On the Size of the Chiral Condensate Generalized Chiral Perturbation Theory and the DIRAC experiment*

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Abstract

In the near future, the DIRAC collaboration will measure $\pi\pi$ scattering lengths with great precision. Those measurements are likely to shed some light on the problem of the size of the chiral $\langle 0|\bar{q}q|0\rangle$ condensate. Although it is usually assumed to be as a large as $\sim(-225\,\mathrm{MeV})^3$, in the last years a more general approach, has been developed to accommodate either a large or a small alternative $\sim(-100\,\mathrm{MeV})^3$. Such a low value would also modify the standard temperature estimate at which the chiral phase transition occurs. In this work we briefly review the basic theoretical ideas related to this issue as well as the experiment that could help to establish any of the two scenarios.

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1 Introduction

In spite of the common belief that QCD is the appropriate theory describing strong interactions, it is very few what this theory can tell us about low energy hadron physics. In this regime the main guiding fact is the chiral symmetry $SU(3)_L \times SU(3)_R$ of the QCD Lagrangian in the massless quark limit ($m_u = m_d = m_s = 0$). For still not quite well understood dynamical reasons, this global symmetry is spontaneously broken to the diagonal group $SU(3)_{L+R}$. Thus the eight lowest mass $J^P = 0^-$ mesons π, K and η emerge as the corresponding Goldstone bosons. Indeed, their relatively low masses can be understood from the fact that quarks do actually have some small masses which, even though they modify the previous picture, still can be treated as small perturbations. Using chiral symmetry and these small quark masses it was possible in the early sixties to derive many important relations between hadron masses [1]. For example, in the exact isospin limit ($m_u = m_d$) it was proposed the celebrated Gell-Mann-Okubo mass formula

$$4M_K^2 - M_\pi^2 - 3M_\eta^2 \simeq 0 \tag{1}$$

Associated with the spontaneous chiral symmetry breaking is the so-called quark condensate $\langle 0|\bar{q}q|0\rangle$. A non-vanishing value of this magnitude breaks the $SU(3)_L \times SU(3)_R$ chiral symmetry, but not the diagonal $SU(3)_{L+R}$ group. Thus it is a good candidate to play the role of the order parameter associated with the chiral symmetry broken phase. At higher temperatures, where the chiral symmetry is expected to be restored, the thermal condensate, $\langle 0|\bar{q}q|0\rangle_T$, should vanish above some critical temperature T_c. This chiral phase transition, together with the deconfinement transition, has raised a considerable interest, both theoretically and experimentally, in the heavy ion physics community. In this paper we will review recent works that have proposed an alternative to the standard scenario we have just described. Indeed, we will discuss about the precise value of the quark condensate, its relation with other low energy hadron parameters and the possibly of measuring it in future experiments. The plan of the paper is as follows. In Sec.2 we make some introductory remarks on the role of the condensate and spontaneous chiral symmetry breaking. In Sec.3 we review the effective Lagrangian formalism. In Sec.4 we introduce the explicit symmetry breaking mass terms and compare Chiral Perturbation Theory with the so called Generalized Chiral Perturbation Theory. In Sec.5 we review the present experimental evidence and the predictions in the different scenarios. In Sec.6 we deal with the chiral condensate evolution with the temperature, which could change if the standard picture has to be modified. In Sec.7 we describe briefly the DIRAC experiment where pionium ground states will be produced in order to get very precise measurements of the pion scattering lengths that could provide decisive information about the quark condensate. Finally in Sec.8 we summarize.

2 The chiral condensate

As it was commented above, in the standard scenario it is assumed that the quark condensate is the order parameter of the chiral phase transition [1]. In the chiral limit ($m_u = m_d = m_s = 0$) it is customary to define the constant

$$B_0 \equiv -\frac{\langle 0|\bar{u}u|0\rangle}{F^2} = -\frac{\langle 0|\bar{d}d|0\rangle}{F^2} = -\frac{\langle 0|\bar{s}s|0\rangle}{F^2} \tag{2}$$

where $F \simeq 90\,\mathrm{MeV}$ is the pion decay constant. Thus the standard wisdom can be simply summarized by the *assumption* that $2\hat{m}B_0$ (with $2\hat{m} = m_u + m_d$) is roughly the pion mass squared M_π^2, so that $B_0 \simeq 1.3\,\mathrm{GeV}$. The resulting condensate is $\langle 0|\bar{q}q|0\rangle \simeq (-225\,\mathrm{MeV})^3$. In this scenario the critical temperature T_c is expected to be around $200\,\mathrm{MeV}$ for two flavors [2, 3]. This large condensate assumption leads naturally to the Gell-Mann-Okubo formula. The effect of the quark masses can be included systematically in this scheme giving rise to the standard Chiral Perturbation Theory (ChPT) [4, 5].

However, in spite of the above argument in favor of the standard scenario, the Gell-Mann-Okubo formula can hold quite independently of the relation between $2\hat{m}B_0$ and M_π^2. In fact, an alternative completely consistent formalism where $2\hat{m}B_0$ is considerably lower than M_π^2 has been recently proposed [6]. In the new scheme the quark condensate could be much smaller than in the standard case, typically around $100\,\mathrm{MeV}$. The corresponding perturbative treatment of the quark mass effects is called Generalized Chiral Perturbation Theory (GChPT).

There is a very illustrative analogy of the above two alternatives with spin systems [7]. Whenever spins are strongly correlated with their nearest neighbors we have an ordered phase at low temperatures and a disordered one above some critical temperature T_c. In a system with just one kind of spins we have two possible cases. On the one hand, if the exchange coupling constant J is positive, the interaction will favor parallel spins. Even in the absence of any external magnetic field, in the ordered phase the spins will be nearly parallel, thus yielding a macroscopic magnetization, which is therefore a good order parameter to distinguish between the ordered and the disordered phase. Such a system is nothing but a ferromagnet and T_c is called the Curie temperature. That would be analogous to the standard ChPT case, where the magnetization would play the role of the quark condensate and the quark masses would play the role of some external magnetic field. (Note however that the analogy cannot be carried too far since, among other things, the symmetry groups are different). On the other hand, when J is negative, and even if $T < T_c$ the spins are nearly antiparallel and no macroscopic magnetization is produced. Thus the magnetization is not a good order parameter to distinguish between the two phases, the system is antiferromagnetic and T_c is called the Néel temperature. That would be similar to the extreme case of GChPT where $B_0 = 0$. Note that general principles do not exclude the possibility that the quark condensate vanishes in the chiral limit. Another interesting possibility appears when different kinds of spins are considered. Then, even for $J < 0$ it is possible to generate a macroscopic magnetization in the antiparallel ordered state for $T < T_c$ and thus the magnetization is still a good order parameter. This kind of systems are called ferrimagnets (natural magnets are of this type). This case would be the most general to establish the analogy with QCD.

3 Effective Lagrangians

We have just seen that pions, kaons and etas can be identified with Goldstone bosons and thus we will parameterize them in an $SU(3)$ matrix as follows:

$$U = \exp(i\Phi/F) \quad ; \quad \Phi = \sqrt{2}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \tag{3}$$

Let us consider first the chiral limit. In this case the GB are massless and therefore they are the most relevant degrees of freedom at sufficiently low energies or momenta. The philosophy of Chiral Perturbation Theory [5] (see also ref.[8]) is to perform a low momentum expansion, or what it is the same, an expansion in the number of derivatives in the Lagrangian. Generically, low momenta means much smaller than the typical hadronic scale of around $\mathcal{O}(1\,\mathrm{GeV})$. Thus, the lowest order (denoted $\mathcal{O}(p^2)$) Lagrangian is

$$\mathcal{L}^{(2)}_{m_q=0} = \frac{F^2}{4}\mathrm{tr}\left(D_\mu U D^\mu U^\dagger\right) \tag{4}$$

Note that we have introduced a covariant derivative $D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu$, where r_μ and l_μ are right and left gauge fields. In order to include the weak and electromagnetic interactions of mesons, they can be substituted by the corresponding gauge bosons of the Standard Model. If we work out the tree level amplitude for, say, $\pi^+\pi^- \to \pi^0\pi^0$ scattering we obtain:

$$A(s,t,u) = \frac{s}{F^2} \qquad (m_q = 0) \tag{5}$$

From that expression it is possible to obtain all other $\pi\pi \to \pi\pi$ scattering processes using isospin and crossing symmetry. Similar expressions can be found for other processes involving other mesons, as, for instance, the $I = 3/2$ amplitude for elastic πK scattering:

$$T^{3/2}(s,t,u) = \frac{s}{2F^2} \qquad (m_q = 0) \tag{6}$$

There are two relevant observations common to the above two and other low order light meson-meson amplitudes: first, that at $s = 0$ the interaction vanishes due to the GB nature of these light mesons. The second is that there is only one parameter, the scale F in the Lagrangian, which means that its predictions are the same for any fundamental theory whose symmetry breaking pattern is the same as in the three flavor QCD chiral limit. As a consequence any other specific feature from QCD, apart from its spontaneous chiral symmetry breaking, shows up at higher orders. Indeed, at $\mathcal{O}(p^4)$ the most general Lagrangian satisfying the symmetry constraints is given by:

$$\begin{aligned} \mathcal{L}^{(4)}_{m_q=0} &= L_1\left(\mathrm{tr}(D^\mu U^\dagger D_\mu U)\right)^2 + L_2\,\mathrm{tr}(D_\mu U^\dagger D_\nu U)\mathrm{tr}(D^\mu U^\dagger D^\nu U) \\ &+ L_3\,\mathrm{tr}(D^\mu U^\dagger D_\mu U D^\nu U^\dagger D_\nu U) - i L_9\,\mathrm{tr}(F^R_{\mu\nu} D^\mu U D^\nu U^\dagger + F^L_{\mu\nu} D^\mu U^\dagger D^\nu U) \\ &+ L_{10}\,\mathrm{tr}(U^\dagger F^R_{\mu\nu} U F^{L\,\mu\nu}) + H_1\mathrm{tr}(F^R_{\mu\nu}F^{R\,\mu\nu} + F^L_{\mu\nu}F^{L\,\mu\nu}) \end{aligned} \tag{7}$$

where F^R and F^L are the usual strength tensors of the r_μ and l_μ gauge fields, respectively. With the two above Lagrangians it is possible to calculate amplitudes for many different processes. Then, comparing with data from just a few processes, we can extract the values of the L_i parameters, which can be used later to obtain predictions for other processes. These constants are not fixed just by the scale and do depend on the specific dynamics of QCD. However, their actual values cannot be computed directly from perturbative QCD, which is not valid at these low energies. Maybe the most relevant fact about these effective Lagrangians is that, even though they are not strictly renormalizable, it is still possible to calculate loops and obtain finite results. For instance, if we calculate diagrams with $\mathcal{L}^{(2)}$ and one loop, it will be possible to absorb all the infinities in renormalized L_i^r parameters. This process can be carried on to arbitrarily high orders, and it ensures that, up to a given order in momenta, we will always get a finite result.

4 Quark masses and Explicit Symmetry Breaking

Let us now turn on the quark masses. In so doing we are breaking *explicitly* the Chiral Symmetry, and, consequently, our GB will become massive pseudo-GB. Nevertheless, the masses of pions, kaons ant etas are still smaller than the typical hadronic scale. We can therefore treat them perturbatively and include mass terms in the Lagrangian. In addition, we will be assuming exact isospin symmetry ($m_u = m_d$). Since the meson mass matrix term in the Lagrangian should vanish when so does the quark mass matrix $\mathcal{M} = diag(\hat{m}, \hat{m}, m_s)$, we can write, for instance:

$$M_\pi^2 = 2B_0\hat{m} + 4A\hat{m}^2 + ... \tag{8}$$

where $B_0, A, ...$ coefficients are, in principle, to be determined phenomenologically. There is, however, an straightforward physical interpretation for B_0. In the chiral limit, it is, up to a normalization factor, the chiral condensate defined in eq.(2) (since it yields a Lagrangian term which is proportional to the mass). That is:

$$\lim_{m_q \to 0} \langle 0|\bar{q}q|0\rangle = -2F^2 B_0 \tag{9}$$

4.0.1 Standard Chiral Perturbation Theory

As discussed above the standard scenario is to assume that B_0 is large, $\mathcal{O}(1\,\mathrm{GeV})$, and thus it dominates the expansion in eq.(8). Since $M_{meson} = \mathcal{O}(p^2)$ we have that $m_{quark} \simeq \mathcal{O}(p^2)$, which is the standard chiral counting rule. If we follow that counting, there is just one term that we can add to our previous $\mathcal{O}(p^2)$ chiral Lagrangian, which now becomes:

$$\mathcal{L}^{(2)} = \frac{F^2}{4}\left\{\mathrm{tr}(D_\mu U D^\mu U^\dagger) + 2B_0\mathrm{tr}(\mathcal{M}(U^\dagger + U))\right\} \tag{10}$$

Therefore, the meson masses are given by:

$$\begin{aligned} M_\pi^2 &= 2\hat{m}B_0 \\ M_K^2 &= (\hat{m} + m_s)B_0 \\ M_\eta^2 &= \frac{2}{3}(\hat{m} + 2m_s)B_0 \end{aligned} \tag{11}$$

Just by cancelling the B_0 terms we get the Gell-Mann-Okubo mass relation,

$$4M_K^2 - M_\pi^2 - 3M_\eta^2 = 0 \tag{12}$$

The fact that within this formalism this formula is obtained at leading order, is an strong argument of plausibility for the initial large B_0 assumption. We can also write the so called Gell-Mann-Oakes-Renner formula in this notation as:

$$\frac{M_\pi^2}{2\hat{m}} - \frac{M_K^2}{\hat{m}+m_s} = \frac{3M_\eta^2}{2(\hat{m}+2m_s)} = -\frac{\langle 0|\bar{q}q|0\rangle}{2F^2} \tag{13}$$

In addition, eq.(11), implies the following ratio of quark masses:

$$r \equiv \frac{m_s}{\hat{m}} = 2\frac{M_K^2}{M_\pi^2} - 1 \simeq 26 \tag{14}$$

Indeed, the light quark masses are estimated using eq.(11), which are obtained under the assumption that there is a large chiral condensate. If the size of $\langle 0|\bar{q}q|0\rangle$ turns out *not* to be that of the standard scenario, then the current estimates of the masses of the three lightest quarks [23] would have to be changed. Concerning the low energy theorems, within this standard formalism, they are modified to:

$$\begin{aligned} A(s,t,u) &= \frac{s - M_\pi^2}{F^2} \\ T^{3/2}(s,t,u) &= \frac{s-(M_\pi^2+M_K^2)}{2F^2} \end{aligned} \tag{15}$$

Note, first, that now the meson elastic interaction does *not* vanish at $s=0$. Indeed, *all the interaction at threshold is due to the explicit chiral symmetry breaking*. Second, once we have assumed the large condensate scenario the $\mathcal{O}(p^2)$ predictions for the amplitudes are *fixed*, since we know F, M_π and M_K from experiment.

We can now again build the most general $\mathcal{O}(p^4)$ terms, following the standard counting, and add them to the $\mathcal{O}(p^4)$ Lagrangian obtained in the previous section. That is, [5]

$$\begin{aligned} \mathcal{L}^{(4)} &= \mathcal{L}^{(4)}_{m_q=0} + 2B_0L_4\,\mathrm{tr}(D^\mu U^\dagger D_\mu U)\mathrm{tr}(\mathcal{M}(U+U^\dagger)) \\ &\quad +2B_0L_5\,\mathrm{tr}(D^\mu U^\dagger D_\mu U\mathcal{M}(U+U^\dagger)) + 4B_0^2L_6\,\mathrm{tr}(\mathcal{M}(U+U^\dagger))^2 \\ &\quad +4B_0^2L_7\,\mathrm{tr}(\mathcal{M}(U-U^\dagger))^2 + 4B_0^2L_8\,\mathrm{tr}(\mathcal{M}U^\dagger\mathcal{M}U^\dagger + \mathcal{M}U\mathcal{M}U) \end{aligned} \tag{16}$$

This $\mathcal{O}(p^4)$ Lagrangian now yields corrections to the pseudo-GB masses which therefore modify the GMO formula to [5]:

$$4M_K^2 - M_\pi^2 - 3M_\eta^2 = \frac{6}{F^2}(M_\eta^2 - M_\pi^2)^2(L_5^r - 6L_8^r - 12L_7^r) - 2(4M_K^2\mu_K - M_\pi^2\mu_\pi - 3M_\eta^2\mu_\eta) \tag{17}$$

where $\mu_a = (M_a^2/32\pi F^2)\log(M_a^2/\mu^2)$. Unfortunately, the correction depends on L_8 whose value cannot be determined with any other independent experimental data. In that sense the Gell-Mann-Okubo relation prediction does not ensure the dominance of the condensate term in eq.(8).

4.0.2 Generalized Chiral Perturbation Theory

As we have just seen, there is still room for an alternative scenario. Namely, to have a small condensate which therefore implies a different counting scheme. In the extreme case, when $B_0 = 0$, we would have $m_q \sim \mathcal{O}(p)$, and this can be generalized to non-vanishing small condensates with $B_0 \sim \mathcal{O}(p)$. Hence, *although we can still build the very same terms, now they will count differently* and their relative importance at low energies will be modified. For instance, at $\mathcal{O}(p^2)$ we find [6]:

$$\begin{aligned}\tilde{\mathcal{L}}^{(2)} &= \frac{F^2}{4}\Big\{\mathrm{tr}(D_\mu U D^\mu U^\dagger) + 2B_0\mathrm{tr}(\mathcal{M}(U^\dagger + U)) \\ &+ A_0\,\mathrm{tr}(\mathcal{M}U^\dagger\mathcal{M}U^\dagger + \mathcal{M}U\mathcal{M}U) + Z_0^S\,\mathrm{tr}(\mathcal{M}(U+U^\dagger))^2 \\ &+ Z_0^P\,\mathrm{tr}(\mathcal{M}(U-U^\dagger))^2 + 2H_0\mathrm{tr}(\mathcal{M}^2)\Big\} \qquad (18)\end{aligned}$$

and it can be noticed that the A_0, Z_0^S, and Z_0^P terms have the same structure as those of L_8, L_6 and L_7, respectively. As a consequence of the new counting we now have, to $\mathcal{O}(p^2)$, the following masses:

$$\begin{aligned}M_\pi^2 &= 2\hat{m}B_0 + 4\hat{m}^2A_0 + 4\hat{m}(2\hat{m}+m_s)Z_0^S \\ M_K^2 &= (\hat{m}+m_s)B_0 + (\hat{m}+m_s)^2A_0 + 2(\hat{m}+m_s)(2\hat{m}+m_s)Z_0^S \\ M_\eta^2 &= \frac{2}{3}(\hat{m}+2m_s)B_0 + \frac{4}{3}(\hat{m}^2+2m_s^2)A_0 \\ &+ \frac{4}{3}(\hat{m}+2m_s)(2\hat{m}+m_s)Z_0^S + \frac{8}{3}(m_s-\hat{m})^2Z_0^P \qquad (19)\end{aligned}$$

The appearance of so many new constants at leading order implies much less predictive power. Indeed, it is not possible to obtain the Gell-Mann-Okubo formula at this order since now

$$4M_K^2 - M_\pi^2 - 3M_\eta^2 = -4(\hat{m}-m_s)^2(A_0 + 2Z_0^P) \qquad (20)$$

and $(A_0 + 2Z_0^P)$ is not expected to vanish [17]. But of course, that does not mean that there is any incompatibility with experimental data.

Note that, using eq.(19), the quark mass ratio now ranges in the interval

$$r_1 \equiv 2\frac{M_K}{M_\pi} - 1 \leq r \leq 2\frac{M_K^2}{M_\pi^2} - 1 \equiv r_2 \qquad (21)$$

That is: $6.3 \leq r \leq 26$, where the upper bound corresponds to standard ChPT with $A_0 = 0$. Once more, the low energy theorems are modified to [18]

$$\begin{aligned}A(s,t,u) &= \frac{\alpha_{\pi\pi}}{3F^2}M_\pi^2 + \frac{\beta_{\pi\pi}}{F^2}\left(s - \frac{4}{3}M_\pi^2\right) \qquad (22) \\ T^{3/2}(s,t,u) &= \frac{\alpha_{\pi K}}{3F^2}M_\pi M_K + \frac{\beta_{\pi K}}{4F^2}\left(t - \frac{2}{3}M_\pi^2 - \frac{2}{3}M_K^2\right) + \frac{\gamma_{\pi K}}{4F^2}(s-u) + \frac{(M_K - M_\pi)^2}{6F^2}\end{aligned}$$

where

$$\alpha_{\pi\pi} = 1 + 6\frac{r_2 - r}{r^2 - 1}\left(1 + 2\frac{Z_0^S}{A_0}\right) \quad ; \quad \beta_{\pi\pi} = 1 \tag{23}$$

$$\alpha_{\pi K} = 1 + 6\frac{r_2 - r}{(r_1 + 1)(r - 1)}\left(1 + 2\frac{Z_0^S}{A_0}\right) \quad ; \quad \beta_{\pi K} = \gamma_{\pi K} = 1 \tag{24}$$

In contrast with standard ChPT, we now deal with more parameters and therefore *it is not enough to know* F, M_π and M_K to determine the GChPT $\mathcal{O}(p^2)$ predictions for the amplitudes [17]. Up to the moment we have limited the analysis of Generalized Chiral Perturbation Theory to the $\mathcal{O}(p^4)$ level. Following closely the philosophy of standard ChPT, it is also possible to write down higher order Lagrangians, whose parameters can absorb the infinities that do appear when computing loop diagrams.

However, the counting scheme has been modified, and we have introduced quantities that count as $\mathcal{O}(p)$, therefore, we will now have terms in the Lagrangian whose order in p is an *odd number.* In general, the GChPT Lagrangian is built of terms like [6, 20]

$$\tilde{\mathcal{L}}^{(d)} = \sum_{k+l+n} B_0^n \mathcal{L}_{(k,l)}, \quad \text{with} \quad \mathcal{L}_{(k,l)} \sim \mathcal{O}(p^k m_q^l) \tag{25}$$

Indeed we have already given $\tilde{\mathcal{L}}^{(2)}$ in eq.(18), and it is easy to see that $\tilde{\mathcal{L}}^{(4)}_{(4,0)}$ is the very same $\mathcal{L}^{(4)}$ Lagrangian of standard ChPT, eq.(16). For the precise expressions of the rest of the terms we refer to [20], but let us notice that we have a Lagrangian at $\mathcal{O}(p^3)$, which is not present in the standard formalism. Only the $\tilde{\mathcal{L}}^{(4)}_{(2,2)}$ and$\tilde{\mathcal{L}}^{(4)}_{(0,4)}$ terms have more than 18 parameters and consequently, the predictive power of the Generalized approach is somewhat weaker than within the standard scenario. Nevertheless, it has been possible to perform calculations, for instance, for the $\pi\pi$ scattering lengths up to two loops, where only six different combinations of chiral parameters appear in the final expressions [20].

5 Present Evidence

Up to now we have just sketched the very basic consequences for the effective chiral formalism, of having either a large or small chiral condensate. Let us now review what support from data or other sources both alternatives have.

The generalized scenario was motivated by some deviations in the πN Goldberger-Treiman relation. For some values of the πN coupling constant $g_{\pi N}$, it even suggested a ratio $r \simeq 10$ [12], twice less than expected in the standard scenario, although the discrepancy was smaller for other values. More accurate data for the πN coupling constant before extracting any final conclusion. On the theoretical side, there are also some calculations which seem to prefer a lower value of the chiral condensate, like a variationally improved perturbation theory [13], and some relativistic many-body approaches [14]. Their low results, however, could be due to the approximations of their respective formalisms. One may think that the determinations of light quark mass ratios could shed some light on this issue. However, due to the ambiguity pointed out in [15] these ratios can only be obtained at lowest order using the standard ChPT counting with the large condensate assumption. In order to obtain the masses themselves

	LET	$ChPT(1l)$	$ChiPT(2l)$	$GChPT(2l)$	Exp
a_{00}	0.16	0.20	0.217	0.263	0.26 ± 0.05
b_{00}	0.18	0.26	0.275	0.25	0.25 ± 0.03
a_{20}	-0.045	-0.045	-0.0413	-0.027	-0.028 ± 0.012
b_{20}	-0.089	-0.085	-0.072	-0.079	-0.082 ± 0.008
a_{11}	0.030	0.037	0.040	0.037	0.038 ± 0.002
b_{11}	$-$	0.043	0.0079	0.054	$-$

Table 1 Scattering lengths a_{IJ} and slope parameters b_{IJ} for different $\pi\pi$ spin and isospin channels. We show the predictions coming from the Low Energy Theorems (LET), one-loop ChPT, two-loop ChPT and the two-loop GChPT *fit* to the scattering lengths, whose experimental values are given on the last column.

some additional dynamical information has to be combined with chiral symmetry. At least, it is possible to obtain a set of light quark masses consistent with ChPT [10]). The recent result $m_s(1\text{GeV}) = 235^{+35}_{-42}$ MeV reported by ALEPH [11], is consistent with both approaches, although the relatively high central value would be preferred by GChPT.

Concerning the standard scenario, we have seen that it was inspired by the Gell-Mann-Okubo and Gell-Mann-Oakes-Renner formula, which emerges very naturally at first order. In addition, the large condensate assumption receives a strong support from lattice calculations. Let us remember that within standard ChPT the pion mass *squared* is proportional to the quark masses. That is indeed found in several lattice calculations [9] which show a linear dependence of M_π^2 on $\hat{m}$. This results have to be interpreted carefully since they involve an extrapolation, a quenched approximation and possible problems with the breaking of chiral symmetry when implementing fermions on a lattice.[1] In addition, using the hypothesis of global QCD-hadron duality, a QCD sum rule technique yields values for the condensate which support the standard framework [16].

Thus there are no significant deviations in the data in conflict with either the standard or the generalized formalism, and the theoretical evidence always contains some approximation or extrapolation that obscures a conclusive statement. Fortunately it is very possible that the definitive answer will be obtained from experiment, by a precise measurement of the $\pi\pi$ scattering lengths. As we have seen in previous sections their non-vanishing values are exclusively due to the explicit symmetry breaking pattern. Whereas the large condensate scenario leads to a very sharp prediction of their values, the generalized framework allows for a wider range of values. Once again, the present experimental values of the scattering lengths are somewhat higher than expected within the standard formalism, but not in a strong disagreement.

In Table 1 we have listed the different values of the $\pi\pi$ scattering lengths, a_{IJ} and slope

[1] When we had finished this review we became aware of two new works related to this subject that give more support to the standard scenario. Another work on the lattice [33] seems to confirm previous results with better statistics. On ref.[34] it is argued that, in the chiral limit, the low condensate scenario is ruled out by QCD inequalities. However, they conclude that "one cannot rigurously rule out" the alternative scenario using QCD inequalities " at finite quark mass" although "the condensate cannot be too small in order the QCD inequalities to be satisfied at all distances".

parameters b_{IJ}. In the last column we show the present experimental values [22]. Note that the errors are rather large. The second column corresponds to the $\mathcal{O}(p^2)$ predictions in the standard formalism (Low Energy Theorems). Columns third and fourth correspond to the values predicted by ChPT at $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ [19]. Surprisingly, the $\mathcal{O}(p^4)$ corrections can be as large as 20% in some cases. Since the values of a_{00} and a_{20} are above these predictions, there was a hope that the two loop $\mathcal{O}(p^2)$ corrections would also turn out to be large and get a better result. Although they contribute in the appropriate direction, they are not enough to get the measured central value.

The fifth column has been obtained using GChPT at $\mathcal{O}(p^6)$ [20]. As we have already mentioned, in this case, the predictions are not unique, since they depend on the values of α and β. Thus, now it is possible to include the scattering lengths as input in the fits. As a matter of fact the values given on the table are obtained from a combined fit to low energy experimental K_{l4} decays *and* the phase shifts themselves, and they correspond to $\alpha = 2.16$ and $\beta = 1.074$ [20].

6 The Chiral Phase Transition

Before discussing future experiments, let us briefly comment how the critical temperature of the Chiral Phase Transition could be changed if the standard scenario has to be modified.

In Fig.1 we show the evolution of the chiral condensate at finite T [21]. These results have been obtained studying the thermodynamics of a pion gas using the virial expansion. In the second virial coefficient the $\pi\pi$ elastic scattering phase shifts are introduced to take into account the interactions of the pions. Thus, there are two effects that can modify the temperature dependence: first, obviously, the value of $\langle 0|\bar{q}q|0\rangle$ itself, which sets the starting point at $T = 0$. Second, the $\pi\pi$ interaction, which is different depending on whether we use the standard or generalized scenario. In Fig.1 we therefore see the interplay of both effects.

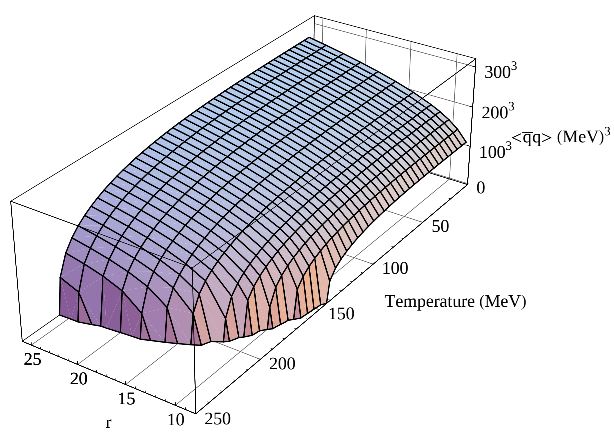

Figure 1: Quark condensate versus temperature and r.

Note that at $r \simeq 24$ the generalized scenario reproduces approximately the standard case.

The condensate is of the order of $(-250\,\text{MeV})^3$and the critical temperature seems to be around 200 MeV. When the higher order corrections are included in GChPT, the minimum value of r which is allowed is shifted from $r_1 \simeq 6.3$ to $r_1^* \simeq 8$ [20]. That is why Fig.1 starts at $r = 8$, which can therefore be interpreted as the extreme non-standard case. All these results have to be taken cautiously, since although the virial expansion could be a very good approximation at those energies, we are neglecting the presence of more massive states in the gas. An equivalent study within standard ChPT, showed that massive states will spoil the approximation at about 150 MeV. However, their effect will always *decrease* the critical temperature. In addition, the plot in Fig.1 corresponds to the central values of all the parameters in GChPT, and some errors bands should also be taken into account (For a more complete analysis we refer to [21]). Nevertheless, the figure can illustrate how the non standard scenario would predict that the condensate could melt at considerably lower temperatures than in the standard picture. In the extreme case, even below 150 MeV. That could have important consequences in the study of quark gluon plasma and heavy ion physics.

7 The DIRAC experiment

In order to bring some light on the above discussed issues it was proposed and now is under construction at CERN the DIRAC [24] experiment. By using the 24 GeV proton beam of the CERN Proton Synchrotron this experiment will produce, among other things, pionia, i.e. $\pi^+\pi^-$ atoms bounded by electromagnetic forces (first observed in [26]). Those relativistic atoms ($\gamma \simeq 10$) will either decay in $\pi^0\pi^0$ pairs, get excited or be ionized. In the last case characteristic charged pions called atomic pairs will emerge. The two pions in these pairs have low relative momentum in their center of mass system, small opening angle and nearly identical energies in the lab system.

The DIRAC experiment will consist on a double arm magnetic spectrometer which will measure the number of atomic pairs above the background of pion pairs produced in free states. For a given target, the ratio of the number of atomic pairs to the total number of pions depends on the pionium lifetime in a unique way. Indeed, a simple calculation shows that the lifetime $\tau_{n,0}$ of a pionium atom state with principal quantum number n and orbital angular momentum $l = 0$, can be written as

$$\frac{1}{\tau_{n,0}} = \frac{8\pi}{9}\sqrt{\frac{2\Delta m}{\mu}}\frac{(a_{00}-a_{20})^2}{M_\pi^2}|\psi_{n,0}(0)|^2 \qquad (26)$$

where $\Delta m = M - 2M_{\pi^0}$, M is the pionium mass, μ is its reduced mass and $\psi_{n,0}$ is its wave function in the coordinate representation. For Coulombian wave functions $\tau_{n,0} = \tau_{1,0} n^3$. For odd l the decay of the pionium in a pair of neutral pions is forbidden and for even $l > 0$ this annihilation is suppressed by the square of the fine structure constant. Therefore pionium annihilation takes place mainly from $l = 0$ states. That is why the total pionium lifetime happens to be proportional to $(a_{00} - a_{20})^2$.

The DIRAC experiment is expected to achieve a 10% precision in the lifetime measurement, whose actual value is of the order of $10^{-15}s$. Thus the above combination of scattering lengths will be determined to 5%. It is hoped that such resolution would be enough to distinguish between the standard and the generalized scenarios.

Further development of the DIRAC experiment could also lead to other interesting measurements. For instance, strong interactions yield a splitting of the energy levels ns and np. In particular it can be found [27] that

$$\Delta E_{2s-2p} = -\frac{2\pi}{3M_\pi}|\psi_{1,0}(0)|^2(2a_{00}+a_{20}) \simeq -0.3\,\mathrm{eV} \tag{27}$$

Therefore, a simultaneous measurement of the pionium lifetime and the strong Lamb shift would be able to separate a_{00} from a_{20} in a model independent way.

In addition, with some modifications, DIRAC may produce πK atoms and measure their lifetime, which is given by

$$\frac{1}{\tau_{n,0}} = \frac{8\pi}{9}\sqrt{\frac{2\Delta m}{\mu}}(b_{1/2}-b_{3/2})^2|\psi_{n,0}(0)|^2 \tag{28}$$

where $\Delta m = M - M_{\pi_0} - M_{k_0}$, M is the πK atom mass and μ is the corresponding reduced mass. Unfortunately, in this case the relevant πK scattering length combination $(b_{1/2}-b_{3/2})$ is *not* sensible to the size of the condensate [18] (note the different unit convention for the scattering lengths), although it could be relevant to measure chiral parameters like L_1, as suggested in [5]. Nevertheless, the present experimental data at threshold are very bad [29] and a better measurement would be interesting to see how well ChPT works at those higher energies. As a matter of fact, it could be used to test possible unitary extensions of the chiral approach like the Inverse Amplitude Method [28] which, by using ChPT complemented with dispersion relations, makes it possible to extend the energy range of applicability of plain ChPT. In particular it describes quite well the available πK scattering data and it would be interesting to compare its predictions with the more accurate data that could come from DIRAC.

What about KK-omnia? The production of such atoms has also been studied for DIRAC [31]. Indeed that reference is a complete study of the whole set of $\pi\pi$, πK and KK atoms. However, at such a high energies, it is necessary to unitarize the ChPT predictions, for instance by using the IAM method in a coupled channel formalism [32]. Nevertheless, the analogous treatment within GChPT is still to be done.

8 Conclusions

In this paper we have briefly reviewed the current status of the quark condensate from the point of view of Chiral Effective Lagrangians. We have paid attention both to the theoretical and the phenomenological role of this magnitude for large and small condensate scenarios including the relevance for future heavy ion experiments. We have seen that a value smaller than the standard one for this magnitude is not ruled out by the present experimental evidence. In particular we have reviewed how the standard Chiral Perturbation Theory can be generalized in order to include the possibility of a small condensate giving rise to the so called Generalized Chiral perturbation Theory. That scenario could modify the standard estimates of the critical temperature at which the chiral phase transition takes place. Finally we have commented on the DIRAC experiment which will provide important data in order to clarify the issue of the precise value of the quark condensate and other related with the unitarization of the ChPT amplitudes.

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