



Temporal demodulation of fringe patterns with sensitivity change

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Abstract

There are many phase measuring experimental setups in which the rate of temporal phase variation cannot be easily determined. In the case of phase stepping techniques, asynchronous phase measuring techniques were developed to solve this problem. However, there are situations for which the standard asynchronous techniques are not appropriated, like experiments with a sensitivity variation in the phase. In this work, we present an asynchronous demodulation technique for which the only requirement is the monotonicity of the phase in time. The proposed method is based in the computation of the quadrature sign (QS) of the fringe pattern and afterwards the demodulation is performed by a simple arccos calculation, that thanks to the QS extends its range from half fringe to a modulo 2π calculation. The presented demodulation method is asynchronous, direct, fast and can be applied to a general n -dimensional case. We have applied the proposed method to a load stepping experimental fringe pattern obtaining good results.

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1. Introduction

Many full-field optical methods of measurement deliver the desired information codified in the modulating phase of a fringe pattern. The objective of the phase demodulation techniques is the computa-

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tion of the modulating phase (continuous or modulo 2π) from the fringe pattern. One possibility to compute the modulating phase for a given fringe pattern is the introduction of controlled phase changes in order to determine the desired phase from the corresponding observed irradiance variations produced by these changes. Among the phase demodulation methods, the most widely used are the phase stepping techniques [1]. In phase stepping methods, a controlled phase change is introduced between temporal samples (temporal phase sampling) or along a spatial direction (spatial techniques). When the phase steps are known we speak of synchronous techniques. There are many synchronous techniques to deal with problems like noise, errors in the phase steps and non-linear response of the detector. The synchronous phase stepping techniques are direct (a fixed number of operations) and fast due to the simple arctan calculation normally used. When the phase changes are not a priori known, the application of asynchronous techniques is necessary. In these methods, the phase changes are introduced as additional unknowns and the algorithms estimate them together with the remaining parameters (background, modulation and phase). The main assumption in standard asynchronous phase stepping techniques is that there exists a constant but unknown piston term among samples [1]. An interesting point is that although the asynchronous techniques are designed to deal with unknown phase steps, they have always an optimum phase step for which they have the better behavior in front of problems as noise and detector non-linearity. For example, the method described in [2] has a better performance when the phase steps are $2\pi/3$, as the phase steps differentiate from $2\pi/3$ the performance of the technique decreases progressively. To alleviate these problems, in [3] the preliminary results obtained with computer simulations of an iterative self-calibrating technique designed for phase shifting are presented. However, this method assumes constant background and modulation among irradiance samples and is based in the use of a generalized phase shifting whose performance with real fringe patterns has also a dependence on the phase steps. In phase stepping, this is not a big drawback because one usually is able to design the experimental setup in such a

way that the phase steps are near the optimum value. However, there are experimental techniques, as photoelasticity and shadow Moiré, where it is experimentally difficult to introduce a piston term in the phase but the simplest thing is to change the set-up sensitivity, that is, the number of fringes one obtains for a fixed amount of the magnitude to be measured. In this case, the phase changes multiplicatively among temporal samples and the actual phase steps depend on the current value of the modulating phase, that is, the phase steps could range from 0 to 2π across the field of view, making the application of standard asynchronous phase stepping techniques not advisable, due to the mentioned bad behavior far from their tuning points.

Thus, in the case of temporal sensitivity variation it will be desirable to have an asynchronous phase demodulation technique with a good behavior independently of the phase change (out of the singular values of $2n\pi$, n being an integer, for which there is no irradiance change).

Recently, it has been demonstrated that the demodulation of a fringe pattern with arbitrary frequency content is reducible to the computation of the fringe quadrature sign (QS) [4]. The QS of a fringe pattern is defined as the sign of the corresponding quadrature signal. In general, the computation of the QS is neither a linear nor a direct process. However, in the case of sensitivity change the fast and direct computation of the QS is possible and the resulting demodulation method will be asynchronous in the sense that the phase changes should not be known in advance. Once the QS is estimated the phase can be demodulated by a normalization process [5] and a further arccos calculation, being both direct and fast. In the next section, we will present a QS method for the direct temporal demodulation of fringe patterns with sensitivity change.

This work is organized as follows; in Section 2 we will present the theoretical bases of the proposed method. In Section 3 experimental results are shown in the case of load stepping photoelasticity and the comparison with standard asynchronous methods demonstrates the better performance of the proposed method. Finally, in Section 4 conclusions are given.

2. Quadrature sign direct demodulation of fringe patterns with temporal variation of the sensitivity

Numerous full-field optical methods of measurement deliver the result as a phase modulated irradiance signal given by

$$I(\mathbf{r}) = b(\mathbf{r}) + m(\mathbf{r}) \cos \phi(\mathbf{r}), \quad (1)$$

where I is the irradiance, m the modulation, b the background, ϕ the modulating phase and $\mathbf{r} = (x_1, \dots, x_n)$ denotes the n -dimensional position vector. For temporal analysis experiments where a set of images is acquired, $n = 3$ and $\mathbf{r} = \mathbf{r}(x, y, t)$ being t the time and x, y the spatial dimensions. In this model, the modulating phase is associated to the physical magnitude to be measured and the background and the modulation are associated to environmental conditions, as illumination setup or object reflectance. As mentioned above, the objective of the phase demodulation techniques is the computation of the modulating phase $\phi(\mathbf{r})$ in all positions of the field of view.

In first instance, the simplest way to demodulate the phase from the irradiance is to normalize it (background filtering and modulation equalization) and afterwards compute the arccos of the normalized image. In the literature there exist several normalization methods, in our case we recommend the use of the n -dimensional isotropic normalization algorithm described in [5]. If we normalize the irradiance signal of Eq. (1), we get

$$I_N(\mathbf{r}) = \cos \phi(\mathbf{r}), \quad (2)$$

from which the demodulation of the phase is possible by

$$\hat{\phi}(\mathbf{r}) = \arccos I_N(\mathbf{r}). \quad (3)$$

However, due to the even nature of the $\cos(\)$ function the phase obtained by Eq. (3) corresponds to the absolute value of the modulo 2π wrapped version of the continuous phase, that is

$$\hat{\phi}(\mathbf{r}) = |W\{\phi(\mathbf{r})\}|, \quad (4)$$

where $W\{\}$ denotes the modulo 2π wrapping operator in the range $[-\pi, \pi]$. Eq. (4) states that the modulating phase can be recovered directly by the arccos operation only if the modulating phase lies within the range $[0, \pi]$, this is the basis of the

demodulation technique denominated Half-fringe photoelasticity [6]. From Eq. (4), it is clear that the wrapped version of the actual modulating phase is given by

$$W\{\phi\} = \text{sgn}(W\{\phi\}) \cdot \hat{\phi}, \quad (5)$$

where $\text{sgn}(\)$ stands for the *signum* function. If we take the sine of both sides of Eq. (5) and taking into account its odd character

$$\sin(W\{\phi\}) = \text{sgn}(W\{\phi\}) \cdot \sin \hat{\phi}, \quad (6)$$

as by definition $\sin \hat{\phi} \geq 0$ ($\hat{\phi} \in [0, \pi]$) and $\text{sgn}(\sin(W\{\phi\})) = \text{sgn}(\sin \phi)$ we obtain

$$\text{sgn}(\sin \phi) = \text{sgn}(W\{\phi\}). \quad (7)$$

Finally, introducing this result in Eq. (5) we obtain

$$\begin{aligned} W\{\phi\} &= \text{sgn}(\sin \phi) \cdot \arccos I_N \\ &= -\text{QS}\{I\} \cdot \arccos I_N. \end{aligned} \quad (8)$$

The term $\text{QS}\{I\} = -\text{sgn}(\sin \phi)$ is denominated the QS of the fringe pattern $I(\mathbf{r})$ [4].

Eq. (8) indicates that if, we can estimate the QS of a fringe pattern, we can demodulate the phase just by a normalization process and a multiplication in the direct space by the QS. As shown in [5] the normalization process can be achieved by a linear filter, thus in case of direct calculation of the QS the technique proposed by Eq. (8) will be direct, fast and asynchronous in the sense that the phase variations (spatial or temporal) should not be known. Also, an interesting point is that Eq. (8) is a general n -dimensional expression. In the case of temporal sensitivity change, $I(\mathbf{r})$ and the QS will be 3D signals as well as the recovered phase map. However, in general the computation of the QS is a non-direct process [4].

In the case of a temporal experiment with sensitivity change, the modulating phase can be written as

$$\phi(x, y, t) = h(x, y)S(t), \quad (9)$$

and the general expression (1) can be rewritten as

$$I(x, y, t) = b(x, y, t) + m(x, y, t) \cos(h(x, y)S(t)), \quad (10)$$

where $h(x, y)$ is the quantity to be measured and $S(t)$ is a scaling factor that relates the modulating

phase, ϕ , and the quantity to be measured. The factor S is known as sensitivity, and its value depends on the nature of the magnitude $h(x,y)$ and the type of experimental setup (Moiré, interferometry, etc.) employed. For example, in Shadow Moiré topography with collimated observation and illumination $\phi = \frac{2\pi}{p}(\tan \alpha + \tan \beta) \cdot z$, $h(x,y) = z$ is the shape of the measured object and $S = \frac{2\pi}{p}(\tan \alpha + \tan \beta)$, where p is the pitch of the used grid and α and β are the observation and illumination angles, respectively.

If the background and the modulation are temporally smooth, the temporal irradiance gradient can be approximated by

$$\frac{\partial I}{\partial t} = \partial I_t = -m \sin \phi \frac{\partial \phi}{\partial t} = -m \sin \phi \cdot \omega_t, \tag{11}$$

where ∂I_t is the temporal component of the irradiance gradient and ω_t is the temporal instantaneous phase frequency. In the case of sensitivity change, the instantaneous temporal frequency is given by

$$\omega_t(x,y,t) = h(x,y)S_t(t), \tag{12}$$

where $S_t(t) = \partial S/\partial t$.

As in phase stepping techniques, in the case of sensitivity change it is usually possible to fix the sign of the sensitivity variation, that is, it is possible to know the sign of the instantaneous temporal frequency ω_t from the experimental setup (given that the sign of h is known from the physics of the problem). Mathematically if the phase is monotonic in time, the sign of the temporal instantaneous frequency, ω_t , will be known a priori. From Eq. (11), sign of the temporal irradiance gradient and the temporal frequency is related by

$$\text{sgn}(\partial I_t) = -\text{sgn}(\sin \phi)\text{sgn}(\omega_t), \tag{13}$$

and thus the QS can be computed as

$$\text{QS}\{I\} = \text{sgn}(\omega_t) \cdot \text{sgn}(\partial I_t). \tag{14}$$

Eq. (14) indicates that if we have a sensitivity change with known behavior (increasing or decreasing) the phase is monotonic in time, the term $\text{sgn}(\omega_t)$ is known, and QS can be computed directly from the temporal irradiance gradient. It has to be mentioned that Eq. (14) only holds if $\omega_t \neq 0$ (that is, $\partial I_t \neq 0$) because in this case the sign of ω_t is not defined (as well as the QS). Once the

QS is estimated, the wrapped modulating phase can be obtained using Eq. (8). As any standard demodulation technique, the estimation of the QS from the temporal gradient of the irradiance implicitly assumes that the image is well sampled in time that is, $|\omega_t| < \pi$ rad. This technique is that we have denominated QS method for direct asynchronous demodulation of fringe patterns. In this case, the term asynchronous refers to the fact that the explicit knowledge of the phase instantaneous frequency is not necessary but only its sign.

The condition of monotonicity is also the necessary condition for the Fourier transform method [7]. If the phase is monotonic in a given direction, by filtering out the corresponding negative frequencies it will be possible to generate the analytic signal associated with the fringe pattern and demodulate the phase from it. However, in the case that the instantaneous frequency is less than 1–3 fringes/field the discrete Fourier Transform will not have enough frequency resolution and the demodulation with the standard Fourier Transform method will be very difficult. In this sense, the QS method is a rather general approach valid for a wider range of phase variations than the standard Fourier Transform method.

Although the monotonicity condition seems too restrictive, in practice the introduction of a spatial carrier is usually possible that makes the temporal phase variation monotonic under a sensitivity variation. However, it is clear that the monotonicity condition limits the application of the QS demodulation method to general cases with an arbitrary temporal phase variation as deformation measurement by speckle interferometry or lens testing by infinite fringe Moiré deflectometry.

Another characteristic of the asynchronous QS method is that the phase variations (for example, the phase steps in phase stepping techniques) do not appear explicitly like in the standard asynchronous algorithms, where both the phase variations and the wrapped phase are a result. When problems as noise or non-linearities are present, the performance of asynchronous techniques depends on the value of the phase variation between samples. In the presence of additive noise one measure of the performance of any demodulation method is the total modulation of the recovered phase map

that, given the noise level, is a measure of the SNR of the recovered phase map. In general, all the phase extraction methods that deliver the wrapped version of the continuous phase compute the phase map as the arctan of a quotient in the form $W[\phi] = \arctan(f(I_n)/g(I_m))$, where f and g are function, of the measured irradiances. In this case, the total modulation of the phase map is defined as $m_\phi = m \cdot m_D = \sqrt{f^2 + g^2}$, being this modulation composed of two terms, the modulation of the fringe pattern, m – coming from the experimental setup as given in Eq. (1), and the response of the demodulation algorithm m_D .

For example, the Carré technique [8] uses four samples of the irradiance given by $I_j = a + m \cos(\phi + j\Delta\phi)$ with $j = \{-3, -1, 1, 3\}$ where $\Delta\phi$ is a constant phase variation in a given dimension. In our case, as we are interested in the temporal demodulation of fringe patterns with variable sensitivity, $\Delta\phi$ will be the temporal phase variation at every point and will be given by Eq. (12). In this case, it is easy to demonstrate that $m_D = 2|\cos 3\Delta\phi - \cos \Delta\phi|$. This function has a maximum for $\Delta\phi \approx 55^\circ$, that coincides with the well known result that the samples in the Carre method should be separated 110° to obtain optimum performance in front of additive noise [1].

Following the notation used for arctan based techniques, the total modulation of the proposed QS method will be given by two terms

$$m_\phi = m(\text{QS}\{I\}) \cdot m(\arccos I_N) = m_{\text{QS}} \cdot m_N, \quad (15)$$

where m_{QS} and m_N are the modulation responses of the QS and normalization calculation methods, respectively. The term corresponding to the modulation response of the normalization method, m_N , depends on the normalization technique used. In [5], it is shown how the computation of the modulation response of the described isotropic normalization method is possible. On the other hand, the modulation m_{QS} is a binary field that equals 1 if the QS is calculated correctly and zero otherwise.

In the case of additive noise, we can model the measured irradiance as $\tilde{I} = I + N$ where I is the irradiance model given by Eq. (1) and N is a random variable representing additive noise. For the noisy irradiance, its temporal variation, ∂I_t , and

the corresponding temporal phase and noise variations, ω_t and ∂N_t , are related by

$$\partial \tilde{I}_t = \partial I_t + \partial N_t = -m\omega_t \sin \phi + \partial N_t. \quad (16)$$

Thus, $\text{sgn}(\partial I_t)$ and therefore the QS – see Eq. (14) – will be computed correctly from $\text{sgn}(\partial \tilde{I}_t)$ if

$$|m\omega_t \sin \phi| > |\partial N_t|. \quad (17)$$

To find a bound for $|\partial N_t|$, we will assume normal distributed additive noise (the usual assumption for the electronic noise of the CCD detectors) with standard deviation σ , as the noise variation in time can be expressed as a $\partial N_t = N(t) - N(t + \partial t)$, if we assume no correlation between temporal samples, the temporal noise variation will have a standard deviation given by $\sigma(\partial N_t) = \sqrt{2}\sigma$. Therefore, we can estimate the upper bound for the temporal noise variations as $|\partial N_t| \approx \sqrt{2}\sigma$. Introducing this result in (17), we conclude that the QS is correctly estimated for the 68% of the cases if

$$|\omega_t| > \sqrt{2}\sigma/|m \sin \phi|. \quad (18)$$

Note that doubling the limit (2σ) would set the limit for the correct estimation for 95% of the cases.

From Eq. (18), we can derive two main conclusions. First, in the presence of additive noise the QS will be correctly estimated if the phase variations are high enough except in the case $\sin \phi = 0$ that corresponds to the maxima and minima of the fringe pattern. And second, the higher the modulation of the original fringe pattern the better will be the estimation of the QS. Then, for example, for a fringe pattern with a temporal carrier and with additive noise with $\sigma = 5$ gray values, and a modulation of $m = 100$ gray levels, for the correct calculation of the QS from the irradiance temporal gradient it will be necessary that $|\omega_t| > 0.33$ rad/sample assuming that $|\sin \phi| > 0.2$. Therefore, for a temporal series of 10 images a total variation in time of about 0.52 fringes will be enough.

This analysis can be extended to the case of multiplicative noise (for example, speckle noise in correlation fringes) assuming that the additive noise level depends on the fringe pattern irradiance. In this case, $|\partial N_t| \approx \sigma I = A + B \cos \phi$, therefore for

a reliable QS computation we need that $|\omega_t| > A + B/|m \tan \phi|$. From this expression, we obtain the same conclusions for the correct calculation of the QS than in the preceding paragraph.

To give an example of the proposed method, we have prepared a computed simulated spatio-temporal fringe pattern with a sensitivity change given by

$$I(i, j, k) = 100\{1 + \cos[26.6\pi S(k)(x^2 + y^2)/M^2]\} + N(x, y), \quad (19)$$

where $M = 256$, $i, j = -M/2 \dots M/2, k = -1, 0, 1$, $S(k) = \{0.85, 1, 1.15\}$ and N is a normally distributed random variable with standard deviation of 15 gray values. In this case, the instantaneous temporal frequency for $k = 0$ is given by $\omega_t(i, j, 0) = 4\pi(x^2 + y^2)/M^2$, that is, the temporal frequency grows from the center of the image toward the borders. Fig. 1 shows the three noisy fringe patterns used, the sensitivity change is visible in the borders of the image. As in this case we know that $\text{sgn}(\omega_t) > 0$ the QS for $I(i, j, 0)$ is computed from the irradiances using a discrete version of Eq. (11) as $\text{QS} = \text{sgn}(I(i, j, 1) - I(i, j, -1))$. Fig. 2 shows the results of the demodulation process. Fig. 2(a) shows the obtained sign map QS. Once the QS for $I(i, j, 0)$ is obtained the irradiance is normalized and the wrapped phase is computed using Eq. (8). Fig. 2(b) shows the obtained phase map from the three images shown in Fig. 1. In Fig. 2(c), a profile of the demodulated and the actual wrapped phases along the line 128 is shown. From Fig. 2(a)–(c), it can be seen how the performance of the method increases with the phase differences between temporal samples, in this case the center corresponds to the lowest temporal frequencies and therefore the SNR of the demodulated phase is low. In Fig. 2(a) and (b), the region where the temporal frequency is near π is clearly visible as the noisy circular ring of radius 128 px. In the surroundings of this region, the SNR decreases again and there is a phase change of π rad at both sides of this region where, as commented above, the QS computation cannot be done with the proposed method.

In conclusion, the computation of the QS from the temporal irradiance gradient has a very low dependence on the phase increments among tem-

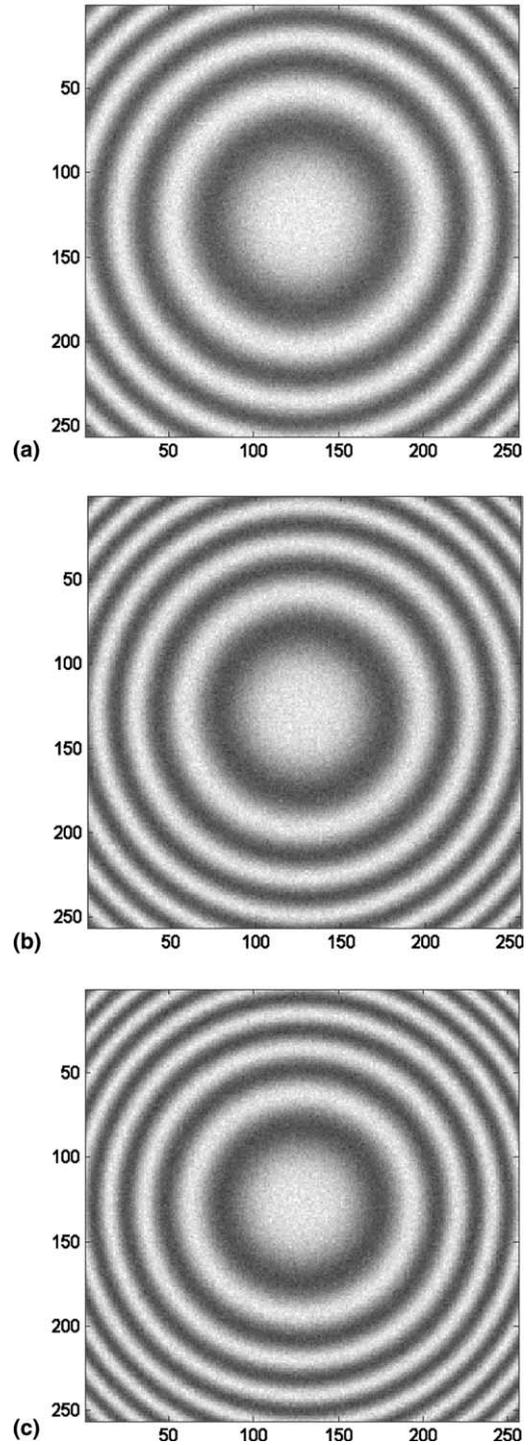


Fig. 1. Simulated fringe patterns to show the performance of the proposed method (see text for details).

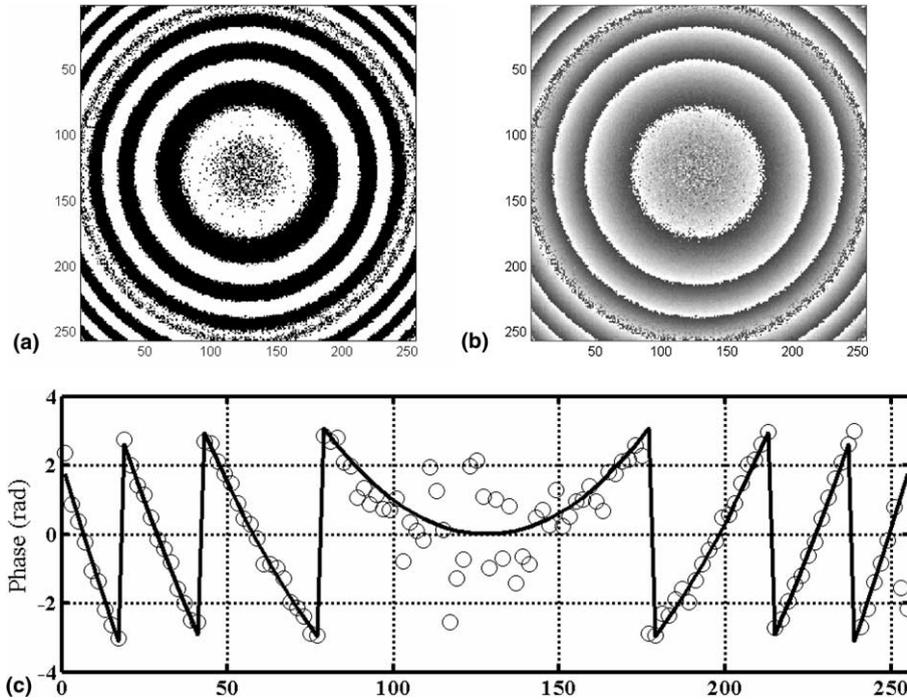


Fig. 2. (a) QS of the fringe pattern of Fig. 1(b) computed from the temporal irradiance gradient of the fringe patterns of Fig. 1. (b) Demodulated wrapped phase map obtained by the proposed method, black and white levels represents, $-\pi$ and π , respectively. (c) Comparison of the actual phase (continuous line) and the estimated phase (circles) for the line 128 px.

poral irradiance samples even in the presence of noise. Also the method does not impose any restriction in the type of phase variation among irradiance samples, as is the case in the standard asynchronous techniques.

3. Experimental results

To test the procedure presented above, we have demodulated the isochromatic phase of a photoelastic fringe pattern in a load stepping experiment [9]. In this kind of experiments, the isochromatic fringe pattern of a photoelastic sample is observed as the load is changing, generating in this way a spatio-temporal 3D fringe pattern with variable sensitivity.

For a circular bright field configuration of a circular polariscope, the isochromatic fringe pattern is given by [9]

$$I(x, y, t) = \frac{I_0}{2} (1 + \cos \delta(x, y, t)), \quad (20)$$

where I_0 is the incident intensity, and δ the isochromatic phase given by

$$\delta(x, y, t) = \frac{2\pi}{\lambda} C_\lambda d (\sigma_1 - \sigma_2), \quad (21)$$

where λ is the wavelength used, C_λ the photoelastic constant for that wavelength, d the sample thickness and $\sigma_{1,2}$ the principal stresses. If the sample is under elastic regimen and the load geometry does not change, it is always possible to write the difference between principal stresses as

$$\sigma_1 - \sigma_2 = G(x, y)F(t), \quad (22)$$

where $G(x, y)$ is a spatial dependent function and $F(t)$ the variable applied load. Thus, the load change is equivalent to a change in the sensitivity of the isochromatic fringe pattern. That is the total phase variation grows (and the number of

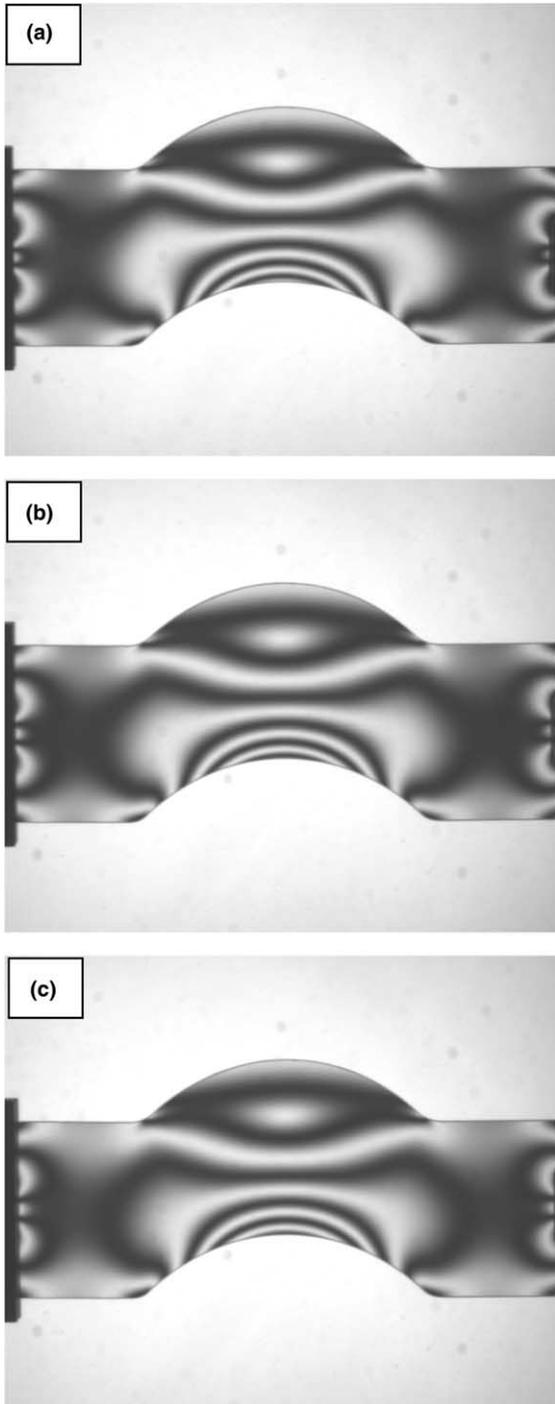


Fig. 3. Isochromatic fringe patterns for a load of (a) 45, (b) 47 and (c) 50 kg.

observed fringes) as the load increases. Thus, if the sign of the variation of the load (sign of the sensitivity variation) is known the sign of the temporal frequency – Eq. (12) – will be a priori known. In the case of load stepping, $\text{sgn}(\omega_t) = \text{sgn}(F_t(t))$ and the QS can be obtained directly from the sign of the temporal component of the irradiance gradient by Eq. (14). It is worth to note that we only need to know the sign of ω_t , but not the exact amount of phase variation the type of phase variation (linear, quadratic).

For the example shown, we have used three temporal samples, $I(k), k = \{-1, 0, 1\}$ of an arc-shaped photoelastic sample subjected to traction load. We have applied an increasing traction load of 45, 47 and 50 kg, thus in this case $\text{sgn}(\omega_t) = \text{sgn}(F_t(t)) > 0$. We have selected a non-linear

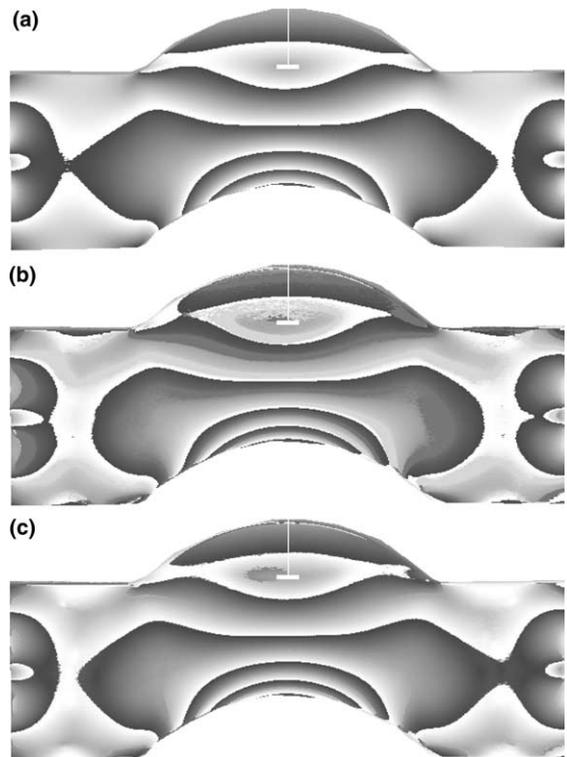


Fig. 4. Wrapped phase maps of the isochromatic image corresponding to Fig. 3(b) (load 47 kg) obtained by a (a) phase sampling method, (b) the three-step asynchronous technique of Servin et al. and (c) the proposed QS technique. Black and white levels represents, $-\pi$ and π , respectively.

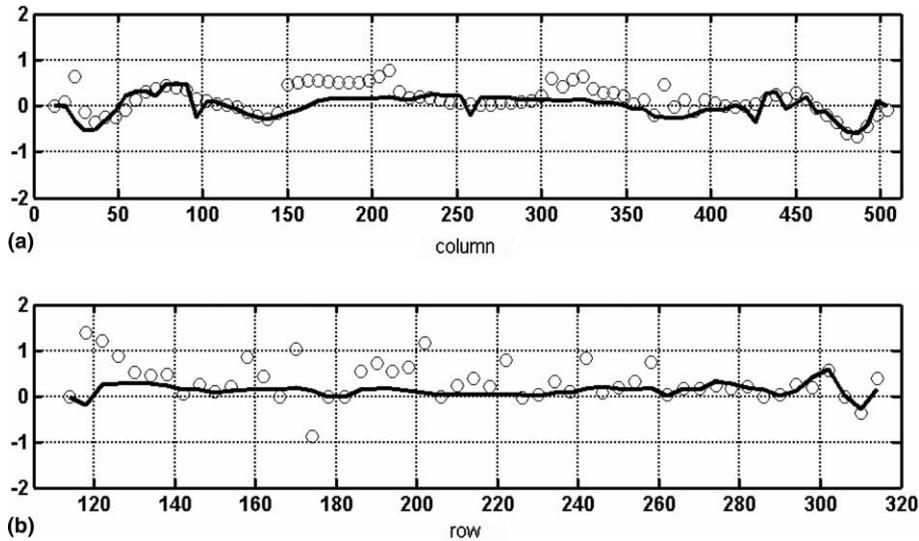


Fig. 5. Profiles along (a) line 262 px and (b) column 262 px of the errors of the three-step (circles) and the proposed QS technique (continuous line) with respect to the phase sampling result shown in Fig. 4(a).

increment of the load to show the performance of the proposed method in front of standard asynchronous techniques that require a constant phase rate. As in the example shown in Fig. 2, we have computed the QS from the temporal variation of the irradiance for $k=0$ as $QS = \text{sgn}(I(1) - I(-1))$.

The sample is an arc-shaped piece under axial traction observed in a circular bright field configuration. The photoelastic constant of the sample is $7000 \text{ Pa m fringe}^{-1}$ and its thickness 3.18 mm . Fig. 3 shows the three isochromatic fringe patterns for each one of the used loads. Fig. 4(a) shows the isochromatic phase map obtained by a phase sampling method [10] used as reference, to obtain this map 6–8 images should be taken in different configurations of the polariscope. Fig. 4(b) shows the phase map obtained from the three load stepping images with a three step asynchronous method [2] and Fig. 4(c) shows the phase map obtained by the proposed method. Fig. 5 shows two profiles along column 262 px and row 262 px of the error with respect to the phase shifting demodulation of the three step asynchronous method and the proposed QS technique. As can be observed in Figs. 4 and 5, the results of the proposed technique are better because of the wider modulation response in front of the temporal frequency and

the better response in front of non-symmetrical phase increment in the irradiance samples. This result is not surprising as the three-step asynchronous (nor any of the standard asynchronous techniques in the literature) is not designed for a sensitivity change, being this application covered the proposed technique.

4. Conclusions

We have presented a method for the demodulation of spatio-temporal fringe patterns with sensitivity change. The method is based in the estimation of the QS of the fringe pattern from the temporal component of the irradiance gradient. The proposed technique is direct, fast and asynchronous in the sense that the temporal frequencies should not to be known in advance. Also, the technique is applicable in any general n -dimensional case just by selecting a dimension for the computation of the QS with a monotonic phase. Although we designed originally the method for the temporal demodulation of variable sensitivity fringe patterns, it can be applied in any general case of phase variation along any dimension for which the phase presents a monotonic behavior, for example phase stepping techniques.

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