Market Integration Dynamics and Asymptotic Price Convergence in Distribution*

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Revised: August 2013

Abstract

In this paper we analyse the market integration process of the relative price distribution, develop a model to analyze market integration, and present a formal test of increasing market integration. We distinguish between the economic concepts of price convergence in mean and in variance. When both types of convergence occur, prices are said to converge in distribution. We present concepts and definitions related to the market integration process, link this to price convergence in distribution, argue that the Law of One Price (LOP) is not a sufficient condition for market integration, and present a formal test of price convergence in distribution. In the empirical analysis, we analyze integration of the inland grains market in 19th Century USA.

Keywords: Regional and global markets, Integration, Asymptotic price convergence, Mean, Variance, Distribution.
JEL: C22, C32, N70, F15

*The authors wish to thank Maria T. González-Pérez, Giovanni Federico and Viktor Todorov for helpful comments and suggestions. Garcia-Hiernaux gratefully acknowledges financial support from Ministerio de Ciencia e Innovación, ref. ECO2011-23972 and Fundación Ramón Areces. McAleer is most grateful to the Australian Research Council and National Science Council, Taiwan for financial support.

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1 Introduction

An analysis of the history of the development and integration of regional and global markets is highly topical in Economics, with an emphasis on the relationship between market and economic development. There is an open question about the determinants and effects of the market integration process. The main contributions of this paper are to develop a model to analyze market integration, and a formal test of increasing market integration.

The empirical perspective is focused on measuring and quantifying market development and related concepts. This paper highlights the distinction between the two main concepts in the literature, namely the Law of One Price (LOP) and the Market Integration Process: the former refers to the Extent of the Market problem, that is, if the prices observed in different places arise from the same market (see Cournot (1897), Stigler and Sherwin (1985) and Treadway (2009)), while the latter refers to the degree of relatedness of different market locations.

Both concepts are closely related, but correspond to different economic situations. The LOP refers to a state of the market, while market integration refers to the internal dynamics of the market. In this paper we argue that Market Integration is a more general concept than LOP. Therefore, LOP is a necessary but not sufficient condition for Market Integration.

It is traditional in economics to use cointegration analysis to conclude if LOP is satisfied. A cointegration relationship is expected under the LOP as it is a particular case of convergence. Bernard and Durlauf (1995, 1996) and Hobijn and Franses (2000) provide definitions and methods for what is called steady state convergence. In this context, García-Hiernaux and Guerrero (2011) define a model for convergence that includes a transition path, so that convergence can be represented as a catching-up convergence.

Market integration studies based on cointegration are not well connected with the notion of the market integration process, as convergence in mean does not necessarily imply greater market integration. A change in trade barriers, such as tariffs, could imply a convergence to parity, but it does not necessarily imply a change in the internal dynamics of the market. In this paper we analyze the market integration process through the relative price distribution, by extending the ideas in Dobado et al. (2012) on the relationship between market integration and relative price dispersion.

When the LOP is satisfied, both prices will have converged in mean, that is, in the first moment of the distribution. For instance, the LOP is not a matter of degree, but is a binary distinction. If two prices are stochasticaly the same, they might came from the same spread market. However, if two prices do not share at least one common nonstationary factor, it is not possible for them to come from the same market. Thus, the LOP is a matter of cointegration, not of correlation.

On the other hand, the relatedness of two different markets, or market locations, is a matter of degree and, hence, of contemporary and lagged cross covariances. This means that it is necessary to check at least the second moment of the price distribution to conduct an appropriate market integration analysis. In order to conclude positive market integration, it is necessary to have prices converge in distribution, and not just the LOP. Under normality, two necessary conditions for market integration are convergence in mean and convergence in variance.

In the empirical exercise, we examine the market integration process through price convergence in distribution under normality. We present the case of the inland grains market integration in 19th Century USA as an illustration. We examine the historical prices of wheat in several cities across the USA, including coastal and inland cities. The results show price convergence in mean and variance for this commodity between many cities in the sample, suggesting a strong market integration process occurred in 19th Century USA. The patterns are not the same, but they are very similar.

The paper is organized as follows. Section 2 introduces the theoretical framework, presents concepts and definitions related to the market integration processes, and links this concept with the notion of price convergence in distribution. Section 3 describes the model. In Section 4 the econometric representation and hypothesis testing are presented. Section 5 presents the empirical results on wheat price convergence in the second half of 19th Century USA. Section 6 gives some concluding comments.

2 Theoretical Framework

In this section we describe the concepts and definitions relating to convergence, structure of markets, market integrations and convergence of prices. We present the assumptions on the relationship between the prices of the goods to be analyzed. All prices are transformed into naperian logarithms to induce linearity, and to avoid heteroskedasticity and non-normality. In what follows, $p_{i,t}$ is the log of the nominal price of a good in place i at time t, and $p_{j,t}$ is the log of the nominal price of a good in place j at time t.

2.1 Market-related concepts

It is assumed that nominal prices need to be differenced at least once to be stationary. Based on economic theory, it is expected that any market-clearing nominal price follows a non-stationary process. This reflects the idea that some shifts in supply (for example, technological breakthroughs or changes in wages) or in demand (for example, changes in consumer preferences or population growth) imply price adjustments to clear the market in the long run.

By definition, a stationary price level cannot change in the long run to clear the market precisely because stationarity implies a long term, constant value. However, as market conditions can change due to many factors, prices need to react to those changes in order to clear the market. Therefore, our analytical framework requires that the log nominal price follows a non-stationary process, otherwise they would not be market-clearing prices. If the price follows a stochastically stationary process, this reveals a poorly developed market, or a market where some participants may have market power.

Our view of market efficiency follows that proposed by Lo (2004, 2005) for the Adaptive Market Hypothesis, in which transitory arbitrage situations in the time domain and under uncertainty are allowed. Specifically, the (log) price series can be represented by an ARIMA$(p, 1, q)$ model, with $p > 0$ and/or $q > 0$. In this case, the persistent behaviour implied in such models represents transitory arbitrage possibilities. This clearing market condition is not implied for the Efficient Market Hypothesis, including "weak-form" efficiency, because a persistent component in prices is permitted.[1]

In this framework, arbitrage possibilities have two dimensions, namely time and space, so the market efficiency notion used is also bi-dimensional. For instance, an extension of the Adaptive Market Hypothesis in the space domain can be used, which means that transitory arbitrage possibilities in the space domain, and under uncertainty, are allowed. In the sense of space efficiency, we say that a *Spread Market* exists when the prices of homogeneous goods traded in different locations come from the same market. In this case, a market clearing price is not a scalar, but a vector. Under market clearing conditions, relative prices in a spread market should follow a stochastically stationary process so that, in a strict sense, prices are statistically the same in the long run.

In this paper we consider the prices of perfectly homogeneous or quasi-homogeneous goods. It is assumed that price similarities or dissimilarities (generated by quality, brand, and consumer perception) are time invariant. In such a case, Cournot's pioneering definition of *market* "an entire territory of which the parts are so united by the relations of unrestricted commerce that prices take the same level throughout with ease and rapidity" (Cournot, 1897, pp. 51-52), applies. Market integration is then the process experienced by two prices in different locations when tending to the previous definition by Cournot. Thus, we state the following definition that follows directly from Cournot's in a extreme case: there is Perfect Market Integration when prices take the same value "exactly and

[1] Other market hypotheses can also be used, depending on the context. In the case of the Efficient Market Hypothesis, for example, the assumption is that prices follow a random walk.

instantaneously":

Definition 1 *Perfect Market Integration (PMI) in a spread market occurs when arbitrage opportunities are zero at any time t, that is, when m prices of the same product in m different locations are adjusted instantaneously, and clear the market for all time t.*

Under this definition, for PMI the m observed time series prices would be exactly the same, at any time t, if the trade cost were zero.[2] Note that the relative prices in this case have a degenerate distribution at zero, and this probability distribution is constant over time, that is, there is no relative price dispersion.

Definition 1 coincides, therefore, with that of the Law of One Price in its strongest version, that is, $p_{it} = p_{jt}$, so that, there is Perfect Market Integration between i and j $\Leftrightarrow p_{it} = p_{jt}$. These concepts are useful for understanding the relationship between relative price dispersion and market integration, although they might appear to be utopian.

Obviously, the strongest version of the LOP is hard to find in practice. Arbitrage opportunities should prevent prices from moving independently of each other, not exactly, but in the long run. Hence, a weaker version of the LOP can be defined as:

$$r_{ij,t} = p_{i,t} - p_{j,t} = \tau_{ij} + \varepsilon_{ij,t} \tag{1}$$

where the constant τ_{ij} denotes trading barriers, transport costs and other transaction costs, and $\varepsilon_{ij,t}$ is a zero mean stationary stochastic process. In short, if two prices arise from the same market, they should be cointegrated of order one, with cointegrating vector $[1, -1]$. Hence, this is a requirement for maker integration, but it says nothing about the direction of the process.

The statistical properties of the random variable, ε_{ijt}, are closely related with market efficiency. For example, under the efficient market hypothesis, ε_{ijt} fulfills $\mathrm{cov}(\varepsilon_{ijt}, \varepsilon_{ijt-k}) = 0$ for $k = 1, 2, \ldots$, and has zero mean and finite variance, $\sigma^2_{\varepsilon_{ij}}$. On the contrary, if the adaptive market hypothesis is assumed, ε_{ijt} will possess a predictable structure. The speed of adjustment, a proxy for market efficiency, can be measured with autoregressive polynomials, but it is inevitably dependent on the frequency of the data. For example, with annual data, an autoregressive process can measure the inter-annual arbitrage possibilities and it is then a measure of inter-annual market efficiency. On the other hand, the intra-annual efficiency of the market would be included in the variance of the shocks in (1), $\sigma^2_{\varepsilon_{ij}}$, reflecting the idea that the smaller is the variance, the smaller are the arbitrage possibilities within a year. To better understand this idea, think about a first-order autoregressive with decreasing persistency in time in monthly data. If we only

[2]For instance, the price vector is a singleton.

observed the annual average of this process we would probably not be able to capture the autoregressive behavior but, instead, we would observe a decreasing residual variance.

Fama (1970) noted that a market is (totally) efficient when there are no opportunities for profit from exploitation of some information. According to this, a totally (in time and space domain) efficient market is characterized by (i) no predictable structure and (ii) $\sigma^2_{\varepsilon_{ij}} = 0$. Therefore, $p_{it} = p_{jt} + \tau_{ij}$ that reflects the fact that no opportunities for profit from exploitation of some information is possible. The only way to obtain temporal profit from this relation would be to reduce transaction costs, τ_{ij}. In contrast, a totally inefficient market is characterized by $\sigma^2_{\varepsilon_{ij}} = \infty$, which means that there is no long run relationship between the nominal prices. In such a case, the information does not flow at all between the markets, and so the LOP, even the weakest version defined in (1), is rejected.[3]

Hence, there are only two fundamental ways in which two markets can become more integrated. These two ways are related in moving from the weakest to the strongest version of the LOP, that is, from $p_{it} = p_{jt} + \tau_{ij} + \varepsilon_{ijt}$ to $p_{it} = p_{jt}$. The first way is through a reduction toward zero of the transaction costs, which will produce an abrupt or smooth shift in the mean, τ_{ij}; for example, those originated by the ending of some protectionist laws or technological improvements. This first way is described in García-Hiernaux and Guerrero (2011). The second way is through an increase in the inter- or intra-market efficiency, that is, a reduction in the persistence of the autoregressive structure (if any) or in the variance of the shocks affecting the relative price, $\sigma^2_{\varepsilon_{ij}}$. With this in mind, we define the "Market Integration Process":

Definition 2 *The Market Integration Process towards PMI occurs when arbitrage opportunities decrease continuously to zero.*

This definition means that greater market efficiency in the time and space domains implies greater market integration. The main assumption in this case is that there is a transitive relationship between arbitrage opportunities, relative price dispersion, and increasing market integration. In other words, if in a certain spread market the relative price dispersion is decreasing continuously, then there is evidence that market integration is increasing for this specific market. Therefore, in a Market Integration Process, the variable $\varepsilon_{ij,t}$ in Model (1) converges in distribution to a degenerate distribution that is equal to zero.

[3]Note that if ε_{ijt} requires one or more differences to be stationary, then $\sigma^2_{\varepsilon} = \infty$.

2.2 Price convergence in distribution

Given our conceptual model of market integration, we have a definition of price convergence in distribution that is consistent with the notion of the Market Integration Process. Price convergence in distribution is more general than a simple notion of relative price dispersion, and is also more feasible in formal testing procedures.

Based on the relation between arbitrage and market integration, and following the stochastic definitions of convergence in output presented by Bernard and Durlauf (1995, 1996) and Hobijn and Franses (2000), we have the following definition, where $\mathcal{F}_t$ denotes the information set of the agents at time t:

Definition 3 *For Asymptotic Price Convergence in Distribution (APCD), the prices of goods i and j converge asymptotically in distribution if:*

$$\lim_{k\to\infty} \mathbb{E}\big[(p_{i,t+k} - p_{j,t+k})^p|\mathcal{F}_t\big] = 0,\ \forall p = 1, 2, ..., n.$$

Corollary 1 *Assuming normality of prices and the asymptotic expected variance of relative prices, $p_{i,t}$ and $p_{j,t}$ converge asymptotically in distribution if Definition 3 holds for $p = 1, 2$.*

Thus, it is necessary to check the evolution of the first two moments of the distribution of relative prices to conclude convergence in distribution under the Gaussian assumption. In contrast, market integration analyses based on cointegration analysis concentrate only on the first moment condition, that is, on convergence in mean. Market integration analyses based on cointegration are also not well connected with the notion of the market integration process, as convergence in mean does not necessarily imply greater market integration. For example, a change in trade barriers, such as tariffs, could imply a level change in relative prices, but this could happen together with an increase in the residuals' relative price dispersion.

For the first moment condition, when $p = 1$ in Definition 3, this coincides with Bernard and Durlauf's (1995) definition of convergence in output. In that case, the definition should be interpreted as the weaker form of the LOP presented in equation (1). In such a case, the long term forecast of the (log) price differential, the relative price, is zero mean stationary and $\tau_{ij} = 0$.

For the second moment condition, when $p = 2$ in Definition 3, notice that $\mathbb{E}\big[(p_{i,t} - p_{j,t})^2|\mathcal{F}_t\big] = \mathrm{var}\big[\varepsilon_{ij,t}|\mathcal{F}_t\big]$, the unconditional variance. Thus, $p_{i,t}$ and $p_{j,t}$ will have converged asymptotically in distribution if $\mathrm{var}\big[\varepsilon_{ij,t}|\mathcal{F}_t\big]$, is zero for all t (which is unlikely), or if it tends to zero as t approaches infinity. Therefore, in order to test the implications of Definition 3, we should relax the assumption of a constant variance and make it depend deterministically on t. The notion behind this requirement is that the fluctuations around a constant mean of a relative price series can be considered as the net idiosyncratic shocks in the two market locations. Market Integration enhances the ability of the two market locations to cope with shocks in nominal prices. Therefore, one would expect that, as integration in a spread market increases, the dispersion of those transitory shocks would decrease.

In summary, assuming the conditions given in Corollary 1, we will have convergence in distribution and, therefore, convergence to the stronger version of the LOP ($p_{it} = p_{jt}$) when: (i) $\tau_{ij} = 0$, and (ii) $\varepsilon_{ij,t}$ converges in probability to zero.

3 Model

The model for representing the convergence process is based on García-Hiernaux and Guerrero (2011). Using equation (1), and including a transition path, the model for the price differential may be written as:

$$\begin{aligned} p_{i,t} - p_{j,t} &= D_{ij,t} + S_{ij,t}, \\ D_{ij,t} &= \mu_{ij} + \nu_{ij}(B)\xi_t^{t^*}, \\ \phi_{ij,p}(B)S_{ij,t} &= \theta_{ij,q}(B)a_{ij,t} \end{aligned} \tag{2}$$

where B is the backshift (lag) operator, such that $Bp_t = p_{t-1}$, and the relative price, $r_{ij,t} = p_{i,t} - p_{j,t}$, has an additive decomposition between a deterministic component, $D_{ij,t}$, and stochastic component, $S_{ij,t}$. In the deterministic component, μ_{ij} is the constant mean, $\nu_{ij}(B)$ represents the convergence operator, and the variable, $\xi_t^{t^*}$, describes the effects of an event that will last permanently after time t^*, as unity whenever $t > t^*$, and zero otherwise. The stochastic component follows a stationary process and has an ARMA(p,q) representation, strictly stationary and invertible (that is, the autoregressive and moving average polynomials have all their zeros lying outside the unit circle), and $a_{ij,t}$ is a weak white noise stochastic process.

In Model (2), the transition path is represented by a combination of the convergence operator with the deterministic variable, $\xi_t^{t^*}$:

$$\nu_{ij}(B)\xi_t^{t^*} := \frac{\omega_s(B)}{\delta_r(B)} B^b \xi_t^{t^*}, \tag{3}$$

where $\omega_s(B) = \omega_0 - \omega_1 B - ... - \omega_s B^s$, $\delta_r(B) = 1 - \delta_1 B - ... - \delta_r B^r$, there are no common factors between $\omega_s(B)$ and $\delta_r(B)$, and s, r, b are non-negative integers. The concept of convergence is closely linked to stability, so that it is assumed $\delta_r(B)$ is stable. Therefore, the condition for a stable convergence process is that the roots of the characteristic equation, $\delta_r(B) = 0$, should lie outside the unit circle.

Observe that the long run gain of the transition path can be estimated as a function of the estimated parameters in (3). The steady state gains, g_{ij}, from a stable convergence process, are defined as:

$$g_{ij} := \sum_{k=0}^{\infty} \nu_{ij,k} = \nu_{ij}(1) < \infty. \quad (4)$$

This definition implies that the back shift operators in (3) are replaced by one to obtain the steady state gains. The estimated steady state gains could be used for testing asymptotic convergence in mean when a transition path is present. With the initial conditions, represented by the constant mean, μ_{ij}, and the long run gains, g_{ij}, having the same value, but with opposite signs, there is evidence of convergence in mean.

4 Representation and Hypothesis Testing

Model (2) could be used for testing convergence in the sense of the APCD in Definition 3, and assuming a Gaussian process for nominal prices. For the requirement of convergence in mean (when $p = 1$ in Definition 3), both prices will have converged asymptotically if $p_{i,t} - p_{j,t}$ is a stationary process and $D_{ij,t}$ is equal to zero, or tends to zero as t approaches infinity. For the requirement of convergence in variance (when $p = 2$ in Definition 3), asymptotic convergence in distribution arises if ε_{ijt} converges in probability to zero. In the subsections below, we describe in detail the procedures proposed for testing asymptotic price convergence in both the mean and the variance.

In all the estimated models that include a convergence transition path (3), we fix t^* at the year in which convergence could have started. Despite historical reasons that justify the use of any year as an initial point for the convergence process, we undertook a thorough search for alternative starting dates.

4.1 Testing asymptotic price convergence in mean

Testing APCD for $p = 1$ or Asymptotic Price Convegence in Mean, hereafter APCM, is equivalent to testing Asymptotic Strong Price Convergence in the García-Hiernaux and Guerrero (2011) framework. It requires the log price differential, corrected or uncorrected by the transition path, to be stationary. Moreover, it requires that $D_{ij,t}$ is equal to zero, or $\tau_{ij} = g_{ij} + \mu_{ij} = 0$, if a transition path is present. Steady state convergence arises when $D_{ij,t}$ is equal to zero. In contrast, we say that there is catching-up convergence if $D_{ij,t}$ tends to zero as t approaches infinity. In the following, we summarize the steps suggested by García-Hiernaux and Guerrero (2011).

As the goods whose prices are analyzed are assumed to be homogeneous, univariate analysis could be used to examine convergence in pairs. As $r_{ij,t} = p_{i,t} - p_{j,t}$, model (2) can easily be estimated as the univariate model of the relative price, $r_{ij,t}$. This not only makes the analysis simpler, but also has gains in terms of the power of the unit root tests as the critical values are closer to zero. In this case, Saikkonen and Lutkepohl (2002), hereafter SL-GLS, present a test for a unit root with different level shifts that includes the transition path (3). They show that the convergence parameters in $\nu_{ij}(B)$, or the time at which the convergence begins, t^*, do not affect the limiting distribution of the non-stationarity test. Furthermore, the Shin and Fuller (1998) test, SF, which is more powerful than ADF-type tests in the case of ARMA structures, can also be used.

When the non-stationarity hypothesis is rejected in the univariate version of model (2), standard inference applies. In this case, all the parameters in the model can be estimated jointly, and the estimates are generally asymptotically normally distributed, so that standard inference can be applied. The representation used has two remarkable advantages for our purposes: (i) simplicity; and (ii) maximum likelihood estimation of the univariate model means that an optimal asymptotic theory of inference applies, so that APCM can be tested using standard asymptotic tests.

In order to test the null hypothesis $\tau_{ij} = 0$, we use two procedures, which are explained in greater detail in García-Hiernaux and Guerrero (2011). Assume that $\hat{g}_{ij}$ and $\hat{\mu}_{ij}$ are consistent and asymptotically normal estimators of g_{ij} and μ_{ij}, respectively, so that $\sqrt{T}(\hat{\tau}_{ij} - \tau_{ij})/\hat{\sigma}_\tau \xrightarrow{d} N(0,1)$, where $\hat{\tau}_{ij}$ and $\hat{\sigma}_\tau$ are calculated using the Delta Method. For the same purpose, the statistic $-2\log l(\Theta_2|p_{1,t}, p_{2,t}, \xi_t^{t^*})/l(\Theta_1|p_{1,t}, p_{2,t}, \xi_t^{t^*})$, where $\Theta_2 = \{\alpha, \omega_0, ..., \omega_s, \delta_1, ..., \delta_r, \phi_{1,ii}, ..., \phi_{p,ii}, \theta_{1,ii}, ..., \theta_{q,ii}, \theta_{ij}\}$, and which follows a χ^2 distribution asymptotically with 1 degree of freedom, can be applied. Whatever the test that is used, when $p_{i,t}$ and $p_{j,t}$ are cointegrated and $\tau_{ij} = 0$ cannot be rejected, then $p_{i,t}$ and $p_{j,t}$ are said to converge asymptotically in mean. Therefore we have APCD when $p = 1$.

4.2 Testing asymptotic price convergence in variance

Assuming the conditions in Corollary 1, testing the APCD now requires testing whether the residual variance in model (2) tends to zero. We propose using the well-known test of Breusch and Pagan (1979), which tests whether the estimated variance of the residuals is unconditionally homoscedastic. We regress the squared residuals on an exogenous variable. The test statistic, LM, is the product of the coefficient of determination (R^2) from this regression and the sample size n, namely $LM = nR^2$, where LM is the Lagrange multiplier statistic. The test statistic is asymptotically distributed as $\chi^2(1)$ under the null hypothesis of homoscedasticity.

If the null hypothesis is not rejected, there is no evidence in favor of the APCD as the variance of ε_{ijt} is constant over time. In that case, APCD and, therefore, increasing market integration through increasing market efficiency, can be rejected. Unfortunately, when the null hypothesis is rejected, we cannot conclude that there is APCD, as APCD implies heteroscedasticity, but the reverse is not always true. In that case, we could have growing integration, disintegration, or both.

In the inconclusive case, we propose to observe the residual standard deviation calculated with a rolling window. In this way, we could use this visual aid to decide on the evolution of the variance, the APCD and, finally, the market integration process. As an alternative, the relative price standard deviation can be used. This can also be estimated with a rolling window by using the residual standard deviation and the estimated parameters of the ARMA model. Additionally, the relative price standard deviation provides a good measure of market integration evolution, and also provides a tool to compare the levels of market integration when more than one case is analyzed as the residual standard deviation cannot be used to compare levels of market integration.

5 Empirical Results on Price Convergence

5.1 Data

The empirical analysis in this section considers the historical annual series of wheat prices in seven cities in the USA, namely New York (NY), Philadelphia (P), Alexandria (A), Cincinnati (CI), Chicago (CH), Indianapolis (I), and San Francisco (SF). All of these cover a common period in the second part of the 19th Century.[4] The sources of data are given in Jacks (2005, 2006). Nominal prices are annual averages, and are expressed in US dollars. The selection of the markets is based on data availability and geographical representativeness. Markets in the coastal zones and inland territories in the 19th Century USA are represented. The series of nominal prices are shown in Figure 1, and their relative prices are given in Figure 2.

In Figure 1, the nominal series clearly wander, showing little or no affinity for a constant mean value. In the observations later in the sample, many of which have a similar level, the cross-sectional dispersion is also much lower than in the rest of the sample. This is confirmed in the corresponding relative price graphs in Figure 2.

Note that there is visual evidence in Figure 2 that the relative price series show a strong affinity for a constant parity value, equal to or very close to one. Note also that

[4] NY covers 1800-1913, P covers 1800-1896, CI covers 1816-1913, CH and I cover 1841-1896, and SF covers 1852-1916.

the convergence paths are the same in all cases. In the case of Chicago and Indianapolis, it seems that the prices for wheat are cheaper than in the other cities, and holds for all of the 19th Century.

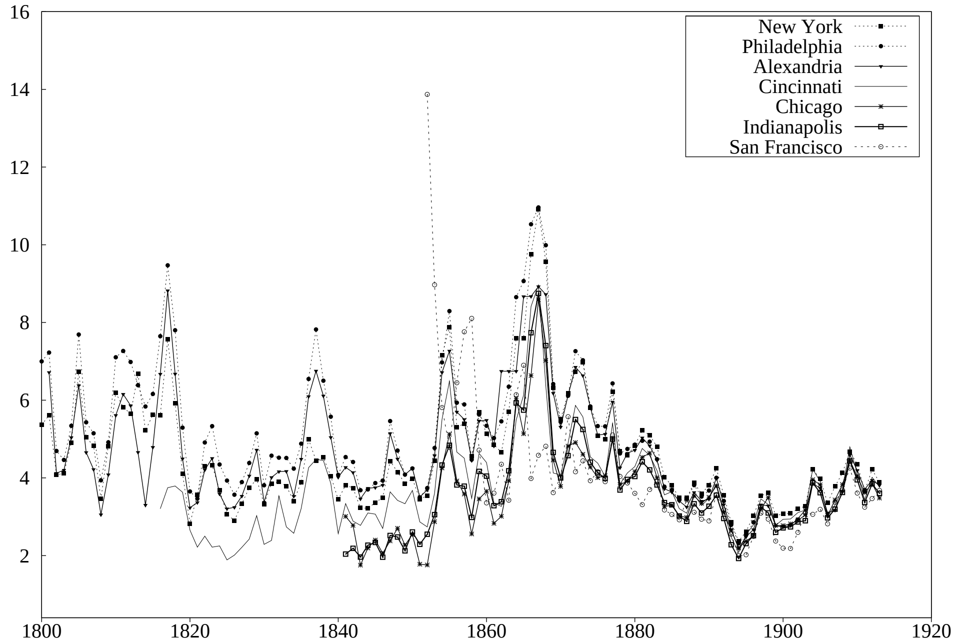

Figure 1: Nominal Prices of Wheat in 19th Century USA

All nominal prices show similar statistical properties, namely they: (i) are integrated of order one; (ii) need to be transformed to natural logs to avoid heteroskedasticity, non-normality and non-linearity; (iii) fit a zero mean ARIMA(2,1,1) model; and (iv) have a small number of impulse interventions due to the American Civil War.[5] The AR(2) structures have two conjugate imaginary roots, leading to damped oscillations with a period of 5-13 years. A damping factor of around 0.5 represents quasi-cyclical behaviour, where the period describes the time elapsed (in years) from peak to trough. There is no evidence of over-differentiation in the univariate models of nominal prices as the null hypothesis of MA(1) noninvertibility is clearly rejected by the Generalized Likelihood Ratio (GLR) test of Davis et al. (1995). Moreover, SF does not reject the null hypothesis of nonstationarity in an alternative ARIMA(3,0,1) model. Consequently, I(1) is confirmed in all cases. These results are summarized in Table 1.

On the contrary, relative prices do not seem to be stationary, especially at the beginning of each sample (see Figure 2). In all cases, adding a convergence component seems to be necessary to represent the transition path, and also for having a stationary representation. The estimates for relative prices are reported in Table 2 for New York and Chicago as numeraire. The rest of the estimates are reported in Table the Statistical

[5]All the interventions are of an impulse type and do not significantly affect the results.

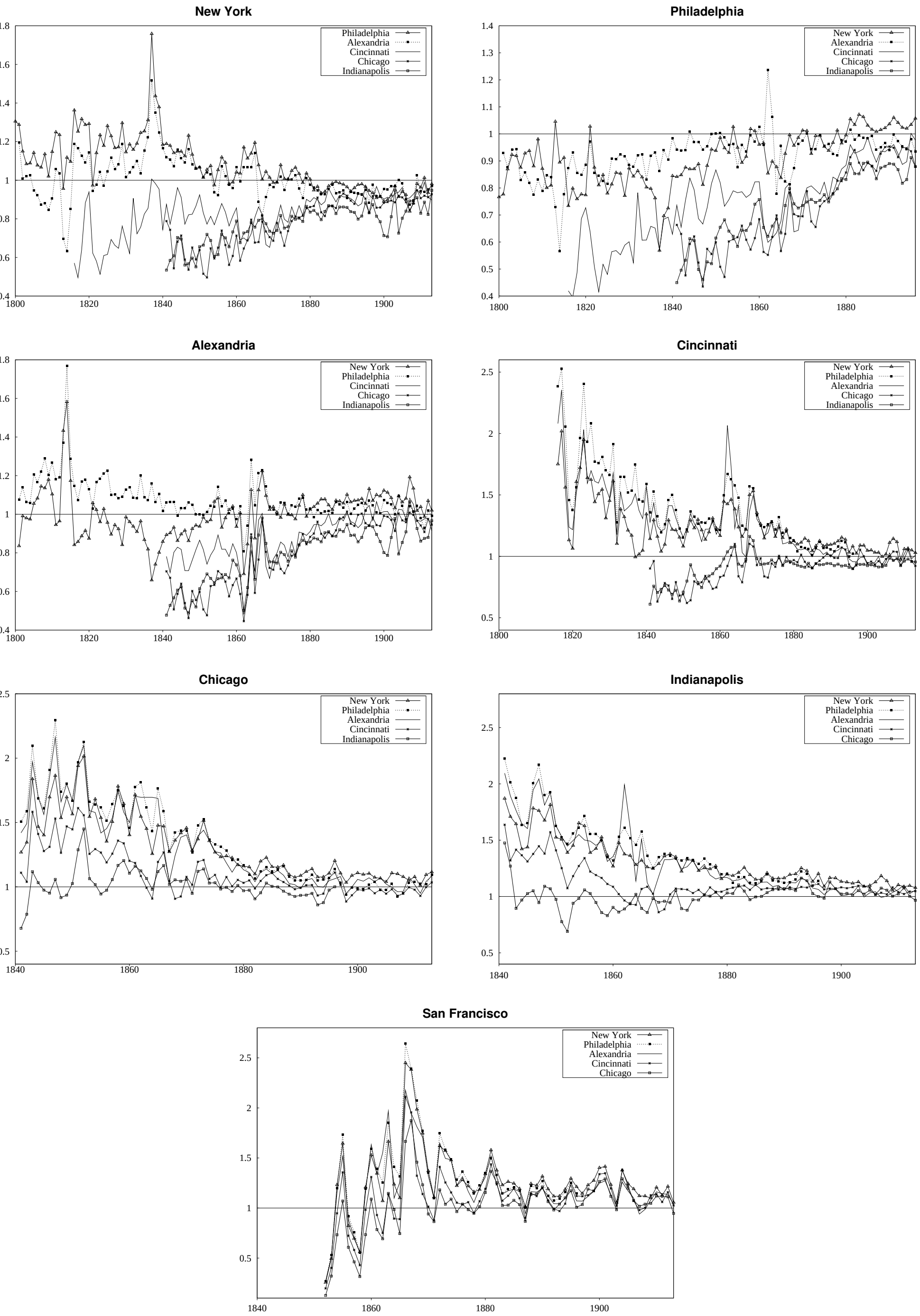

Figure 2: Relative Prices of Wheat in 19th Century USA

Appendix in Table 6.[6]

The model identified and estimated in each case is relatively simple: (i) an AR(1) process for the stochastic part; (ii) a mean, μ; and (iii) a gradual and monotone convergence

[6] All the estimated parameters are statistically different from zero, including the steady-state gains, g, and the convergence operator is stable in most cases. Q statistics by Ljung and Box (1978) show no sign of poor fit, except in a few cases, where an AR(2) operator might fit better. For simplicity, only AR(1) models are shown. The conclusions do not change significantly if a second-order representation is used. The initial specification for the stochastic part is according to the correlogram, AIC (Akaike, 1974) and HQ (Hannan and Quinn, 1979), which agree on the same initial specifications.

Table 1: Estimated Univariate Models of Wheat Prices in Log Differences[1]

Sample	Variable (Mnemonics)	AR(2) $\hat{\phi}_1$ (s.e.)	$\hat{\phi}_2$ (s.e.)	MA(1) $\hat{\theta}$ (s.e.)	Resid. Std.Dev. (%)	ACF[2] $Q_{(9)}$	SF[3] $H_0 : \phi = 1$	GLR[4] $H_0 : \theta = 1$
Univariate Models for Nominal Wheat Prices in Ag./liter of grain								
1800-1913	New York	0.63	-0.34	0.62	15.7	7.1	1.28	6.85
	(NY)	(0.14)	0.09	0.13				
1841-1913	Chicago	0.69	-0.31	0.70	18.0	3.3	0.6	2.4
	(CH)	0.18	0.12	0.16				
1800-1896	Philadelhia	0.78	-0.38	0.67	17.0	6.3	0.7	5.0
	(P)	0.15	0.10	0.14				
1801-1913	Alexandria	0.71	-0.44	0.64	16.1	10.9	0.7	12.0
	(A)	0.12	0.09	0.11				
1816-1913	Cincinatti	0.72	-0.26	0.78	17.5	3.5	0.8	3.5
	(CI)	0.15	0.11	0.13				
1841-1913	Indianapolis	0.62	-0.23	0.62	16.1	7.1	0.1	2.4
	(IN)	0.22	0.12	0.21				
1852-1913	San Francisco	0.49	-0.38	0.58	18.5	17.0	0.0	10.1
	(SF)	0.18	0.13	0.15				

Notes: (1) Eighteenth and Nineteenth Centuries yearly prices in gr.Ag./liter. (2) Q is the Ljung and Box (1978) statistic for the autocorrelation function (ACF). H_0 is no autocorrelation in the first nine lags. (3) SF: Shin and Fuller (1998) statistic tests if an AR(1) operator is nonstationary against an alternative ARIMA(3,0,1) model. (4) GLR: Generalized Likelihood Ratio (GLR) test of Davis, Chen and Duismuir (1995) for the null hypothesis of noninvertibility of an MA(1) operator.
*(**)Rejects the null hypothesis at the 10% (5%) level.

path, $\omega_0/(1 - \delta_1 B)$, for the deterministic component. The estimated parameters, with their standard deviations and some diagnostic tools, are also presented. The estimation results for the Alexandria/New York prices are not included in Table 2 as it was not possible to estimate a stable convergence path in this case. The probable reason is that this convergence process was too slow, such that the appropriate representation is very close to a straight line with a positive ramp.

In all the analyses reported in Tables 2 and 6, we fix t^* optimally as the year in which the convergence could have started. We seek t^* such that this value maximizes the log-likelihood function from the beginning of the sample, in each case, until the end of the sample, minus 25 observations. As is reported in Table 3, there is a different optimal starting date for the convergence path in each case. The range of starting dates begins in 1836, with the pair Alexandria and Philadelphia, which are located very close to each other. The prices in New York and Philadelphia also start the transition to parity early, relative to other cases, in 1849. For the remaining cities, the convergence process to parity starts at around the American Civil War. A special case is San Francisco, given

Table 2: Models of the Relative Prices Including a Convergence Path

Sample	Variable	AR(1)	Convergence Paramenters				Mean	Resid.	ACF[(1)]	SF[(2)]
	(Mnemonics)	$\hat{\phi}_1$	$\hat{\omega}_0$	$\hat{\delta}_1$	$\hat{l}$	$\hat{g}$	$\hat{\mu}$	Std.Dev.	$Q_{(9)}$	$H_0 : \phi = 1$
		(s.e.)	(s.e.)	(s.e.)	(s.e.)	(s.e.)	(s.e.)	(%)		
Panel A: Relative prices with New York as Numeraire										
1841-1914	Chicago	0.20	0.026	0.94	16.6	0.46	-0.45	7.7	8.9	37.5**
	(CH/NY)	(0.11)	(0.007)	(0.03)	(8.6)	(0.11)	(0.01)			
1800-1896	Philadelphia	0.48	-0.076	0.52	17.2	-0.16	0.17	6.3	7.1	15.2**
	(P/NY)	(0.09)	(0.025)	(0.14)	(12.8)	(0.03)	(0.01)			
1801-1913	Alexandria	-		-		-		-	-	-
	(A/NY)									
1816-1914	Cincinatti	0.44	0.016	0.91	10.5	0.18	-0.21	6.1	13.2	21.2**
	(CI/NY)	(0.10)	(0.011)	(0.08)	(11.0)	(0.06)	(0.01)			
	Indianapolis	0.45	0.023	0.94	17.2	0.42	-0.46	5.6	8.4	15.2
	(IN/NY)	(0.11)	(0.009)	(0.03)	(12.8)	(0.13)	(0.02)			
1852-1914	San Francisco	0.46	-0.84	0.47	0.92	-1.6	1.4	20.0	7.5	16.6**
	(SF/NY)	(0.10)	(0.21)	(0.10)	(0.40)	(0.2)	(0.2)			
Panel B: Relative prices with Chicago as Numeraire										
1800-1896	Philadelphia	0.25	-0.039	0.92	12.6	-0.54	0.53	8.6	17.2	23.2**
	(P/CH)	0.13)	(0.010)	(0.03)	(5.8)	(0.10)	(0.02)			
1801-1913	Alexandria	0.60	-0.080	0.81	4.4	-0.43	0.45	8.6	6.9	13.8**
	(A/CH)	(0.09)	(0.039)	(0.09)	(2.9)	(0.06)	(0.04)			
1816-1914	Cincinatti	0.28	-0.10	0.59	1.5	-0.25	0.29	8.0	9.4	29.0**
	(CI/CH)	(0.11)	(0.04)	(0.19)	(1.1)	(0.03)	(0.02)			
	Indianapolis	0.39	-0.021	0.80	4.2	-0.11	0.067	6.6	17.3	23.0**
	(IN/CH)	(0.11)	(0.017)	(0.16)	(4.5)	(0.03)	(0.019)			
1852-1914	San Francisco	0.51	-0.97	0.53	1.2	-2.1	2.1	21.2	7.6	14.3**
	(SF/CH)	(0.11)	(0.26)	(0.11)	(0.6)	(0.2)	(0.2)			

Notes: (1) Q is the Ljung and Box (1978) statistic for the autocorrelation function (ACF). H_0 is no autocorrelation in the first nine lags. (2) SF: Shin and Fuller (1998) statistic tests if an AR(1) operator is nonstationary.
*(**)Rejects the null hypothesis at the 10% (5%) level.

that the beginning of the sample started in 1852, so that the convergence started at the beginning of the sample. However, the starting gap in these cases was significant at the beginning of the respectives samples.

5.2 Testing asymptotic price convergence in mean

Perfect homogeneity of wheat across markets is assumed as it simplifies the analysis, improves the performance of the unit root tests, and seems a realistic assumption. Tables 2 and 6 (last column) show the results of the unit root tests, using the SF test. In all cases, non-stationarity is clearly rejected when a transition term is introduced from t^*. However, the same tests generally do not reject non-stationarity at any standard level

Table 3: Optimal Starting Time t^* for the Convergence Path

City	New York	Chicago	Philadelphia	Alexandria	Cincinnati	Indianapolis	San Francisco
New York	X	1866	1849	–	1874	1856	1853
Chicago	1866	X	1866	1874	1860	1863	1853
Philadelphia	1849	1866	X	1836	1875	1866	1853
Alexandria	–	1874	1836	X	1874	1870	1853
Cincinnati	1874	1860	1875	1874	X	1856	1853
Indianapolis	1856	1863	1866	1870	1856	X	1853
San Francisco	1853	1853	1853	1853	1853	1853	X

when there is no convergence in the model. This reveals strong evidence of asymptotic convergence in mean as catching-up (see García-Hiernaux and Guerrero, 2011).

These empirical results are also self evident in the graphs of nominal prices (Figure 1), and is confirmed in the relative price graphs (Figure 2). As the relative prices are transition-stationary, which fulfills the first requirement for asymptotic mean convergence, we performed the formal test for APCM using the models presented in Tables 2 and 6.

The results of the tests for APCM (H_0: $\tau = g_{ij} + \mu_{ij} = 0$) are presented in Table 4. Both the student-t and LR tests strongly confirm that the wheat price series converge in APCM for one half of the possible pairs in our data set. In the pairs with New York as the numeraire, prices converge in this APCM strong sense, except for prices in San Francisco. One reason could be that these cities are on opposite sides of the country, and the Panama Canal was only available in 1914. Surprisingly, relative prices between San Francisco and Chicago have converged, in the APCM sense, at the end of the sample. The rail connection between these two cities might have made this possible. However, the Chicago, Cincinnati and Indiana prices had not converged until towards the end of the sample, in the APCM sense, even though the price gaps are very small.

5.3 Testing asymptotic price convergence in variance

In order to examine APCD when $p = 2$, we use the BP test, as explained in Section 4.2. The residuals series are obtained from the models presented in Tables 2 and 6.

The results of the BP test are reported in Table 5.[7] The statistics for the joint null hypothesis that the residuals are homoscedastic are rejected in most cases at the 5% level. Therefore, asymptotic price convergence in variance cannot be rejected. The only exception is the case for relative prices between Alexandria and Philadelphia. In this case, the relative price dispersion, corrected for autocorrelation, is around 4%, and is one of the lowest in relative prices. This means that the level of market integration between

[7] The parameter estimates of this model are available from the authors upon request.

Table 4: Testing Asymptotic Price Convergence in Mean by Pairs

Panel A: Long Run Gap Estimation Results and t-student test for convergence in mean[1]

City	New York	Chicago	Philadelphia	Alexandria	Cincinnati	Indianapolis	San Francisco
New York	X	0.01	0.015	–	0.05	-0.05	-0.24**
Chicago	0.01	X	-0.01	0.017	0.034**	-0.043**	-0.043
Philadelphia	0.015	-0.01	X	-0.042**	-0.045	-0.21	-0.043
Alexandria	-	0.017	-0.042**	X	0.028	-0.034	-0.20**
Cincinnati	0.05	0.034**	-0.045	0.028	X	-0.047**	-0.097**
Indiana	-0.05	-0.043**	-0.21	-0.034	-0.047**	X	-0.061
San Francisco	-0.24**	-0.043	-0.043	-0.20**	-0.097**	-0.061**	X

Panel B: LR test for convergence in mean[2]

City	New York	Chicago	Philadelphia	Alexandria	Cincinnati	Indianapolis	San Francisco
New York	X	0.02	0.27	–	0.18	0.09	10.8**
Chicago	0.02	X	0.01	0.12	3.9**	4.1**	0.50
Philadelphia	0.27	0.01	X	4.4**	0.45	4.8**	7.1**
Alexandria	-	0.12	4.4**	X	0.40	0.01	8.1**
Cincinnati	0.18	3.9**	0.45	0.40	X	4.4**	3.8*
Indiana	0.09	4.1**	4.8**	0.01	4.4**	X	1.33
San Francisco	10.8**	0.50	7.1**	8.1**	3.8*	1.33	X

Notes: (1) The Tau test is a student-t test of Asymptotic Price Convergence in Mean, where $H_0 : \tau_{ij} = g_{ij} + \mu_{ij} = 0$ is that the long run gap between nominal prices is zero. (2) Likelihood Ratio (LR) test of Asymptotic Price Convergence in Mean, where H_0 is the same as for the Tau test.
*(**)Rejects the null hypothesis at the 10% (5%) level.

Alexandria and Pennsylvania was already probably very high by this time.

For the rest of the cases, we draw the evolution of the residual standard deviations of the natural log of relative prices in order to see whether the heteroscedasticity detected is generated by a decreasing variance. The residual standard deviations are calculated using rolling windows with a span of $t = 35$, and are shown in Figure 3. The figure clearly suggests that the standard deviation decreased over time, with both growing market efficiency and integration. However, it is not so clear that the standard deviations tend to zero, which does not support APCD, at least, in this sample.

These results show: (i) there was a unique wheat spread market in 19th Century USA; and (ii) the strength of the market integration process in the distribution sense in 19th Century USA. One-half of the pairs examined come to parity at the end of the 19th Century. In the rest of the cases the gap is very small, although statistically different from zero.

Table 5: Testing Asymptotic Price Convergence in Variance by Pairs

Breusch-Pagan Modified Statistics[1]

City	New York	Chicago	Philadelphia	Alexandria	Cincinnati	Indianapolis	San Francisco
New York	X	14.2**	4.3**	–	10.2**	12.0**	12.3**
Chicago	14.2**	X	14.0**	6.6**	9.6**	5.45**	11.6**
Philadelphia	4.3**	14.0**	X	0.2	4.6**	5.7**	8.3**
Alexandria	-	6.6**	0.2	X	3.4*	6.3**	16.5**
Cincinnati	10.2**	9.6**	4.6**	3.4*	X	8.6**	11.2**
Indiana	12.0**	5.45**	5.7**	6.3**	8.6**	X	12.2**
San Francisco	12.3**	11.6**	8.3**	16.5**	11.2**	12.2**	X

Notes: (1) BP modified test is a Likelihood Ratio test of Asymptotic Price Convergence in Variance, where H_0 is homoscedasticity. If the null hypothesis is rejected, there is conditional heteroscedasticity, with variance decreasing with time starting at t^*.
*(**)Rejects the null hypothesis at the 10% (5%) level.

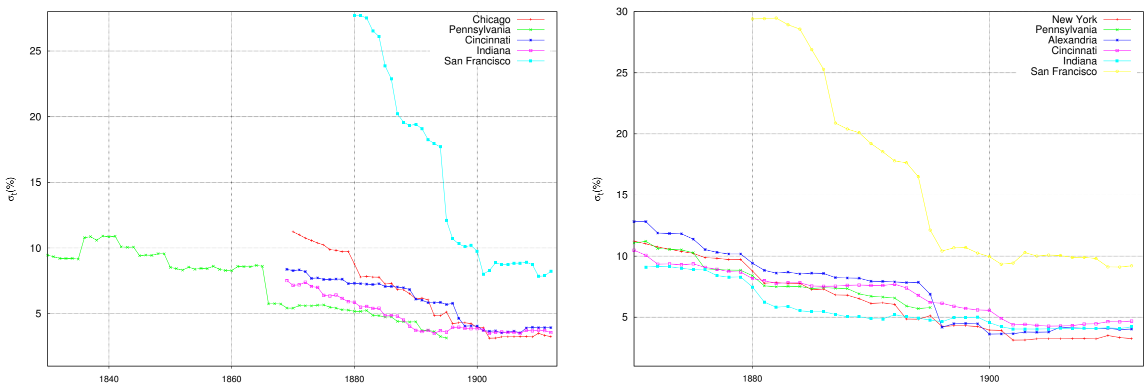

Figure 3: Market Integration in 19th Century USA
The series are the residual standard deviations of the natural log of relative prices calculated using rolling windows with a span of $t = 35$.

6 Concluding Remarks

This paper presented a general framework for the analysis of price convergence in distribution according to the econometric tradition, including assumptions, definitions, model building, econometric representations, and hypothesis testing. The approach is based on cointegration and conditional variance analyses, but is flexible and, consequently, is compatible with either steady state or catching-up convergence. Furthermore, it enables one to distinguish between asymptotic convergence, as steady state or catching-up, and describes completely a convergence process by representing its transition path and measuring its speed.

The definitions and methods presented here are useful not only for applied microeconomics and economic history. Macroeconomic aggregates, such as monetary markets or price levels, can be analyzed in the same way regarding economic integration. The methods used here could also be helpful to understand related macro-market and economic integration problems, and also can be extended for the study of the convergence clubs, in the sense of Quah (1997).

The empirical analysis reported here shows how to use the proposed methodology, leading to an interesting conclusion, especially for economic historians, namely that the inland grains market integration triggered the price convergence process as catching-up during the second half of 19th Century USA.

Finally, at least two main subjects related to this paper could be the object of future research. First, the methodology is flexible enough for different data frequencies and so has great potential not only for prices, but also for output, productivity, and macro and finance variables. Second, a more efficient procedure to identify endogenously the time when the convergence process begins would be very helpful for users who do not have extra-sample information. The latter issue is closely related to the existing literature on unit roots with shifts at unknown dates.

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Table 6: Models of Relative Prices Including Convergence Path

Sample	Variable	AR(1)	Convergence Paramenters				Mean	Resid.	ACF[(4)]	SF[(5)]
	(Mnemonics)	$\hat{\phi}_1$	$\hat{\omega}_0$	$\hat{\delta}_1$	$\hat{l}$	$\hat{g}$	$\hat{\mu}$	Std.Dev.	$Q_{(9)}$	$H_0 : \phi = 1$
		(s.e.)	(s.e.)	(s.e.)	(s.e.)	(s.e.)	(s.e.)	(%)		
Panel C: Relative prices with Philadelphia as Numeraire										
1801-1913	Alexandria	0.39	0.024	0.73	2.8	0.09	-0.13	4.3	5.0	32.2**
	(A/P)	(0.09)	(0.015)	(0.16)	(2.3)	(0.02)	(0.01)			
1816-1914	Cincinatti	0.51	0.046	0.81	4.4	0.25	-0.30	6.9	23.8	13.1**
	(CI/P)		(0.11)	(0.024)	(0.12)	(3.5)	(0.06)	(0.02)		
	Indianapolis	0.83	0.19	0.41	0.70	0.25	-0.52	6.2	16.2	1.5*
	(IN/P)	(0.09)	(0.06)	(0.25)	(0.74)	(0.06)	(0.02)			
1852-1914	San Francisco	0.52	-1.7	0.48	0.91	-3.3	3.1	22.4	4.9	9.6**
	(SF/P)	(0.12)	(0.9)	(0.13)	(0.46)	(1.0)	(1.0)			
Panel D: Relative prices with Alexandria as Numeraire										
1816-1914	Cincinatti	0.38	0.030	0.89	8.1	0.28	-0.25	8.6	13.6	24.6**
	(CI/A)	(0.10)	(0.016)	(0.07)	(5.8)	(0.06)	(0.02)			
	Indianapolis	0.67	0.043	0.89	8.1	0.41	-0.44	8.8	7.0	9.0**
	(IN/A)	(0.09)	(0.028)	(0.09)	(5.8)	(0.10)	(0.05)			
1852-1914	San Francisco	0.51	-0.80	0.47	0.90	-1.5	1.3	18.9	10.2	14.5**
	(SF/A)	(0.10)	(0.19)	(0.11)	(0.04)	(0.2)	(0.2)			
Panel E: Relative prices with Cincinatti as Numeraire										
1816-1914	Indianapolis	0.57	0.096	0.64	1.8	0.27	-0.31	5.4	13.2	10.8**
	(IN/CI)	(0.10)	(0.033)	(0.12)	(1.0)	(0.03)	(0.03)			
1852-1914	San Francisco	0.40	-0.84	0.50	1.0	-1.7	1.6	20.8	13.4	19.4**
	(SF/CI)	(0.12)	(0.23)	(0.12)	(0.5)	(0.2)	(0.2)			
Panel F: Relative prices with Indianapolis as Numeraire										
1852-1914	San Francisco	0.40	-0.54	0.68	2.1	-1.7	1.6	20.5	12.5	10.8
	(SF/P)	(0.12)	(0.21)	(0.10)	(1.0)	(0.2)	(0.2)			

Notes: (1) Q is the Ljung and Box (1978) statistic for the autocorrelation function (ACF). H_0 is no autocorrelation in the first nine lags. (2) SF: Shin and Fuller (1998) statistic tests if an AR(1) operator is nonstationary.
*(**)Rejects the null hypothesis at the 10% (5%) level.