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Simple model of compound waveguide structures used as fiber-optic sensors

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Abstract

In this work we present an application of a simple quasi-geometrical model to analyze the behavior of compound waveguide structures used as fiber-optic sensors. This theoretical model is based on the adjustment of the parameters of the structure from the experimental measures to predict the observed behavior of the device. It also takes into account the non-monocromaticity of the used source. In this way, it can be used as a design criterion for this kind of structures. It is applied to a refractive index fiber-optic sensor based on the excitation of surface plasmon in a metal layer by the light guided by a monomode fiber. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Fiber optics; Surface plasmons; Guided waves; Optical sensors

1. Introduction

Surface plasmons can be excited in thin metallic layers by light guided by a monomode optical fiber through evanescent field coupling when the cladding of the fiber is polished and the layer is deposed on it, so that the distance between the core and the layer is small enough [1]. Optical devices based on this principle have been developed in recent years and used for many different applications, including directional couplers [2,3], in-line polarizers [4], sensors [5,6], etc.

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When used as sensors, the structural parameters of the device must be chosen so that the behavior is the expected and the accuracy and range of measurements are guaranteed. In this way, a good knowledge of that behavior is specially important at the design stage.

Different theoretical models have been proposed for these structures, based on field matching [7], mode-coupling [8] or quasi-geometrical approximations [9]. These analyses do not always provide satisfactory results since the agreement with experiment is not always good enough. On the other hand, the most accurate treatments are usually complicated, so they are not well suited to the purpose of establishing design criteria. We present in this paper a theoretical model that permits to overcome both difficulties. It is based on a quasi-geometrical model, thus implying simple calculations, which is convenient for our purposes. Also, it takes into account two of the possible sources of uncertainty that make other models inaccurate. First, it includes an optimized choice for the values of the magnitudes characterizing the structure, these values are usually affected by experimental uncertainties due to the deposing process. Control of thickness during deposition is made by gravimetry and may be thought as accurate enough in principle. But refractive indices are not well known, because the optical properties of thin films are not the same as those of bulk materials, and optical constants of the films can be affected by the deposition process [10]. Another important point that we take into account is the spectral distribution of the light source that is employed. Lasers are commonly used, but for many applications it is most convenient to use LEDs, since these structures are very sensitive to polarization and we can obtain more stable responses if we use depolarized light.

Our model adequately predicts the observed behavior of the devices and can, therefore, be used as a basis for establishing criteria of design for specific uses of the structures. We illustrate its application to a fiber-optic refractive index sensor.

2. Theoretical model

2.1. Basic aspects

The studied structure is shown in Fig. 1. A monomode optical fiber is curved (usually by introducing it in a mould) and polished until the thickness of the cladding is of a few microns. Then, one or more planar layers are deposed. Evanescent coupling between the fiber and the multilayer structure is produced and eventually surface plasmons are excited on the metal layer.

In our case, we use this kind of devices for refractive index measurements [5,6] and for that purpose two layers, a metal layer of aluminum and a dielectric layer of titanium dioxide are deposed. For this or any other use of this kind of structures, the optical and geometrical parameters of these layers must be adjusted to assure that the operating range of the sensor coincides with the desired one. Since the measured parameter is the attenuation of the guided mode of the fiber, we must work in the region where the coupling between the structures is most efficient and we must select the parameters to obtain this. For this, some theoretical calculations must be

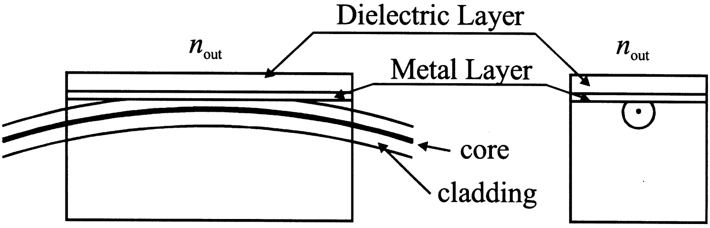

Fig. 1. Scheme of the structures studied in this work. On the polished fiber two layers, one metallic and one dielectric, are deposed.

performed to permit a right design of the device, and these calculations must be supported by a model of the behavior of the structure.

This theoretical problem is not simple, because the structure does not possess any simple symmetry (except for the invariance in the direction of propagation). The resolution of the field equations and the application of the boundary conditions lead to difficult calculations [7] and is not a good approach if we are only interested in obtaining some previous information about the behavior of a structure for designing purposes. Two approximate techniques have been proposed to simplify the problem. First, Marcuse [8] uses modal decomposition in the two simple structures (cylindrical waveguide and planar slab) and studied mode coupling between them. Some modifications of this technique have been proposed, like the one of Ctyrocký et al. [11] applied to planar waveguides.

Another approach is to consider a quasi-ray-optics approximation and to use a planar equivalent for the cylindrical guide and then use common matrix treatment of planar multilayers [3,9].

The performance of all these models depends strongly on the accuracy on the knowledge of the initial parameters that must be introduced in the calculations, namely, the refractive indices and thickness of the different layers. As we have said before, we may trust on gravimetric measurements to know thickness (nominal errors for deposition in vacuum chambers are of the order of 1 nm) and then use tables to determine the refractive indices. Optical methods of controlling deposition usually proceed the other way round, starting from the value of refractive index to deduce thickness. In any case, we must consider as exact the values of the tables. But, usually, what appears in the tables is the value of the refractive index for the bulk material, derived from the conventional refractometric measurements, and it is well known that these values are not valid when working with thin films, since they depend strongly on the thickness of the sample [10].

Another question to be taken into account is the spectral width of the source. All the previous studies consider monochromatic sources, but, as we have said before, the dependence on the polarization of the behavior of the structure makes it interesting, when used as sensors, to employ LEDs as light sources. Since only TM-mode reacts to the presence of the metal layer, we are obliged to control polarization if we are to obtain stable, repeatable results when using lasers as light sources. This is not too convenient if we want to simplify the setup, for instance, in refractive index measuring devices. The use of LEDs provide very good results but, obviously, the spectral width of a LED is not negligible and must be introduced into the calculations. This is very important, since the dependence of the behavior of the device on wavelength is strong.

2.2. Theoretical model

In this paper we present a modification of the quasi-geometrical approach by Tseng et al. [9] that permits to overcome the problems related to the uncertainty in the parameters of the structure and that includes the non-monochromaticity of the source and is, therefore, well suited for its use when dealing with optical sensors of this kind.

We model the compound structure as a multilayer structure by obtaining the planar equivalent of the cylindrical waveguide [3] (Fig. 2) In this way, we face a simpler problem in which the usual matrix treatment for reflectivity calculations in multilayered structures may be used [12]. However, we must then introduce into the matrices the structure parameters, which are, as we have said, difficult to obtain. We need not only the refractive indices and thickness of the deposed layers, but also the remaining thickness of the cladding of the fiber and the length of the zone where significant interactions between fiber and planar layers take place.

To perform the first calculations, we introduce, then, some, say, *nominal* values. These values will afterwards be adjusted by using experimental results, as we will explain later.

The initial value of thickness of the cladding is calculated following Leminger and Zengerle [13] from the value of light attenuation before layers are deposed, that must be measured during the manufacturing process. Effective length of interaction is calculated from the value of the radius of curvature of the fiber in the zone where layers are deposed [2]. This value can be deduced from the measures of the mould employed. The thickness of the deposed layers is obtained from the readings during

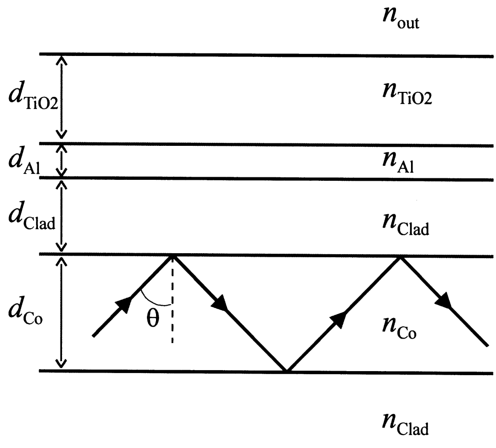

Fig. 2. Planar multilayer equivalent structure.

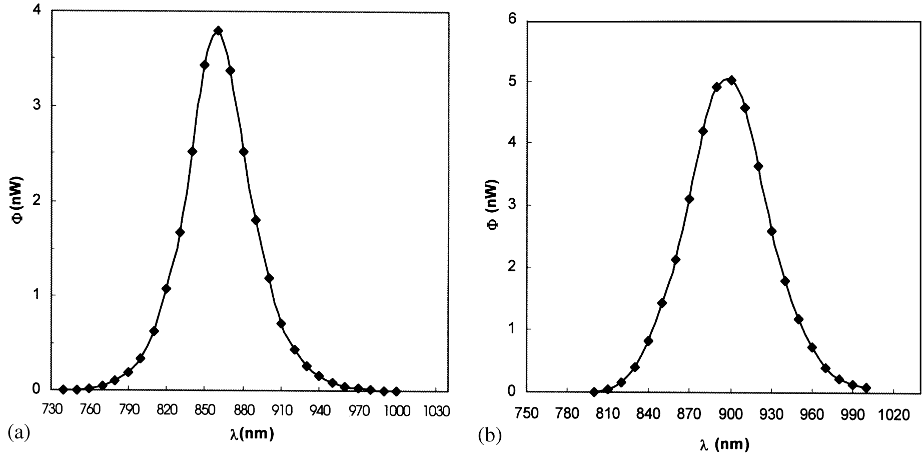

Fig. 3. Experimental setup. Details are given in the text.

the deposition in the vacuum chamber, where it is measured by gravimetry. Finally, the refractive index of aluminum is taken from the table given by Ordal et al. [14] and for the titanium dioxide, the bulk value is take as the initial value [15].

Since we need initial values of refractive indices in a wide range of wavelengths, because the source is not monochromatic, we interpolate these values from the tables when it is necessary. We show in Fig. 3 the spectral emission curve for the LEDs employed. The central wavelengths were 855 and 890 nm and the corresponding spectral widths were 55 and 66 nm.

With this set of initial values, we calculate the reflection coefficient of the structure for the different wavelengths by using the matrix treatment. Details are given in Appendix A. From the matrix of the whole structure,

$$M(\lambda) = \begin{pmatrix} m_{11}(\lambda) & m_{12}(\lambda) \\ m_{21}(\lambda) & m_{22}(\lambda) \end{pmatrix}, \tag{1}$$

the reflection coefficient is given by

$$r(\lambda) = \frac{m_{12}(\lambda)}{m_{22}(\lambda)}. \tag{2}$$

Reflectivity should be, simply, the squared modulus of the reflection coefficient. But, to take into account the finite length of the interaction zone we must estimate a *number of reflections*, N, which would correspond, in this geometrical model, to the number of times that a ray would be reflected between the core and the cladding of the equivalent structure within the interaction zone, so the reflectivity corresponding to one reflection, is powered to N to give account of those successive reflections:

$$R(\lambda) = |r(\lambda)|^{2N}. \tag{3}$$

Finally, to calculate the power reflected in the structure, that is, the amount of light that is reflected back to the fiber and will emerge from it at the end, we multiply the spectral reflectivities by the corresponding optical powers for each wavelength

$$\Phi_r = \sum_i R_i(\lambda_i)\Phi_i(\lambda_i).. \tag{4}$$

From these calculations we obtain a theoretical curve of dependence of reflected power with the refractive index of the outer medium, which is taken as the variable (this is the parameter upon which we are interested if we are to design an optical sensor to measure it). If we assume that the optical power at the output of the fiber is proportional to that value, we will have a theoretical curve that relates the measurable parameter to the variable to be determined.

As we see, the modifications introduced in the approach followed by Tseng et al. [9] are the consideration of the spectral distribution of the source and the use of a number of reflections larger than one. These modifications permit to improve the performance of the method, but are not enough to give a good picture of the observed experimental behavior of the system. This is the reason to use a technique of experimental fitting of parameters that is depicted in the following paragraph.

2.3. Experimental fitting of parameters

To improve the performance of the above depicted theoretical approach we use, then, the experimental behavior of one structure. The obtained results are then applicable to any other structures built with the same materials, at least if the deposition conditions are similar.

To obtain this experimental behavior, we have used the setup shown in Fig. 4. We have used as light source commercial LEDs. The first one (Fig. 3a) was a GaAlAs LED, model Hamamatsu L3989-01, with a central wavelength of 855 nm and a spectral width 55 nm. These values were experimentally obtained by us (using a monochromator and a silicon photodiode) and do not coincide with the nominal values. The other one (Fig. 3b) was also a GaAlAs LED, from opto-diode, model OD 8810, measured a central wavelength of 890 nm, and a measured spectral width of 66 nm.

A polarizer and a collimating lens are used to launch the light into the fiber. After the passing through the sensor, optical power is measured with a Hewlett-Packard optical average power meter, model HP 8152. The sensor is immersed in a solution of ethylene glycol in water whose refractive index depends on the concentration of the components according to the following empirical law [16]:

$$n(T) = n_{H_2O}(T) + 0.111\frac{V_{eth}}{V_{tot}}, \tag{5}$$

where n is the refractive index of the mixture, n_{H_2O} is the refractive index of water, V_{eth} is the volume of ethylene glycol and V_{tot} is the total volume. T stands for absolute

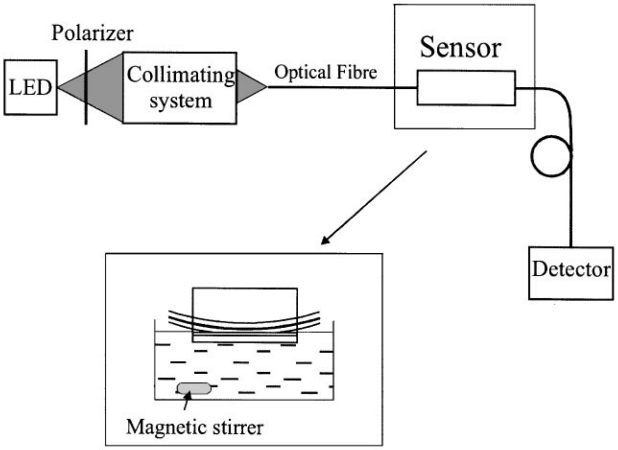

Fig. 4. Spectral distribution of the emission of the two LEDs employed in the work.

temperature, which is controlled during the process. We can then vary the outer refractive index of the structure simply by adding ethylene glycol in a controlled way. A magnetic stirrer is used to ensure a good mixing.

In this way, we can obtain experimental curves of transmitted optical power across the fiber versus outer refractive index.

If we compare these experimental results with the theoretical calculations obtained by considering $N = 1$ and $n_{TiO_2} = 2.5$ (bulk value) we can see (Fig. 5, discontinuous line) that the agreement is extremely poor. Transmitted power has been normalized by dividing it by the value of the transmitted power when the outer medium is air. An analogous normalization has been employed for the theoretical calculations: reflected power is calculated for outer medium with $n = 1$ to obtain the normalization constant in this case.

If we now change the values of N and of the refractive index of TiO_2, taking the refractive index of aluminum as correct, the theoretical curve (Fig. 5, continuous line) significantly changes and begins to approximate the experimental values (dots). We have observed that the change in the refractive index of TiO_2 produces a shift in the position of the minimum of the curve, while an increase in N makes the well wider and lowers the minimum.

In this way, if we use the experimental values, we can perform an optimization process to obtain the best-fitting values for N and n_{TiO_2}. If we use then these values for any structures manufactured in an analogous way, assuming that they are going to be essentially the same, we will obtain a good agreement between the experimental values obtained for these new structures and the results predicted by the theoretical model with the best-fitting parameters.

We complete in this way our theoretical model with the inclusion of reliable initial values in the algorithm. Obviously, this procedure is not limited to the case depicted here and can be used with any other structure employed as fiber-optic sensor, in which

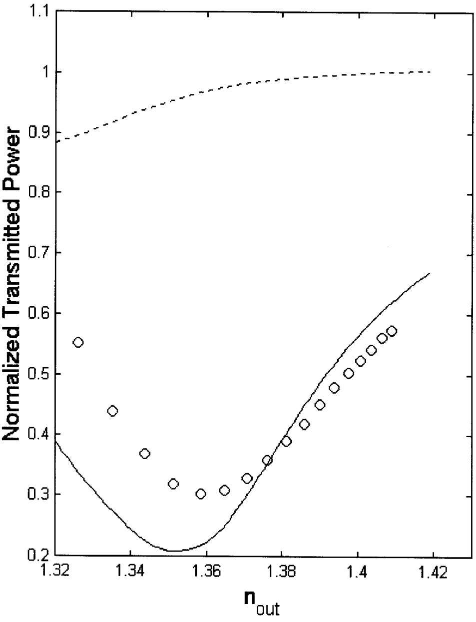

Fig. 5. Performance of the theoretical model. In discontinous line the results for $N = 1$ and $n_{TiO_2} = 2.5$ (bulk value) are shown. In continuous line an improved result obtained with $N = 20$ and $n_{TiO_2} = 2.3$ are shown. Experimental values appear as dots.

any parameter is measured, given that a characterization curve like the one performed by us is obtained.

3. Results

Following the procedure depicted in the previous paragraph we have obtained good adjustments to the experimental values for different structures. In Fig. 6a we show the theoretical curve and the experimental values for a structure with 8 nm of aluminum and 56 nm of TiO_2, taking a remaining cladding thickness of 1.74 μm. The LED employed was the one showed in Fig. 3b. The best fitting was obtained for $N = 20$ and $n_{TiO_2} = 2.25$ for the peak wavelength of 890 nm. In Fig. 6b we use the same LED and we have the same thickness for the layers, but now the cladding thickness is changed to 2.52 μm. We have obtained the best fitting with the same values of N and n_{TiO_2}. This shows that the method works properly, and suggests that a value of 2.25 for the refractive index of the titanium dioxide layer is a better choice than the bulk value for that thickness and wavelength.

Finally, in Fig. 6c we show the results for a structure with 8 nm of aluminum and 47 nm of TiO_2, using the LED whose spectral curve is shown in Fig. 3a. The thickness of the remaining cladding is 2.37 μm. Best fitting is obtained with $N = 20$ and

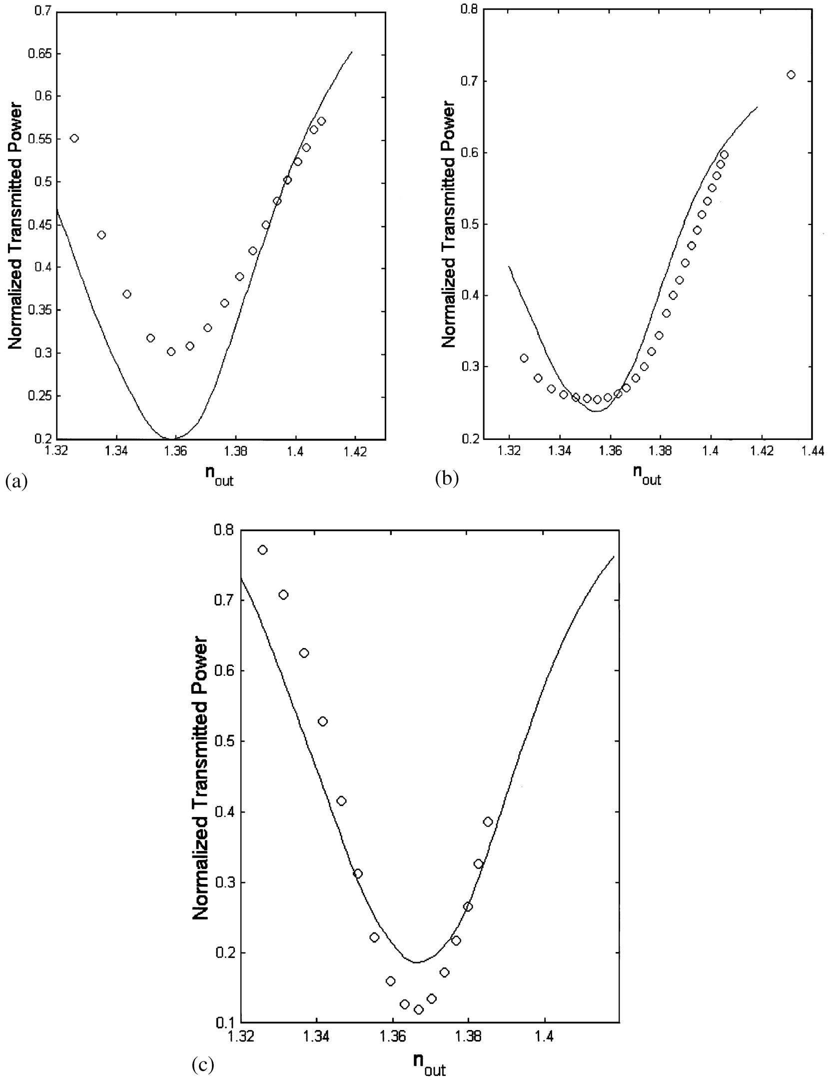

Fig. 6. Experimental results and theoretical curves obtained using experimental fitting for three different cases. (a) $d_{Al} = 8\,\text{nm}$; $d_{TiO_2} = 56\,\text{nm}$; $d_{clad} = 1.74\,\mu\text{m}$; $N = 20$; $n_{TiO_2} = 2.25$. (b) $d_{Al} = 8\,\text{nm}$; $d_{TiO_2} = 56\,\text{nm}$; $d_{clad} = 2.52\,\mu\text{m}$; $N = 20$; $n_{TiO_2} = 2.25$. (c) $d_{Al} = 8\,\text{nm}$; $d_{TiO_2} = 47\,\text{nm}$; $d_{clad} = 2.37\,\mu\text{m}$; $N = 20$; $n_{TiO_2} = 2.3$.

$n_{TiO_2} = 2.3$ for the peak wavelength of 855 nm. It is interesting to note that the value of N is the same in both cases and approximately corresponds to the value that is obtained by calculation using the geometrical model, taking into account the effective length of interaction.

We can observe that the agreement between theory and experiment is good enough in the sense that the main features of the experimental behavior, namely the point of maximum attenuation and the range of outer indices where significant coupling appears. That means that the theoretical model can be used as an aid for the design of this kind of optical sensors. Also, the results can be taken as the starting points for more sophisticated treatments, based on modal decomposition [7,8]. Finally, the modifications introduced by us to the method of Tseng et al. [9] permits to improve their results.

4. Conclusions

We have presented a theoretical and experimental analysis of some compound waveguide structures formed by a cylindrical optical fiber and a series of planar layers deposed onto it. These structures have different applications, but in our case the discussion is centered on optical sensors for refractive index measurements. For these kind of structures, where a metal and a dielectric layers are employed the theoretical, quasi-geometrical, model can adequately predict the experimental behavior when best-fitting parameters from experimental values are obtained. These parameters can be taken as initial values for successive measures with other sensors produced in analogous conditions. The algorithms are easy to implement, since only conventional matrix treatment for multilayers is used.

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Appendix A. Calculation of the matrix of the multilayered structure

From the boundary conditions for the fields in the interfaces between layers we can obtain a transmission matrix between the jth and the $(j+1)$th layers given by

$$T(j,j+1) \equiv \begin{bmatrix} \left(\dfrac{n_{j+1}}{n_j}\right)^2 + \dfrac{V_{j+1}}{V_j} & \left(\dfrac{n_{j+1}}{n_j}\right)^2 - \dfrac{V_{j+1}}{V_j} \\ \left(\dfrac{n_{j+1}}{n_j}\right)^2 - \dfrac{V_{j+1}}{V_j} & \left(\dfrac{n_{j+1}}{n_j}\right)^2 + \dfrac{V_{j+1}}{V_j} \end{bmatrix}, \tag{A.1}$$

where n_i is the refractive index of the ith medium and

$$V_i = k_0\sqrt{n_z^2 - n_i^2}, \tag{A.2}$$

with k_0 the vacuum wavenumber of the incident light and n_z the effective propagation index of the fiber given by the product of the refractive index of the core and the incidence angle, considered as the critical angle in our geometrical model.

Within the jth layer we obtain a propagation matrix given by

$$P(j) \equiv \begin{bmatrix} e^{-V_j d_j} & 0 \\ 0 & e^{V_j d_j} \end{bmatrix} \tag{A.3}$$

where d_j is the thickness of the layer.

Then, the total matrix of the structure is calculated as

$$M = T(3,4)P(3)T(2,3)P(2)T(1,2)P(1)T(0,1), \tag{A.4}$$

where the numbers identify the different media as follows: 0, core; 1, cladding; 2, metal layer; 3, dielectric layer; 4, outer medium.

All the refractive indices and the values V_i are dependent on wavelength, although this dependence is not made explicit here. The refractive indices used for core and cladding are the equivalent ones, as stated in the text.

References

[1] Zervas MN. Surface plasmon-polariton waves guided by thin metal films. Opt Lett 1991;16:720–2.

[2] Digonnet MJF, Shaw HJ. Analysis of a tunable single mode optical fiber coupler. IEEE J Quantum Electron 1982;QE-18:746–54.

[3] Sharma A, Kompella J, Mishra PK. Analysis of fiber directional couplers and couplers half-blocks using a new simple model for single-mode fibers. IEEE J Lightw Technol 1990;8:143–51.

[4] Zervas MN. Optical-fibre surface-plasma-wave polarizers. In: Arditty HJ, Dakin JP, Kersten RT, editors. Optical fiber sensors, Springer Proceedings in Physics, vol. 44. Berlin: Springer, 1989. p. 327–33.

[5] Alonso R, Villuendas F, Tornos J, Pelayo J. New 'in-line' optical-fibre sensor based on surface plasmon excitation. Sensors and Actuators A 1993;37–38:187–92.

[6] Esteban O, Navarrete MC, González-Cano A, Bernabeu E. Measurement of the degree of salinity of water with a fiber-optic sensor. Appl Opt 1999;38(25):5267–71.

[7] Dinleyici MS, Patterson DB. Vector modal solution of evanescent coupler. J Lightw Technol 1997;15:2316–24.

[8] Marcuse D. Investigation of coupling between a fiber and an infinite slab. IEEE J Lightw Technol 1989;7:122–30.

[9] Tseng S, Hsu K, Wei H, Chen K. Analysis and experiment of thin metal-clad fiber polarizer with index overlay. IEEE Photon Technol Lett 1997;9:628–30.

[10] Alvarez-Herrero A, Fort AJ, Guerrero H, Bernabeu E. Ellipsometric characterization and influence of relative humidity on TiO_2 layers optical properties. Thin Solid Films 1999;349:212–9.

[11] Ctyrokỳ J, Homola J, Skalskỳ M. Modelling of surface plasmon resonance waveguide sensor by complex mode expansion and propagation method. Opt Quantum Electron 1997;29:301–11.

[12] Born M, Wolf E. Principles of optics, 6th corrected ed. Oxford: Pergamon Press, 1986.

[13] Leminger OG, Zengerle R. Determination of single-mode fiber coupler design parameters from loss measurements. J Lightw Technol 1985;LT-3:864–7.

[14] Ordal MA, Long LL, et al. Optical properties of the metals Al, Co, Cu, Au, Fe, Pb, Ni, Pd, Pt, Ag, Ti and W in the infrared and far infrared. Appl Opt 1983;22:1099–119.
[15] Palik ED. Handbook of optical constants of solids. New York: Academic Press, 1985.
[16] Alonso R. Estudio teórico y experimental de dispositivos ópticos basados en el acoplamiento entre el modo guiado por una fibra óptica y estructuras multicapa incluyendo medios metálicos. Ph.D. thesis, Spain: Universidad de Zaragoza, 1995.