A Helicopter Control based on Eigenstructure Assignment

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Abstract

In this paper a controller based in the eigenvalues assignment technique is designed. The system to be stabilized is an unmanned helicopter. This eigenstructure based controller is designed considering all the measurable states. A LQ controller has also been applied to the same system in order to compare the responses regarding the robustness. The results are encouraging and prove that the eigenstructure assignment technique is useful for these nonlinear systems.

1. Introduction

Controlling a helicopter is a difficult task due to the complexity of the system. It is strongly non-linear and highly unstable. Besides, it has many parameters to be fixed and they are interrelated. It is an example of a MIMO system where the different inputs and the states are coupled.

Many different controllers have been investigated and some of them give good results [1]. But due to the fact that the number of states is greater than the number of inputs, for practical considerations many of the controllers in the literature are designed to govern only some specific outputs, such as position or velocity tracking [2, 3, 4, 8, 9].

The eigenstructure assignment technique (EA) allows us to design a robust controller [5, 10]. At the same time, it provides the engineer with the degree of freedom that he needs to place the poles in the closed-loop and to decouple some of the modes of the system. In this way, it is possible to fulfil the control requirements.

However, the engineer needs to deeply know the behaviour of the system, i.e., the dynamic of the helicopter in this case. The main difficulty of the controller design lies in the selection of the modes that are to be coupled and which ones need to be decoupled.

The article is structured as follows. Section 2 describes the model of the helicopter. In Section 3 the eigenstructure technique is introduced. In Section 4 a LQR controller is designed in order to obtain some initial values for the EA controller. Section 5 shows the application of the EA strategy to control the helicopter. Some simulation results are shown proving the efficiency of the stabilizer and comparing the responses of both controllers in terms of robustness. The conclusions end the paper.

2. Model of the helicopter

The system is represented by an approximate non-linear model of the helicopter dynamics derived in [6], considering hover and slow flight. Based on the Newton-Euler equations for a rigid solid, the equations of its behaviour are obtained. They are referred to the centre of gravity.

The variables that describe the model are the position vector, $\mathbf{P} \in R^{3,}$ which consists of the three components $\mathbf{P} = [Px\ Py\ Pz]^T$ where [x, y, z] are the scaled Cartesian coordinates of the helicopter's centre of mass. Another model variable is the linear velocity vector, whose components are $\mathbf{V} = [Vx\ Vy\ Vz]^T$.

Furthermore, the rotations are defined by the Euler angles, i.e., $\Theta = [\varphi\ \theta\ \psi]^T$ (*roll*, *pitch* and *yaw* motion, respectively). Finally, it is necessary to consider the helicopter's angular velocity, $\omega = [p\ q\ r]^T$.

The state vector, **x**, of the model of the helicopter is then,

$$\mathbf{x} = [V^T\ \Theta^T\ \omega^T]^T$$

where the positions **P** are obtained by integrating the velocity vector.

The input vector is defined as,

$$\mathbf{u} = [T_M, T_T, a_{1S}, b_{1S}]^T$$

where T_M and T_T are the normalized main and tail rotor thrust respectively, while a_{1s} and b_{1s} are the longitudinal and lateral tilt of the main rotor.

Figure 1 shows the inputs of the system and the different states.

The output vector **y** consists of the three velocities, Vx, Vy, Vz and the heading, ψ. That is,

$$\mathbf{y} = [Vx\ Vy\ Vz\ \psi]^T$$

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The control action is based on these outputs.

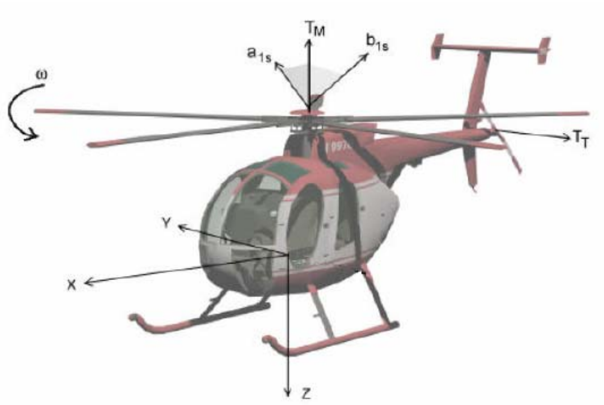

Figure 1. System inputs and states

This non-linear system is driven by the inputs. The inputs contain both the linear and the rotational forces; that means that the linear and the rotational dynamics of the helicopter are highly coupled [11, 12]. This is the main difficulty of the design of a controller for this system. In this sense, the eigenstructure assignment method can help the control engineer in this task.

3. Eigenstructure assignment

The linearized model in hover and slow flight conditions can be expressed in the state space as,

$$\mathbf{x}' = \mathbf{A}x + \mathbf{B}u \qquad (1)$$
$$\mathbf{y} = \mathbf{C}x + \mathbf{D}u$$

where the matrix A represents the internal dynamic of the system. The B matrix expresses the relationship between the inputs and states of the helicopter. The C matrix describes the observable states. In this case, the D matrix is zero.

Initially, the eigenvalues in the open loop give information about which of the modes are coupled and also about the measurement of this coupling [5, 7]. Based on that it is possible to decide which modes are going to be decoupled and it is also possible to construct the eigenvector in the closed loop.

Following the linear matrix algebra, let the n eigenvalues and eigenvectors of the system defined by,

$$\Lambda = [\lambda_1 \ldots \lambda_i \ldots \lambda_n]$$
$$V = [v_1 \ldots v_i \ldots v_n]$$
$$AV = V\Lambda$$

The set of the eigenvectors V is a basis set for the state space, i.e., any vector **x** in the state space can be expressed as a linear combination of the eigenvectors of the system. These eigenvectors are also called the right eigenvectors of the system. The left or dual eigenvectors are given by W, where,

$$W^T = [w_1 \ldots w_i \ldots w_n]$$

$$WA = \Lambda W$$

Solving the state space equations given in (1), the time response of the system is obtained,

$$y(t) = \sum_{i=1}^{n} Cv_i w_i^T e^{\lambda_i t} x_0 + \sum_{i=1}^{n} Cv_i w_i^T \int_0^t e^{\lambda_i (t-\tau)} Bu(\tau) d\tau \qquad (2)$$

In this equation there are two components. The first term depends on the initial conditions of the system. It is called the homogeneous component because it contains the homogeneous term x_0. The second is dependent on the inputs to the system and it is denoted the forced component. In brief, the entire time response of the linear system depends on four variables,

- The eigenvalues
- The eigenvectors
- The initial conditions
- The system inputs

The homogeneous component of equation (2) can be written as,

$$y(t) = \sum_{i=1}^{n} C\alpha_i e^{\lambda_i t} v_i \qquad (3)$$

where α_i are the scalars given by,

$$w_i^T x_0,\ i = 1 \ldots n$$

Therefore, the output of the system is composed of a linear combination of the eigenvalue-eigenvector sets of the matrix A. Each of this set is called a *mode*. The eigenvalue of each mode determines the decay/growth rate of the response and the eigenvector gives the strength of the coupling of this mode with the outputs.

From (3) we can see that the coupling of the i^{th} mode with the j^{th} output is given by $C_j v_i$, where C_j is the j^{th} row of C. If $C_j v_i = 0$ then equation (3) shows that the i^{th} mode does not contribute to the j^{th} output, i.e., they are decoupled.

This analysis is realized in open loop. Then, the control designer can determine the desired eigenstructure. The poles in the closed loop will be placed so that the system fulfils some specifications as stability, etc. At the same time, the modes can be coupled or decoupled by using the eigenvectors.

Let define Λ_d and V_d as the sets of desired eigenvalues and eigenvectors in the closed loop, respectively

$$\Lambda_d = [\lambda_{d1} \ldots \lambda_{di} \ldots \lambda_{dp}]$$
$$V_d = [V_{d1} \ldots V_{di} \ldots V_{dn}]$$

Each one of the closed loop eigenvector indicates, if it is zero, that the parameter is wanted to be decoupled in the closed loop eigenstructure; if it is one, it will be coupled. Finally, "*x*" means that the parameter will not vary. For instance, the configuration of a desired eigenvector in the closed loop is shown in Table I.

Note that the position of the helicopter has not been taken into account as they can be obtained by integrating the velocities.

Table I. Configuration of a desired closed loop eigenvector

States	Mode 1
Vx	1
Vy	x
Vz	x
Pitch	1
Roll	0
Yaw	0
p	x
q	x
r	x

Given a set of desired eigenvalues Λ_d and the corresponding set of desired eigenvectors V_d, the control problem consists of finding the feedback matrix **K** (mxp) of the control equation, being p the number of outputs and m the number of inputs,

$$\mathbf{u} = -\mathbf{Ky},$$

such that the eigenvalues of the closed loop system matrix (**A - BKC**) include Λ_d as a subset.

4. LQ Controller

First of all, in order to compare the proposed EA based controller in terms of its performance and robustness, and also to obtain some initial values of the poles for the EA regulator, a LQ controller has been developed. The LQR has been designed applying the expression,

$$J[u] = \int_0^\infty \left(x^T Qx + u^T Ru\right)dt \tag{4}$$

where Q is the quitar matrix. In its diagonal, the values of the velocities are weighted by 10 and the rest are equal to one.

The weighting matrix R of the LQ controller is shown in table II. It has been obtained taking into account the range of the control signal, i.e, T_M, T_T, a_{1S} and b_{1S}.

Table II. R matrix of the LQ controller

0.011069	0	0	0
0	0.095057	0	0
0	0	1.146	0
0	0	0	1.4323

The eigenvalues of the LQR are listed below (Table III).

Table III. LQR eigenvalues

P_i, $i = 1 .. 9$
-233.17
-137
-10.53
-2.21 + 2.25i
-2.21 - 2.25i
-2.25 + 2.22i
-2.25 - 2.22i
-1
-1.92

The system response when using the LQ controller is shown in Figures 2 and 3. The response is satisfactory as the weighting matrix R has been rightly tuned.

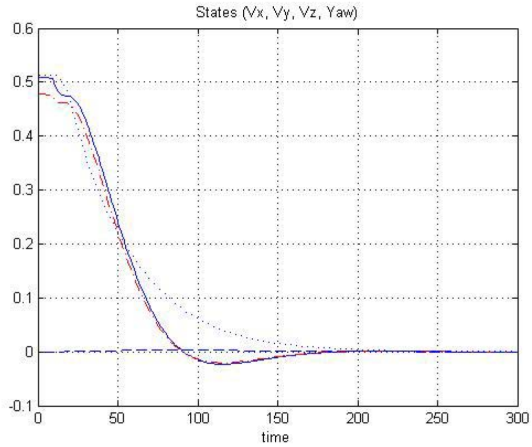

Figure 2. System response with the LQR (Velocities and *yaw*)

Figure 2 shows how the LQR stabilizes the system when it is disturbed from the initial conditions. The states variables that are represented are Vx (solid), Vy (lines), Vz (dots) and the *yaw* angl (line-dot).

In Figure 3 it is possible to see how the system follows the references when they are abruptly changed. The response is satisfactory but it presents overshoot. It is difficult to see the *yaw* behaviour because of the order of magnitude.

Finally, the value of the robustness is computed for the system controlled with the LQR (Table IV) by calculating the gain and phase margins for inputs and outputs [7, 13].

To obtain the robustness the following functions can be used: the sensitivity function $S = (I + L)^{-1}$, and the complementary sensitivity function $T = L(I + L)^{-1}$ or the balanced sensitivity function (S + T), where L is the open loop gain matrix. This sensitivity may be calculated at the inputs and output of the actuator. At the inputs, L = HG and at the outputs, L = GH, where H is the controller transfer function matrix and G is the plant transfer function matrix. We will use the peak value of the maximum singular value (σ). Then, to obtain the gain and the phase margins, the parameter named *a* is calculated by the expression a=1/σ, where $a \leq 1$.

The phase margin is computed by $\pm 2sin^{-1}$ (a/2).

Table IV. LQR robustness

Actuators	σ	a = 1/ σ
Input	3.8922	0.2569
Output	3.9756	0.2515
	Gain Margin (dB)	Phase Margin (deg)
Input	[-1.9862,2.5793]	± 14.7613
Output	[-1.9489,2.5166]	± 14.4502

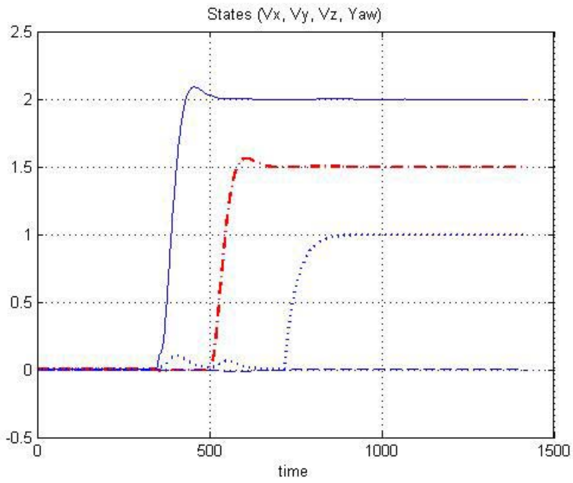

Figure 3. System response with the LQ controller (Velocities and *yaw* for some step changes in the references)

5. Design of the controller based on eigenstructures

The main contribution of this paper is the implementation of the Eigenstructure assignment method to stabilize a helicopter.

The first step in the design of a controller based on eigenstructures is to analyze where in the plane the eigenvalues in open loop are. The eigenvalues obtained are zero in this case so it means that the system is completely unstable and it offers a good opportunity to place all the system eigenvalues wherever we want in order to stabilize the system and to produce a good response according to some specifications.

On the other hand we also need to know which of the modes are coupled and how the performance of the inputs and the states is. So, we have calculated the right eigenvectors in open loop (Section 3) and determined the products Cv_i that are listed in Table V.

Table V. Absolute values of the components of the vectors Cv_i

	φ	θ	ψ	p	q	r
Px	0	1	0.1903	0	1	0
Py	1	0	0.9817	1	0.0008	0
Pz	0	0	0	0	0	1
ψ	0	0	0	0	0	0

It is possible to see in Table V the coupling between the different states and the outputs of the helicopter. For example, the *yaw* motion is strongly coupled with the *y* position, Py, and also with Px. We can also observe that the pitch angle θ, Vx and *q* have a strong influence on the output Px.

In addition to the absolute values calculated by the product Cv_i of Table V, we have also obtained w_i^TB to see the influence of the different inputs on the system states (Table VI).

Table VI. Absolutes values of the vectors w_I^TB

	T_M	T_T	A_{1S}	B_{1S}
Vx	0.0188	0.8103	15.1313	278.3217
Vy	0.0157	0.7521	15.0143	278.3322
Vz	0.1699	3.2050	6.3122	3.1013
φ	0.0442	0.1858	146.0277	7.7991
θ	0.0442	0.1858	146.0277	7.7991
ψ	0.0157	0.7521	15.0143	278.3322
p	0.0157	0.7521	15.0143	278.3322
q	0.0465	0.3361	139.8406	64.7390
r	0.0157	0.7521	15.0143	278.3322

Table VI shows the complexity of the control problem we are dealing with. For example, it is possible to see that the longitudinal tilt of the main rotor is strongly coupled with the roll and pitch states variables. It is also coupled with *q*, the ratio of the angular speed in the *y* axis. This means that when trying to increase the longitudinal tilt to increment the pitch, for example, the rpm of the main rotor decreases and therefore, so does the angular speed.

The solution could be to increase the flow of the fuel in order to increment the torque of the fuselage, and then it will increment the thrust of the tail rotor.

The desired eigenvectors and eigenvalues of the system were chosen knowing part of the dynamic of the system and applying an iterative algorithm [10, 14]. We initially select as desired poles in closed loop the 9 poles that have been obtained with the LQ controller. After that we have applied the iterative algorithm that slightly changes them. At each iteration, the obtained eigenstructure is tested and if the response could be improved on (that is, the response of the closed-loop system is too slow or not very robust) a new iteration begins.

The final desired values of the poles for the EA controller are listed in Table VII.

Table VII. Desired poles in closed loop

P_i, $i= 1 .. 12$
-15
-10
-8
-1.75 + 1.75i
-1.75 – 1.75i
-1.5
-1.25
-1
-1.1

Logically, each desired eigenvalue has its corresponding eigenvector. Besides, we desire to decouple all states.

Once the desired eigenvalues and the eigenvectors have been obtained, the proportional gain matrix K can be calculated by using the eigenstructure assignment algorithm [10]. The values are shown in Table VIII.

Table VIII. Feedback control matrix

	T_T	T_M	a_{1s}	b_{1s}
Vx	1.0988	0.0725	-0.0182	-0.0008
Vy	0.1004	-0.2157	-0.0010	0.0072
Vz	-39.1591	-2.0919	0.0092	-0.0029
φ	1.1405	-3.2700	-0.0149	0.1057
θ	-11.2080	-0.8781	0.2959	0.0139
ψ	-0.0188	0.5102	0.0006	0.0015
p	0.4141	-1.2216	-0.0055	0.0428
q	-4.1740	-0.3056	0.1207	0.0057
r	0.1691	0.7856	-0.0046	0.0032

In table IX we can see the achievable eigenvalues. It can be notice that desired eigenvalues of Table VII were practically achieved and therefore all the poles are now shifted to the left semi plane and the system is stable.

Table IX. Achievable closed loop poles

P_i, $i= 1 .. 9$
-14.9919
-10.0001
-8.0006
-1.7409 + 0.0090i
-1.7409 – 0.0090i
-1.5045
-1.0044
-1.0995
-1.2577

Even though the resulting gain matrix gives a good modal decoupling, the system response is unsatisfactory because of the input matrix B. So, a precompensation gain was added using the following equation,

$$N = -\mathrm{inv}[C(A-BK)^{-1}B]$$

With this matrix the system response is now better and the steady-state error is zero.

The system may be disturbed by any disturbance value Δ that satisfies $1/(1+a) < \Delta < 1/(1-a)$, without destabilising the closed loop system. This can be proved by disturbing the system as it is shown in Figure 4.

Indeed, Figure 4 shows the response when the system starts with different initial conditions from the operation point. As it is possible to see, the EA controller stabilizes the system very quickly, specially the Vz state.

In Figure 5 we can see how the system follows the references values when they are changed by some steps and how it is stabilized for these new values.

The response with the EA controller is smoother than with the LQR as it does not present overshoot but it is still a quick response.

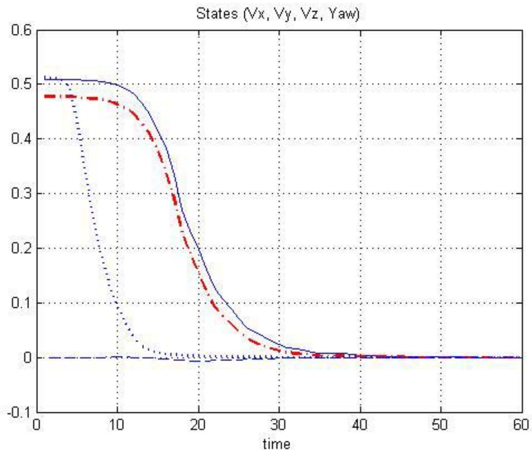

Figure 4. System response with the EA controller (Velocities and *yaw* angle)

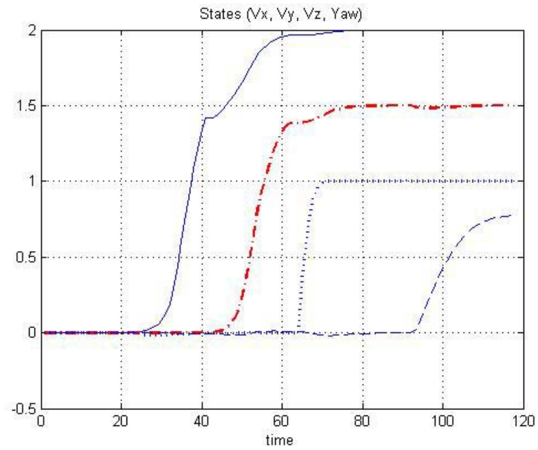

Figure 5. System response using the EA controller (Velocities and *yaw* for some step changes in the references)

Comparing the behaviour of the EA controller (Figures 4 and 5) and the LQ one (Figures 2 and 3) we can see that both of them give good responses but the LQR's is slower, and in addition is less smooth.

The values of the gain and phase margins for the system controlled with the EA regulator are shown in Table X.

Table X. Full states robustness

Actuators	σ	a = 1/ σ
Input	1.4473	0.6910
Output	4.7616	0.2100
	Gain Margin (dB)	Phase Margin (deg)
Input	[-4.5626,10.1995]	±40.4216
Output	[-1.6558,2.0476]	±12.0551

In terms of robustness, it possible to see that the gain margin at the outputs of the LQR (Table IV) is quite similar to the EA one. However, at the inputs, this gain margin is much smaller. The phase margin of the LQR is smaller at the outputs but larger at the inputs than the obtained with the EA controller.

6. Conclusions

In this paper a controller based on eigenstructure assignment has been designed. It has been applied to an unmanned vehicle. The system which was initially unstable has been stabilized. The poles of the system have been placed so that the desired specifications were achieved. In addition, thanks to the flexibility of eigenstructure assignment it is possible to improve the system response using a recurrent algorithm for selecting new eigenvalues and/or eigenvectors for decoupling.

Other controllers such as a LQR have been applied to the system. LQR has a similar response but, in this case, it is slower and more abrupt than the EA controller.

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