Calibration of a Shack–Hartmann wavefront sensor as an orthographic camera

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We demonstrate a method to calibrate a Shack–Hartmann sensor as an orthographic camera. This calibration method permits us to obtain the distance, the rotation matrix between the microlens array and CCD imaging planes, and the projection matrix, which models the projection of the incoming rays to the CCD imaging plane. The proposed calibration method introduces a very compact matrix notation and allows wavefront reconstruction without an explicit centroid search between the reference and distorted spot diagrams. We show a set of simulations in code V that prove the effectiveness of the proposed method.

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A Shack–Hartmann (SH) wavefront sensor consists of a 2D microlens array focusing on a CCD camera. A necessary previous step in reconstructing a wavefront error with an SH sensor is the calibration process. Typically, the calibration method consists in obtaining the light spot distribution of a high-quality aberration-free beam, which is used as a reference. Additionally in the calibration process, it is necessary to determine the distance between the microlens and CCD planes. The slope distribution of the wavefront error—the difference between the incoming and reference wavefronts—is determined by comparison between the reference and measured centroids:

$$\nabla[\Delta W]_{ij} = \left[\begin{pmatrix} \frac{\partial \Delta W}{\partial x} \\ \frac{\partial \Delta W}{\partial y} \end{pmatrix}\right]_{ij} = \left[\frac{1}{d}\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}\right]_{ij}, \tag{1}$$

where ΔW is the wavefront error, d is the distance between the microlens and CCD planes, i and j denote the ith and jth microlens, and $(\Delta x, \Delta y)$ are the displacements between corresponding centroids of the distorted and reference beams. With the wavefront slope distribution obtained by Eq. (1), the wavefront error is computed by a zonal or modal reconstruction [1].

The SH calibration method mentioned above does not consider other misalignment errors between the microlens array and CCD planes; furthermore, in order to obtain an accurate wavefront reconstruction, the distance between the lens and CCD planes must be precisely known to avoid a calibration offset. This distance is difficult to measure and usually is calibrated by recording a number of known wavefronts. The systematic wavefront reconstruction errors caused by various alignment and calibration errors are considered in [2], and it is shown that these misalignment errors can have a dramatic effect in the accuracy of an SH sensor.

In this Letter, we propose a novel calibration method for an SH sensor. The calibration method needs to obtain the Hartmann spot diagram of at least two collimated beams. The relative direction between the different collimated beams has to be known. The direction of light propagation can be easily controlled by diffraction using a spatial light modulator (SLM) [3], with an accurate moving stage, or with an autocollimator, for example.

The proposed SH calibration method is based on calibrating the SH sensor as an orthographic camera. Orthographic projection is a form of parallel projection. It can be understood as a perspective projection where the camera lies an infinite distance away from the object and has an infinite focal length. As shown in [4], Chap. 6, from p. 166 to p. 170, the orthographic projection of a 3D point, $M = [X \;\; Y \;\; Z \;\; 1]^T$ to a camera 2D point $m = [x \;\; y \;\; 1]^T$, both in homogeneous coordinates, is given by

$$m = HM = \begin{bmatrix} \kappa_x & 0 & \mu_x \\ 0 & \kappa_y & \mu_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r^{1T} & 0 \\ r^{2T} & 0 \\ 0^T & 1 \end{bmatrix} M, \tag{2}$$

where H is a homography and will denote the projection matrix, κ_x and κ_y are scale factors, μ_x and μ_y are offset parameters, and r^{1T} and r^{2T} are the first and second row of the rotation matrix (R) relating the world and camera reference frames, respectively. It is possible to use this mathematical representation to model the projection of the microlens centers in the microlens array plane to the SH imaging plane by the chief rays when they are illuminated by a collimated beam. In this case, the world and camera reference systems are a 3D frame fixed to the microlens array plane and a 2D frame containing the imaging plane, respectively. The coordinates m and M are the spot centroid coordinates in the image plane and the microlens centers spatial coordinates, referred with respect to the camera and world reference systems, respectively; R is a rotation matrix that takes into account possible tip-tilt errors between the microlens array and SH CCD planes. This rotation matrix R can be described in terms of the Euler angles (α, β, γ) as $R = R_Z[\gamma]R_X[\beta]R_Z[\alpha]$, where R_Z and R_X denote a rotation around the Z and X axes, respectively. We assume without loss of generality that the microlens array plane and the Z axis of the world coordinate system are perpendicular, and, therefore, for all microlens centers, $M = [X \;\; Y \;\; 0 \;\; 1]^T$. As m depends on the relative orientation between the SH sensor and the

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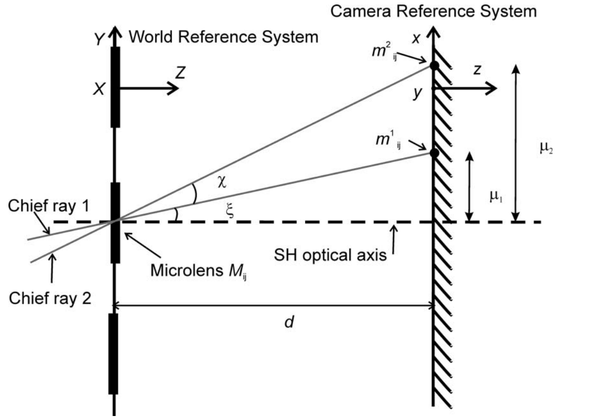

Fig. 1. Scheme of the SH imaging process of two chief rays passing through a microlens with different implying angles.

collimated beam, the notation of m has to be extended as $m_{ij}^k = [x_{ij}^k, \quad y_{ij}^k, \quad 1]^T$—the resultant spot centroid coordinates of the ith and jth microlens, given by $M_{ij} = [X_{ij}, \quad Y_{ij}, \quad 1]^T$, when illuminated by a collimated beam with orientation k. Note that the Z_{ij} coordinate has been eliminated because it always is equal to zero. Equation (2) can be rewritten as

$$\begin{bmatrix} x_{ij}^k \\ y_{ij}^k \\ 1 \end{bmatrix} = \begin{bmatrix} \kappa_x r_{11} & \kappa_x r_{12} & \mu_x^k \\ \kappa_y r_{21} & \kappa_y r_{22} & \mu_y^k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{ij} \\ Y_{ij} \\ 1 \end{bmatrix}. \qquad (3)$$

From a set of different Hartmann diagrams (at least two) obtained with collimated beams implying in the SH with different orientations, is possible to calculate the different homographies (H^k) as shown in [4], Chapter 4, pages 87–127. The value of κ_x and κ_y can be obtained from the SH sensor specifications as $\kappa_x = d_x{}^{-1}$ and $\kappa_y = d_y{}^{-1}$, where d_x and d_y are the lateral size of the SH sensor pixel. From the H^k matrix and the κ_x, κ_y coefficients, the r_{11}, r_{12}, r_{21}, and r_{22} parameters of R rotation matrix can be obtained from Eq. (3); the rest of the R parameters can be determined, taking into account that for a rotation matrix it is satisfied that $R \cdot R^T = I$ and $\det(R) = 1$, where I is the identity matrix and $\det(R)$ denotes the determinant operation of the R matrix. To determine the distance between the microlens array plane and the SH CCD plane, it is necessary to know the relative orientations between two incident collimated beams. Note that, if this distance is previously known, the calibration can be performed using a unique collimated beam without need of an SLM device or any additional tool. A scheme of the measuring principle is shown in Fig. 1, which shows the imaging process of two chief rays (chief rays 1 and 2) passing through the microlens M_{ij}. The angle between these rays is χ. These incident rays reach the SH CCD imaging plane at m_{ij}^1 and m_{ij}^2 points, respectively. The distance d in Fig. 1 can be obtained from simple trigonometric relations, taking into account that $\tan(\chi + \xi) = \frac{\mu_2}{d}$ and $\tan(\xi) = \frac{\mu_1}{d}$. If it is assumed that χ or ξ is small, d is given by

$$d = \frac{\mu_2 - \mu_1}{\tan(\xi)}. \qquad (4)$$

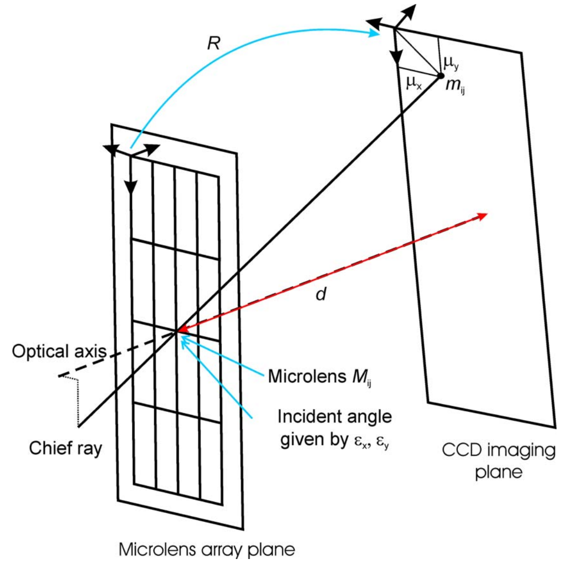

Fig. 2. (Color online) 3D scheme of the SH imaging process.

In Fig. 2 is given a 3D expansion of Fig. 1. Figure 2 shows the rotation matrix R that relates the world and camera

Table 1. Computed Projection Matrices and Root Mean Square Errors

$(\varepsilon_x, \varepsilon_y)(°)$		$(\alpha, \beta, \lambda) = (0, 0, 0)(°)$	$(\alpha, \beta, \lambda) = (0.12, 0.3, 0.5)(°)$
(0, 0)	H^1	$\begin{pmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 99.996 & 0.8732 & -1.2450 \\ -0.8796 & 99.998 & 1.0275 \\ 0 & 0 & 1 \end{pmatrix}$
	rms (mm)	1.0×10^{-18}	2.5×10^{-10}
(0.23, 0)	H^2	$\begin{pmatrix} 100 & 0 & 0.3849 \\ 0 & 100 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 99.998 & 0.8722 & -0.7735 \\ -0.8706 & 99.995 & 2.6823 \\ 0 & 0 & 1 \end{pmatrix}$
	rms (mm)	3.8×10^{-18}	2.3×10^{-10}
(0.12, 0.34)	H^3	$\begin{pmatrix} 100 & 0 & 0.2008 \\ 0 & 100 & 0.569 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 99.999 & 0.8718 & -0.5923 \\ -0.8738 & 99.996 & 2.1053 \\ 0 & 0 & 1 \end{pmatrix}$
	rms (mm)	4.10×10^{-18}	2.3×10^{-10}
(−0.15, −0.65)	H^4	$\begin{pmatrix} 100 & 0 & -0.251 \\ 0 & 100 & -1.071 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 99.997 & 0.8726 & -0.9816 \\ -0.8737 & 99.996 & 2.1087 \\ 0 & 0 & 1 \end{pmatrix}$
	rms (mm)	5.4×10^{-18}	2.2×10^{-10}

Table 2. Recovered α, β, and γ Tilt Errors in Euler Notation

$(\varepsilon_x, \varepsilon_y)(°)$	$(\alpha,\beta,\lambda)=(0,0,0)(°)$	$(\alpha,\beta,\lambda)=(0.12,0.3,0.5)(°)$
(0, 0)	(0 0 0)	(0.4122 0.0320 0.5022)
(0.23, 0)	(0 0 0)	(0.2600 0.4030 0.4992)
(0.12, 0.34)	(0 0 0)	(0.1223 0.4768 0.5000)
(−0.15, −0.65)	(0 0 0)	(0.1213 0.2994 0.5003)

reference frames and the chief ray passing through the microlens center M_{ij} in terms of its direction cosines $(\varepsilon_x, \varepsilon_y)$.

Once the SH sensor has been calibrated as an orthographic camera, it is possible to obtain a wavefront measurement from an aberrated beam. The spot centroids of the Hartmann diagram obtained from this aberrated beam are denoted $\hat{m}_{ij} = [\hat{x}_{ij}, \quad \hat{y}_{ij}, \quad 1]^T$. If a spot centroid $\hat{m}_{ij}$ is arbitrarily assigned to a microlens center M_{ij}, its spot neighbors can be assigned from this initial guess. From this neighbor's expansion procedure, the corresponding identification process can be achieved. Note that if the first corresponding points $\hat{m}_{ij} \leftrightarrow M_{ij}$ are not the true corresponding points, an additional unknown tilt aberration will appear in the wavefront reconstruction. These corresponding points are related by

$$\begin{bmatrix} \hat{x}_{ij} \\ \hat{y}_{ij} \\ 1 \end{bmatrix} = \begin{bmatrix} \kappa_x r_{11} & \kappa_x r_{12} & \hat{\mu}_{x,ij} \\ \kappa_y r_{21} & \kappa_y r_{22} & \hat{\mu}_{y,ij} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{ij} \\ Y_{ij} \\ 1 \end{bmatrix}, \qquad (5)$$

where $\hat{m}_{ij}$ and M_{ij} are previously known and κ_x, κ_y, r_{11}, r_{21}, and r_{22} have been obtained in the previous calibration approach explained above. From Eq. (5), $\hat{\mu}_{x,ij}$ and $\hat{\mu}_{y,ij}$ are determined. Observe that Δx_{ij} and Δy_{ij} in Eq. (1) are equivalent to $\hat{\mu}_{x,ij}$ and $\hat{\mu}_{y,ij}$ in Eq. (5) if the reference is a collimated beam parallel to the SH optical axis. As the distance d has been previously obtained in the calibration procedure, the wavefront error, up to an unknown tilt aberration, is given by

$$\nabla[\Delta W]_{ij} = \left[\begin{pmatrix} \frac{\partial \Delta W}{\partial x} \\ \frac{\partial \Delta W}{\partial y} \end{pmatrix}\right]_{ij} = \left[\frac{1}{d}\begin{pmatrix} \hat{\mu}_x \\ \hat{\mu}_y \end{pmatrix}\right]_{ij}. \qquad (6)$$

To show the effectiveness of the proposed method, we have performed two simulations in code V. One simulation reproduce an SH sensor without tip-tilt error between the microlens and CCD planes. The other simulation simulates an SH with a tip-tilt error with values $(\alpha,\beta,\gamma)=(0.12,0.3,0.5)°$ in the typical Euler notation. The simulated SH sensor is formed by a 5×5 microlens array with a pitch between the microlenses of 100 μm. The distance between the microlens array and the CCD planes is 9.59 mm. In both configurations (with and without tip-tilt error), the Hartmann spot diagram of four collimated beams with different incident angles has been obtained. These incident angles are given in terms of the direction cosines $(\varepsilon^k{}_x, \varepsilon^k{}_y)$, where k denotes the different beam orientations. Table 1 shows the computed projection matrixes and root mean square (rms) errors for the different incident angles and tip-tilt errors. The rms parameter is obtained by the difference between known and computed spot centroid coordinates through the obtained projection matrixes. As can be seen from Table 1, the largest rms value obtained is 2.5×10^{-10} mm. Note that the computed rms values in Table 1 are larger in the tilt error case because of truncation errors. Table 2 shows the recovered α, β, and γ tilt errors in Euler notation. As can be seen from Tables 1 and 2, the values of α, β and γ tilt closest to the true values coinciding with the smallest computed rms error in Table 1. The mean values of α, β, and γ tilt obtained from Table 2 are $(0.22, 0.30, 0.50)°$. Finally, Table 3 gives the computed distances d using Eq. (3). Observe in Table 3 that $\chi^k{}_x = \varepsilon^{k+1}{}_x - \varepsilon^k{}_x$ and $\chi^k{}_y = \varepsilon^{k+1}{}_y - \varepsilon^k{}_y$. Table 3 shows a good agreement between theoretical and computed distance values.

Table 3. Computed Distances d

$(\chi_x, \chi_y)(°)$	$(\alpha,\beta,\lambda)=(0,0)(°)$	$(\alpha,\beta,\lambda)=(0.12,0.3,0.5)(°)$
(0.23, 0)	9.59	9.59
(0.12, 0.34)	9.59	9.69
(−0.15, −0.65)	9.59	9.68

In conclusion, we have presented a novel calibrating method of an SH wavefront sensor. The technique consists of modeling the SH sensor as an orthoscopic camera. This method permits us to obtain the distance, the possible rotation matrix between the microlens array and CCD imaging planes, and the projection matrix H. Calibration of an SH sensor as an orthographic camera introduces a very compact matrix notation and allows the wavefront reconstruction without a corresponding centroid search between the reference and distorted spot diagrams. We have shown a set of simulations in code V that prove the effectiveness of the proposed method.

References

1. D. N. Neal, J. Copland, and D. Neal, Proc. SPIE **4779**, 148 (2002).
2. J. Pfund, N. Lindlein, and J. Schwider, Appl. Opt. **37**, 22 (1998).
3. D. O'Shea, T. J. Suleski, A. D. Kathman, and D. W. Prather, *Diffractive Optics: Design, Fabrication, and Test* (SPIE, 2004).
4. R. Hartley and A. Zisserman, *Multiple View Geometry in Computer Vision* (Cambridge U. Press, 2004).