha\beta}}{4}\Big[J^{1413}_{\mu\lambda\sigma\nu}+J^{3141}_{\mu\lambda\sigma\nu}+J^{1314}_{\mu\lambda\sigma\nu}+J^{4131}_{\mu\lambda\sigma\nu}-J^{1314}_{\mu\sigma\lambda\nu}-J^{1314}_{\mu\sigma\nu\lambda}$$
$$-J^{1413}_{\mu\sigma\lambda\nu}-J^{1413}_{\mu\sigma\nu\lambda}\Big]\Bigg\}. \quad (A.26)$$

Here, J^{ij}_{p+k}, $J^{ijk}_{\mu\nu}$ and $J^{ijkl}_{\mu\nu\sigma\lambda}$ are the one-loop integrals given in [34].

The asymptotic results of the above contributions in the decoupling limit can be written as:

$$\Delta\Gamma^{AAW^+W^-}_{\mu\,\nu\,\sigma\,\lambda\;\;H} = \frac{e^2g^2}{16\pi^2}\Bigg\{-\text{ß}_{\mu\nu\sigma\lambda}\frac{1}{6}\Delta_\epsilon+g_{\mu\,\nu}\,g_{\sigma\,\lambda}\,g_1(m_{H^+},m_{H^o},m_{A^o})$$
$$+(g_{\mu\,\sigma}\,g_{\nu\,\lambda}+g_{\mu\,\lambda}g_{\nu\,\sigma})\,g_2(m_{H^+},m_{H^o},m_{A^o})\Bigg\} \quad (A.27)$$

$$\Delta\Gamma^{AZW^+W^-}_{\mu\,\nu\,\sigma\,\lambda\;\;H} = -\frac{eg^3}{16\pi^2}\frac{1}{2\,c_W}\Bigg\{\text{ß}_{\mu\nu\sigma\lambda}\frac{c^2_W}{3}\Delta_\epsilon+g_{\mu\,\nu}\,g_{\sigma\,\lambda}$$
$$\times g_3(m_{H^+},m_{H^o},m_{A^o})$$
$$+(g_{\mu\,\sigma}\,g_{\nu\,\lambda}+g_{\mu\,\lambda}g_{\nu\,\sigma})\,g_4(m_{H^+},m_{H^o},m_{A^o})\Bigg\} \quad (A.28)$$

$$\Delta\Gamma^{ZZW^+W^-}_{\mu\,\nu\,\sigma\,\lambda\;\;H} = -\frac{g^4}{16\pi^2}\frac{1}{2\,c^2_W}\Bigg\{\text{ß}_{\mu\nu\sigma\lambda}\frac{c^4_W}{3}\Delta_\epsilon+g_{\mu\,\nu}\,g_{\sigma\,\lambda}$$
$$\times g_5(m_{H^+},m_{H^o},m_{A^o})$$
$$+(g_{\mu\,\sigma}\,g_{\nu\,\lambda}+g_{\mu\,\lambda}g_{\nu\,\sigma})\,g_6(m_{H^+},m_{H^o},m_{A^o})\Bigg\} \quad (A.29)$$

$$\Delta\Gamma^{W^+W^-W^+W^-}_{\mu\,\nu\,\sigma\,\lambda\;\;\;\;\;\;\;H} = \frac{g^4}{16\pi^2}\frac{1}{4}\Bigg\{\text{ß}_{\mu\sigma\nu\lambda}\frac{2}{3}\Delta_\epsilon+g_{\mu\,\sigma}\,g_{\nu\,\lambda}\,g_7(m_{H^+},m_{H^o},m_{A^o})$$
$$+(g_{\mu\,\nu}\,g_{\sigma\,\lambda}+g_{\mu\,\lambda}g_{\nu\,\sigma})\,g_8(m_{H^+},m_{H^o},m_{A^o})\Bigg\} \quad (A.30)$$

where the $g_i(m_{H^+},m_{H^o},m_{A^o})\,(i=1\ldots8)$ functions are given by,

$$g_1=\frac{1}{2}\log\frac{2m^2_{H^+}+m^2_{A^o}}{3\mu^2_o}+\frac{1}{2}\log\frac{2m^2_{H^+}+m^2_{H^o}}{3\mu^2_o}$$
$$-3\frac{1}{3}\log\frac{3m^2_{H^+}+m^2_{A^o}}{4\mu^2_o}-\frac{1}{3}\log\frac{3m^2_{H^+}+m^2_{H^o}}{4\mu^2_o},$$

$$g_2=-\frac{1}{4}\log\frac{m^2_{H^+}+m^2_{A^o}}{2\mu^2_o}-\frac{1}{4}\log\frac{m^2_{H^+}+m^2_{H^o}}{2\mu^2_o}$$
$$+\frac{1}{2}\log\frac{2m^2_{H^+}+m^2_{A^o}}{3\mu^2_o}+\frac{1}{2}\log\frac{2m^2_{H^+}+m^2_{H^o}}{3\mu^2_o}$$
$$-\frac{1}{3}\log\frac{3m^2_{H^+}+m^2_{A^o}}{4\mu^2_o}-\frac{1}{3}\log\frac{3m^2_{H^+}+m^2_{H^o}}{4\mu^2_o},$$

$$g_3=-\frac{c_{2W}}{2}\log\frac{2m^2_{H^+}+m^2_{A^o}}{3\mu^2_o}-\frac{c_{2W}}{2}\log\frac{2m^2_{H^+}+m^2_{H^o}}{3\mu^2_o}$$
$$+\frac{c_{2W}}{3}\log\frac{3m^2_{H^+}+m^2_{A^o}}{4\mu^2_o}+\frac{c_{2W}}{3}\log\frac{3m^2_{H^+}+m^2_{H^o}}{4\mu^2_o}$$
$$-\frac{1}{3}\log\frac{2m^2_{H^+}+m^2_{H^o}+m^2_{A^o}}{4\mu^2_o},$$

$$g_4=-\frac{s^2_W}{2}\log\frac{m^2_{H^+}+m^2_{A^o}}{2\mu^2_o}-\frac{s^2_W}{2}\log\frac{m^2_{H^+}+m^2_{H^o}}{2\mu^2_o}$$
$$+\left(s^2_W-\frac{1}{4}\right)\log\frac{2m^2_{H^+}+m^2_{H^o}}{3\mu^2_o}+\left(s^2_W-\frac{1}{4}\right)$$
$$\times\log\frac{2m^2_{H^+}+m^2_{A^o}}{3\mu^2_o}+\frac{c_{2W}}{3}\log\frac{3m^2_{H^+}+m^2_{H^o}}{4\mu^2_o}+\frac{c_{2W}}{3}$$
$$\times\log\frac{3m^2_{H^+}+m^2_{A^o}}{4\mu^2_o}+\frac{1}{2}\log\frac{m^2_{H^+}+m^2_{H^o}+m^2_{A^o}}{3\mu^2_o}$$
$$-\frac{1}{3}\log\frac{2m^2_{H^+}+m^2_{H^o}+m^2_{A^o}}{4\mu^2_o},$$

$$g_5=\frac{1}{4}\log\frac{m^2_{H^o}}{\mu^2_o}+\frac{1}{4}\log\frac{m^2_{A^o}}{\mu^2_o}-\frac{1}{4}\log\frac{2m^2_{H^o}+m^2_{H^+}}{3\mu^2_o}$$
$$-\frac{1}{4}\log\frac{2m^2_{A^o}+m^2_{H^+}}{3\mu^2_o}-\frac{c^2_{2W}}{4}\log\frac{2m^2_{H^+}+m^2_{A^o}}{3\mu^2_o}-\frac{c^2_{2W}}{4}$$
$$\times\log\frac{2m^2_{H^+}+m^2_{H^o}}{3\mu^2_o}-\frac{1}{4}\log\frac{m^2_{H^o}+2m^2_{A^o}}{3\mu^2_o}-\frac{1}{4}$$
$$\times\log\frac{2m^2_{H^o}+m^2_{A^o}}{3\mu^2_o}+\frac{c^2_{2W}}{6}\log\frac{3m^2_{H^+}+m^2_{H^o}}{4\mu^2_o}+\frac{c^2_{2W}}{6}$$
$$\times\log\frac{3m^2_{H^+}+m^2_{A^o}}{4\mu^2_o}+\frac{1}{6}\log\frac{m^2_{H^+}+m^2_{H^o}+2m^2_{A^o}}{4\mu^2_o}$$
$$+\frac{1}{6}\log\frac{m^2_{H^+}+2m^2_{H^o}+m^2_{A^o}}{4\mu^2_o}$$
$$-\frac{c_{2W}}{3}\log\frac{2m^2_{H^+}+m^2_{H^o}+m^2_{A^o}}{4\mu^2_o},$$

$$g_6=\frac{s^4_W}{2}\log\frac{m^2_{H^o}+m^2_{H^+}}{2\mu^2_o}+\frac{s^4_W}{2}\log\frac{m^2_{A^o}+m^2_{H^+}}{2\mu^2_o}-s^2_W$$
$$\times\log\frac{m^2_{H^+}+m^2_{H^o}+m^2_{A^o}}{3\mu^2_o}+s^2_W\frac{c_{2W}}{2}\log\frac{2m^2_{H^+}+m^2_{A^o}}{3\mu^2_o}$$
$$+s^2_W\frac{c_{2W}}{2}\log\frac{2m^2_{H^+}+m^2_{H^o}}{3\mu^2_o}+\frac{c^2_{2W}}{6}\log\frac{3m^2_{H^+}+m^2_{H^o}}{4\mu^2_o}$$
$$+\frac{c^2_{2W}}{6}\log\frac{3m^2_{H^+}+m^2_{A^o}}{4\mu^2_o}-\frac{c_{2W}}{3}$$
$$\times\log\frac{2m^2_{H^+}+m^2_{H^o}+m^2_{A^o}}{4\mu^2_o}$$
$$+\frac{1}{6}\log\frac{m^2_{H^+}+m^2_{H^o}+2m^2_{A^o}}{4\mu^2_o}$$
$$+\frac{1}{6}\log\frac{m^2_{H^+}+2m^2_{H^o}+m^2_{A^o}}{4\mu^2_o},$$

$$g_7 = -\frac{2}{3}\log\frac{m_{H^o}^2+m_{H^+}^2}{2\mu_o^2} - \frac{2}{3}\log\frac{m_{A^o}^2+m_{H^+}^2}{2\mu_o^2}$$

$$\begin{aligned} g_8 = &-\log\frac{m_{H^+}^2}{\mu_o^2} - \frac{1}{2}\log\frac{m_{H^o}^2}{\mu_o^2} - \frac{1}{2}\log\frac{m_{A^o}^2}{\mu_o^2} \\ &+\log\frac{2m_{H^o}^2+m_{H^+}^2}{3\mu_o^2} + \log\frac{2m_{H^+}^2+m_{H^o}^2}{3\mu_o^2} \\ &+\log\frac{2m_{A^o}^2+m_{H^+}^2}{3\mu_o^2} + \log\frac{2m_{H^+}^2+m_{A^o}^2}{3\mu_o^2} \\ &-\frac{2}{3}\log\frac{m_{H^+}^2+m_{H^o}^2}{2\mu_o^2} - \frac{2}{3}\log\frac{m_{H^+}^2+m_{A^o}^2}{2\mu_o^2}\,. \end{aligned} \quad \text{(A.31)}$$

Notice that all these g_k functions behave in the decoupling limit generically as,

$$g_k(m_{H^+}, m_{H^o}, m_{A^o}) = O\left(\log\frac{m_{A^o}^2}{\mu_0^2}\right) + O\left(\frac{\Delta m^2}{\Sigma m^2}\right) \quad \text{(A.32)}$$

and the differences Δm^2 vanish in the present case of the heavy MSSM Higgs sector in the asymptotic limit.

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