Applicability constraints of the equivalence theorem

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In this work we study the applicability of the equivalence theorem, either for unitary models or within an effective Lagrangian approach. There are two types of limitations: the existence of a validity energy window and the use of the lowest order in the electroweak constants. For the first kind, we consider some methods, based on dispersion theory or the large N limit, that allow us to extend the applicability. For the second, we obtain numerical estimates of the effect of neglecting higher orders in the perturbative expansion. [S0556-2821(97)04421-4]

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I. INTRODUCTION

Probably the main challenge for high-energy physics in the next decades will be to establish the nature of the symmetry breaking sector (SBS) of the standard model (SM). The SBS can either be weakly or strongly interacting. In the first case, it is expected that one or more Higgs bosons will be found at energies below 1 TeV. In contrast, if the SBS is strongly interacting, new states are expected above that energy, mostly in form of resonances.

In this work we will only deal with an strongly interacting SBS, whose experimental signature will be a production enhancement of longitudinally polarized gauge bosons (V_L), at energies higher than 1 TeV [1,2]. The link between V_L's and the SBS is the so-called equivalence theorem (ET) [2–7], which relates amplitudes containing V_L's with those amplitudes where every V_L is replaced by its associated Goldstone boson (GB). Indeed, the theorem is used in two ways: to connect the hidden SBS with physical observables but also to simplify calculations, since GB are scalars, and hence much easier to handle. It is therefore quite usual to evaluate the GB amplitudes and then translate them using the ET.

Based only on the SBS group structure, it is possible to derive some identities known as low-energy theorems (LET's) [8], which predict the behavior of GB's at very low energies, irrespective of the precise mechanism that yields the symmetry breaking. During the last few years, a very powerful formalism [9] has been developed in order to deal with the strongly interacting SBS in a model-independent way. Such a description is inspired in chiral perturbation theory (ChPT) [10], which has been remarkably succesful in describing low-energy pion physics. The idea is to build an effective Lagrangian as a derivative expansion, whose first order term (with two derivatives) reproduces the LET. The next order includes operators up to dimension 4 [11] and encodes the details of the undelying SBS in a set of parameters. Just by changing the values of these parameters, we can differentiate alternative SBS models [9,12] (such as a heavy-Higgs SM [13], or a QCD-like model [14]). It is expected that some of them will be measured at future accelerators such as the CERN Large Hadron Collider (LHC) or Next Linear Collider (NLC) [15].

However, the fact that chiral amplitudes are obtained as an expansion in powers of $p/(4\pi v)$, where $v \simeq 250$ GeV and p is the external momenta, spoils the usual proof of the ET [16]. Only very recently has a new version of the ET for effective Lagrangians been derived [17–18]. Nevertheless, the lack of unitarity of chiral amplitudes apparently imposes severe constraints on the validity of the theorem. The purpose of this work is to study in detail such constraints, how they affect the usefulness of the ET approximation and how they could be avoided.

The plan of the paper is as follows. In Sec. II we state the ET with a brief sketch of its derivation. Then, in Sec. III, we identify the approximations performed to obtain the theorem. In Sec. IV we study the case of V_L scattering and, in Sec. V, we analyze some unitarization procedures that extend the ET applicability range. In Sec. VI we present some brief conclusions.

II. THE EQUIVALENCE THEOREM

As it is well known, the GB and the V_L are related through a gauge transformation. Indeed, the GB, denoted as ω^a, disappear from the Lagrangian if we use a unitary gauge. However, most of the calculations are performed in the so-called R_ξ or renormalizable gauges, where the gauge-fixing term has the following generic form:

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2}(\partial^\mu V_\mu^a + \xi M \omega^a)^2, \tag{1}$$

where M is the mass of V^a. The gauge-fixing term is nothing

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THE PROOF OF THE EQUIVALENCE THEOREM

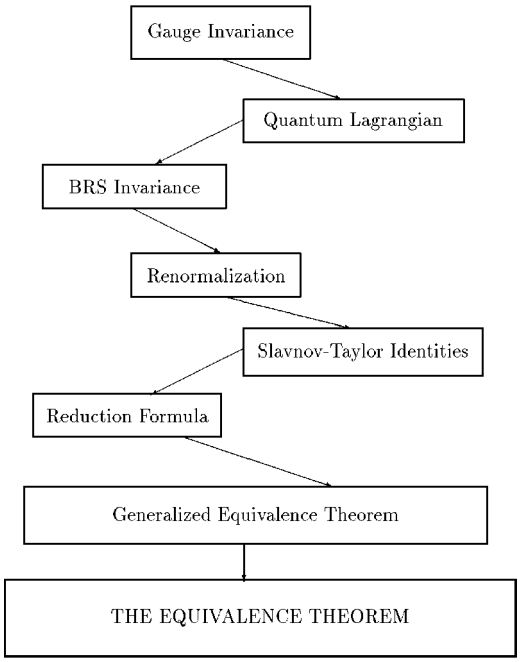

FIG. 1. The proof of the equivalence theorem.

but a δ function in functional space and thus, intuitively it is telling us, in momentum space, that $V_\mu^a p^\mu/M \sim \omega^a$. But, as far as

$$\epsilon_L^\mu = \frac{p^\mu}{M} + v_\mu \quad \text{with} \quad v_\mu = O\left(\frac{M}{E}\right), \tag{2}$$

we can replace longitudinal gauge bosons by their associated GB at energies $E \gg M$. Naively, that is the equivalence theorem [2,3].

Rigorously, we have to reformulate the ET within quantum field theory in terms of amplitudes or S matrix elements. The first formal proof of this kind was given by Chanowitz and Gaillard in [4] in the context of the minimal standard model (MSM). The complete derivation of the ET is rather complicated and we will not go through it in here, although in Fig. 1 we have included a simple sketch of the most important steps. Indeed, they used the Slavnov-Taylor identities which are obtained from the Becchi-Rouet-Stora (BRS) invariance of the quantum Lagrangian, which now has the role that gauge invariance played in the classical reasoning. That proof was later simplified in [5]. We say it is formal because one also has to take into account renormalization and some correction factors K have to be included in the final ET statement [6]. These factors have been calculated in the most usual renormalization schemes as well as in those where their value is just one [7]. The Slavnov Taylor identities yield relations among renormalized Green functions which, using the reduction formula, can be translated to amplitudes. Once all these are done, the momentum factors can be replaced using Eq. (2). After a little bit of algebra [4,18] it is possible to write separately all the powers of v_μ. That expression is nowdays known as the generalized equivalence theorem (GET) and it reads

$$T(V_L^{a_1}, \ldots, V_L^{a_n}; A) = \sum_{l=0}^{n} (-i)^l \left(\prod_{j=1}^{l} K_{\alpha_j}^{a_j} \right) \left(\prod_{i=l+1}^{n} v_{\mu_i} \right) \times \overline{T}(\omega_{\alpha_1} \cdots \omega_{\alpha_l}, V_{a_{l+1}}^{\mu_{l+1}} \cdots V_{a_n}^{\mu_n}; A) \tag{3}$$

This formula deserves some comments (see [18] for details).

(a) A stands for any set of other *physical* fields, i.e., not V_L's.

(b) On the left-hand side we have the amplitude we were looking for, containing an arbitrary number n of V_L's. On the right-hand side we have a sum over all amplitudes T where we have replaced one, two, ... , up to n V_L's by their associated GB, ω^α. The bar over the amplitudes means that we are considering all permutations of indices. Observe that the only energy-dependent factors that multiply the amplitudes are the v_μ, one per each V_L which has not been substituted by its GB.

(c) The GB can be written in any parametrization. That is why the K correction factors have two indices, since the V_L^a carry indices of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, whereas the ω^α belong to the broken quotient space. Thus, the K factors also choose the right GB combination that is ''eaten'' by each V_L^a.

From this expression it is fairly simple to obtain the ET using Eq. (2) and the fact that unitarity constrains how fast the amplitudes can grow with the energy. In the limit $E \gg M$,

$$T(V_L^{a_1}, \ldots, V_L^{a_n}; A) \simeq \left(\prod_{j=1}^{n} K_{\alpha_j}^{a_j} \right) T(\omega_{\alpha_1} \cdots \omega_{\alpha_n}; A) + O\left(\frac{M}{E}\right). \tag{4}$$

That is the equivalence theorem in its most common version, but unfortunately it is not valid when dealing with chiral Lagrangians. It is easy to understand why: on the one hand, the ET is a high-energy limit, since we need $E \gg M \simeq gv$ in order to neglect the v_μ terms in Eq. (2). On the other hand, the chiral approach is a low-energy limit, and we are expanding the amplitudes as follows:

$$T(V_{\mu_1}^{a_1} \cdots V_{\mu_l}^{a_l}, \omega_{\alpha_{l+1}} \cdots \omega_{\alpha_n}; A) \simeq \sum_{k=0}^{N} a_l^k \left(\frac{E}{4\pi v} \right)^k + \sum_{k=1}^{\infty} a_l^{-k} \left(\frac{M}{E} \right)^k \tag{5}$$

(we do not display the μ_i and a_i indices in the a_l^k factors). Thus, we are taking a high- and low-energy limit at the same time, which may be possible or not.

Nevertheless the proof in Fig. 1, can be modified for the effective formalism. First, considering the nonlinear character of the symmetry, which only affects the K factors [17]. Second, taking into account that chiral amplitudes grow with the energy, as in the above equation. As a consequence, the $O(M/E)$ terms contained in the v_μ factors mess up with the chiral $E/4\pi v$ powers. Fortunately, between the scale $4\pi v$ and $M\sim gv$ there is a $g/4\pi$ factor, which finally allows us to extract the leading order in g of every coefficient of the chiral expansion. Thus, the ET reads

$$T(V_L^{a_1},\ldots,V_L^{a_n};A)\simeq\left(\prod_{j=1}^{n}K_{\alpha_j}^{a_j(0)}\right)T_L^N(\omega_{\alpha_1}\cdots\omega_{\alpha_n};A)+O(g\text{ or }g'-\text{ suppressed})+O\left(\frac{M}{E}-\text{ suppressed}\right)+O\left(\frac{E}{4\pi v}\right)^{N+1},\quad(6)$$

where T_L^N is the $O(p^N)$ chiral amplitude at lowest order in the electroweak couplings g or g'. Note that the K factors have also been expanded and their only contribution comes from the lowest order in g or g', denoted by $K^{(0)}$.

The ET may have not changed very much in its form, but we have to neglect more terms, which will limit the validity of the whole approach. To which extent these approximations are consistent and useful will be the topic of the next sections.

As a last remark, we also want to point out that, very recently, it has appeared a new derivation of the ET, based on Lorentz invariance [19], which reaches similar results for both cases, although it clarifies the interpretation of the ET in different Lorentz frames. Here it has been assumed that we are at the c.m. in a collision where all initial or final V_L's have comparable momenta, generically $O(p_i)=O(E)=O(\sqrt{s})$. Notice that we have also been considering $O(M_W)=O(M_Z)$.

III. APPLICABILITY CONSTRAINTS

Up to now, the ET has been considered as a formal mathematical statement. We now want to know when we are allowed to neglect all the higher order terms that appear in its expression. From the previous section, there are two kind of restrictions: those related with the energy regime, and those connected with neglecting higher orders in the electroweak expansion. We will address them separatedly.

A. Conditions due to the energy expansion

As we have already seen, there are two formulations of the ET and each one is used depending on the unitarity properties of the amplitudes. As a consequence we have two sets of applicability conditions.

(i) For models respecting unitarity. That is, for instance, the case of the MSM. As far as the amplitudes cannot grow as a power of the energy, we can extract the ET leading contribution by simply neglecting the v_μ terms in Eq. (2). Thus we can use the first formulation of the ET in Eq. (4) and we only have to ask for $M\ll E$.

(ii) When dealing with effective Lagrangians. The amplitudes are obtained as truncated series up to a given power $(E/4\pi v)^N$. Hence, in order to neglect higher $E/4\pi v$ terms we have to demand that $E\ll 4\pi v$. But we still want to approximate $p/M\sim\epsilon_L$ to obtain the ET, so that we need

$$M\ll E\ll 4\pi v=\frac{8\pi}{g}M.\quad(7)$$

But that is not all in Eq. (6), we are neglecting the $O(M/E-$ suppressed) terms while keeping those of $O(E/4\pi v)^N$. Thus, we also need

$$O\left(\frac{M}{E}-\text{ suppressed}\right)\ll O\left(\frac{E}{4\pi v}\right)^N.\quad(8)$$

Notice that the last condition is more strict for higher N.

Nevertheless, even in unitary models such as the MSM, amplitudes are obtained perturbatively. Then it could be possible that some high order terms could also be hidden by the $O(M^2/E^2)$ neglected contributions [19]. In that case we would have similar conditions to those in Eq. (8). In this work we will only study such conditions within the chiral formalism.

B. Conditions due to the expansion in g and g'.

It has been shown that the ET only yields the lowest order in the g expansion within the effective formalism. But even in the unitary cases it is still very frequent to work at lowest order in the electroweak couplings g and g', since then there are no internal lines of gauge bosons and one does not have to deal with complicated propagators inside loops. Once more we have to demand that the terms we drop should be smaller that those we keep, that is,

$$O(g\text{ or }g'-\text{ suppressed})\ll O\left(\frac{E}{4\pi v}\right)^N.\quad(9)$$

In practice, if one wants to work at a higher g or g' order, one has to calculate the following.

(a) The amplitude $T(\omega,\ldots,\omega)$ to next order.

(b) The K factors, whose expressions are rather complicated but that have been obtained to one loop in several renormalization schemes [7].

Within the chiral formalism, we are using Eq. (6) and thus, in order to work at higher orders of g or g', it is not enough with steps (a) and (b) but we also have to obtain

(c) The lowest order contribution in the electroweak couplings of $T(V_L,\omega,\ldots,\omega)$. In other words, we need the amplitudes where one of the longitudinal gauge bosons has not been replaced by its corresponding GB. (Indeed, we have to calculate the amplitudes with the V_L in all the different positions.)

This complication shows up in the last step of Fig. 1 because the above amplitudes appear in the generalized equivalence theorem multiplied by just one v_μ factor (for details see [18]). Notice that for our purposes, v_μ counts as $\sim gO(v/E)$. Indeed, we have explicitly checked at tree level that without this last contributions one does not get the same results on both sides of the ET. Very recently a work has been published [21] where these corrections have been in-

cluded, again at tree level, for the $W^+W^+\to W^+W^+$ process and a perfect agreement was found. In addition, they obtained the K renormalization factors in the on-shell scheme.

C. Special Kinematic Regions

The constraints in the previous sections are obtained on very general grounds. However, the numerical values of the coefficients of the chiral expansion also determine the size of the different contributions. If they become to big or too small they can spoil the previous naive counting. Indeed that is the case in some special kinematic regions.

It is well known that the exchange of massless particles, such as photons or GB, in t or u channels could lead to infrared divergencies at low scattering angle θ. If those singularities appear in the terms we are neglecting, then the ET would be a very bad approximation, although the formal mathematical statement will still hold. When dealing with $V_LV_L\to V_LV_L$, those divergencies coming from photon exchange, are at least $O(e^2)\sim O(g^2)$. As a consequence, if we just keep the lowest order in g, we would be droping the divergent (dominant) term at low scattering angle, and in that region the approximation will be completely inadequate. It is always possible, however, to keep precisely that part of the $O(g^2)$ contributions which is $O(e^2)$. Mathematically, it may seem at first not very consistent to keep just part of the $O(g^2)$ contributions, but physically it can be accepted, as far as it is the dominant part at low θ [20].

It could also happen that in a given channel the term obtained from the ET may be proportional to, let us say, u [20]. It is then possible to make that term as small as we want by approaching $\theta\to\pi$. Therefore, even though the mathematical expression of the ET is still correct, we are not allowed to neglect the $O(M/E)$ term.

IV. PHYSICAL PROCESSES

The above constraints on the applicability of the ET can be relaxed for most of the processes that have been proposed to probe the SBS in future colliders:

$$V_LV_L\to V_LV_L \overset{\text{ET}}{\leftrightarrow} \omega\omega\to\omega\omega,$$

$$qq'\to V_LV_L \overset{\text{ET}}{\leftrightarrow} qq'\to\omega\omega,$$

$$\gamma\gamma\to V_LV_L \overset{\text{ET}}{\leftrightarrow} \gamma\gamma\to\omega\omega.$$

As far as only an even number of V_L appear, the expressions of the amplitudes will only contain *even powers* of momenta or the energy. Consequently, we just need $M^2\ll E^2$, instead of $M\ll E$, and when dealing with the effective Lagrangian formalism we need $(N=2n)$

$$M^2\ll E^2\ll(4\pi v)^2, \tag{10}$$

$$O\left(\frac{M^2}{E^2}-\text{ suppressed}\right)\ll O\left(\frac{E}{4\pi v}\right)^N, \tag{11}$$

$$O(g\text{ or }g'-\text{ suppressed})\ll O\left(\frac{E}{4\pi v}\right)^N. \tag{12}$$

In order to obtain numerical values for the above applicability limits, we have to keep in mind that the numbers that we will give are not strict mathematical bounds, but rather some rough estimates.

We will then start with the lowest energy bound, which is common to both formulations of the ET, namely, $M^2\ll E^2$. If we demand, for example, E^2 to be one order of magnitude bigger than M^2, that means that $E>300$ GeV.

Within the effective Lagrangian formalism we also have the $4\pi v$ upper bound, but in practice that is too optimistic. One possible way to estimate the upper validity range, is to compare it with that of ChPT and pion physics. As a matter of fact, that is an analogous formalism to the one we are using in this work, although it is scaled down to energies of the order of 1 GeV. More precisely, in ChPT, the scale $4\pi f_\pi\simeq 1$ GeV plays the very same role as $4\pi v\sim 3$ TeV here. The $O(p^4)$ ChPT amplitudes are known to work reasonably well only up to 500 MeV$\simeq 2\pi f_\pi$ in the best cases [10]. Therefore, we should not trust the chiral formalism beyond energies of the order of $2\pi v\simeq 1.5$ TeV. In brief, in a first approximation we find that the applicability of the ET within the $O(p^4)$ chiral Lagrangian formalism is restricted to the following energy range:

$$0.3\text{ TeV}<E<1.5\text{ TeV}, \tag{13}$$

maybe less. Indeed, in some models, we expect resonances to appear below 1.5 TeV, and for that reason the chiral approach is not valid. Notice that the high-energy constraint *is inherent to the effective Lagrangian approach*, it is not really due to the usage of the ET.

Concerning Eqs. (11) and (12), they have been obtained by demanding that the neglected terms should be smaller than the effects we want to observe. Therefore, Eqs. (11) and (12) are telling us which energies we have to reach in order to see the desired $O(p^N)$ effects, but below that energy the rest of the amplitude will be enough to describe the relevant physics. These constraints are much more process dependent than the others above, and we will study them for $V_LV_L\to V_LV_L$.

Scattering of longitudinal gauge bosons. This process is one of the most interesting at LHC since one of the signatures of a strongly interacting SBS is the enhancement of the V_L interaction. Within the chiral Lagrangian formalism, the first term, which is $O(E^2/v^2)$, is universal and it just reproduces the LET. Therefore, if we want to differentiate between alternative SBS, we have to look at the next order, which, in general is $O[E^2/v^2\cdot E^2/(4\pi v)^2]$. Hence Eq. (11) can we written as

$$\frac{M^2}{E^2}\ll\frac{E^4}{v^2(4\pi v)^2}\Rightarrow E>4\pi v\sqrt[3]{\frac{g}{32\pi^2}}\simeq 0.4\text{ TeV}. \tag{14}$$

But we are also neglecting a g-suppressed term in the tree amplitude, which is $O(E^2/v^2 \cdot M^2/E^2)$, and thus Eq. (12) yields

$$\frac{M^2}{v^2} \ll \frac{E^4}{v^2(4\pi v)^2} \Rightarrow E > 4\pi v^2 \sqrt{\frac{g}{8\pi}} \simeq 0.5 \text{ TeV}. \quad (15)$$

(Note that if we do not keep track of the 2 in $M = gv/2$, the lower bound is 0.7 TeV.) This lower bound does *not* mean that the whole $O(p^4)$ amplitude cannot be used with the ET below 0.5 TeV. It is just telling us that the $O(p^4)$ terms will only be seen above that energy. Notice, however, that the $O(p^4)$ amplitude also contains $O(p^2)$ terms, whose consistency condition (12) reads

$$\frac{M^2}{v^2} \ll \frac{E^2}{v^2}, \quad (16)$$

which is again the usual constraint of the ET. These $O(p^2)$ terms dominate the chiral expansion at low energies, so that they are enough to obtain the correct result between 0.3 and 0.5 MeV, but they do not differentiate alternative SBS's.

To summarize, within the effective Lagrangian formalism and for V_L elastic scattering, we can expect the ET to be a reasonable approximation when $0.3 \text{ TeV} < E < 1.5$ TeV, although the truly $O(p^4)$ operators can only be tested whenever

$$0.5 \text{ TeV} \leqslant E \leqslant 1.5 \text{ TeV}. \quad (17)$$

Notice that the high-energy bound is only due to the use of the effective formalism. This restriction is very dissappointing since the enhancement of the V_L interaction is expected to be bigger at higher energies. Moreover, one of the main features of strongly interacting systems are resonances, which are expected to appear around 1 TeV. Indeed, the existence of such resonant states implies the saturation of unitarity bounds and a breakdown of the chiral expansion. Thus, in some cases, the 1.5 TeV bound could be even very optimistic (as for instance for the MSM with a heavy Higgs boson, that could appear as a very broad scalar resonance at energies near 1 TeV). For all these reasons, it would be very interesting to extend the applicability of the ET to much higher energies. We will refer to this possibility in Sec. V.

Let us remark once again that the numbers in the above equations should be understood as orders of magnitude, since we do not know what are the actual values of the chiral parameters and the coefficients in the chiral expansion. Nevertheless, there are some analysis of the LHC capabilities to measure the $O(p^4)$ parameters, although they have been performed just at tree level with the $O(p^2)$ and the $O(p^4)$ Lagrangian. It seems likely to determine their values within the order of magnitude expected for a generic strong SBS [15]. These analyses do not make use of the ET, but it could be very helpful in case the whole one-loop calculation was performed.

Numerical estimates. We have seen that for longitudinal gauge boson scattering, the biggest terms that we are neglecting seem to be those of $O(g^2)$. However, their size has been estimated comparing energy powers, although it could well happen that the actual values of the coefficients in the expansion may spoil the power counting arguments. With that purpose, we have calculated the amplitudes of several processes with and without the ET, by considering all the graphs obtained, at tree level, from both the $O(p^2)$ and $O(p^4)$ terms in the electroweak chiral Lagrangian. On the one hand, the amplitude containig external GB has only been calculated at lowest order in g and g', since that is how the ET should be applied. On the other hand, the amplitude obtained without the ET contains the higher order g and g' contributions. In such a way, by comparing both amplitudes, we get an estimate of the error caused when using the ET.

The results can be found in Fig. 2, where we have plotted the cross sections for $ZZ \to ZZ$, $W^+W^- \to ZZ$, and $W^\pm Z \to W^\pm Z$. The solid lines represent the calculation with the ET and the dashed lines without it. Their difference is only due to higher order g and g' effects, and that we have checked by letting $g, g' \to 0$. In that limit, the two lines superimpose.

Let us now remember that the point in using chiral Lagrangians is to reproduce, within the same formalism, any symmetry-breaking mechanism that follows the SM symmetry-breaking pattern. That is enough to fix the $O(p^2)$, but the operators at $O(p^4)$, whose form is also dictated by symmetry requirements, are all affected by some multiplying parameters, usually denoted as α_i [11]. For our purposes we are only interested in α_5 and α_4, since they are the only ones appearing in the above amplitudes. Their values depend on the underliying symmetry-breaking mechanism and, by varying them, we are able to mimic any strong SBS. We have indeed performed our calculations for many different models, but in Fig. 2 we have only displayed the two which are usually present in the literature.

(i) The SM with a heavy Higgs boson (on the left column), whose values for the α_i can be obtained by matching the Green functions of the Higgs model with those coming from the effective Lagrangian approach [13]. At the tree level, the matching yields $\alpha_5 = v^2/8M_H$ and $\alpha_i = 0$ if $i \neq 5$. Therefore, by choosing $M_H \simeq 1$ TeV, we have $\alpha_5 \simeq 0.008$

(ii) A QCD-like model (on the right column), whose estimated values for the chiral parameters are $\alpha_5 = -0.0016$ and $\alpha_4 = 0.0008$. They have been obtained from a large-N_c expansion of the QCD effective action [14].

Observe that, in Fig. 2, we have represented the SM-like model only up to 1 TeV, since that is the mass we have chosen for the Higgs boson and therefore we expect it to show up as a resonance at that energies. Such resonant states cannot be reproduced within the chiral approach, although it can be noticed that there is a huge increase in the cross section. The QCD-like model cross sections have been plotted up to 1.5 TeV; that is the energy up to which the chiral approach works for QCD, but rescaled to the electroweak SBS.

In Fig. 2 it can be noticed that the typical cross sections for longitudinal gauge boson scattering is of the order of 1 nb. In contrast the contributions from higher orders in g or g' are typically of the order of 0.1 nb. Only in those channels which almost vanish, these corrections may be important. In those channels with a significant signal, the effects of neglecting them are just corrections of the order of $\simeq 10\%$.

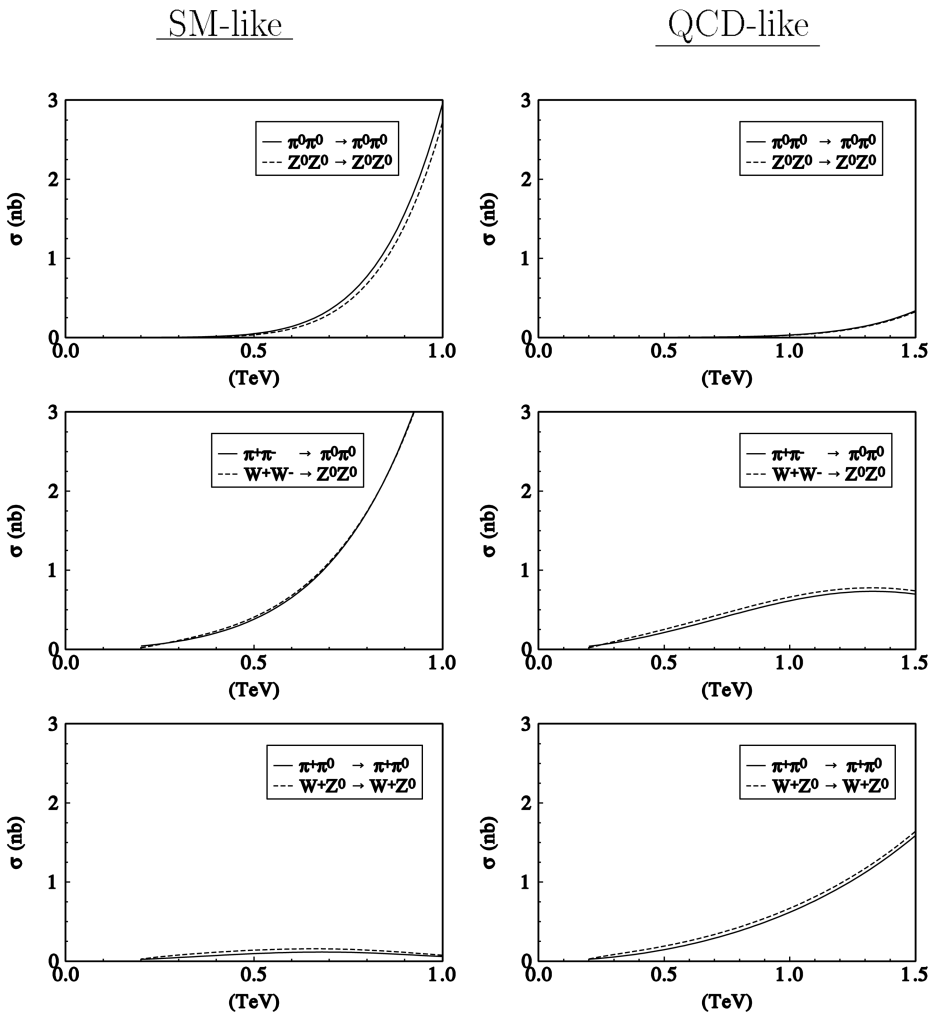

FIG. 2. The cross sections in this figure have been obtained at the tree level from $\mathcal{L}_2$ and $\mathcal{L}_4$. With different values of the α_i parameters in $\mathcal{L}_4$ we mimic either a heavy Higgs model (left column) or a QCD model (right column). The dashed lines are the calculation without the ET. The solid lines are obtained from the ET. Hence they are calculated at lowest order in g. The differences between both lines are therefore numerical estimates of the effect of neglecting higher order electroweak corrections.

V. DISPERSION RELATIONS

Up to now we have seen that the most severe constraint to the applicability of the ET appears when it is applied within the effective Lagrangian approach. Even though the $O(g,g')$ effects may be negligible, the lack of unitarity strictly limits how high in energy we can trust the whole approach. Indeed, the most typical feature of strong interactions is the appearence of resonances, which are deeply related to the saturation of unitarity, where we do not expect the effective approach to work properly.

Our purpose in this section is not to review all the unitarization methods proposed in the literature, but only those that have been succesfully tested with pion data in the framework of ChPT. Notice that QCD has the very same symmetry-breaking pattern, $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$, that we have in the electroweak SBS. Indeed, there is a similar chiral lagrangian formalism for pions, but rescaled from $v \simeq 250$ GeV down to $f_\pi \simeq 92$ MeV. The only differences are the actual values of the α_i parameters, the small masses of the pions (which are pseudo GB's) and the fact that we do not couple electroweak gauge bosons. However, we can use the ET to translate the rescaled pion amplitudes into longitudinal gauge boson amplitudes. That is why we use pion physics as a reference. Apart from that, we also look for methods that do not require the explicit introduction of resonances as additional fields. That may be useful in pion physics since the resonances are already known, but that is not the case of the electroweak SBS. Thus we have restricted ourselves to illustrate the dispersive approach, since it is able to unitarize the chiral amplitudes and reproduce resonances just from the knowledge of their low-energy behavior.

The general idea is to reproduce the unitarity cut which is present when we extend the partial waves of a given process to the complex s plane. This cut is due to the existence of a threshold. In addition, the partial waves also have a left cut due to crossing symmetry and possibly some poles in the second Riemann sheet, which are closely related to the resonances. If we apply Cauchy's theorem to this analytic structure, we can obtain the value of the amplitude at any s in terms of integrals of the imaginary part of the amplitude over the cuts, plus maybe some polynomials of the energy. The low-energy behavior is given by the polynomials, which can

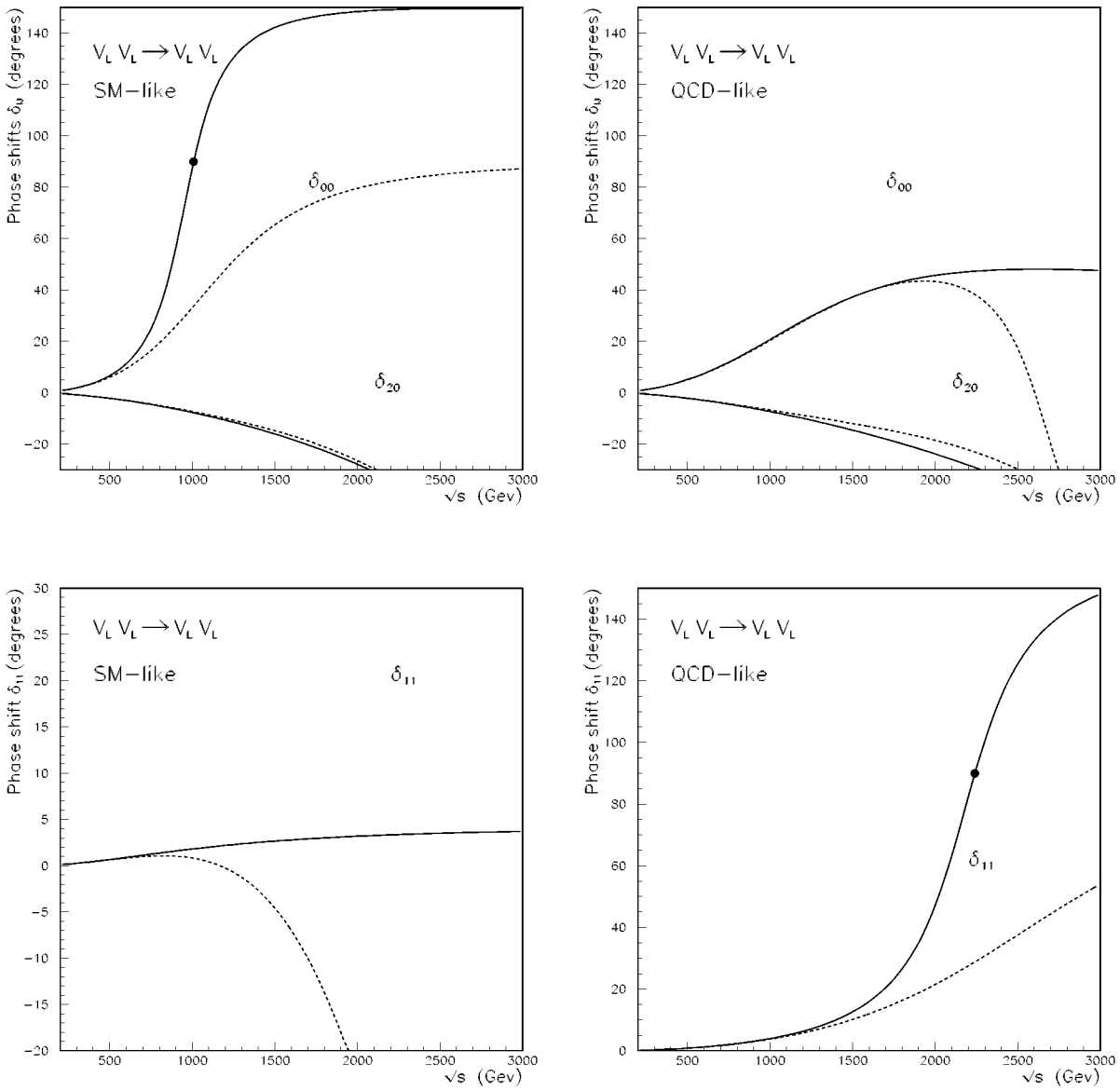

FIG. 3. Phase shifts δ_{IJ} for $V_L V_L \to V_L V_L$ scattering. The pictures on the left correspond to the heavy Higgs SM-like case and those on the right to the QCD-like model. The dashed lines are the predictions of the effective Lagrangian and the solid lines are the unitarized results. The black dots stand at $\delta_{IJ}=90°$ pointing the existence of a resonant state.

be approximated using the chiral approach. The integrals carry the high-energy and resonant behavior although they also correct the bad high-energy behavior of the polynomials. The interesting point is that sometimes some of these integrals can be calculated exactly.

A. The inverse amplitude method

In the elastic scattering of GB we have partial waves of definite (weak)-isospin I and angular momentum J. In the chiral approach they are obtained as $t_{IJ}=t_{IJ}^{(0)}+t_{IJ}^{(1)}+O(s^3)$. The LET are reproduced by $t_{IJ}^{(0)}$, which is an $O(s)$ polynomial. Formally, $t_{IJ}^{(1)}$ can be seen as an $O(s^2)$ polynomial whose coefficients can contain logarithmic terms which provide a first approximation of the cuts. The unitarity constraint, for real values of s above threshold (the elastic cut), is nothing but

$$\mathrm{Im}t_{IJ}=\sigma|t_{IJ}|^2. \tag{18}$$

The σ factor is the GB two-body phase space and for pure GB is 1 [for pions, which are massive, it is $\sigma(s)=\sqrt{1-4m_\pi^2/s}$]. Obviously the polynomials cannot satisfy this quadratic constraint and they violate unitarity. Nevertheless, the chiral amplitudes satisfy unitarity *perturbatively*: $\mathrm{Im}t_{IJ}^{(1)}=\sigma|t_{IJ}^{(0)}|^2$.

The important remark is that we can obtain the imaginary part of the inverse amplitude exactly: $\mathrm{Im}1/t_{IJ}=\sigma$. Thus, it is possible to calculate the dispersive integral of $1/t$ over the elastic cut and then solve for t [22]. The result is [23]

$$t_{IJ}=\frac{t_{IJ}^{(0)}}{1-t_{IJ}^{(1)}/t_{IJ}^{(0)}}. \tag{19}$$

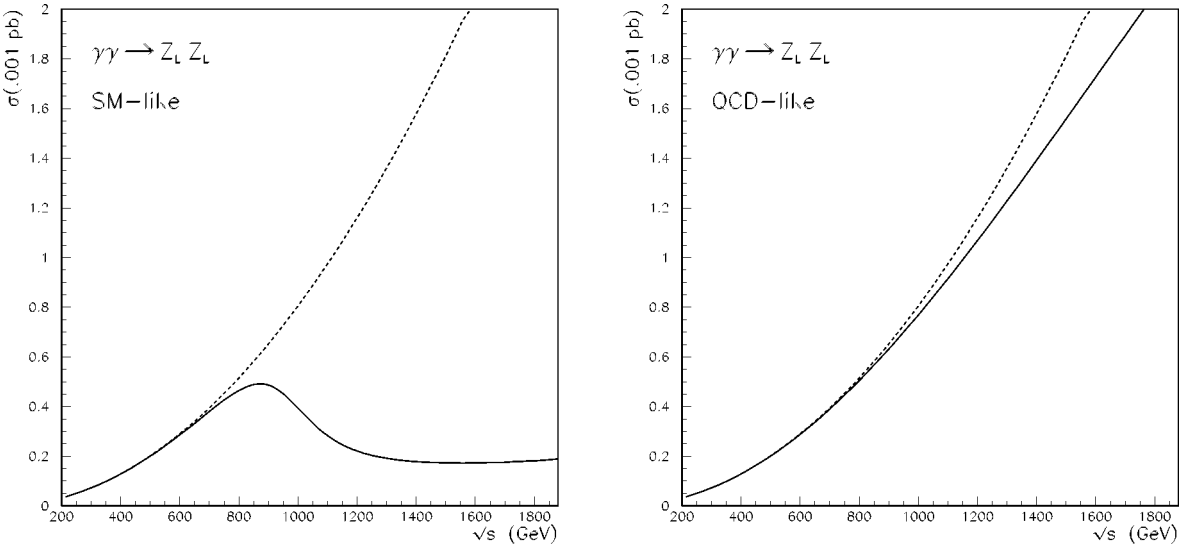

FIG. 4. Cross sections for $\gamma\gamma \to Z_L Z_L$ both for a heavy Higgs SM or a QCD-like model. The dashed lines come from the effective Lagrangian approach and the solid lines are the unitarized results.

This formula presents several interesting properties.

It can be easily checked that the above formula satisfies the unitarity constraint in Eq. (18).

If we expand it again at small s, we recover the original chiral amplitude. Thus, at low energies we have not spoiled the good behavior given by the effective Lagrangian.

It is able to accomodate poles in the second Riemann sheet and reproduce resonances.

Indeed, it has been shown that the inverse amplitude method (IAM) is able to reproduce the $\rho(770)$ resonance in $\pi\pi$ scattering [23], and the $K^*(892)$ in πK scattering [22]. Their correct masses and witdths are obtained with chiral parameters which are compatible with those coming from a fit of pure ChPT to the low-energy data. As it should be, the resonances always appear related to poles in the second Riemann sheet, by the formula $\sqrt{s_{\rm pole}} \simeq M + \Gamma/2$, where M and Γ are, respectively, the mass and width of the resonance [22].

The IAM was first applied to longitudinal gauge boson scattering in [24]. In Fig. 3, we are showing the results as continuous lines. We represent the phase shifts δ_{IJ} for the three lowest angular momentum channels $(I,J) = (0,0),(1,1),(2,0)$. The dashed lines correspond to the pure chiral Lagrangian approach with the ET, and, as we have already seen, should not be trusted above 1.5 TeV or even before if there is a resonance. For illustrative purposes, we have chosen two typical scenarios for the electroweak SBS.

Those graphs on the left have been obtained with the chiral parameters that mimic the SM. As we commented before, they are estimated from a matching of the SM Green functions and those coming from the chiral Lagrangian [13]. This time, however, we are calculating at one loop and thus, for $M_H = 1.2$ TeV, we have to use $\alpha_5 = 0.0045$ and $\alpha_4 = -0.0020$. Notice that the expected scalar resonance in a heavy-Higgs scenario does indeed appear, in contrast with the nonunitarized approach. As a matter of fact, we have chosen M_H so that the mass of the scalar resonance is 1 TeV.

Those graphs on the right column are just a rescaled version of pion-pion scattering in the limit $m_\pi = 0$. Therefore they represent a QCD-like SBS. The parameters we have used are obtained directly from the IAM applied to elastic pion scattering data: $\alpha_5 = -0.00103$ and $\alpha_4 = 0.00105$ [22]. Once again, and in contrast to the pure effective approach, there is a ρ-like resonance in the (1,1) channel.

B. Inelastic case

Dispersive techniques can also be applied to the inelastic case [25] or to calculate form factors of inelastic processes [26]. As far as the reaction contains two GB, we can expect strong rescattering effects to become relevant, and then unitarization would come into play. We will illustrate such effects using the $\gamma\gamma \to Z_L Z_L$ process, since the dispersive unitarization techniques have also been succesfully applied to $\gamma\gamma \to \pi^0\pi^0$. We refer to [25] for further details. We can therefore apply the very same techniques of that work since both reactions are related through the ET in the $m_\pi = 0$ limit once we rescale $f_\pi \simeq 92$ MeV up to $v \simeq 250$ GeV.

The dashed curves in Fig. 4 correspond to the cross section of the reaction calculated within the pure effective formalism [27]. This process is forbidden at the tree level, and the one-loop contributions become dominant.

As far as the effective amplitude does not depend on α_i, the predictions of the effective approach are the same for every underlying SBS. However, by looking at the continuous lines, which correspond to the unitarized calculation, we see how the cross section differ from a SM-like to a QCD-like model. The unitarization has been performed by imposing the elastic phase shifts of Fig. 3 as the phases of the $\gamma\gamma \to V_L V_L$ amplitudes. For the absolute value of the amplitude we keep the pure result from the effective Lagrangian. In the SM case, it can be noticed that the corrections become important as soon as the resonance appears, and they modify the result at the qualitative level.

C. The large-N limit

Finally, we will also like to comment that there is another approach that yields unitary amplitudes, which has also been

tested with pion data and ChPT. The idea is to take the large-N limit [28], N being the number of GB, and keep the dominant terms in a $1/N$ expansion. The resulting amplitudes do not behave as polynomials in the energy and they satisfy unitarity up to $O(1/N)$. Therefore it is possible to use them together with the simplest version of the ET, Eq. (4), which has less severe applicability bounds and allows us to trust the calculations at higher energies. This approximation is specially well suited for the scalar channel (but not so well for the others since they are subdominant and would require higher orders in $1/N$). Indeed, it has been shown that the large-N approximation is able to reproduce an scalar resonance in a heavy-Higgs model, together with its associated pole in the second Riemann sheet [29].

VI. CONCLUSIONS

Usually, the ET has been applied to the SM or other unitary models. Except in special kinematic regions, once the renormalization effects are taken into account, it is enough to demand that $E \gg M_W$.

In contrast, when dealing with nonunitary models such as in the electroweak chiral approach, further limitations appear. First, there is the typical constraint $E \ll 4\pi v$, of the effective Lagrangian formalism. But apart from that, the version of the ET valid in this case does not include higher order electroweak corrections. Even more, if one is interested in probing the $O(p^N)$ terms of the amplitudes, there is an additional condition

$$O\left(\frac{M}{E}, g \text{ or } g' - \text{ suppressed}\right) \ll O\left(\frac{E}{4\pi v}\right)^N. \qquad (20)$$

In this work we have obtained, on the one hand, some estimates of the above constraints for the processes of interest in future accelerators. It seems that the ET applicability window to test the truly $O(p^4)$ (model-dependent) contributions is roughly $0.5\ \text{TeV} \leqslant E \leqslant 1.5\ \text{TeV}$, *if there are no resonances in that range*, otherwise it will be much smaller. On the other hand, we have obtained, from detailed numerical calculations, that the effect of neglecting higher order electroweak corrections is typically $\simeq 10\%$, in the cross sections of the relevant channels.

Therefore, it seems that neglecting the electroweak effects will not be very relevant when applying the ET. In contrast, the constraints on the energy range do severely limit the simultaneous application of the ET and the chiral approach. Such applicability range can be considerably enlarged using dispersive techniques, or nonperturbative chiral expansions (such as the large-N limit), which improve the unitarity behavior of the amplitudes, without changing their low-energy properties. In conclusion, and in view of our results, it seems that the simultaneous application of the ET and the effective chiral approach to the physically relevant cases is severely limited unless it is complemented with some nonperturbative techniques.

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