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de Análisis
Económico**

A new approach to the unconditional measurement of default risk

Alex Ferrer

Department of Quantitative Economics
Universidad Complutense de Madrid
Spain

José Casals

Department of Quantitative Economics
Universidad Complutense de Madrid
Spain

Sonia Sotoca

Department of Quantitative Economics
Universidad Complutense de Madrid
Spain

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JEL Classification: C46, C58, G21, G32



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A new approach to the unconditional measurement of default risk

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ICAE Working Papers

Alex Ferrer^{†,§}, José Casals[†], Sonia Sotoca[†]

Abstract

This paper analyzes the unconditional measurement of default risk and proposes an alternative modeling approach. We begin the analysis by showing that when conducted under non-stationarity, the objective of the unconditional measurement changes and that some relevant problems appear as a consequence of the sample dependence. Based on this result, we introduce our approach and discuss its consistency, practical advantages, and the main differences from the conventional static framework. An empirical analysis is also conducted. Under non-stationarity, the regulatory model for the unconditional probability of default distribution performs badly when compared to our approach. Results also show that the capital figure presents a determinant and non-trivial dependence on the homogeneity and severity of the economic scenario represented in the sample.

Keywords: default risk, probability of default, unconditional measurement, conditional measurement

JEL Classification: C46, C58, G21, G32

1. Introduction

It has been acknowledged that banks and the economies in which they operate benefit from stable capital requirements over the business cycle. Thus, capital stability reduces adjustment costs for banks, Estrella (2004), and helps to smoothe the business cycle, Drumond (2009). The latter is considered appropriate from a macro-prudential perspective, so both the Basel II, BCBS (2006), and Basel III, BCBS (2011), frameworks assume the principle of capital stability, although with questionable success, Repullo and Suarez (2008).

Regarding default risk, the ideal of stability has led to a long-term approach to the probability of default (PD) of each type of debtor included in the portfolio, BCBS (2009). This kind of PD attempts to represent a neutral risk profile instead of the real spot default risk prevailing in each point of the business cycle, Gordy and Howells (2006). Due to the cyclical nature of default risk, Festic et al. (2011), the former is not, in terms of creditworthiness, as good as the latter in expansions nor as bad in recessions. The use of a long-term PD results in a non-cyclical or

unconditional default risk measurement, and therefore in unconditional capital requirements for the credit portfolio. Unconditional means here that the risk profile represented in the PD is not subjected to any specific state of the business cycle, as opposed to the conditional measurement, which does reflect a specific economic environment.

In spite of its central role in banking regulation and its widespread use by the industry, the fundamentals of the unconditional measurement are not yet entirely clear, especially if the portfolio exhibits non-stationary default risk. Resorting to intuition, it is natural to think that if there is no stationary default risk, the word unconditional cannot have the same meaning in risk terms than it has in probabilistic terms, and that some limitations may appear. Once this issue arises, the contiguous problems of (i) modeling an unconditional PD distribution coherently with such limitations, and (ii) understanding the effects of the non-stationarity over the resulting capital estimate, immediately surface. Again, intuitively, the standard techniques for characterizing the marginal distributions of stationary processes must be avoided in (i), while the irregularity and sample dependence produced by the non-stationarity are expected to be relevant in (ii).

The aim of this paper is to explore these issues from the perspective of the conditional measurement, whose fundamentals are clear and do not present blind spots under non-stationarity. In other words, we follow a conditional approach to the unconditional measurement. This novel view allows us to contribute to each of these three issues.

[†]Department of Quantitative Economics, Universidad Complutense de Madrid, Campus de Somosaguas, 28223 Madrid, Spain. Tel.: (+34) 91 394 23 83. Email addresses: af.econres@gmail.com (A. Ferrer), jmcasals@ccee.ucm.es (J. Casals), sotoca@ccee.ucm.es (S. Sotoca).

[§]Corresponding author.

First, we discuss the concept of unconditional measurement under non-stationarity with focus on the unconditional PD distribution. We conclude that it has to be interpreted as a measurement that relies on a balanced economic scenario and that is, in fact, an unconditional measurement conditional on the sample used. Such sample dependence generates problems of comparability, stability, and even capital arbitrage opportunities. The static modeling approach, which implicitly obviates this fact, is questioned.

Second, we propose an alternative modeling approach to the unconditional PD distribution. It is based on the conditional PD distributions and avoids any explicit or implicit assumption of stationarity. The term “approach” instead of “model” indicates its general nature, which is not restricted to any specific statistical formulation. We discuss its relationship with the literature, flexibility, and coherence.

Third, we conduct an empirical analysis based on American data. Results show that the non-stationarity induces multimodal PD distributions. The model of Vasicek, which underpins the regulatory framework, handles this feature badly, while our approach captures it properly and parsimoniously. On the other hand, the unconditional capital presents a significant sensitivity to the homogeneity and severity of the economic scenario captured in the sample. The former produces lower capital figures under the Great Recession than in a more balanced scenario, while the latter is caused by the cyclical nature of the conditional PD distributions’ variance.

On the basis of these results—conceptual discussion, methodological proposal, and empirical study—we suggest rethinking the unconditional measurement of default risk, especially regarding the regulatory framework.

The rest of the paper is organized as follows. Section 2 briefly reviews the analytical framework used, and Section 3 discusses the meaning and limitations of the unconditional measurement under non-stationarity. Section 4 presents the proposed approach to the unconditional PD distribution, Section 5 discusses its properties, and Section 6 describes the empirical exercises and their results. Section 7 provides a conclusion. Appendix A presents the proof for a result obtained in the body of the paper.

2. Loss model review

Discussing unconditional measurement requires the formulation of a loss model and a brief review of its main features. For this purpose, we assume a standard framework in which time is measured discretely at regular intervals that, for simplicity, match the time horizon used by the bank to measure the default risk of its credit portfolio¹.

¹However, our analysis can easily be extended to the multi-period framework, Duffie et al. (2007).

In practice, this time horizon is usually a year, but no specific assumptions about it are made. We also assume that the only source of loss for the bank is its credit portfolio.

As a portfolio model, we consider a default-event, constant-exposure, conditional-independence formulation, see Gordy (2000) and Frey and McNeil (2003). This model is formulated as follows:

$$L = \sum_{j=1}^N L^j = \sum_{j=1}^N \sum_{i=1}^{M^j} L^{ji} = \sum_{j=1}^N \sum_{i=1}^{M^j} Ber^{ji}(F^j) e^{ji} \quad (1)$$

where N is the number of risk units, formed by homogeneous groups of debtors, e^{ji} is the net exposure of each debtor, $e^{ji} > 0$, and Ber^{ji} is the Bernoulli random variable that takes a value of 1 if the debtor defaults and 0 otherwise. M^j is the number of debtors assigned to the risk unit j , with $M = \sum_{j=1}^N M^j$ being the total number of debtors of the portfolio. L^{ji} , L^j and L are the loss distributions of debtor i from risk unit j , the loss distribution of the risk unit j and the loss distribution of the entire portfolio, respectively. F^j is the PD of risk unit j during the time horizon. It is a continuous random variable with support in $(0, 1)$, with $\mathbf{F} = (F^1, \dots, F^N)$ being the continuous multivariate PD random vector with support in $(0, 1)^N$. The loss experienced by the bank during the time horizon is then a fraction of $E = \sum_{j=1}^N \sum_{i=1}^{M^j} e^{ji}$.

The capital, or unexpected loss, k , of the portfolio is assumed to be $k = \eta - \mu$, where $\mu = E[L]$, and η is the Value at Risk (VaR) of L , $P(L \leq \eta) = u$, with u , $0 < u < 1$, being its pre-defined coverage level².

The capital estimate strongly depends on the choice of PD distribution. \mathbf{F} is, in fact, the core of the loss model presented in Eq. 1 since it establishes its asymptotic properties, Gordy (2003). In practice, two main types of \mathbf{F} distribution are employed, each related to a different type of default risk measurement.

On the one hand, the conditional PD distribution, $\mathbf{F}_t = (F_t^1, \dots, F_t^N)$. \mathbf{F}_t produces a conditional measurement that represents the economic situation prevailing³ in period t by incorporating all the available information up to $t - 1$.

On the other hand, the unconditional PD distribution, $\mathbf{F}^* = (F^{*1}, \dots, F^{*N})$. \mathbf{F}^* produces an unconditional measurement that represents a full-business-cycle scenario. This kind of scenario, as opposed to the conditional one, is a synthetic construction mixing periods of both growth and recession in a proportion similar to that observed during a standard business cycle.

While the conditional measurement relies on a dynamic modeling, the unconditional measurement usually makes

²Unexpected loss can be defined in terms of other risk measures, like the expected shortfall (ES), see Tasche (2002). However, we base our discussion, without loss of generality, on the VaR due to its prominent position in risk management.

³Although we do not consider them, hypothetical conditional scenarios can also be used, as occurs in stress test exercises, see Sorge and Virolainen (2006).

use of a more static approach. However, both \mathbf{F}_t and \mathbf{F}^* are identified on the grounds of the portfolio hazard rates, $\mathbf{h}_t = (h_t^1, \dots, h_t^N)$. h_t^j represents the default frequency rate observed in risk unit j at period t .

Thus, \mathbf{F}_t is given by the conditional distribution of \mathbf{h}_t in t given all the information up to $t-1$, \mathbf{H}_t . Therefore, the identification of \mathbf{F}_t usually requires the prior identification of a dynamic model for \mathbf{h}_t , as in [Pesaran et al. \(2006\)](#).

The identification of \mathbf{F}^* relies on the observation of \mathbf{h}_t over a long period, or *time window*, represented hereinafter by the interval $t = 1, \dots, T$. The time window should include, under the full-business-cycle principle, recessions and expansions to avoid biases. Related to this time window there is a matrix of observed hazard rates per period and risk unit. The conventional static approach is then to fit a distribution with support in $(0, 1)^N$ to this matrix. A well-known unconditional PD distribution is the model of Vasicek, [Vasicek \(2002\)](#), denoted by $\mathbf{V} = (V^1, \dots, V^N)$:

$$V^j = N \left(\frac{N^{-1}(\theta^j) - \sqrt{\rho^j} z^j}{\sqrt{1 - \rho^j}} \right) \quad (2)$$

where $0 < \theta^j < 1$, $0 < \rho^j < 1$ and $\mathbf{z} = (z^1, \dots, z^N) \sim N_N((0, \dots, 0); \mathbf{\Sigma})$ with $\mathbf{\Sigma}(j, j) = 1$, $j = 1, \dots, N$.

This model enjoys a notorious position among the PD models used in the industry, [BCBS \(2009\)](#), and several extensions have been proposed, [Burtshell et al. \(2007\)](#). It is also the basis of the regulatory model for default risk, this time presented as the Asymptotic Single Risk Factor (ASRF) model, which is equivalent to [Eq. 2](#) setting $z^j = z$, $j = 1, \dots, N$.

3. Unconditional measurement and non-stationarity

In probabilistic terms, the unconditional PD distribution can be understood as the marginal distribution of \mathbf{h}_t , $\mathbf{H}^* = (H^{*1}, \dots, H^{*N})$. Since the marginal distribution is not subjected to the information available at any period, but reflects the long-term behavior of the process, it can be informally seen as a mix of the different stages that form the business cycle.

Obviously, if \mathbf{h}_t is not stationary, then such marginal distribution does not exist, so it is not possible to define the unconditional PD distribution in this way. In other words, \mathbf{F}^* is an unconditional PD distribution conditional on the hazard rate sample observed during the time window, and only if \mathbf{h}_t is stationary and the time window is long enough that \mathbf{F}^* can be considered truly unconditional. If this happens, \mathbf{F}^* is expected to converge⁴ to the marginal distribution \mathbf{H}^* .

If this is not the case⁵, then there is no convergence and therefore explicit or implicit assumptions about the

existence of a long-term representation of the hazard rate series should be avoided. This fact translates directly into the capital figure, for which there is no marginal representation either.

This fact means that under non-stationarity, the meaning and objective of the unconditional measurement change. Thus, the idea of capturing the portfolio's long-term default risk must now be substituted with the idea of representing the default risk related to an equilibrated economic scenario, one incorporating both expansions and recessions in a balanced mix. In other words, unconditional capital simply means capital derived from a balanced economic scenario. The adjective "unconditional" is, in fact, an informal, or even allegorical, use and abuse of a term that loses its true meaning under non-stationarity.

Although it could be argued that the sample dependence problem has long been a concern in risk modeling, the truth is that regarding unconditional measurement, it produces relevant effects that are not yet properly understood or even fully acknowledged. This is the case with the static approach: Fitting a distribution to a large realization of a stochastic process only makes sense if such process is stationary. Otherwise, this kind of identification process for \mathbf{F}^* , although still applicable, lacks any theoretical basis. Yet the static approach is no other but the one on which the regulatory framework relies. In fact, neither Basel II nor Basel III have addressed the stationarity issue in detail, although it affects the estimation of the TtC (Through the Cycle) PD, which is the parameter θ^j in [Eq. 2](#).

Regarding the literature, [Daniélsson \(2002\)](#) highlights the general problems that any risk management process encounters due to the recurrent structural changes exhibited by financial variables, but it focuses on the problem of endogenous response and does not discuss the relationship between unconditional measurement and non-stationarity. [Lucas and Klaassen \(2006\)](#), [Rösch and Scheule \(2010\)](#), [Carey \(2002\)](#) and [Bangia et al. \(2002\)](#) observe that the risk metrics depend on the economic scenario reflected in the sample, or, equivalently, in the time window, but do not discuss the implications of this dependence for the unconditional measurement. [Jarrow and Turnbull \(2000\)](#) note that the implicit assumption of stationarity behind the commercial solution of KMV, [Crosbie and Bohn \(2003\)](#), can be seen as a deficiency, which is a criticism that can be generalized to other commercial solutions, like those analyzed by [Crouhy et al. \(2000\)](#). However, this remark is not subjected to further analysis. In other works this issue is virtually dodged, even if a dynamic modeling for the hazard rates is considered, as in [Wilson \(1997a,b\)](#) and [Lee and Poon \(2014\)](#).

this controversial issue. Moreover, hazard rate series are often defined in terms of another unbounded process, \mathbf{x}_t , being $h_t^j = \varphi(x_t^j)$, with $\varphi: \mathbb{R} \rightarrow (0, 1)$ being a continuous function. Our discussion can, in turn, be extrapolated to x_t^j .

⁴We assume throughout the paper that \mathbf{h}_t is ergodic.

⁵Since $0 < h_t^j < 1$, the hazard rate cannot be purely non-stationary although it can behave as if it is, [Nicolau \(2002\)](#). For the sake of simplicity, however, we follow [Hall et al. \(1992\)](#) and omit

The influence of the time window in the unconditional measurement constitutes a marked difference between stationarity and non-stationarity. Thus, if \mathbf{h}_t is stationary there is no inconsistency as the longer the time window, the closer the unconditional PD distribution is to the hazard rates' marginal distribution. That means that choosing the largest available time window is a simple and optimal criterion to represent a full-business-cycle scenario. Moreover, due to this convergence, the lower and upper limits of the time window are not relevant beyond the total length that they determine.

But if \mathbf{h}_t is not stationary, then it is not clear that the best choice to achieve a balanced time window is to use the largest available one. Banks then must choose the proper mix of expansion and recession periods that results in a full-business-cycle time window. This critical decision influences the resulting unconditional capital as it does any other modeling option, although in a less evident way.

In addition to the difficulty of dealing with the vague concept of full-business-cycle time window, two practical problems appear under non-stationarity.

First, if setting the time window is not as simple as just considering the largest sample available, a more elaborate updating process is required with the observation of new hazard rate values. These can be added to the time window, which increases its length, or be used to replace the oldest values. The goal is to keep a proper full-business-cycle representation as the prevailing time window becomes obsolete. Updating the time window involves modifying the initial and final periods that define it, which, under non-stationarity, may introduce instability in the unconditional capital estimate, thereby harming the ultimate objective of stability.

Second, comparing unconditional capital figures over time, and, especially, across different banks, is of little usefulness. Thus, different banks will probably follow different criteria to establish the full-business-cycle time window, which distort the comparison.

4. Proposed approach to the unconditional PD distribution

We denote our approach as $\mathbf{\Pi} = (\Pi^1, \dots, \Pi^N)$. $\mathbf{\Pi}$ is based on the collection of conditional PD distributions $\{\dots, \mathbf{F}_t, \dots\}$ generated by the realization of \mathbf{h}_t and it is formed as the equally weighted mixture, or simply, mixture, of those \mathbf{F}_t included in the time window considered by the bank. Thus, $\mathbf{\Pi}$ can be expressed as follows:

$$\mathbf{\Pi} = \{\mathbf{F}_t\}_{t=1, \dots, T} \circ I \quad (3)$$

where I is a random variable that takes values $1, \dots, T$ with probability $1/T$. $\{\mathbf{F}_t\}_{t=1, \dots, T}$ is the set of conditional PD distributions included in the time window. By relying on them, the proposed approach (i) avoids any implicit assumption of stationarity for \mathbf{h}_t , (ii) follows a dynamic modeling, and (iii) reflects explicitly the evolution of the

portfolio's default risk during the time window. These three features make $\mathbf{\Pi}$ radically different from the traditional static framework.

5. Discussion of the proposed approach

$\mathbf{\Pi}$ constitutes a distinctive approach to unconditional PD modeling. Still, some links to the literature can be drawn. Thus, Pederzoli and Torricelli (2005) and Bania et al. (2002) form the unconditional distribution as a weighted aggregation of distributions subjected to two states of the economy, expansion and recession, each receiving a weight equal to the relative frequency of such states over time. $\mathbf{\Pi}$ can be understood as an extreme case of this framework that considers as many discrete states for the economy as periods form the time window, all having a weight $1/T$. These periods implicitly represent recession, expansion, or stagnation if that is what is exhibited by the hazard rate series during the time window.

The proposed approach also resembles the resampling methodology, Carey (1998), a non-parametric approach to the PD distribution that relies on historical simulation. Thus, the realization of I can be understood as a resampling process over the set of T conditional scenarios implicitly formed by the time window. According to this interpretation, $\mathbf{\Pi}$ extends the re-sampling methodology by adding uncertainty about the realization of the conditional PD distributions themselves, this time in a stochastic, rather than historical, simulation framework.

$\mathbf{\Pi}$ is a modular formulation, since it is obtained by adding to the mixture those components \mathbf{F}_t that best represent the economic scenario considered. This is a marked difference from the flat static PD distributions, which are directly fitted to the matrix of observed hazard rates. Thus, the problem of representing a full-business-cycle scenario is now the problem of choosing a proper combination of conditional PD distributions related to expansions and recessions. This modularity allows the bank to consider even non-connected time windows⁶.

Another advantage of the modular structure is the possibility of easily representing economic scenarios other than the full-business-cycle one. For stress test analysis, for example, forming $\mathbf{\Pi}$ only with the distributions \mathbf{F}_t related to the downside of the business cycle can be useful. $\mathbf{\Pi}$ would then be a PD distribution conditioned on a general downturn environment but not conditioned on any particular type of recessive scenario.

$\mathbf{\Pi}$ dissociates the long- and short-term dependence between risk units. So, while the former is captured in a non-parametric way through I , the latter is induced at each period of the time window by the conditional PD distributions \mathbf{F}_t , $t = 1, \dots, T$. Since \mathbf{F}_t is determined by the dynamic model fitted to \mathbf{h}_t , this is only accountable for the short-term dependence. Therefore, a dynamic model

⁶That is, time windows formed by non-consecutive time periods.

that does not fully capture the long-term relationship between the h_t^j series can still be appropriate and preferable to an excessively complex one.

The dynamic model fitted to the hazard rate series can be understood as the “metaparameter” of the proposed approach, since there is no restriction on its formulation. Consequently, Π is a general approach to the unconditional PD distribution rather than a certain model. It is worth noting that changing the time window does not require re-identifying the dynamic model, but simply modifying the collection of conditional PD distributions that form Π . By contrast, in the static approach any variation of the time window involves a variation in the observed hazard rates matrix, and therefore demands the re-identification of the distribution fitted to it.

Π produces an unconditional measurement coherent with its natural probabilistic interpretation. Thus, under some general conditions, the mixture of conditional PD distributions converges to the marginal distribution of the hazard rates if these are stationary. Proposition A states this result at the univariate level, which can easily be extended to the multivariate case.

Proposition A.

Let h_t be a stationary and ergodic univariate process with support in $(0, 1) \subset \mathbb{R}$. h_t has marginal distribution F^* with density function f^* , and conditional distribution F_t with density function f_t satisfying the following:

$$f_t(x) = \varphi(x; z_t^1, \dots, z_t^K) = \varphi(x; \mathbf{z}_t) \quad (4)$$

for any $x \in (0, 1)$, with $\varphi \in \mathcal{C}((0, 1) \times \mathbb{R}^K)$. \mathbf{z}_t is a stationary and ergodic vector process with support in \mathbb{R}^K . The marginal distribution of \mathbf{z}_t is $\mathbf{Z}^* = (Z^{*1}, \dots, Z^{*K})$ with density function g^* .

Then, the equally weighted mixture $\Pi_{\{1, \dots, T\}}$ of the collection of conditional distributions $\{F_1, \dots, F_T\}$ generated by the realization $\{\dots, \mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_T, \dots\}$ of \mathbf{z}_t converges in distribution to F^* as T approaches infinity:

$$\Pi_{\{1, \dots, T\}} \xrightarrow[T \rightarrow \infty]{D} F^* \quad (5)$$

Proof.

See Appendix A. ■

Proposition A states the convergence for a varied family of dynamics for h_t , and for \mathbf{h}_t in the multivariate extension, including any linear model with constant coefficients and normal innovations.

6. Empirical analysis

6.1. Data, dynamic models and credit portfolio

We consider as a proxy of hazard rate series the quarterly series of charge-off provided by the FDIC⁷ for “Mort-

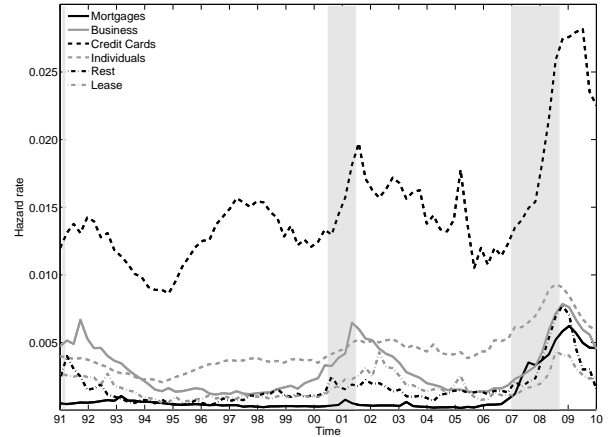


Figure 1: Hazard rate series. Shaded areas indicate U.S. recessions, according to NBER (*National Bureau of Economic Research*).

gages” (1-4 Family Residential Real Estate Loans), “Business” (Commercial & Industrial Loans to U.S. Addressees), “Credit Cards” (Credit Cards), “Individuals” (Other Loans to Individuals), “Rest” (All Other Loans) and “Lease” (Lease Financing Receivables) between 1991Q1-2010Q4.

These series are similar to those employed by Röscher and Scheule (2004, 2010) and Lee and Poon (2014). As discussed therein, they present some shortcomings, like vulnerability to normative distortions⁸ or the fact that they represent a money default ratio instead of a frequency one. However, due to their systemic nature, they adequately capture the default risk that any standard commercial bank could have experienced. Because the aim of the analysis is not the estimation of any capital figure *per se* but the study of the unconditional measurement, they are considered appropriate.

Table 1 summarizes the main statistics of the hazard rate series, Figure 1 presents them and Figure 2 displays their histograms. Series are not stationary, which makes them convenient for the purpose of the analysis, and exhibit asymmetric cyclicity, a pattern also noted by Marcucci and Quagliariello (2009). They also present moderate heterogeneity, with some visible differences across series, although they evolved in a very similar way during the Great Recession.

As dynamic model, we chose an ARIMA formulation with probit link function for each hazard rate series. We do so to focus the analysis on the unconditional PD distribution and not on the underlying econometric formulation. Thus, a univariate ARIMA model with normal innovations is fitted to each series $x_t = N^{-1}(h_t)$. Pesaran et al. (2006) and Bonfim (2009) also consider the probit link function. This dynamic model generates conditional PD distribu-

⁷Federal Deposit Insurance Corporation, see <http://www.fdic.gov/>.

⁸In order to reduce this presumable noise, series are seasonally adjusted, and 8 out of 480 values are smoothed due to their extreme outlier condition.

<i>Series</i>	<i>Mean</i>	<i>Std. Dev.</i>	<i>Min</i>	<i>Median</i>	<i>Max</i>	<i>JB test</i>	<i>ADF test</i>
Mortgages	0.0010	0.0015	0.0002	0.0004	0.0062	0.0010	0.9667
Business	0.0030	0.0018	0.0012	0.0023	0.0078	0.0133	0.3653
Credit cards	0.0148	0.0044	0.0086	0.0138	0.0282	0.0010	0.8222
Individuals	0.0044	0.0016	0.0021	0.0039	0.0093	0.0016	0.7740
Rest	0.0017	0.0014	0.0004	0.0013	0.0077	0.0010	0.1569
Lease	0.0017	0.0009	0.0005	0.0014	0.0043	0.0075	0.2294

Table 1: Main statistics for the hazard rate series. The p-value is presented for the Jarque-Bera test (JB test) and the Augmented Dickey-Fuller test (ADF test).

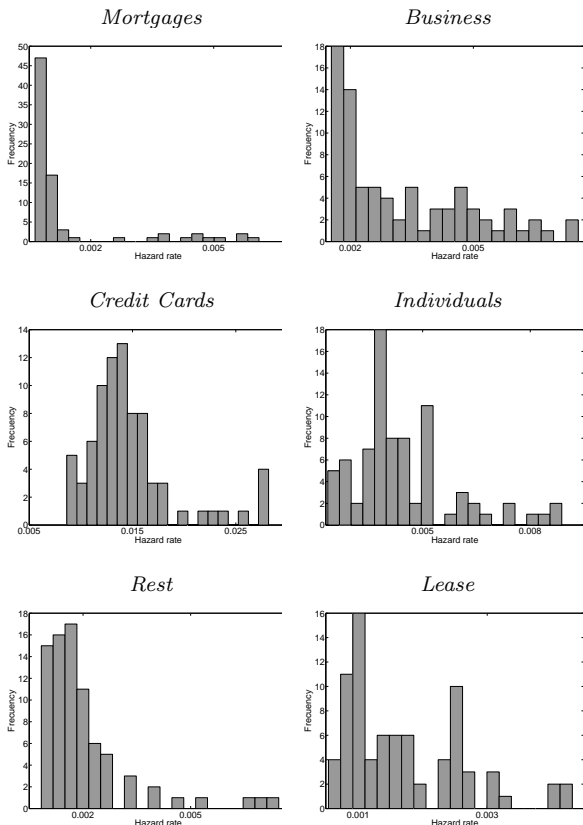


Figure 2: Histograms of the hazard rate series.

tions of the form $F_t = H_t = N(X_t)$, with X_t being the conditional distribution of x_t given all the available information up to $t - 1$. Table 2 summarizes the univariate ARIMA models, which are identified using the whole sample, 1991Q1-2010Q4.

Each hazard rate series represents a stand-alone portfolio formed by a single risk unit, so that there are six portfolios. We assume that all have infinitely fine-grained exposures, Gordy (2003), so there is no need to characterize debtors individually. Thus, Eq. 1 is reformulated as follows:

$$L = eF^* \quad (6)$$

where e is the total exposure of the single risk unit, $e > 0$. In order to better study the PD distributions, we assume $e = 1$, which means that $L = F^*$ has support in $(0, 1)$.

These synthetic portfolios only bear systemic risk, which facilitates the interpretation of the results without harming their generality⁹.

We consider five different types of PD distribution:

- V , estimated from the realization of \mathbf{h}_t between 1991Q1 and 2010Q4, see Rösch and Scheule (2004).
- Π_C , formed by the conditional PD distributions included in the *complete* time window, 1991Q1-2010Q4.
- Π_D , formed by the conditional PD distributions included in the *downturn* time window, 2008Q1-2010Q4.
- Π_I , formed by the conditional PD distributions included in the *initial* time window, 1991Q1-2000Q4.
- Π_F , formed by the conditional PD distributions included in the *final* time window, 2001Q1-2010Q4.

For each portfolio and PD distribution, we estimate the capital figure at the regulatory coverage level¹⁰, $u = 99.9$. Since the hazard rate series are quarterly, the capital also has a quarterly time horizon. Capital estimates are obtained by Monte Carlo simulation with 1,000,000 realizations, a number large enough to ensure accuracy. On the basis of these simulations we also characterize the density function of the corresponding loss distributions.

Based on these series, models, and portfolios, we study the performance of the proposed approach and the influence of the time window on the capital figure.

6.2. Results

Table 3 shows the capital figures, while Figure 3, Figure 4 and Figure 5 present the comparison between the loss distribution densities generated by V and Π_C , Π_C and Π_D , and Π_I and Π_F , respectively.

We first analyze the differences between V and Π_C , which means comparing different PD models under the same time window, and then the differences between Π_C , Π_D , Π_I and Π_F , which means comparing different time windows under the same modeling approach to the unconditional PD distribution.

⁹They can easily be extrapolated to a more general portfolio exhibiting both systemic and idiosyncratic risk, since the latter is not tied to the business cycle.

¹⁰Coverage values are expressed in percentage points.

<i>Risk Unit</i>	ρ_1		ρ_2		θ_1		θ_2		$\hat{\sigma}_a$	<i>LBQ(16)</i>	<i>AIC</i>	<i>SBC</i>
	$\hat{\rho}_1$	$\hat{\sigma}_{\hat{\rho}_1}$	$\hat{\rho}_2$	$\hat{\sigma}_{\hat{\rho}_2}$	$\hat{\theta}_1$	$\hat{\sigma}_{\hat{\theta}_1}$	$\hat{\theta}_2$	$\hat{\sigma}_{\hat{\theta}_2}$				
Mortgages	-	-	0.2120	0.1070	-	-	-	-	0.0670	0.0913	-2.44	-2.38
Business	0.7602	0.1137	-	-	0.4882	0.1456	-	-	0.0435	0.4473	-3.25	-3.16
Credit Cards	-	-	-	-	-	-	-	-	0.0359	0.1160	-3.72	-3.69
Individuals	0.6230	0.1443	-	-	0.5208	0.2090	-0.2886	0.1183	0.0208	0.1218	-4.69	-4.57
Rest	-	-	-	-	-	-	-	-	0.0869	0.9121	-1.98	-1.95
Lease	-	-	-	-	-	-	-	-	0.0712	0.8821	-2.37	-2.34

Table 2: Univariate ARIMA models fitted to x_t^d , $x_t^d = x_t - x_{t-1}$, being $x_t^d = \rho_1 x_{t-1}^d + \rho_2 x_{t-2}^d - \theta_1 a_{t-1} - \theta_2 a_{t-2} + a_t$ and $V[a_t] = (\sigma_a)^2$. $\hat{\beta}$, estimated parameter. $\hat{\sigma}_{\hat{\beta}}$, estimated standard deviation of $\hat{\beta}$. *LBQ(16)*, Ljung–Box Q test p-value with 16 lags. *AIC*, Akaike Information Criteria. *SBC*, Schwarz Information Criteria.

<i>Portfolio</i>	<i>V</i>	Π_C	Π_D	Π_I	Π_F
Mortgages	0.0085	0.0078	0.0056	0.0011	0.0077
Business	0.0107	0.0072	0.0057	0.0064	0.0069
Credit Cards	0.0164	0.0182	0.0132	0.0066	0.0168
Individuals	0.0069	0.0064	0.0037	0.0013	0.0054
Rest	0.0074	0.0097	0.0093	0.0051	0.0099
Lease	0.0050	0.0046	0.0045	0.0031	0.0046

Table 3: Capital estimate at the 99.9 coverage level. *V*, Vasicek PD distribution for the complete time window (1991Q1-2010Q4). Π_C , proposed PD distribution for the complete time window (1991Q1-2010Q4). Π_D , proposed PD distribution for the downturn time window (2008Q1-2010Q4). Π_I , proposed PD distribution for the initial time window (1991Q1-2000Q4). Π_F , proposed PD distribution for the final time window (2001Q1-2010Q4).

Regarding the former, differences between the loss density functions related to *V* and Π_C are evident in all risk units. Thus, while the former shows, as expected, a unimodal shape, the latter presents a bimodal or even multimodal shape consistent with the histograms shown in [Figure 2](#). This notorious discrepancy shows that the static approach, when based on a unimodal parametric distribution, may offer a worse fitting than the proposed approach, as corroborated in [Table 4](#), which presents the p-value of the Kolmogorov–Smirnov goodness-of-fit test conducted under both models. In fact, the proposed approach is able to capture such multimodality without introducing any *ad hoc* feature in the underlying dynamic model—e.g., a regime switching mechanism, [Hamilton \(1989\)](#), as in [Bruche and González-Aguado \(2010\)](#). In other words, the mixture of conditional PD distributions is flexible enough to capture, in a parsimonious way, irregular morphologies when they are present in the data.

Moreover, the inability of the model of Vasicek to properly capture multimodality leads to heavier PD distribution tails under this formulation. [Figure 6](#) presents evidence of this effect. It compares the increases in the VaR figure when the coverage level passes from 99.9 to 99.97 under both *V* and Π_C . The former generates significantly higher increases in all portfolios. This result challenges the conventional idea that presents *V* as a model that generates too light tails due to its inner assumption of normality: It can lead to inflated tails if it is applied under non-stationarity.

The divergence in density functions generates significant differences between their respective capital figures, es-

<i>Portfolio</i>	<i>V</i>	Π_C
Mortgages	0.0010	0.9289
Business	0.0631	0.9071
Credit Cards	0.1605	0.8232
Individuals	0.5783	0.9944
Rest	0.5085	0.8268
Lease	0.1091	0.7581

Table 4: P-value of the Kolmogorov–Smirnov goodness-of-fit test applied to the hazard rate sample during the complete time window (1991Q1-2010Q4). The null hypothesis is that the sample comes from the specified distribution. *V*, Vasicek PD distribution for the complete time window. Π_C , proposed PD distribution for the complete time window.

pecially in Business and Credit Cards, and also in Rest and Mortgages. These discrepancies indicate that the rigidity of the model of Vasicek does affect the estimation of capital requirements. Therefore, if the assumption of stationarity is not met, the model of Vasicek may offer misleading capital estimates even if the hazard rate series are assumed to follow a simple linear dynamic of constant coefficients and normally distributed innovations. It can be concluded that the non-stationarity generates model risk, [Kerkhof et al. \(2010\)](#).

In comparing the different time windows using the proposed approach, [Figure 4](#) and [Figure 5](#) show a displacement to the right of the loss densities generated by Π_D and Π_F with respect to those generated by Π_C and Π_I , respectively. This shift is consistent with the role that the

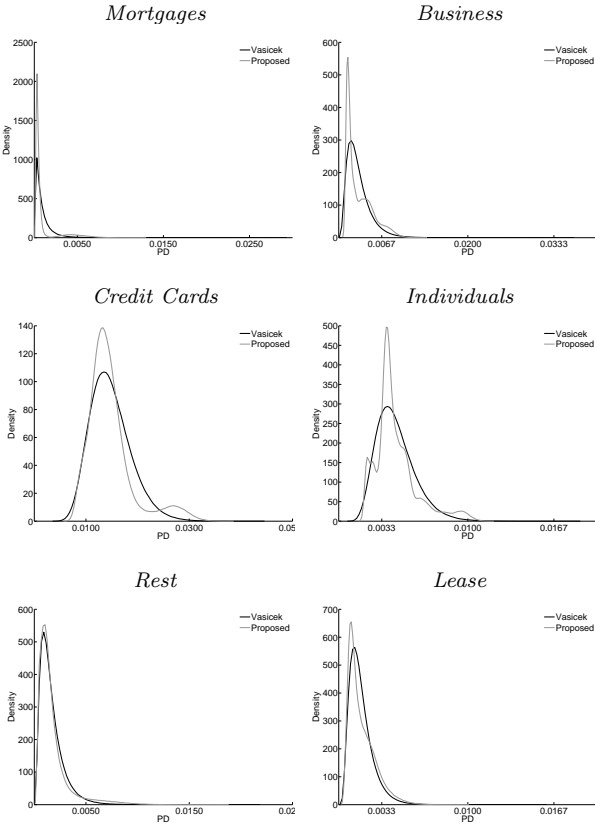


Figure 3: Density functions of the loss distributions generated by the Vasicek PD model, V , and the proposed approach, Π_C , both related to the complete time window (1991Q1-2010Q4).

Great Recession, the most severe economic crisis included in the sample, played in each time window. Thus, Π_D is exclusively devoted to the Great Recession, while Π_C also captures the rest of the sample. Similarly, Π_F includes the Great Recession, while Π_I does not. This result shows the significant effect over the loss distribution of diluting or omitting the Great Recession from the time window.

The capital figure presents, however, a more peculiar behavior, being lower in the downturn time window than in the complete time window, and higher in the final time window than in the initial time window.

Higher capital figures under Π_C than under Π_D mean that the increase exhibited by η when the economic scenario is restricted to the Great Recession is smaller than that experienced by μ , so that $k = \eta - \mu$ decreases. This result can be analyzed in terms of the different homogeneity of the complete and downturn time windows. Thus, the downturn time window contains a uniform, enclosed economic scenario (i.e., the Great Recession), so the set of conditional PD distributions F_t^j are less dispersed along the interval $(0, 1)$. On the contrary, the complete time window also includes expansion periods, so the dispersion is higher. Consequently, Π_D has lower variance than Π_C ,

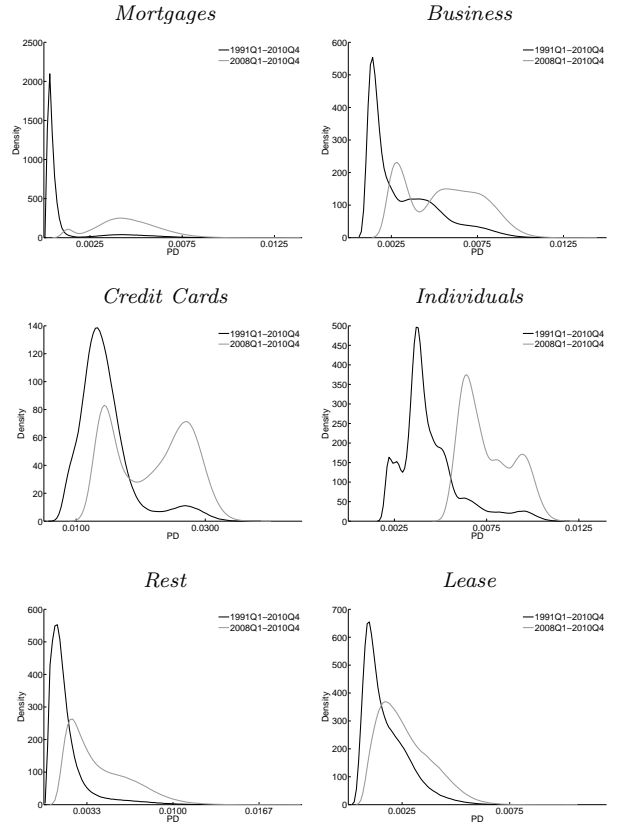


Figure 4: Density function of the loss distributions generated by the PD distributions Π_C , related to the complete time window (1991Q1-2010Q4), and Π_D , related to the downturn time window (2008Q1-2010Q4).

which means a shorter distance between the mean and the VaR of the loss distribution—i.e., a lower capital.

Therefore, increasing the homogeneity of the time window may reduce the resulting capital estimate even in the case of the Great Recession. This fact questions the suitability of calculating capital requirements under a general downturn scenario with the objective of obtaining a conservative capital figure. It is worth noting that this result critically depends on the definition of capital as unexpected loss, $k = \eta - \mu$. This is the Basel definition and makes k strongly sensitive to the variance of L . On the contrary, if the capital is directly defined as η , then it is higher in the downturn time window than in the complete time window, which is the intuitive effect already reported by [Bangia et al. \(2002\)](#) and [Bruche and González-Aguado \(2010\)](#), among others.

The homogeneity effect, however, does not fully explain the higher capital figure obtained under the final time window than under the initial time window, since both have the same number of periods and include both expansions and recessions. Thus, in this case it is also relevant the fact that under the dynamic model fitted to $h_t = N(x_t)$, the variance of the conditional distributions $F_t = N(X_t)$

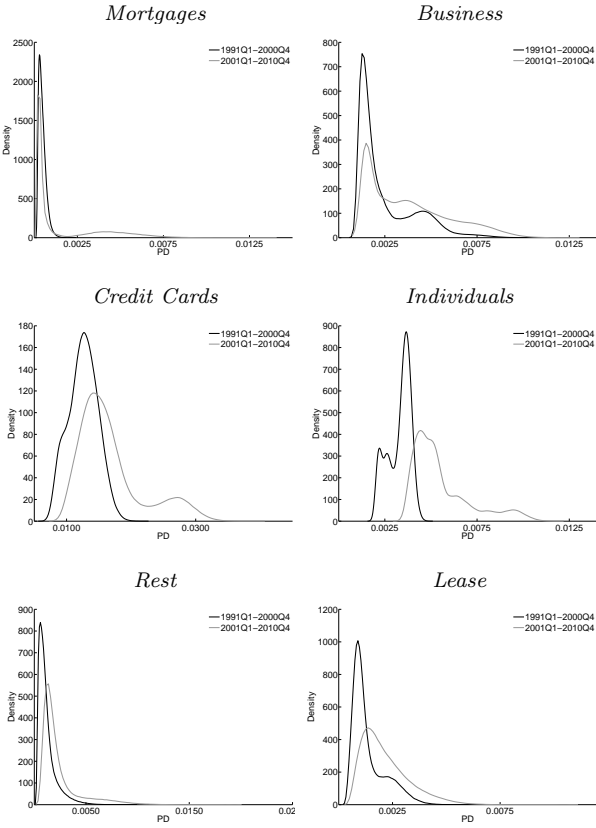


Figure 5: Density function of the loss distributions generated by the PD distributions Π_I , related to the initial time window (1991Q1-2000Q4), and Π_F , related to the final time window (2001Q1-2010Q4).

shows a cyclical evolution over time. In other words, it is higher in those periods related to higher hazard rates, and vice versa. Since the hazard rate series take higher values in the final time window than in the initial one, as is shown in Figure 7, higher conditional PD variances are observed between 2001Q1 and 2010Q1 than between 1991Q1 and 2000Q4. This results in a higher variance for Π_F than for Π_I , and consequently in a higher capital estimate for the former.

Figure 7 also illustrates the difficulty of choosing a full-business-cycle time window due to the non-stationarity: Two disjoint time windows, both having a mixed composition of expansion and recession periods, present quite different hazard rate series and, as observed in Table 3, produce quite different capital figures.

The explanation for the cyclical nature of the variance of the conditional PD distributions can be found in the probit link function used in the dynamic models fitted to the hazard rate series. Thus, all series have a unit root and its additional dynamic structure is weak, so it can be assumed that $E[X_t] \approx x_{t-1}$. Since the conditional distributions X_t have constant variance over time, $V[X_t] = v$, the next delta-approximation to the variance of $F_t = N(X_t)$

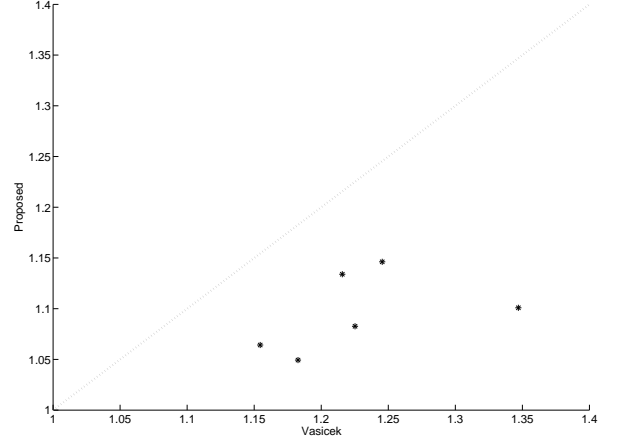


Figure 6: Increment in the VaR estimate when passing from $u = 99.9$ to $u = 99.97$, $\Delta^j = \eta_{u=99.97}^j / \eta_{u=99.9}^j$, under the model of Vasicek, displayed at the horizontal axis, and the proposed approach, displayed at the vertical axis. Each portfolio is represented by an unidentified point.

can be obtained:

$$\begin{aligned}
 V[F_t] &= V[N(X_t)] \\
 &\approx n(E[X_t])^2 V[X_t] \\
 &\approx n(x_{t-1})^2 V[X_t] \\
 &= n(N^{-1}(h_{t-1}))^2 v
 \end{aligned} \tag{7}$$

where $n(\cdot)$ is the density function of the standard normal.

Figure 8 depicts the delta-approximation to the standard deviation of X_t , $g(h) = n(N^{-1}(h))\sqrt{v}$, with h taking values between the minimum and the maximum of each hazard rate series over the complete time window. It is a monotonous increasing function. Figure 9 confirms empirically this cyclical behavior, showing that the variance of the conditional PD distributions increases in recessions and decreases in expansions.

This result is a clear example of the influence that the conditional PD distributions have on the unconditional PD distribution and, in turn, the influence that the dynamic model for the hazard rate series has on the conditional PD distributions themselves.

Other things being equal, it also indicates that the conditional capital, $k_t = \eta_t - \mu_t$, is cyclical, as it depends on the variance of F_t . Figure 10 corroborates this hypothesis in all the portfolios. Although conditional capital cyclicity has been noted by others, like Koopman et al. (2005), Pederzoli and Torricelli (2005), Truck and Rachev (2005) and Rösch and Scheule (2010), it had never been presented as a mere and direct consequence of a modeling decision—that of resorting to a probit link function for the dynamic of the hazard rate series.

Therefore, it can be concluded that two forces drive the capital generated by a time window: The distance between the conditional PD distributions that form it, which

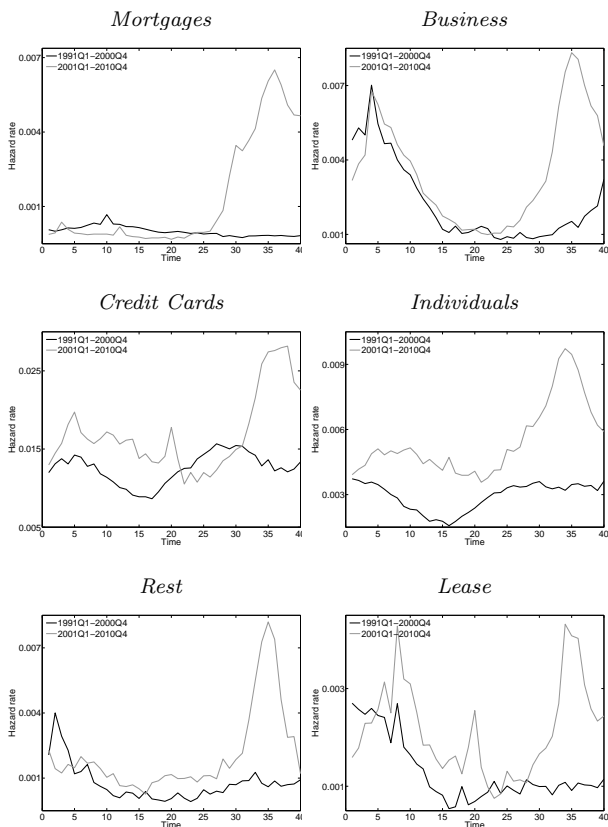


Figure 7: Hazard rate series during the forty periods that form the initial time window (1991Q1-2000Q4) and the final time window (2001Q1-2010Q4).

depends of the homogeneity of the scenario represented, and their respective variances, which also depend on the scenario but are shaped by the underlying dynamic model. These are, in fact, the two forces that drive the variance of a mixture, [McLachlan and Peel \(2000\)](#). They can present opposite effects, as happens in the comparison of the complete and downturn time windows, where the homogeneity prevails.

7. Concluding remarks

This paper addresses the unconditional measurement of default risk. It contributes to the existing literature by (i) analyzing the meaning and limitations of the unconditional measurement when conducted under non-stationarity, (ii) proposing a new modeling approach to the unconditional PD distribution, and (iii) studying empirically both the consequences of such limitations and the properties of the proposed approach.

These three actions have led to relevant results.

We have shown that the existence of a true unconditional PD distribution is subjected to the dynamic of the hazard rate series, and only if they are stationary can it be properly defined and consistently implemented. If this

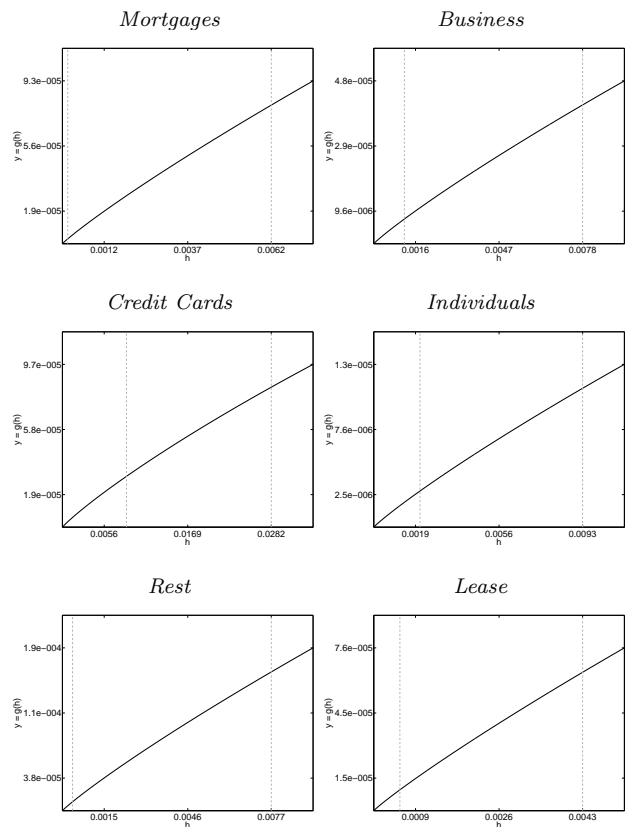


Figure 8: Graph of the function $g(h) = n(N^{-1}(h)) \sqrt{v}$, with v being the variance of X_t and $n(\cdot)$ the density function of the standard normal. The vertical lines indicate the minimum and maximum values taken by h_t during the complete time window (1991Q1-2010Q4).

is not the case, the concept must be understood in a more general sense regarding the use of a balanced time window in its identification. The choice of such a balanced time window affects the comparability and stability of the unconditional capital.

The proposed approach to the unconditional PD distribution differs significantly from the traditional static modeling. It is defined as the equally weighted mixture of the conditional PD distributions included in the time window, which grants it a dynamic veneer. We have discussed its main properties, including its generality, flexibility and coherence.

The empirical analysis has shown that non-stationarity generates multimodal PD distributions. The model of Vasicek, which is the regulatory model, does not capture this feature properly and leads to misleading capital estimates and inflated loss distribution tails. On the contrary, the proposed approach adequately captures multimodality even if a simple dynamic model for the hazard rate series is used. Regarding the time window, the capital figure is very sensitive to its homogeneity and severity, the latter effect directly caused by the dynamic model fitted to the

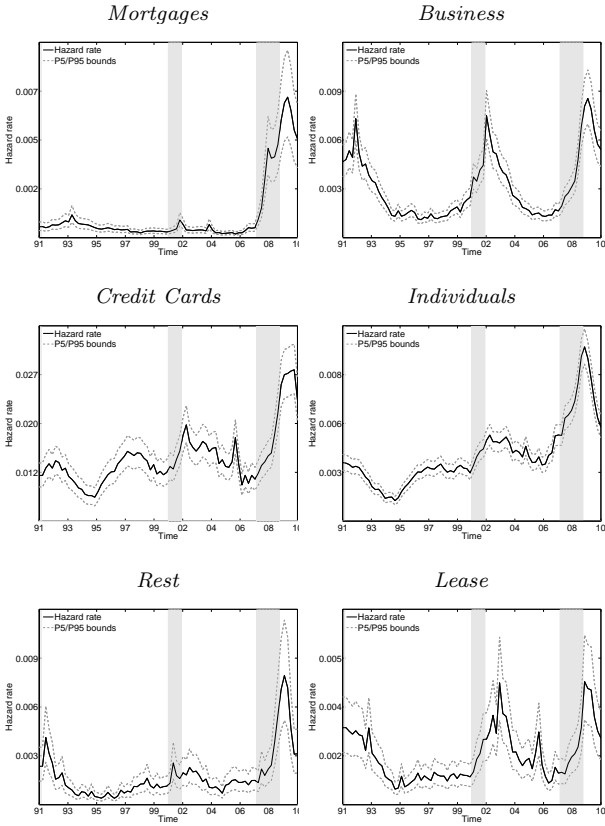


Figure 9: Hazard rate series and percentiles 5th and 95th of F_t during the complete time window (1991Q1-2010Q4). Shaded areas indicate U.S. recessions, according to NBER (*National Bureau of Economic Research*).

hazard rate series.

A direct implication of these results is the convenience of undertaking a critical review of the unconditional measurement of default risk, especially in the regulatory framework. Thus, the serious flaws that the model of Vasicek presents when applied under non-stationarity are a real concern. A different PD model, or, more generally, a modeling approach different from the static one, should be considered.

Another reason for rethinking the unconditional measurement is the marked difference in capital estimates that can be obtained under different full-business-cycle time windows, as observed in Section 6. Determining such a time window is a delicate process requiring subjective, although not arbitrary, criteria. Moreover, this subjectivity suggests that banks may use the time window as a lever with which to manipulate the resulting capital figure to their advantage¹¹. Thus, it is of major importance to evaluate the time window chosen by the bank in the vali-

¹¹The arbitrage opportunities that banks can exploit when calculating their capital requirements, especially in the regulatory framework, “regulatory capital arbitrage”, have been a recurrent concern since Basel I, see Jones (2000).

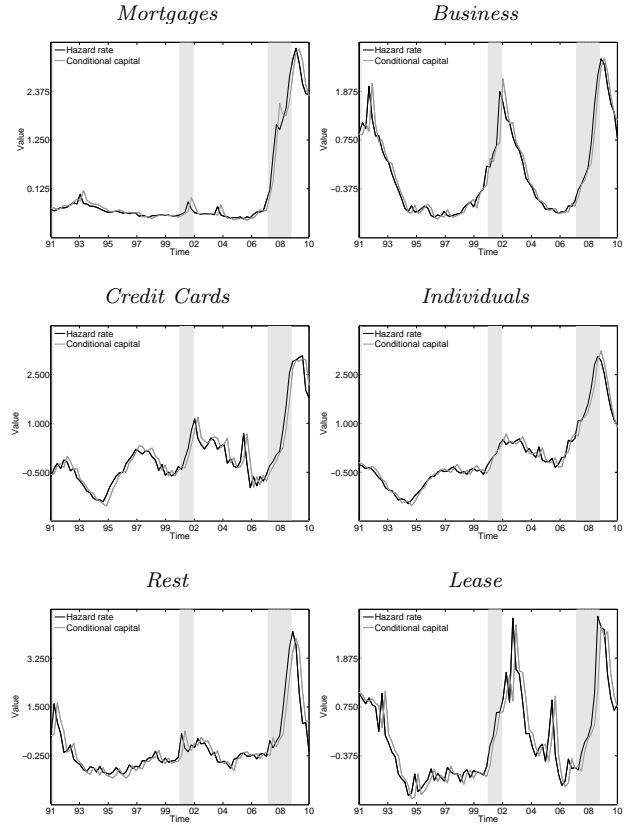


Figure 10: Hazard rate, h_t , and conditional capital, k_t , series. Both are presented standardized. Shaded areas indicate U.S. recessions, according to NBER (*National Bureau of Economic Research*).

dation of any unconditional loss model, especially if h_t is non-stationary.

Appendix A.

Proof of Proposition A.

Under the conditions of Proposition A, the following is satisfied for $x \in (0, 1)$:

$$\begin{aligned}
 f^*(x) &= E[f_t(x)] \\
 &= \int_{\mathbb{R}^K} \varphi(x; z^1, \dots, z^K) g^*(z^1, \dots, z^K) dz^1 \dots dz^K \\
 &= E_{Z^*1, \dots, Z^*K} [\varphi(x; Z^*1, \dots, Z^*K)]
 \end{aligned} \tag{A.1}$$

On the other hand, given $x \in (0, 1)$ and the realization $\{\dots, z_t, \dots\}$, the stationarity and ergodicity of z_t produce the following convergence, Doob (1953):

$$\frac{1}{T} \sum_{t=1}^T \varphi(x; z_t^1, \dots, z_t^K) \xrightarrow{T \rightarrow \infty} E_{Z^*1, \dots, Z^*K} [\varphi(x; Z^*1, \dots, Z^*K)] \tag{A.2}$$

Given Eq. A.1 and Eq. A.2, it follows that:

$$\frac{1}{T} \sum_{t=1}^T \varphi(x; \dot{z}_t^1, \dots, \dot{z}_t^K) \xrightarrow{T \rightarrow \infty} f^*(x) \quad (\text{A.3})$$

Or, equivalently,

$$\frac{1}{T} \sum_{t=1}^T f_t(x) \xrightarrow{T \rightarrow \infty} f^*(x) \quad (\text{A.4})$$

Eq. A.4 holds for every $x \in (0, 1)$, which means, according to Scheffé's theorem, Scheffé (1947), that:

$$\Pi_{\{1, \dots, T\}} \xrightarrow{T \rightarrow \infty} F^* \quad (\text{A.5})$$

since the density function of $\Pi_{\{1, \dots, T\}}$ is $\frac{1}{T} \sum_{t=1}^T f_t$ due to its condition as an equally weighted mixture, McLachlan and Peel (2000). ■

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