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A NOTE ON LINDAHL EQUILIBRIA AND INCENTIVE COMPATIBILITY

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ABSTRACT

We show that if there are Constant Returns to Scale in the production of the public good a) Any Lindahl equilibrium (L.E) is a Nash equilibrium (N.E.) in a price-setting game, b) not all N.E. are L.E., but just those for which the production of the public good is positive and c) the set of L.E. and Strong Equilibria coincide. However if the supply function is continuously differentiable, L.E. is never a N.E. We end the paper with some general comments about the nature of the incentive problem.

(*) An earlier version of this paper benefited from comments of A. Mas-Colell and R. Repullo.

I. INTRODUCTION

Since Samuelson classical dictum (see Samuelson, 1954,p.388), Lindahl equilibrium (L.E.) has been considered as non incentive compatible, since ".. it is in the selfish interest of each person to give false signals". In this note I prove that if constant returns to scale in the production of public goods are assumed, L.E. is incentive compatible. This assumption is often made in some pepers concerned with the existence of a L.E. (see Roberts, 1974,p.33),or with the properties of mechanisms implementing the L.E. (see Hurwicz, 1979 and Vega 1985).

We first prove that any L.E. is a Nash Equilibrium (N.E.) of a price-setting game. The idea behind our proof is very simple. Under constant returns to scale the sum of prices payed by consumers must be equal to the average cost of producing the public good. Hence if a consumer pretends to pay a lower price, no production of the public good is undertaken. Therefore he will end up being worse. So no one exploits the monopoly power that the mechanism gives him. Of course this phenomenon is closely related to limit theorems in Monopolistic Competition. As Hart (1979) proved, in large economies, monopolistic competition is Pareto Efficient because demand function becomes a horizontal line. Hence, even if firms retain monopoly power, it does not pay to exploit it. In our case consumers have no incentive to deviate from Lindahl prices because the supply function is horizontal for any positive production of public good.(+) These reasonings are summarized in Proposition 1.

We also prove that any N.E. with positive production of the public good must be a L.E. (Proposition 2). Therefore in our economy, there are only two types of N.E. Those for which the results is Pareto Efficient, so the incentive problem in fact does not arises and those for which the result is pure free riding, so the incentive problem is quite severe.(*)

An interesting question is how to supplement Nash competition in prices in order to get full Pareto Efficiency in any case. The Strong Equilibrium (S.E.) notion appears to be a natural candidate in these circumstances. In this way we bring cooperation in order to remedy failures of Nash-Competition.

(+) A related phenomenon has been studied by Makowsky -Ostroy (1984) in a different framework (a Walrasian economy).

(*) It may be argued that this last kind of N.E. is not very relevant since it is Pareto dominated by any N.E. which achieves L.E.

We show that L.E. is a S.E. (Proposition 3), i.e. no coalition can upset the L.E. allocation. Moreover if utility functions are strictly quasi-concave, then any S.E. is a L.E. (Proposition 4).

The precedent results may be understood in an intuitive sense as follows: The technological side fulfils the conditions of the so-called "No-Substitution Theorem "(*) (see for instance Arrow- Hahn, ch.2). Therefore consumers are price-takers since prices are independent of demand (however a pure free-rider N.E. usually exists). This may be used to illustrate the danger of an aparently simplificatory assumption, i.e. the usual simple model of a public good economy may well be misleading in order to understand the incentive problem.

The simplest way to depart from the "Non-Substitution Theorem" is to consider non constant returns to scale in the production side. In fact we show that if the supply function of the public good is continuously differentiable, results change dramatically. Any interior L.E. (under differentiability assumptions) cannot be a N.E. (Proposition 5).

Putting together Propositions 1 to 5 we get the impression that continuous differentiability may be a strong requirement in the incentive compatibility issue. (See the Satterwhite-Sonnenschein paper about Dominat Strategies and implementation in a Smooth framework).

However a mechanism has been proposed (see Walker (1981)) which implements Lindahl allocations in Nash Strategies. This mechanism appears to work also for the differentiable case. (i.e. non constant returns to scale). However it must be said that the equilibrium notion proposed by Walker is, in spirit, very different form our's. In the approach presented in this paper, equilibrium is of the variety used in the industrial organization area: strategic agents (here, consumers) maximize their utility functions each assuming that the others will not change their strategies. On the other hand passive agents (here the firm producing the public good) are represented by a function (here the supply function of the firm). Strategic agents maximize taking as given this function which represents the reaction of passive agents.(+)

(*) This means that the assumption of constant returns to scale is by itself not sufficient in order to get results similar to our Propositions 1-4. The assumptions o no joint production and an unique non-produced factor are also needed.

(+) In the industrial organization literature, roles are reversed. Active players are firms and passive agents are consumers.

Therefore equilibrium here has some Stackelberg-like flavour. In Walker approach the equilibrium notion is purely a Nash one. In particular, consumers do not incorporate to their maximization programs the supply functions of the firm producing the public good, as if happens in our approach. Moreover if they do so, Walker mechanism ceases to achieve Lindahl allocations as simple examples can show.

Of course this discrepancy of approaches calls for a discussion on what the issues are in incentive theory. This is done in the last section.

The rest of the paper goes as follows. In section 2 we explain the basic economy and the main definitions. In section 3 we treat the constant returns to scase case and in section 4 we treat the smooth case. Final comments are gathered in section 5.

II;THE MODEL

We follow a standard model in the literature (see Laffont (1984) Hurwicz (1979) or Vega (1985)).There are n consumers,1 private good and 1 public good.The utility function of the i^{th} consumer is $U_i=U_i(y,x_i)$ where y (resp. x_i) is the quantity of the public (resp. private) good consumed by consumer i.Every consumer has a consumption set $X_i \subseteq R^2_+$,an initial endowment of the private good R_i and a share in the profits of the firm producing the public good θ_i.Therefore his income is $=R_i + \theta_i \Pi(p)$ where $\Pi(p)$=maximum profits of the firm and $p = \sum_{i=1}^{n} p_i$,p_i being the Lindahl price of consumer i.There is a unique firm offering the public good.The cost function of the firm is c(y).The competitive supply correspondence is $y(p)=\{y \in R_+ | py-c(y) \geq py'-c(y') \; \forall y' \in R_+\}$. $\Pi(p)=py(p)-c(y(p))$.We will assume that $U_i()$ is strictly increasing on x_i.

Now we will state our main definitions.

Definition 1; A tuple $(x_i^L,y^L,p_i^L)_{i=1...n}$ is said to be a L.E. if $\forall i=1...n$

a) $p_i^L \in R_+$

b) (x_i^L,y^L) maximises $U_i(x_i,y)$ over $x_i+p_i^L y \leq R_i+\theta_i \Pi(p^L)$

c) $y^L \in y(p^L)$

Notice that equilibrium in the private good market is implied by equilibrium in the public good market and Walras Law.

Definition 2; A tuple $(x_i^N,y^N,p_i^N)_{i=1...n}$ is said to be a N.E. if $\forall i=1...n$ (x_i^N,y^N,p_i^N) maximises $U_i(x_i,y)$ over $x_i+p_i y \leq R_i+\theta_i \Pi(p_i+\sum_{j\neq i} p_j^N)$ and $y \in y(p_i+\sum_{j\neq i} p_j^N)$

In words,a N.E. is a list of strategies such that any consumer assuming that the rest of consumers are going to maintain their prices (i.e. they are price-setters) and that the firm is price-taker,finds no profitable to deviate from N.E. Of course other definitions are possible.For instance we can assume that consumers are quantity setters,so they conjecture that any deviation from his part is followed by a reaction which keeps the level of y constant.In this case it is clear that no L.E. can be a N.E. However we feel that Definition 2 captures the Samuelsonian idea about incentives to misrepresents preferences. Indeed a possible interpretation of this Definition is that consumers send marginal rates of substitution.A N.E. is a list of consumptions and marginal rates of substitutions such that no consumer can improve his utility if the rest maintain their marginal rates of substitution.

In the rest of the paper we will assume that $y^L > 0$.This is a purely simplificatory assumption in the case of Propositions 1-4.However Proposition 5 requires $y^L > 0$,but since this Proposition asserts that under smooth requirements no L.E. can be a N.E.,this is not an astringent requirement there.

Finally we define a Strong Equilibrium (S.E.).

A coalition C is a non-empty set of consumers.

<u>Definition 3;</u> $(p_i^s, y^s)_{i=1,\dots,n}$ is said to be a S.E. if $\forall$ coalition C $\nexists$ $(p_j', y')_{j \in C}$ such that

a) $y' \in y(\sum_{j \in C} p_j' + \sum_{i \notin C} p_i^s)$

b) $U_j(R_j + \Pi(\sum_{j \in C} p' + \sum_{i \notin C} p_i^s) - p_j' y', y') > U_j(R_j + \Pi(p^s) - p_j^s y^s, y^s)$ $\forall j \in C$.

c) $(p_i^s, y^s)_{i=1,\dots,n}$ satisfy the budget set of each consumer and $y^s \in y(p^s)$.

III; THE CONSTANT RETURNS TO SCALE CASE

In this section we will assume that $c(y)=c>0$. Therefore $y(p)=0$ if $p<c$, $y(p)=[0,\infty)$ if $p=c$, and it is undefined if $p>c$

Proposition 1; If the technology displays constant returns to scale, then L.E. is a N.E.

Proof; Consider consumer i. For prices higher than p_i^L the supply correspondence of the firm is not defined.(*) So if the L.E. is not a N.E. it must be that he finds profitable to decrease p_i^L. But then only $0\in s(\sum_{j\neq i} p_j^L + p_i')$ for $p_i' < p_i^L$. So it must be that $U_i(x_i^L, y^L) < U_i(R_i, 0)$ contradicting that L.E. is individually rational.

We may also prove a partial converse to Proposition 1.

Proposition 2; Any N.E. with $y^N>0$ is a L.E.

Proof; Since (p_i^N, y^N) is a N.E. it should maximise $U_i(R_i - p_i y, y)$ over $y\in y(p_i + \sum_{j\neq i} p_j^N)$, $\forall i=1\ldots n$.

However if $p_i^N \neq c-\sum_{j\neq i} p_j^N$ we get either y=0 or undefined. Therefore $p_i^N = c-\sum_{j=1}^{n} p_j$ (i.e. is like if agent i were a price-taker).

So in a N.E. y maximises $U_i(R_i - p_i^N y, y)$ over $y\in Y(c)$

But the last constraint is identical to $y\in[0,\infty)$.

Therefore a N.E. is a L.E.

Now we give a simple example in which N.E. is not a L.E.

(*) If we bound the economy in the usual way in order to have y(p) well defined, consumers will not find profitable to increase p_i^L since they are already obtaining all they want at Lindahl prices.

Example 1; Let us assume that the utility function for each consumer is as pictured in Figure 1. Then if all players except i play $p_j = 0$, $y=0$ is an optimal choice for i.
In other words, if prices were at their Lindahl levels, i should pay c/n in order to have y^L units of the public good. However if the rest of consumers pay nothing, i must pay c in order to have some public good, and under some circumstances this may lead to $y=0$

Next Propositions will stablish the identity between the set of L.E. and S.E.

Proposition 3; Any L.E. is a S.E.
Proof. Suppose it is not, so $\exists$ C, $(p_i', y')_{i\in C}$ such that
$y' \in Y(\sum_{i\in C} p' + \sum_{j\notin C} p_j^L)$ and
$U_i(R_i - p_i' y', y') > U_i(R_i - p_i^L y^L, y^L)$ $\forall i \in C$
By similar reasonings to Proposition 1 we have that $\sum_{i\in C} p_i' = \sum_{i\in C} p_i^L$
So if $p_j' \neq p_j^L$ some j, $\exists$ i with $p_i' > p_i^L$.
But then a grafic argument shows that i cannot improve his utility (see Figure 1 and take $p_i' = c$, $p_i^L = c/n$). Therefore $p_i' = p_i^L$ $\forall$ $i \in C$. But then $\nexists$ y such that all agents in C can be made better off.

In order to prove the converse we will need a Lemma.

Lemma; If $U_i()$ i=1...n are strictly quasi-concave and $y^L > 0$, then $y^s > 0$.
Proof; Suppose it is not, so $y^s = 0$. But since L.E. is individually rational we have that $U_i(R_i - p_i^L y^L, y^L) \geq U_i(R_i, 0)$ $\forall$ i=1...n.
If inequality occurs for some agent taking ay^L, $a<1$, he must be better off, so $y^L > 0$ is not his best choice in his budget set.
Therefore all inequalities are strict. But this contradicts

that S.E. is Pareto Eficient.

Proposition 4; If all utility functions are strictly quasi-concave, then any S.E. is a L.E.

Proof; Since $y^L > 0, y^S > 0$ (by Lemma 1). But Proposition 2 implies that any N.E. is a L.E. Since S.E. is a N.E. the proof is complete

IV; THE SMOOTH CASE

We assume the following.

1; $U_i()$ and $y()$ are continuously differentiable functions. $i=1...n$

2; $p_i^N>0$ i=1...n. $y^N>0$ $x_i^N>0$ $i=1...n$

3; $\frac{\partial c()}{\partial y}$ is either a strictly increasing or a stricly decreasing function.

4; If $\frac{\partial c()}{\partial y}$ is strictly decreasing $p=\frac{\partial c()}{\partial y}$ characterizes Pareto Efficient Allocations if $y>0$.

5; $n>1$.

Assumption 4 requires that increasing returns to scale are not "large" relative to the curvature of utility functions. Then we have the following

Proposition 5; Under assumptions 1-5 no N.E. can be Pareto Efficient (P.E.) (so no N.E. can be a L.E.)

Proof; In any interior N.E. we have that

$$\frac{\partial u_i()}{\partial y}\Big/\frac{\partial u_i}{\partial x_i} = \left(\frac{\partial y}{\partial p} p_i^N + y^N - \theta_i y^N\right)\Big/\frac{\partial y}{\partial p}$$

(since $\frac{\partial \Pi}{\partial p} = y$.)

Adding over i we have that $\sum_{i=1}^{n} \frac{\frac{\partial u_i()}{\partial y}}{\frac{\partial u_i()}{\partial x_i}} = \left(\frac{\partial y}{\partial p} p^N + (n-1)\, y^N\right)\Big/\frac{\partial y}{\partial p}$

On the other hand the neccesary condition for P.E. (see Laffont ch 2) is

$$\sum_{i=1}^{n} \frac{\partial u_i}{\partial y}\Big/\frac{\partial u_i}{\partial x_i} = \frac{\partial c()}{\partial y}$$

And both equations give the same solution only if $n=1, y\neq 0$ or $\frac{\partial y()}{\partial p}=\infty$ (i.e. constant returns to scale).

V:FINAL COMMENTS

The essence of our procedure in order to treat the incentive compatibility (I.C.) of the L.E. is to construct a game in which consumers maximize their utility over the supply correspondence of the firm. If we apply this procedure to Walker mechanism we find that in the smooth case, it is not I.C. (in our sense). Indeed Walker approach requires that consumers take the price of the public good as parametric.(*) However I.C. questions arose precisely because this price-taking assumption was questioned! For example in the old discussions about the I.C. of Walrasian Equilibrium, agents were assumed to know not just prices, but supply functions of the other consumers. Indeed Walker acknoledges the point when he writes "..the question whether a potential outcome is I.C. cannot be answered by simply determining whether that outcome is attainable as a Cournot-Nash equilibrium" (p.71). Indeed, as the usual proof of the existence of a Walrasian Equilibrium (W.E.) makes clear, W.E. is some kind of a N.E. in which an auctioneer is introduced in order to balance supply and demand. Moreover, the question of the existence of an auctioneer is not a feature of W.E. with respect to "artificial" mechanisms (i.e. those by Hurwicz, Walker, etc).. In this last kind of mechanisms someone has to send outcome functions to the players. In this sense incentive theory replaces the Walrasian auctioneer (who sends prices) by a more sophisticated kind of auctioneer (who sends functions).

(*) In other words his tax function has p as an argument. Curiously Walker writes these functions as if they were independent of p (q_K in his notation, see eq. (9) pg. 68.).

Returning to the incentive question, in any N.E. we can ask the following question; Given the behavior (not the actions) of the other agents, can any player profitably deviate? Of course, if the answer for each agent is no, we are not only in a N.E. but in a Stackelberg equilibrium (S.E.). However, as Dasgupta-Hammond-Maskin have proved, these generalized S.E. (i.e. S.E. which are N.E.) are essentially equivalents to Dominant Strategies. Therefore in the public good case, the Gibbard-Satterthwite theorem applies (for a proof in the smooth case see Satterthwite-Sonnenschein (1981)). Thus it is apparent that the construction of mechanisms with -weak- I.C. properties is essentially impossible in our case.

The above discussion suggest that I.C. problems occur when some agent-or group of agents- can change profitably his behavior. This change may be caused by the fact that this agent posses a piece of information which was not suppose to be at their hands. Thus, in the Walrasian theory, if consumers knew only prices, price-taking behavior appears to be reasonable. However, if consumers knew supply functions of other consumers, price-taking behavior cannot be considered as rational. In this way, I.C. problems may be related to the lack of privacy (i.e. agents know more than their own characteristics).

On the other hand results in this paper suggest that a comparison of pros and cons between Walker mechanism and the notion of S.E. discussed in this paper may be useful.

(*) These authors have constructed a mechanism which for $n > 2$, implements succesfully Walrasian allocations. However they assume constant returns to scale. Our results suggest that this may be a

In the constant returns to scale case,both mechanisms implement (strongly) the set of Lindahl allocations (with minimal dimensionality of the message space).S.E. requires no auctioneer, it is more in tune with real life market institutions and at every point it is individually feasible.However it requires coalitions-in order to get rid of the pure free-riding N.E.- it is not globally feasible at every point,and it is not clear (in a non monetary framework as our´s) why agents should restrict themselves to consumption points inside their budget sets.

On the other hand Walker approach requires an auctioneer in order to tell people the outcome functions,it represent a break with market institutions (so a lot of practical problems are likely to occur),and it is not individually feasible.Moreover it is not inmune to manipulation via coalitions.However,global feasibility is assured at every point.

In the smooth case both the Walker mechanism and our N.E. do not yield efficient allocations.(Therefore S.E. does not exist)

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FIGURE 1