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Relativistic many-body approach to exotic and charmed hybrid mesons*

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A relativistic many-body calculation for an effective QCD Hamiltonian is reported for exotic and charmed hybrid mesons. The nonexotic $J^{PC} = 1^{--}$ charmonium spectrum now resolves the anomalous overpopulation of observed J/Ψ states. The exotic hybrid states are near but above 2 GeV, in agreement with lattice and flux-tube calculations, indicating the recently observed $J^{PC} = 1^{-+}$ resonances at 1.4 and 1.6 GeV are not hybrids.

1. INTRODUCTION AND MODEL HAMILTONIAN

A hallmark but unconfirmed prediction of QCD is hadronic bound states consisting of fermions (quarks) and bosons (gluons). A subset of these states, called exotics, have quantum numbers not possible in pure $q\bar{q}$ or qqq quark systems. Accordingly, recent observation by the E852 collaboration [1] of two exotic $J^{PC} = 1^{-+}$ states with masses 1.4 and 1.6 GeV has generated significant interest. These states are definitely not glueballs (oddballs), since they have isospin $I = 1$. They could be hybrid-meson candidates, especially since vintage bag-model calculations [2] predict exotic gluonic excitations in this mass range. However, their structure is still unclear, since more contemporary lattice-gauge [3] and flux-tube theories calculate the lightest exotic hybrid mass to be about 2.0 GeV. The current work provides an additional prediction, using an alternative model to determine if these exotics can be interpreted as hybrids, within a many-body approach that has successfully described both conventional meson [4,5] and glueball [6] systems.

Our model is based upon an effective QCD Coulomb-gauge Hamiltonian

$$H = \int d\mathbf{x}\Psi^\dagger(\mathbf{x})(-i\alpha\cdot\nabla + \beta m)\Psi(\mathbf{x}) + Tr\int d\mathbf{x}(\mathbf{\Pi}^a\cdot\mathbf{\Pi}^a + \mathbf{B}^a_A\cdot\mathbf{B}^a_A) \\ -\frac{1}{2}\int d\mathbf{x}d\mathbf{y}\rho^a(\mathbf{x})V(|\mathbf{x}-\mathbf{y}|)\rho^a(\mathbf{y}) \ . \qquad (1)$$

The respective quark and gluon fields are Ψ and $\mathbf{A}$, $\mathbf{B}^a_A = \nabla\times\mathbf{A}^a$, and $\rho^a = \Psi^\dagger T^a\Psi + f^{abc}\mathbf{A}^b\cdot\mathbf{\Pi}^c$ is the quark plus gluon color density. Our current quark mass, m, has values $m_u = m_d = 5$ MeV and $m_c = 1200$ MeV for the u, d and c flavors, respectively. Confinement and leading canonical interactions are represented by the instantaneous potential, $V = -\frac{\alpha_s}{r} + \sigma r$, with $\alpha_s = 0.2$, and, $\sigma = 0.135$ GeV^2, as determined by the string tension from lattice and Regge fits. We also use a cut-off, $\Lambda = 4 - 5$ GeV, to regularize

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a logarithmic divergence in the mass-gap equation. The parameters produce reasonable hadronic descriptions, including the Regge-trajectory slopes for the mesons (e.g. ρ tower) and glueballs (pomeron) [7].

2. APPROXIMATE DIAGONALIZATIONS USING MANY-BODY TECHNIQUES

We first develop an improved vacuum (ground state) through a BCS transformation (rotation) to a new quasiparticle basis: $\alpha_i^a(\mathbf{k}) = \cosh\Theta_k\, a_i^a(\mathbf{k}) + \sinh\Theta_k\, a_i^{a\dagger}(-\mathbf{k})$, $B_{c\lambda}(\mathbf{k}) = \cos\frac{\theta_k}{2} b_{c\lambda}(\mathbf{k}) - \lambda\sin\frac{\theta_k}{2} d^{\dagger}_{c\lambda}(-\mathbf{k}), D_{c\lambda}(-\mathbf{k}) = \cos\frac{\theta_k}{2} d_{c\lambda}(-\mathbf{k}) + \lambda\sin\frac{\theta_k}{2} b^{\dagger}_{c\lambda}(\mathbf{k})$, where Θ_k, $\theta_k/2$ are the BCS angles, further specified below, and $a(\alpha), b(B)$ and $d(D)$ are bare (dressed) gluon, quark and antiquark Fock operators, respectively. The indices $a = 1, 2...8$ and $c = 1, 2, 3$ denote color, while λ represents spin projection. The BCS vacuum, $|\Omega\rangle$, is given by minimizing the ground-state expectation value of the Hamiltonian variationally with respect to the specific variational parameters ϕ_k, the quark gap angle related to the BCS angle by $tan(\phi_k - \theta_k) = m/k$, and the gluon self-energy, ω_k, satisfying $\omega_k = ke^{-2\Theta_k}$. The variations yield a quark and gluon gap equation (equivalent to the Schwinger-Dyson) with mass gaps of about 100 MeV for the u/d quarks and 800 MeV for the gluon as well as quark and gluon condensates (Cooper pairs), in reasonable agreement with QCD sum rules and lattice measurements. For more complete details, consult Refs. [4–6].

The Tamm-Dancoff approximation (TDA) is utilized for two constituent applications in which a glueball is represented by Fock states $g^\dagger g^\dagger|\Omega\rangle$ and mesons by $q^\dagger\overline{q}^\dagger|\Omega\rangle$. We have also performed random-phase-approximation (RPA) calculations with essentially identical results, except for the light pseudoscalar sector, where only the RPA correctly reproduced the Goldstone-boson nature of the pion due to spontaneous chiral-symmetry breaking by our BCS vacuum. This is also further documented in Refs. [4,5].

The hybrid meson is then formulated as a three-body constituent system, $|hybrid\rangle = q^\dagger\overline{q}^\dagger g^\dagger|\Omega\rangle \equiv [[q^\dagger \otimes \overline{q}^\dagger]_8 \otimes g^\dagger]_0|\Omega\rangle$, now involving quark color-octet states. The resulting TDA equation for the hybrid mass M is $\langle hybrid|[H, q^\dagger\overline{q}^\dagger g^\dagger]|\Omega\rangle = M\langle hybrid|q^\dagger\overline{q}^\dagger g^\dagger|\Omega\rangle$. We also performed an RPA analysis, but again found no difference from the TDA, since the $q\overline{q}$ pair is now in a color octet and our Hamiltonian, as well as exact QCD, does not conserve the chiral color-octet current. The lightest hybrid states will be S waves leading to $J^{PC} = 1^{+-}$, 0^{++}, 1^{++} and 2^{++}, which are all nonexotic. These states can mix with conventional mesons, which will complicate hybrid identification. The first exotic states involve a P-wave excitation and are heavier (note there are two independent orbital excitations in a three-body problem). Imposing the transversality condition, $\hat{\mathbf{k}} \cdot \alpha(\mathbf{k}) = 0$, from the Coulomb-gauge constraint yields 1^{-+}, 3^{-+} and 0^{--}.

As the TDA nonlocal equations are very formidable (a 12-dimensional problem in momentum space), the hybrid mass is evaluated variationally using exponential radial wavefunctions for each of the two independent momentum variables. The problem then reduces to performing 9-dimensional integrals (in the CM frame) that we evaluate numerically using the Monte Carlo code VEGAS. Results are summarized in Figs. 1 and 2.

Notice for both light and charmed hybrid states there is good agreement between our predictions and the lattice and flux-tube results, and that all three disagree with the BNL data. This suggests that the recently measured exotic states are not hybrids. We

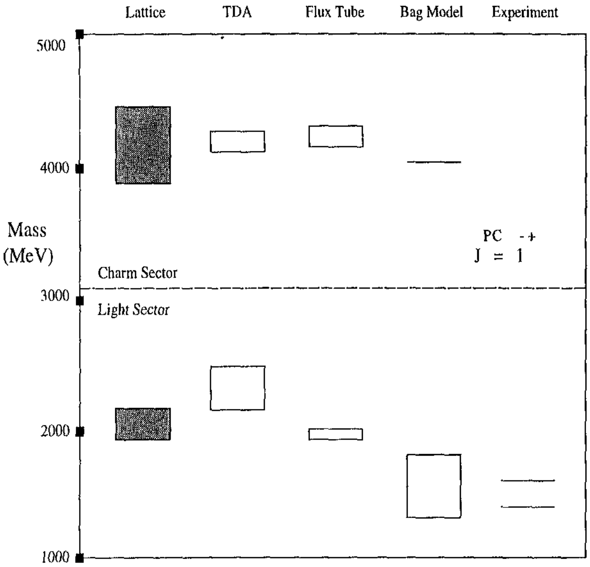

Figure 1. TDA and alternative model exotic 1^{-+} hybrid mesons.

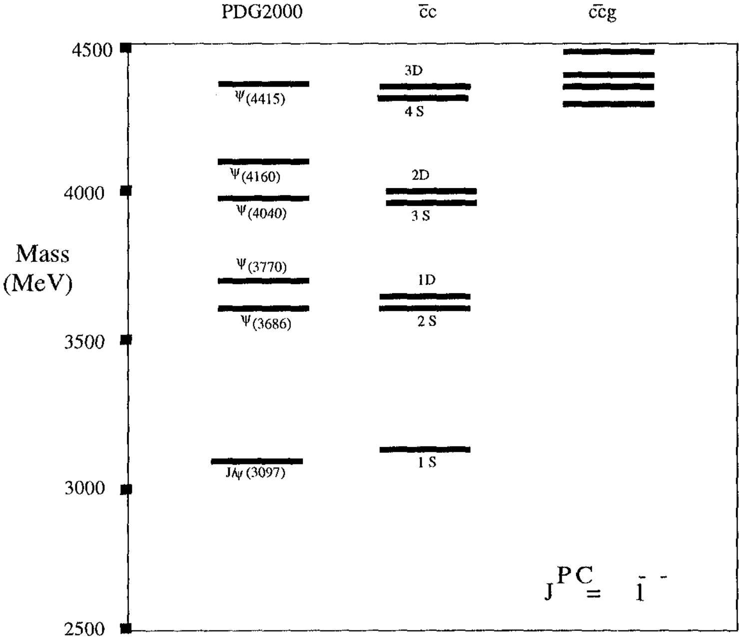

Figure 2. TDA ($c\bar{c}$) and non-exotic ($c\bar{c}g$) states, and the observed J/ψ spectrum.

have performed several checks on our variational calculation including reproducing our conventional meson and glueball exact TDA spectra. Also, in our RPA variational calculation we took the chiral limit ($m_u = m_d \to 0$) producing only a miniscule change in the hybrid mass, consistent with the nonconservation of the chiral color-octet charge discussed above. Finally, we varied the least constrained model parameter, α_s, from 0.2 to 0.4, which yielded at most a 10 % hybrid mass-variation. The box in Fig. 1 represents the maximum hybrid-mass uncertainty from our sensitivity study.

Four-quark states $q\bar{q}q\bar{q}$ can also have exotic quantum numbers and simple estimates yield exotic masses between 1 and 2 GeV when all quarks are in color-singlet configurations. A recent unitary quark-model calculation [8] is also consistent with this, and concluded that the observed 1^{-+} states are indeed predominately meson-meson resonances.

Figure 2 details our TDA $c\bar{c}$ and $c\bar{c}g$ prediction for the, believed to be gluon rich, 1^{--} J/ψ system. Historically there has been an anomalous overpopulation of observed states with respect to S-wave quark-model predictions, which only account for 3 of the known 6 charmonium levels. Interestingly, we have also included D waves and now predict 7 $c\bar{c}$ states in addition to 4 $c\bar{c}g$ hybrids. Further, the most recent PDG2000 summary lists an additional $\psi(3836)$ level assigned $J^{PC} = 2^{--}$, which also agrees with our D-wave prediction (not shown). Hence we have converted the "overpopulation" problem to an "underpopulation" problem, more characteristic of the presently known meson spectrum. Although such states can mix, our current result indicates that simple level-counting arguments will probably not be sufficient to identify charmed hybrid states.

3. CONCLUSIONS

Our key finding is to reaffirm lattice and flux-tube 1^{-+} results, which strongly implies that recent observed exotic states are not hybrids. It is more likely that these resonances are four-quark states and more rigorous, higher-quark-Fock-state calculations are in progress. Perhaps as important, our TDA S- and D-wave J/ψ spectrum is in much better agreement with data, especially in terms of number of states. It is important to note that our quark/gluon unified approach essentially entails only one pre-determined dynamical parameter, and that a more rigorous effective interaction will produce quantitative improvements, but not qualitatively affect our results.

REFERENCES

1. G. S. Adams et al. (E852 Collaboration), Phys. Rev. Lett. **81**, 5760 (1998); D. R. Thompson et al. (E852 Collaboration), Phys. Rev. Lett. **79**, 1630 (1997).
2. T. Barnes, F. E. Close, F. de Viron, and J. Weyers, Nucl. Phys. **B224**, 241 (1983).
3. C. McNeile, hep-lat/9904013 (1999).
4. F. J. Llanes-Estrada and S. R. Cotanch, Phys. Rev. Lett. **84**, 1102 (2000).
5. F. J. Llanes-Estrada and S. R. Cotanch, hep-ph/0009191(2000).
6. A. P. Szczepaniak, E. S. Swanson, C.-R. Ji, and S. R. Cotanch, Phys. Rev. Lett. **76**, 2011 (1996).
7. F. J. Llanes-Estrada, S. R. Cotanch, P. J. Bicudo, J. E. F. T. Ribeiro, and A. P. Szczepaniak, hep-ph/0008212 (2000).
8. S. M. Gerasyura and V. I. Kochin, Phys. Rev. D **62**, 014008 (2000).