

A model of ball lightning as a magnetic knot with linked streamers

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Abstract. We develop a topological model of ball lightning which explains its stability by the coupling of an air ball to a magnetic knot, a magnetic field with linked magnetic lines. Assuming that currents flow inside the ball, along short-circuited linked streamers following the lines of $\nabla \times \mathbf{B}$, the lifetime, energy, and radiated power of the average ball are correctly accounted for. The model explains why some witnesses do not feel heat, while others are burnt, and why filaments are seen to trail the ball in some cases.

1. Introduction

Ball lightning is often referred to as the only natural phenomenon still lacking a scientific explanation. It appears usually as a beautiful flaming ball near the electric discharge of a normal lightning, maintaining its shape, brilliance, and size during a time of the order of 10 s, or even longer, after which it disappears suddenly. Typically, its diameter is of 20–30 cm and its radiance less than 200 W; it can be red, orange, bright white, bluish, or even green. Several explanations have been proposed, but none is generally accepted.

Here we develop a topological model of ball lightning which explains its stability by the coupling of an air ball at 16,000–18,000 K to a magnetic knot, i.e., a magnetic field with linked magnetic lines. Assuming that currents flow inside the ball, along short-circuited linked streamers following the lines of the curl of the magnetic field, the lifetime, energy, and radiated power of the average ball are correctly accounted for.

The main difficulty in understanding what happens in the balls is to find a reason for their surprising stability which makes them last so long [Singer, 1971; Turman, 1977; Barry, 1980a, b; Ohtsuki, 1989; Singer, 1991]. Besides, the emission of light suggests that something is hot inside them, but while hot air expands and moves upward, ball lightnings do not change their size and have a tendency to move horizontally at a leisurely pace. Furthermore, there is a curious contradiction in the re-

ports of witnesses. For some, ball lightnings are cold, since they did not feel heat when one passed nearby [Jennison, 1969], while others were burned and had to receive medical care after touching one, fires being also produced in some cases. Because of this, the view has been expressed that there are different types of phenomena under the same heading, so that no single model would be able to account for all of them. However, this model gives an explanation to this discrepancy.

In a recently proposed topological model of ball lightning [Rañada and Trueba, 1996], this phenomenon is assumed to be a ball of completely ionized plasma coupled to a magnetic knot, i.e., to a magnetic field with linked magnetic lines (in this paper “linked magnetic lines” refers to situations in which any pair of magnetic lines has nonzero linking number); the streamlines of the plasma are also linked, so the system is very tangled. This model (hereinafter referred to as I or the topological model) explains qualitatively the long duration of the fireball as a consequence of the stabilizing effect of the linking or, equivalently, of the constraint imposed by the conservation of the magnetic helicity.

Since it accounts for the main difficulty, the model seems promising as a first approach; however, it has two drawbacks. First, it assumes that the temperature is at least 30,000 K in order for the ball to be completely ionized and that it radiates according to the Stefan law; however, the corresponding radiance turns out to be much larger than observed. Second, although the ball turns out to be stable, it expands somewhat in the model.

This paper proposes a new and more realistic version of the same model which is free from these two prob-

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lems. It explains also the above mentioned contradicting reports of witnesses: although a very small part of the ball is indeed hot, the overall radiation is small, no more than that of a home electric bulb. Consequently a fire can be started or a person be burnt if there is contact, but they do not produce any feeling of warmth if there is not, even if the observer is particularly close.

2. The Model

The air does not conduct as a continuous medium. Quite on the contrary, it is well known that lightning or arc discharges proceed along lines, separated from one another. When a spark jumps across the air between two conductors, very narrow channels of highly ionized air, the so-called streamers, are formed, the charges moving inside them [Raether, 1939; Gallimberti, 1988]; they are, in fact, thin tubes of highly conducting plasma. The current is then very large inside the streamers but zero outside. Their diameters are in the range 50 μm –100 μm .

In this version of the topological model, the stability of the ball is still due to the linking of the force lines (to any pair of them having nonzero linking number or, equivalently, to the conservation of the magnetic helicity), but there are some important differences with I: (1) electric currents flow inside the ball, concentrated in well-separated linked streamers, which occupy a very small fraction of the ball volume; (2) these streamers are hot, in the temperature range of 16,000 K–19,000 K, the air being very ionized and the resistivity very low inside them; the rest of the ball, most of it in fact, is cold and not ionized: it remains at ambient air temperature; (3) the air and the plasma are poor radiators, so the streamers do not follow the Stefan law; they radiate instead a combination of bremsstrahlung and emission from atomic transitions, as is shown experimentally to happen in arc discharges [Evans and Tankin, 1967].

More precisely, we assume that ball lightnings are formed as follows: Near an ordinary lightning discharge, where many streamers are formed in a very rapid process, the joint effect of powerful electric and magnetic fields can cause some streamers to short-circuit, forming closed linked loops which behave, in fact, as highly conducting coils. This leads, at time zero, to a structure characterized by (1) a magnetic knot with linking number n coupled to (2) a set of linked streamers along the lines of the current $\mathbf{j} = \nabla \times \mathbf{B}/\mu_0$, which are, in fact, filamental linked tubes of current ($\mu_0 = 4\pi \times 10^{-7}$ Wb/Am is the vacuum magnetic permeability).

The actual formation of closed streamers of conducting plasma might have seemed a weak, unproved assumption. However, after developing this model, we learned about some experiments in which closed streamers have been observed. Alexeff and Rader produced ultrahigh voltage discharges and observed that above ~ 10 MV, closed loops were formed (see their photographs in the work of Alexeff and Rader [1995]). They consider them as possible precursors of ball lightnings. That experiment is thus a support of the ideas we propose here. Note also that this could explain why ball

lightnings are so rare: they need specially strong discharges above some definite high threshold. In the case of the present model, the closed loops are furthermore linked.

When one looks to a bulb filament, it is not possible to distinguish it clearly; quite on the contrary, one sees a diffuse luminous patch without precise borders along it. A ball of linked streamers should appear therefore as a uniformly diffuse shining ball if enough of them are present.

An electromagnetic knot is a solution of the Maxwell equations in which any pair of magnetic (and any pair of electric) lines is a link, a pair of linked curves [Rañada, 1990; Rañada and Trueba, 1995, 1997]. A knot is thus characterized by two integers, which are the corresponding linking numbers, but for simplicity, we will consider here only magnetic knots (without electric field).

As an example, we will use the following magnetic field inside a sphere, whose lines are linked n times; it vanishes outside the ball of radius L , and inside it is equal to (in spherical coordinates r, ϑ, φ)

$$\mathbf{B} = -\frac{\sqrt{a} \sin^2(\pi R)}{\pi L^2 R^2} \times [n \cos \vartheta \mathbf{e}_r - n\pi R \cot(\pi R) \sin \vartheta \mathbf{e}_\vartheta + \pi R \sin \vartheta \mathbf{e}_\varphi], \quad (1)$$

where \mathbf{e}_k are the unit vectors along the coordinate lines, $R = r/L$, L being the radius of the ball (note that $\mathbf{B} = 0$ at $R = 1$) and \sqrt{a} a normalizing constant measured in tesla times square meter. This expression has been obtained by the method expounded in references [Rañada, 1995; Rañada and Trueba, 1997], in such a way that the magnetic lines are the level curves of the scalar function $\Phi = \sin \vartheta e^{in\varphi} / (\cos \vartheta - i \cot(\pi R))$.

If this field is coupled to a current, its value is $\mathbf{j} = \nabla \times \mathbf{B}/\mu_0$, or

$$\mathbf{j} = \frac{\sqrt{a} \sin^2(\pi R)}{\pi \mu_0 L^3 R^3} \times [-2\pi R \cos \vartheta \mathbf{e}_r + 2\pi^2 R^2 \cot(\pi R) \sin \vartheta \mathbf{e}_\vartheta + n \sin \vartheta (\pi^2 R^2 \cot^2(\pi R) - \pi^2 R^2 - 1) \mathbf{e}_\varphi]. \quad (2)$$

It is easy to show that the lines of \mathbf{j} are also linked n times. The system is thus very tangled.

However, the air does not conduct as a continuous medium but along streamers, unless it is completely ionized what requires a higher temperature. Consequently, a lightning cannot result in a magnetic field characterized exactly by (1) and (2). Nevertheless, one knows that magnetic fields produced by a continuous distribution of currents can be quite similar to those produced by a set a close filamentary current tubes. Of course, it could be difficult to start constructing a simple distribution of streamers leading to a satisfactory model, although such distributions must exist, the magnetic field deviating little from (1) and the streamers from the lines of (2). Thus we assume that some streamers of a normal lightning short-circuit and form closed linked loops, which behave as highly conducting coils,

producing a magnetic field very similar to (1) with the same linking number.

However, this state cannot be really stationary, since the magnetic energy

$$\begin{aligned} E &= \int \frac{B^2}{2\mu_0} d^3x \\ &= \frac{a\pi}{6\mu_0 L} \left(n^2 \left(1 + \frac{4Is(2\pi) - 2Is(4\pi)}{\pi} \right) + 3 \right) \\ &= \frac{(0.9716n^2 + \pi/2)a}{\mu_0 L}, \end{aligned} \quad (3)$$

where $Is(x)$ is the sine-integral function, decreases by expanding the radius of the ball L .

In the case of infinite conductivity, the evolution of the ball is constrained by the linking number, since the helicity integral, a term coined by Moffatt, is then a constant of the motion [Moffatt, 1969, 1978; Moffatt and Ricca, 1992; Marsh, 1996]

$$h = \int \mathbf{A} \cdot \mathbf{B} d^3x = na = \text{constant}. \quad (4)$$

This constraint blocks many channels for the ball decay, precluding, for instance, the relaxation with $a = a(t)$ going to zero. However, when the resistivity η is not nil, the time derivative of the helicity is

$$\frac{dh}{dt} = -2 \int \eta \mathbf{j} \cdot \mathbf{B} d^3r. \quad (5)$$

For practically stationary configurations with slowly varying magnetic fields \mathbf{B} as is our case, $\mathbf{E} = \eta \mathbf{j}$; moreover, we have assumed above the simplest case in which $\mathbf{E} = 0$. Therefore, h is conserved in this case which corresponds to the plausible situation in which, after a discharge caused by the very strong electric fields of a normal lightning and the formation of a fireball, \mathbf{E} dies out and vanishes rapidly. The physical situation then resembles that of a superconducting coil which can maintain the current for a long time because η is practically zero in the conductor (where \mathbf{j} is large), and $\mathbf{E} = 0$ outside (since \mathbf{j} vanishes). In the ball lightning, the highly conducting streamers take the place of the superconducting coil, while no current flows in the space outside the streamers. Therefore h is conserved to a high degree, this being the stabilizing factor which explains the long lifetime of the ball in this model.

Looking for allowed decay processes, we find, as mentioned in I, the natural expansion $L = L(t)$ in which \mathbf{B} would decrease with $L(t)$, so according to (4) the conservation of h is not violated. This is the decay process we consider below, and we find that it is slow and compatible with the observations.

Other more involved kinds of decay which would be in principle possible, such as substituting in (1) L_0^k/L^{k+2} for $1/L^2$ with $L_0 = L(0)$ and take L to evolve with time, would provide the same initial magnetic field but a violation of helicity conservation if it is nonzero. However, if $h = 0$, that is if the linking number is zero, the helicity is conserved in all these expansions. Therefore the lifetime must be shorter if $h = 0$, since there are

then many more open decay modes. More precisely, we will show below that a ball with lines of zero linking number is very unstable against the expansion with k near $-1/2$, the expansion proceeding very rapidly with a very short lifetime.

Note that in knot theory there are some configurations with zero linking number but without absence of "linkage", as the Whitehead link or the Borromean rings [see Marsh, 1996, p. 66; Trueba, 1997]. Should such structures be formed, they would have zero linking number and zero magnetic helicity and could be treated as any other nonlinked configuration with respect to the decay processes.

Consequently, the knot expands to lower its energy by increasing its radius $L = L(t)$ (note, however, that the expansion turns out to be very small, just a few percent, as will be seen). We further assume that the expansion is adiabatic; as the air inside the streamers is a monoatomic gas at the temperature that we are considering, its adiabatic parameter is $\gamma = 5/3$, the temperature varying then as $T = T_0 x^{-2}$, with $x = L/L_0$.

3. Discussion of the Model

According to Smirnov [1989] the average values of the diameter, power emitted, and lifetime of ball lightning are $2L = 28 \pm 4$ cm, $P = 113 \pm 16$ W, and $\tau = 10^{0.95 \pm 0.25}$ s, respectively. To test the model, we will consider therefore the case of a ball of radius $L = 15$ cm, emitting a power $P = 100$ W, and calculate its lifetime. We assume radiation emission at local thermodynamic equilibrium (LTE) and take conveniently the data from Argon plasma torch measurements, the most extensively studied case, where the experimental results are best known [Evans and Tankin, 1967], as described in Figure 1. Equivalent data in air are known to differ no more than 10%, which is acceptable at our precision level. A part of the radiation is bremsstrahlung; the rest comes from atomic lines between excited states, from excited to the ground state, and transitions from the continuum. Note the shoulder between about 15,500 K and 18,000 K where the power is almost independent of the temperature. Also note that 1 cm³ of air at this temperature range emits about 5500 W.

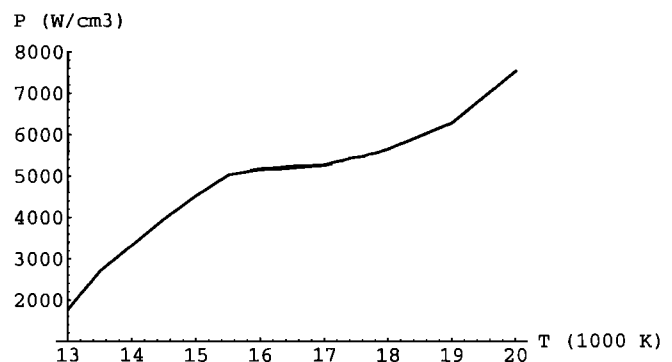


Figure 1. Power density versus temperature $P(T)$ emitted by a plasma torch, according to [Evans and Tankin, 1967].

Assuming that the streamers inside the ball stay within that temperature range, the power radiated will be almost constant as far as the system remains in the shoulder, even while the streamer temperature decreases. This explains in our model the amazing constancy of the brightness of ball lightnings.

The streamers occupy a very small part of the ball volume. Assuming a temperature of 18,000 K, as 1 cm³ of air emits 5500 W, if the power is 100 W, the volume of the streamers must be 1/55 cm³: just a proportion of about 1.2×10^{-6} of the ball volume is ionized. As the streamer diameter is in the range 50 μm–100 μm, their total length is between about 200 cm and 800 cm. For a radius of 15 cm we can estimate their average length at about 30 cm, so there are between about 7 and 27 closed loop streamers; each one will emit between about 15 W and 4 W.

Taking all this into account, this model explains ball lightning as follows: Let a system, as is described here, be formed at $t = 0$. We take as characteristic magnetic field $B_0 = ((0.97n^2 + \pi/2)a)^{1/2}/L_0^2$; the average field is then $(B(0) + B(L))/2 = B_0 n \pi / \sqrt{4n^2 + 2\pi}$ (this is close to B_0 for $n = 1$ and $\pi B_0/2$ for large n). The energy (3) can be written then as $E = B_0^2 L_0^3 / \mu_0 x$, where $x = L(t)/L_0$ is the radius divided by its initial value. Because of the condition of adiabatic expansion $T = T_0 x^{-2}$, the energy can be written as

$$E = \frac{B_0^2 L_0^3}{\mu_0} \left(\frac{T}{T_0} \right)^{1/2}. \quad (6)$$

The system loses energy according to $dE/dt = -P(T)V$, $V = 4\pi L_0^2 x^2/3$, from which it follows that $TdT/P(T) = -dt/(\gamma B_0^2)$, with $\gamma = 3/(8\pi\mu_0 T_0^2)$.

In other words, the temperature evolves in time according to the law

$$-\gamma B_0^2 \int_{T_0}^T \frac{TdT}{P(T)} = t. \quad (7)$$

Note that this equation does not depend on the initial radius L_0 .

The resulting curve $P(t)$, power radiated versus time, is plotted in Figure 2, for $T_0 = 18,000$ and three values of the magnetic field B_0 . The lifetime of the ball is $\tau = 2.5B_0^2$, since the power remains at less than 10% of the initial value 100 W until that time and falls quickly thereafter, as it does happen with the observed balls. As is known, the magnetic field can reach several tesla near the discharge of a lightning. If $B_0 = 1.9$ T, the lifetime in this model for radius equal to 15 cm is 9 s, precisely equal to the observed average value according to *Smirnov* [1989].

The expansion until $t = 2.5B_0^2$ turns out to be only $x = 1.06$; this means that the diameter passes from 30 cm to less than 32 cm, a change hardly noticeable since the ball rim is slightly diffuse, not a clear-cut line.

The average energy of the ball is about 20 kJ, according to *Smirnov* [1987, 1989]. In this model, the initial energy of the average case is $E = 2685 B_0^2$ J. For $B_0 = 2$ T, this is about 11 kJ; for $B_0 = 3$ T, near 24 kJ; the

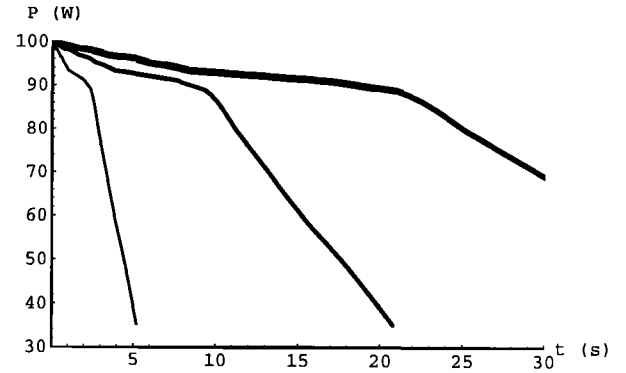


Figure 2. Shape of the curve $P(t)$, power of the ball, versus time, for three values of the magnetic field: $B_0 = 1$ T (thin line), $B_0 = 2$ T (medium line), and $B_0 = 3$ T (thick line). The lifetimes are approximately 2.5 s, 10 s, and 22 s.

agreement is thus good. Only a part of this energy will be radiated during the time in which the ball shines.

In the calculation of $P(t)$ it was assumed, for simplicity, that the ionization is constant, although it is known to vary with temperature according to the so-called Saja formula [*Chen*, 1974]. It turns out that as the streamer temperature remains in the shoulder between 18,000 K and 15,500 K, when the power is nearly constant and remains close to 100 W, the ionization decreases from 13% to 6% approximately; beyond that point it falls down more rapidly. The resistivity enters then into play producing a helicity dissipation according to (5); this accelerates the end of the structure, making the decrease of the power steeper and more abrupt than what is shown in the Figure 2 and improving thus the agreement with what is observed by the witnesses.

To understand better the effect of the helicity, let us consider the case $h = 0$. All the expansions considered above are then compatible with equation (4). Repeating the calculation with L_0^k/L^{2+k} instead of $1/L^2$ in (1), we find instead of (7) the relation $-\gamma T_0^{-k} B_0^2 (1 + 2k) \int_{T_0}^T T^{1+k} dT/P(T) = t$. As is seen, $t \rightarrow 0$ if $k \rightarrow -1/2$. This means that the expansion is instantaneous in that limit. In other words, without linking and helicity there will be no ball lightning, since the system decays too fast to be seen.

Another point must be mentioned. The solutions of the MHD equations taken in I, to represent the state of the system, imply that the ions move along the magnetic lines with velocity $\mathbf{v} = \pm \mathbf{B}/\sqrt{\mu_0 \rho}$, ρ being the fluid density, while the electrons travel along the lines of $\nabla \times \mathbf{B}$; but because of the Coulomb attraction, ions and electrons cannot separate. This would imply that the ions drag the streamers into moving along the lines of the magnetic field. The resulting motion of the streamers would contribute to their not being perceived separately and to the ball being seen as a diffuse sphere of light.

To summarize, the stabilizing effect of the linking number of the magnetic lines shown in reference [*Rañada and Trueba*, 1996] can be used to construct a realistic model of ball lightning, in which its radiation is due

to streamers of ionized air with high conductivity along the lines of $\nabla \times \mathbf{B}$ that are hot, in the temperature range 16,000–18,000 K. They occupy only a very small part of the volume, of the order of 10^{-6} . A ball of radius 15 cm emitting a power of 100 W, which are average values, has in this model a lifetime τ of about $2.5B_0^2$, which for $B_0 = 1.9$ T gives $\tau = 9$ s, just the estimated average value. The ball increases its size about 6% during this time, too small a variation for the witnesses to be aware of. The energy of the ball is $2685 B_0^2$ J, in good agreement with the estimation of 20 kJ for the average case. In the case studied here, the evolution was found to be independent of the initial radius L_0 , but by changing the intensity of the magnetic field B_0 , the initial temperature, or the particular form of the magnetic knot, a wide range of evolution can be found, in agreement with the observed variability of the phenomenon.

This model explains also two meaningful and significant observations. (1) In some cases, filaments are observed trailing a ball; they must be streamers which break open and follow behind (see the photographs of [Ohtsuki, 1989, p. 10] and [Barry, 1980a, chap. 5]). (2) Second, and this is important, there is a controversy on whether the balls are cold or hot, due to a contradiction in the reports of witnesses. Some say that the balls are cold, others that they are hot. More precisely, some report that the balls are cold, because they did not feel heat when one passed nearby; however it must be stressed that other witnesses feel that the balls are hot because they were burned and had to receive medical attention; furthermore, fires were also produced in some cases. This model solves the controversy by saying that the balls are hot and cold at the same time. This is so because a very small part of each ball (a set of streamers, of the order of one millionth of the volume in the example presented here) is indeed hot, but the rest is at ambient temperature, so that the overall radiation is small, no more than that of a home electric bulb. Consequently, a fire can be started or a person be burned if there is contact, but the balls do not produce any feeling of warmth if there is not, even if the observer is particularly close. We believe that this model is the first to explain this curious discrepancy.

An important and difficult question is the production of fireballs in the laboratory. This has been attempted by several means, combustion of mixtures of gases for instance; the best results in air have been the fireballs produced by Ohtsuki and Ofuruton, [1991] by interference of microwaves. They are similar to ball lightnings, but it is not certain that they are the same. This model suggests a way: producing two discharges orthogonal or at least transverse to one another and strong enough according to the data of reference [Alexeff and Rader, 1995]. The combination of the magnetic fields around the discharges should make easier the formation of linked lines. The probability could be enhanced by rotating very rapidly the electrodes.

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