Initial field and energy flux in absorbing optical waveguides.

II. Implications

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We present here numerical estimates of the density and flux of energy based on physical parameters associated with the absorption properties of certain types of absorbing optical waveguides. These results are based on a theoretical formalism established previously for the initial field incident upon the entrance pupil of an absorbing optical waveguide. Based on these results, the mechanisms of confined energy transmission and possible cross talk between neighboring waveguides are discussed. Some comments on the behavior of Bessel functions with complex arguments are also included in the discussion.

1. INTRODUCTION

In an earlier paper we presented the theoretical framework for the analysis of the initial field and energy flow in cylindrical optical waveguides with absorbing properties.¹ In that paper we treated both monomode and multimode (two sets of modes) waveguides. A major primary source of attenuation in optical fibers, such as those used in telecommunication or image transmission, besides the strong effect owing to scattering, is the presence of materials with nonnegligible extinction (or absorption) coefficients. In some cases, the absorption is due to the presence of metallic ion impurities (Fe^{2+} , Fe^{3+} , Cr^{3+} , etc.) in the dielectric material (silicon or germanium) used in the fabrication of the waveguide.² In other cases, the absorption might be due to the effect of irradiation of the silicon by heavy particles such as thermal neutrons. This would induce color centers in the material, with the resultant nonnegligible absorption coefficient.³ In all these cases, the absorption edges can be located at different regions of the optical spectrum, ranging from the ultraviolet to the infrared, depending on the absorption spectrum of the material. Photochromic glasses, which are used in the manufacture of planar optical waveguides, have different absorptions for different regions of the visible optical spectrum owing to their photochemical properties.⁴

In addition to the above-mentioned induced (and some times undesirable) absorbing properties, in certain biological situations we must consider organic materials containing certain types of molecules that present nonnegligible absorption properties. We refer, in particular, to retinal photoreceptors that act as optical waveguides.⁵ Here, owing to the presence of the pigment molecules in photoreceptors, we must deal with nonnegligible absorption properties.⁶ Finally, it has been found that plant tissues act as optical fibers. Molecular pigments such as phytochrome, located inside each cell, react with the light incident upon the cell and absorb in the red part of the spectrum. This light is then transmitted through plant tissues under strict confinement conditions.⁷

Because of all these possibilities for real absorbing waveguides, we have carried out some numerical calculations based on the theoretical formalism.¹ In this paper we

present results for the fraction of energy flow and density of energy in absorbing waveguides with a nonnegligible extinction coefficient. In Section 2, in addition to the background on the necessary theoretical results, we also comment on the behavior of Bessel functions with complex arguments. A number of different experimental parameters that provide information on absorption properties can be used in any discussion of absorbing materials, and these are discussed in Section 3. Our intention is to give the reader a better understanding of these parameters and their interrelationships. Some comparisons are established in Section 4 with respect to nonabsorbing dielectric optical waveguides. Section 5 includes some results for multimode behavior. Here, we consider a waveguide supporting the fundamental mode and the first excited mode. The energy density in a weakly guiding waveguide and the effect of the oscillatory term in the expression for the total fraction of confined power is studied. We conclude with a summary in Section 6.

2. BACKGROUND: FRACTION OF POWER CONFINED WITHIN MONOMODE ABSORBING WAVEGUIDES

The fraction of power η confined within a monomode absorbing waveguide with a core radius R can be established as the ratio¹

$$\eta_0 = P_0^F / P_0^{\text{tot}}, \quad (1)$$

where

$$P_0^F = 2\pi |C_{-}^{(0)}|^2 \exp(-2 \text{Im } \hat{\beta}_0 Z) \text{Re} \left[\beta_0 \int_0^R r dr |J_0(\hat{\chi}_0' |x|)|^2 \right], \quad (2)$$

β_0 is the complex wave vector ($\text{Re } \hat{\beta}_0 + i \text{Im } \hat{\beta}_0$), and $\hat{\chi}_0'$ denotes the complex modal parameter ($\text{Re } \hat{\chi}_0' + i \text{Im } \hat{\chi}_0'$). Note that for a given $\text{Re } \hat{\chi}_0'$, $\text{Im } \hat{\chi}_0'$, and R , the final result for the P_0^F contribution is a function of Z , where Z is the length of the finite waveguide. This Z dependence comes from the damping term $\exp(-2 \text{Im } \hat{\beta}_0 Z)$.

Also,

$$P_0^{\text{tot}} = 2\pi \exp(-2 \operatorname{Im} \hat{\beta}_0 Z) \operatorname{Re} \left\{ \hat{\beta}_0 \left[|C_{<}^{(0)}|^2 \int_0^R r dr |J_0(\hat{\chi}_0' r)|^2 + |C_{>}^{(0)}|^2 \int_R^\infty r dr / H_0(i \hat{\chi}_0' r) |J_0(\hat{\chi}_0' r)|^2 \right] \right\}, \quad (3)$$

where $\hat{\chi}_0$ is the complex modal parameter and $\hat{\chi}_0 = \operatorname{Re} \hat{\chi}_0 + i \operatorname{Im} \hat{\chi}_0$. Both $\hat{\chi}_0$ and $\hat{\chi}_0'$ are related by the expression

$$\hat{\chi}_0'^2 + \hat{\chi}_0^2 = (\omega/c)(\epsilon - \epsilon_c), \quad (4)$$

where ϵ is the complex dielectric permittivity of the waveguide core and ϵ_c represents the real dielectric permittivity of the cladding. The V parameter can be expressed as

$$V_0^2 = \hat{U}_0^2 + \hat{W}_0^2, \quad (5)$$

where $\hat{U}_0 = \hat{\chi}_0' R$ and $\hat{W}_0 = \hat{\chi}_0 R$. The quantities $C_{<}^{(0)}$ and $C_{>}^{(0)}$ are numerical coefficients that are determined by the boundary conditions. The reader may consult Ref. 1 for more details.

The integral in Eq. (2),

$$I = \int_0^R r dr |J_0(\hat{\chi}_0' r)|^2 = \int_0^R r dr J_0(\hat{\chi}_0' r) J_0(\hat{\chi}_0'^* r), \quad (6)$$

can be directly evaluated by virtue of the properties of Bessel functions, assuming that $\hat{\chi}_0'^* = \operatorname{Re} \hat{\chi}_0' - i \operatorname{Im} \hat{\chi}_0'$ is acting as a second variable,⁸ and I can be shown to be

$$I = [R/(\hat{\chi}_0'^2 - \hat{\chi}_0'^*)][\hat{\chi}_0' J_1(\hat{\chi}_0' R) J_0(\hat{\chi}_0'^* R) - \hat{\chi}_0'^* J_0(\hat{\chi}_0' R) J_1(\hat{\chi}_0'^* R)], \quad (7)$$

with

$$\hat{\chi}_0'^2 - \hat{\chi}_0'^{2*} = 4i \operatorname{Re} \hat{\chi}_0' \operatorname{Im} \hat{\chi}_0'. \quad (8)$$

It is interesting to compare this result with the one corresponding to a dielectric waveguide with nonabsorbing properties.

By performing suitable manipulations, we can arrive at the following expression for I :

$$I = R(1/2 \operatorname{Re} \hat{\chi}_0' \operatorname{Im} \hat{\chi}_0') \operatorname{Im} \hat{\chi}_0' [J_0^r J_1^i + J_0^i J_1^r] + \operatorname{Re} \hat{\chi}_0' [J_0^r J_1^i - J_0^i J_1^r]. \quad (9)$$

This is the general expression for any $\hat{\chi}_0'$. Note that this expression is real, as would be expected for an energy contribution.

For numerical purposes, we will have to estimate the following functions: $J_0^r(\hat{\chi}_0' R)$, $J_0^i(\hat{\chi}_0' R)$, $J_1^r(\hat{\chi}_0' R)$, and $J_1^i(\hat{\chi}_0' R)$. The superscript r denotes the real part, and the superscript i denotes the imaginary part, for both J_0 and J_1 . It is important that even for cases with weak absorption, no approximations are allowed, even with the assumption that R is small (as is usually the case in monomode waveguides). This fact, which will be discussed below in Section 4, is due mainly to the fact that $\operatorname{Re} \hat{\chi}_0' \gg \operatorname{Im} \hat{\chi}_0'$, which makes $|\hat{\chi}_0'|R$ nonnegligible. Expansion for small arguments is therefore not allowed.

Also, the general property for Bessel functions with complex argument,

$$J_n[R|\hat{\chi}_n'| \exp(i \arg \hat{\chi}_n')] = \exp(in \arg(\hat{\chi}_n')) J_n(R|\hat{\chi}_n'|)$$

which could, in principle, considerably simplify the expres-

sion for the fraction of the confined power in monomode waveguides, is not applicable. In fact, let us write $J_n(\hat{a}) = J_n[|\hat{a}| \exp(i \arg \hat{a})]$, with $\hat{a} = \hat{\chi}_n' R$. The property that allows us to express $J_n(\hat{a})$ as $\exp(in \arg \hat{a}) J_n(|\hat{a}|)$ implies that $\arg \hat{a} = m\pi$ (where m is an integer), which is not true in general for real values of $\operatorname{Re} \hat{\chi}_n'$ and $\operatorname{Im} \hat{\chi}_n'$. More details on values for $n = 0$ and $n = 1$ are given in the following sections.

3. ABSORPTION PROPERTIES AND PHYSICAL PARAMETERS FOR A MONOMODE WAVEGUIDE

As was discussed in the introduction, the absorption properties of materials that are used in the fabrication of optical waveguides are in general due to the presence of metallic ions and/or hydroxyl groups resulting from the intrusion of water into the structure of the silicon or the germanium during the melting process. As a result of these transition-metal impurities, there is a resultant attenuation in the transmitted signal (as we are neglecting attenuation that is due to scattering).⁹ In both vertebrate and invertebrate optical photoreceptors, the absorption properties depend on the pigment concentration in their structure.¹⁰ In all cases there exists a dependence on the wavelength of the incident radiation, and, when the finite length of the waveguide is considered, it becomes necessary to establish some dispersion relations also.¹¹

To estimate values for the absorption properties in a material, several parameters can be used. In order to establish the possible relations among these parameters, we shall specify the ones most commonly used in the literature.

A. Extinction Coefficient

In an absorbing material the complex refractive index can be defined as¹²

$$n_1 = n(1 + i\kappa), \quad (10)$$

where κ is the extinction coefficient. In general, values of this parameter are obtained theoretically, based on the molar extinction coefficient.⁶ As an example, for a typical vertebrate photoreceptor pigment, $\kappa \sim 1.35 \times 10^{-3}$, which implies that light intensity in such a medium will fall to 37% of its initial value within a length of $\sim 30 \mu\text{m}$. Note that $\kappa \ll 1$.

B. Absorption Coefficient

The absorption coefficient is related to the attenuation of the density of energy and can be defined in terms of the extinction coefficient as

$$\alpha = [(4\pi)/\lambda]\kappa. \quad (11)$$

For example, for a fixed wavelength, $\lambda = 0.550 \mu\text{m}$, $\kappa = 1.35 \times 10^{-3}$ (from above), and $\alpha = 3 \times 10^{-2}/\text{cm}$. Some authors also refer to values of the absorption coefficient in terms of percent absorption per unit length. For vertebrate photoreceptors, a value of α of about 3%/μm has been reported.¹³

C. Absorbance

Absorbance represents the percentage of the energy that is absorbed in a medium that has only transmission and absorption properties. It is defined as¹⁴

$$I^A = (1 - I^t/I^i) = (1 - 10^{\text{OD}}). \quad (12)$$

The ratio I^t/I^i is called the transmittance (some authors refer to this as the transmittivity) and depends on the amplitude transmission coefficient,¹⁵ defined as

$$t = 2/(n' + 1), \quad (13)$$

with $n' = n/n_0$, where n_0 is the refractive index of the material in front of the entrance pupil of the waveguide. Also,

$$I^t/I^i = (n/n_0)(\cos \theta^t/\cos \theta^i)t^2, \quad (14)$$

for which θ^i is the angle of incidence of the incoming radiation and θ^t is the transmission angle that must satisfy the condition $\theta^t > 90^\circ$ for total internal reflection within the waveguide. In Eq. (12), OD is the optical density, defined as

$$\text{OD} = \log_{10}(I^t/I^i). \quad (15)$$

The optical density is proportional to the product of the molar extinction coefficient, the molar concentration, and the path length of light in the absorbing medium. Some authors refer to this parameter as absorbance or simply as density.

D. Bulk Loss Coefficient

The bulk loss coefficient has been introduced by some authors^{16,17} to calculate the power absorbed in an absorbing waveguide. It is proportional to the attenuated power or the power-attenuation coefficient and the fraction of the confined power of a mode propagating within an optical waveguide. Its values do not coincide with the absorption spectrum, and it gives some information on the influence of the absorption properties in cross talk between waveguides. It is defined as

$$\alpha(\lambda) = 2\omega n\kappa/(1 - \kappa^2)^{1/2}, \quad (16)$$

in terms of the angular frequency $\omega = (2\pi/\lambda)c$ (where c is the speed of light in vacuum and λ is the wavelength of the incoming radiation), the real part of the complex refractive index n , and the extinction coefficient.

The absorption spectrum can be obtained directly through optical density values experimentally measured in a conventional spectrophotometer. Absorption spectrophotometry operates on the basic premise that the light missing from the test beam has been absorbed by the medium under consideration. The difficulties in the comparison of different experimental values obtained from different sources arise because of the diversity of the physical conditions of the measuring apparatus. The variables involved include (1) optical properties of the apparatus, e.g., reflection and transmission coefficients of compound lenses and the presence of chromatic aberration; (2) characteristics of the photometer, e.g., dark current; and (3) waveguide properties of the materials being compared. Other experimental methods are based on calorimetric and photothermal techniques, which provide values of the absorption coefficient.¹⁸⁻²⁰

E. Parameters Used in the Analysis

For the numerical estimates, we have chosen the extinction coefficient as the representative parameter for the absorbing waveguide. κ is taken to be approximately equal to 1.35×10^{-3} . Other parameters introduced in the analysis are $n = 1.6$, the real part of the complex refractive index of the core;

$n_2 = 1.5$, the refractive index of the cladding; and $\lambda = 0.9 \mu\text{m}$, wavelength of the incoming radiation (with a corresponding wave number $k = 7.0 \mu\text{m}^{-1}$).

With the above parameters, the range for the complex wave number within the waveguide is

$$10.5 < \text{Re } \beta_0 < 11.2,$$

$$0 < \text{Im } \beta_0 < 0.015.$$

The modal parameters $\hat{U}_0$ and $\hat{W}_0$ are now complex values defined by

$$\hat{U}_0^2 = R^2[k^2n^2(1 - \kappa^2) + 2ik^2n^2 - \beta_0^2] = \hat{\chi}_0'^2 R^2, \quad (17)$$

$$\hat{W}_0^2 = R^2[\beta_0^2 - k^2n_2^2] = \hat{\chi}_0'^2 R. \quad (18)$$

From Eq. (17) we obtain the following system:

$$\begin{aligned} (\text{Re } \hat{\chi}_0') - (\text{Im } \hat{\chi}_0') &= k^2n^2(1 - \kappa^2) - (\text{Re } \beta_0)^2 \\ + (\text{Im } \beta_0)^2 \text{Re } \hat{\chi}_0' \text{Im } \hat{\chi}_0' &= -\text{Re } \beta_0 \text{Im } \beta_0 + k^2n^2\kappa. \end{aligned} \quad (19)$$

Solution of the system of equations above for fixed $\text{Re } \beta_0$, $\text{Im } \beta_0$, n , κ , and k provides values for $\text{Re } \hat{\chi}_0'$ and $\text{Im } \hat{\chi}_0'$. We have chosen from the range of β_0 the following values: $\text{Re } \beta_0 = 10.8 \mu\text{m}^{-1}$ and $\text{Im } \beta_0 = 0.08 \mu\text{m}^{-1}$; then

$$\text{Re } \hat{\chi}_0' = 2.8 \mu\text{m}^{-1}, \quad \text{Im } \hat{\chi}_0' = 0.03 \mu\text{m}^{-1}.$$

With these values the condition $|\hat{\chi}_0'|^2 \ll k^2$ for weak waveguides holds for both transmission and absorption properties.

The waveguide operates in a single-mode regime for all $\lambda < 0.9 \mu\text{m}$. Let us assume that the operating wavelength is $\lambda_c = 0.9 \mu\text{m}$. For fixed K , n , and n_2 we check values of R for which $V < 2.405$ (monomode cutoff condition). According to the definition of the waveguide parameter, $V = KRn \sin \theta_c = KR(n^2 - n_2^2)^{1/2}$. For $R = 0.6 \text{ m}$, we obtain $V \simeq 2.33 < 2.405$. This is an appropriate value of the core radius for the purpose of our study. For example, optical photoreceptors exhibit an average diameter of $\simeq 1.0 \mu\text{m}$. With these particular values, we can get the critical angle for incoming radiation as $\sin \theta_c = V/KRn \simeq 20^\circ$.

4. THE FRACTION OF CONFINED POWER IN AN ABSORBING MONOMODE WAVEGUIDE

With the physical parameters given above, we have estimated Eq. (2) and Eqs. (7)–(9). For the Bessel functions J_0^r , J_0^i , J_1^r , and J_1^i , use of the polynomial expansion for Bessel functions of zero and first order²¹ has been made.

We were particularly interested in the distribution P_0^F/Ω ($\Omega = \pi R^2$ is the area of the circular cross section of the waveguide) to obtain information on the flow of energy per unit area, which is proportional to the confined density of energy. In the polynomial expansion we have kept a second-order approximation, since we are dealing with small absorption and hence a small extinction coefficient (see Appendix A). The radial distribution for the real and imaginary parts of $J_0(\hat{\chi}_0'R)$ and $J_1(\hat{\chi}_0'R)$ and the cross products (broken lines) are displayed in Fig. 1. As expected for small absorption, we see the correct behavior for both J_0^r and J_1^r and the small contributions coming from J_0^i and J_1^i . In Fig. 2, the same functions are displayed with respect to the dimensionless parameter $a = \text{Re } \hat{\chi}_0'R$. The scalar product $J_0^r J_1^r +$

Fig. 1. Radial distribution of the real and imaginary parts of $J_0(\hat{\chi}_0'R)$ and $J_1(\hat{\chi}_0'R)$ (solid lines). Also shown are $J_0^r J_1^r + J_0^i J_1^i$ (---), and $J_0^r J_1^i - J_0^i J_1^r$ (-·-·-·-). These terms contribute to the fraction of the confined power of an absorbing monomode waveguide. [See Eqs. (2), (6), and (9).]

Fig. 2. Variation of J_0^r and J_1^r versus the dimensionless parameter $a = \text{Re } \hat{\chi}_0'R$. $\kappa = 1.35 \times 10^{-3}$; $|C_{<}^{(0)}|^2 = 1.0$. Dashed line represents $J_0^r J_1^r + J_0^i J_1^i$.

$J_0^i J_1^i$ exhibits divergent behavior for points close to $a \approx 3.0$, so that for values of $a \geq 3.0$, a third-order approximation would be required. The variation of the dimensionless parameter $b = \text{Im } \hat{\chi}_0'R$ versus the parameter a is shown in Fig. 3. It is found that there is attenuation of the b parameter such that for values of $a > 0.3$ the b parameter takes on values close to zero. The variation of energy density as a function of the a parameter is shown in Fig. 4. The value of the coefficient $|C_{<}^{(0)}|^2$ is fixed at 1.0. Two distances for the penetration of the field have been considered: $Z_0 = 0 \mu\text{m}$, which gives the distribution at the entrance pupil of the waveguide, and $Z_0 = 20 \mu\text{m}$. For the second case an attenuation in the central peak of $\sim 70\%$ is observed. For fixed $C_{<}^{(0)}$

($|C_{<}^{(0)}|^2 = 1.0$) and fixed R ($R = 0.6 \mu\text{m}$) we have studied the attenuation of the fraction of confined power with the distance of the penetration of the field (Fig. 5). For a distance of penetration of the field, $Z > 60 \mu\text{m}$, almost all the energy is

Fig. 3. Distribution of the dimensionless parameter $b = \text{Im } \hat{\chi}_0'R$ versus $a = \text{Re } \hat{\chi}_0'R$. For $a > 3.0$, b is almost negligible.

Fig. 4. Fraction of the confined power per area ($\Omega = \pi R^2$) within a monomode absorbing waveguide versus the a parameter. The following values have been considered ($\kappa = 1.35 \times 10^{-3}$; $|C_{<}^{(0)}|^2 = 1.0$): —, $Z_0 = 0 \mu\text{m}$ (gives the distribution at the entrance pupil of the waveguide); - - - - , $Z_0 = 20 \mu\text{m}$ (shows the attenuation). Other parameters are $R = 0.6 \mu\text{m}$ (radius of the core) and $a = 1.7$.

Fig. 5. Attenuation of the fraction of the confined power in a monomode absorbing waveguide versus the distance Z of the penetration of the field. Variables: —, $|C_{<}^{(0)}|^2 = 1.0$; ---, $|C_{<}^{(0)}|^2 = 0.25$. Other parameters are $R = 0.6 \mu\text{m}$ and $\lambda = 0.9 \mu\text{m}$.

Fig. 6. Fraction of the confined power per unit area, P_0^F/Ω versus $\hat{x}_0'R$. Here, $\hat{x}_0'$ is a real parameter, since $\kappa = 0$ (zero absorption). $|C_{<}^{(0)}|^2 = 1.0$.

completely absorbed by the waveguide with a resultant loss of signal. We notice the important effect of the axial distribution even for a small extinction coefficient. To compare these results with a situation corresponding to an absorptionless dielectric waveguide, we have estimated P_0^F/Ω with a null extinction coefficient ($\kappa = 0$; Fig. 6). We observe the same distribution as in the $\kappa = 0$ case, with some slight variation up to $a > 3.0$. It is important to remark that the shape for the distribution of the density of the energy is

practically independent of the value of the extinction coefficient, which is equivalent to saying that the most important influence on such a distribution comes from the transmission properties, that is, the values of the real part of the complex refractive index n_1 . The attenuation effect produces a decrease in the confined energy with a rapid loss as $\exp(-2 \text{Im } \hat{\beta}_0 Z)$ decreases.

Interesting results can be also obtained by studying the effect of two neighboring waveguides, separated by a distance of the order of the wavelength. If we suppose that they are separated by a distance of the order of a wavelength (a situation found, for example, in photoreceptors forming the retinal mosaic in the eye), we can observe the effect of the overlapping of both energy distributions (Fig. 7). As we are dealing with the intensity functions, we can assume an incoherent superposition of both distributions: $|\psi|^2 = |\psi_1|^2 + |\psi_2|^2$, with $|\psi_1|^2$ and $|\psi_2|^2$ being the energy confined in the first and second waveguides, respectively. Note that these results are derived from the condition that the fraction of confined power in the finite radius of the waveguide P_0^F be a measure of energy (given in terms of the square modulus of J_0), so that we give information solely in terms of the percentage of energy remaining outside the waveguides in a region close to their boundaries where the overlapping takes place. For the case of larger separation between neighboring waveguides, this description would be irrelevant. It is important to remark as well that the percentage of remaining energy outside the core is a pure effect of addition of energy. In this particular case a surplus of energy reaching a minimum of 20% remains in the intermediate region between the two neighboring waveguides. Recall that we are considering monomode waveguides; no cross-talk phenomena can be described by this method for obvious reasons.

Fig. 7. Effect of two neighboring absorbing waveguides in the fraction of the confined power per unit area, P_0^F/Ω . The waveguides are separated by a distance of the order of one wavelength. The bars denote the region in which the energy of both waveguides is cooperative. Physical parameters: $\kappa = 1.35 \times 10^{-3}$, $Z_0 = 0 \mu\text{m}$ (entrance pupil), $|C_{<}^{(0)}|^2 = 1.0$.

Fig. 8. Flow of the transmitted energy in a monomode waveguide: ---, $R_1 = 0.45 \mu\text{m}$; - - - - -, $R_2 = 0.75 \mu\text{m}$. $|C_{<}^{(0)}|^2 = 1.0$. Note the formation of efficient waveguides with larger radii, as predicted by the theory.

The effect shown comes solely from an incoherent superposition of both intensity distributions. Finally, we have studied the radial distribution of the flow of the transmitted energy (Fig. 8). Here, two different values of the core radius are considered: $R_1 = 0.45 \mu\text{m}$ and $R_2 = 0.75 \mu\text{m}$. It is found that 70% of the energy is still confined within the waveguide boundary for both cases, and this value seems to be independent of the core radius. Some percentage of the transmitted energy also flows into the cladding, producing an efficient waveguide with a radius of the order of $0.75 \mu\text{m}$ in the first case and of about $0.95 \mu\text{m}$ in the second one. This effect, together with the incoherent superposition described above, implies a mechanism wherein, as light propagates along the waveguide, there is a two-way transverse distribution of energy from both sides of the waveguide boundary, thus providing a continuous energy flow while avoiding complete attenuation of the transmitted signal.²²

5. WAVEGUIDE SUPPORTING TWO SETS OF CONFINED MODES

As described in the theoretical formalism,¹ the fraction of the confined power within an absorbing waveguide supporting two sets of confined modes, namely, the fundamental and the first excited one, can be decomposed into two contributions: (1) P_{01}^{1F} , representing the fraction of power for the two set of modes incoherently superimposed and affected by damping terms of the form $\exp(-2 \text{Im} \beta_n Z)$ ($n = 0, 1$), and (2) P_{01}^{2F} , which is a second contribution, representing the interference term that can produce cross-talk phenomena

for a fixed phase shift between both modes. We are especially interested in the numerical behavior of this term for specific values of the optical properties of the waveguides and the cutoff condition. For the purpose of this numerical study we shall rewrite Eq. (28) of Ref. 1 as

$$P_{01}^{2F} = \text{Re}[\hat{\beta}_1 K_2(Z) F(\phi_1) I + \hat{\beta}_0 K_2^*(Z) F^*(\phi_1) I^*], \quad (20)$$

with

$$I = \int_0^R r dr J_0(\hat{\chi}_0' r) J_1(\hat{\chi}_1' r) \quad (21)$$

as the integral containing the cross product between the Bessel functions of zero and first order. We note that the importance of this term is related to the specific behavior of $\hat{\beta}_1 K_2$ and $\hat{\beta}_0 K_2^*$ [$F(\phi_1)$ acts always as a constant factor].

The condition for two mode groups propagating inside the waveguide is $2.405 < V_1 < 3.832$. If we assume a core radius $R = 0.6 \mu\text{m}$ and a critical angle $\theta_c \approx 20^\circ$, we obtain for the upper limit of V_1 a wave number $K = V/Rn \sin \theta_c = 11.67 \mu\text{m}^{-1}$ (corresponding wavelength $\lambda_1 = 0.538 \mu\text{m}$). We have chosen $K_1 < 11.67 \mu\text{m}^{-1}$. Assuming, for example, that $K_1 = 11.1 \mu\text{m}^{-1}$ (corresponding to $\lambda_1 = 0.566 \mu\text{m}$), we obtain $V_1 \approx 3.644$. With these values we obtain the following range for β_1 :

$$16.6 < \text{Re} \hat{\beta}_1 < 17.7,$$

$$0.0 < \text{Im} \hat{\beta}_1 < 0.024.$$

For the numerical estimates we have chosen $\text{Re} \beta_1 = 17.15 \mu\text{m}^{-1}$ and $\text{Im} \beta_1 = 0.012 \mu\text{m}^{-1}$.

By proceeding as for the monomode case and resolving a system of equations similar to Eqs. (19), we get the values for $\text{Re} \hat{\chi}_1'$ and $\text{Im} \hat{\chi}_1'$. For the present case we have obtained $\text{Re} \hat{\chi}_1' = 4.3 \mu\text{m}^{-1}$ and $\text{Im} \hat{\chi}_1' = 0.07 \mu\text{m}^{-1}$. The numerical behavior of the real and imaginary parts of $\hat{\beta}_1 K_2$ and $\hat{\beta}_0 K_2^*$ versus the penetration distance of the field Z is shown in Fig. 9. We observe that $\text{Re}(\hat{\beta}_1 K_2)$ and $\text{Re}(\hat{\beta}_0 K_2^*)$ are both terms tending monotonically to zero after reaching a maximum at $50 \mu\text{m}$. The contributions from $\text{Im}(\hat{\beta}_1 K_2)$ and $\text{Im}(\hat{\beta}_0 K_2^*)$ are less important, as is expected for a weak absorption material; $\text{Im}[\hat{\beta}_0 K_2^*]$ exhibits negative values for the whole interval under consideration. The relative maximum of $\text{Im}[\hat{\beta}_1 K_2]$ is only 9% of that obtained for $\text{Re}[\hat{\beta}_1 K_2]$.

Fig. 9. Numerical estimate of the axial distribution of the real and imaginary parts of the coefficients $\hat{\beta}_1 K_2$ and $\hat{\beta}_0 K_2^*$ affecting the interference term in the fraction of power confined within a multi-mode absorbing waveguide supporting two sets of bound modes. Symbols: ---, $\hat{\beta}_1 K_2$; - - - - -, $\hat{\beta}_0 K_2^*$.

Fig. 10. Total fraction of the confined power per unit area, P_{01}^F/Ω , in the two sets of confined modes. Symbols: $\bullet$, contribution of the set of the fundamental modes; $\circ$, contribution of the first set of excited modes; Δ , the total fraction of power. The shaded region represents the interval where there is only the contribution of the fundamental modes [$z = 0 \mu\text{m}$ (entrance pupil), $\kappa = 0$].

In order to interpret the implications of these results, we have also studied the density of energy for the multimode behavior in a fashion similar to that described in Section 4 (see Figs. 4 and 6). Figure 10 shows the radial distribution for the density of energy at $Z = 0 \mu\text{m}$, that is, at the entrance pupil of the waveguide. We have considered here the case of a weakly guiding waveguide with weak absorption. Even though $\Delta(\chi R) = \text{Re } \hat{\chi}_0' R - \text{Re } \hat{\chi}_1' R = 0.9$, for simplicity we have assumed it to be approximately 1.0. The function P_0^F/Ω (filled circles) represents the contribution of the set of fundamental modes. The function P_1^F/Ω (open circles) is the contribution that is due to the first set, and P_{01}^F/Ω is the total function for the multimode waveguide. Both P_0^F and P_1^F are superimposed with a shift; therefore their superposition in the region $\chi_0' R < 1.0$ is noncooperative, the maximum of P_{01}^F obtained at the origin coincides with one associated with the monomode case, and the total P_{01}^F/Ω is slightly modulated by the P_1^F/Ω function. From Fig. 9 we see that for $Z_0 = 0 \mu\text{m}$ the effect of the oscillating term is null over P_{01}^F/Ω . Nevertheless, at $Z_0 = 50 \mu\text{m}$ the function P_0^F will be affected by $\exp(-2 \text{Im } \hat{\beta}_0 50)$, and the total result for the density of energy will be an attenuation in that function; the same effect will occur for P_1^F/Ω (open circles), so the final result would be an attenuation of the density of energy. But as P_{01}^{2F} now has a maximum, an increment of the total density of energy takes place, consequently diminishing the effect of the attenuation. For $Z > 50 \mu\text{m}$ this process does not take place any more; then, at certain fixed distances on the axial direction of the waveguide, some percentage of the energy can flow transversally to the axis, giving rise to interference phenomena, which will take place in the cladding region within the efficient diameter of the efficient waveguide.

6. SUMMARY AND CONCLUSIONS

From a theoretical formalism¹ for the initial field penetrating an absorbing optical waveguide, we have estimated the distribution of the fraction of energy and the density of

energy in both monomode and multimode waveguides. The influence of the nonzero extinction coefficient on the cutoff conditions requires a new formulation of the modal parameters. For fixed λ , n_1 , and n_2 , these parameters have been estimated numerically for a monomode (V_0 -parameter) waveguide and a multimode (V_1 -parameter) waveguide. Since these parameters are now defined by complex values, we must deal with solutions for the modal field in terms of Bessel functions with complex arguments, whose behavior has also been studied. Interesting results have been obtained for the effect of two neighboring waveguides. A certain percentage of the confined energy acts cooperatively in the cladding region, together with a flow of the transmitted energy from the core to the cladding through the waveguide boundary. The shape of the radial distribution for the density of energy is independent of the absorption properties of the waveguide. For a multimode case, the effect of the second term, which contributes to the total fraction of confined power as an oscillatory term, has been also investigated, together with the radial distribution of the density of energy. It is concluded that for specific values of the modal parameters, some energy can escape in a direction transverse to the waveguide axis, at specific points of the boundary, for which the interference term exhibits maxima. This energy is confined to the cladding, specifically inside the efficient diameter, making possible the presence of cross-talk phenomena. This particular mechanism seems to avoid the complete attenuation of the transmitted signal. The above results are in agreement with earlier results obtained by Sammut and Snyder,²³ who concluded that only a portion of light interacting at the aperture pupil of an ideal waveguide remains trapped for the entire length of the waveguide. The rest of the energy escapes as a radiation field. Our results are complementary to their results. We conclude also that bound modes contribute to the existence of the remaining energy outside the core of the waveguide. Our results were obtained through the exact formulation of the initial field at the entrance pupil of the absorbing waveguide.

APPENDIX A

To compute the numerical behavior of J_0 and J_1 with complex arguments, we made use of the polynomial expansion for J_0 and J_1 .²⁴ In the special case considered here, we can write

$$-3 \leq \hat{x} \leq +3, \quad \text{with} \quad |\hat{x}| = |\hat{\chi}_0' R| = [(\hat{\chi}_0'')^2 + (\hat{\chi}_0'')^2]^{1/2} R.$$

By assuming weak absorption and transmission conditions, we have kept a second order of approximation. Separating out the real and imaginary parts, we can write

$$J_0'(\hat{x}) = 1 - 0.25[(\hat{\chi}_0'')^2 - (\hat{\chi}_0'')^2]R^2, \quad |\hat{\chi}_0' R| \leq 3,$$

$$J_0^i(\hat{x}) = -0.5(\hat{\chi}_0'' \cdot \hat{\chi}_0' \cdot R^2), \quad |\hat{\chi}_0' R| \leq 3,$$

and similarly, for J_1 ,

$$J_1'(\hat{x}) = \hat{\chi}_0'' R \{0.5 - 0.0625 R^2 [(\hat{\chi}_0'')^2 - 3(\hat{\chi}_0'')^2]\}, \quad |\hat{\chi}_0' R| \leq 3,$$

$$J_1^i(\hat{x}) = \hat{\chi}_0' R \{0.5 - 0.0625 R^2 [3(\hat{\chi}_0'')^2 - (\hat{\chi}_0'')^2]\}, \quad |\hat{\chi}_0' R| \leq 3.$$

The stability of these functions is ensured for points close to 3.0. For $\hat{x} \geq 3.0$ it would be necessary to use third (and higher) orders of approximation, resulting in tedious calculations for deriving explicit expressions for the real and imaginary parts of the functions under consideration.

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