Generation of Optical Reference Signals Robust to Diffractive Effects

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***Abstract*—In grating measurement systems, a reference signal is needed to achieve an absolute measurement of the position. The zero reference signals are normally obtained illuminating two identical superimposed zero reference codes (ZRCs) and registering the transmitted light by means of a photodiode. As one ZRC moves with respect to the other, the two codes overlap and the signal registered is the autocorrelation of the ZRC transmittance. In high resolution systems, the diffraction effects degrade the geometrical shadow of the first ZRC as it propagates to the second one. As a result, the autocorrelation is also degraded and the amplitude of the reference signal is greatly reduced. In this letter, we present a method for designing ZRCs with minimum diffractive effects. The method is based on the optimization of ZRCs by means of a genetic algorithm.**

***Index Terms*—Gratings, measurement, optical device fabrication, optical transducers, optimization methods.**

I. Introduction

GRATING measurement systems are widely used for position measurements in microtechnology and precision engineering. The increasing demand of high resolution systems has created a strong incentive for the design of systems that generate zero reference signals (ZRSs). A ZRS consists of an isolated and well distinct pulse, which is used by grating measurement systems to obtain a home position or absolute position measurement. The absolute measurement systems use gratings with adjacent zero reference codes (ZRCs). These gratings have been developed since 1986 [1]. A ZRC is a group of unequally and especially spaced slits. In Fig. 1, we show a scheme of a reference signal generation system. The system consists of two identical ZRCs, which stand opposite and they are separated by a distance z. As one moves respect to the other, it produces a change in the overlapping area between them. A parallel ray beam propagates through both codes and the transmitted light is registered by means of a photodiode. The signal registered in the photodiode is the ZRS. In grating measurement systems, the most important parameters that characterize a ZRS are the width of the central maximum and the "effective signal amplitude." The effective signal amplitude is the height difference between the central and second maximum. The width of the central maximum of the signal is inversely proportional of the system resolution and the effective amplitude is related to the stability and robustness of the system.

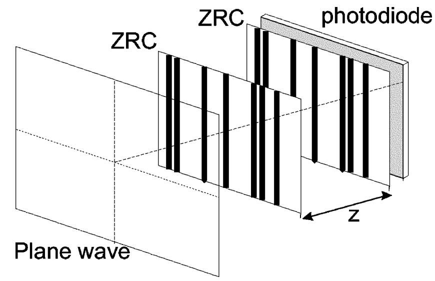

Fig. 1. Scheme of the system of generation of ZRS.

II. Description and Modeling of the Problem

In a practical implementation of a grating measurement system with a reference signal, the signal must be electronically converted into a rectangular pulse of width equal to the width of the slit of the ZRC and the grating period. For that, an electronic threshold is provided at half the height of the effective amplitude. Electronic comparison between the signal and the threshold generates an electronic pulse which width is the full-width at half-effective-amplitude of the signal. If the diffractive effects of the beam as it propagates between the codes are neglected, the ZRS is the autocorrelation of the ZRC. This condition is named the autocorrelation approximation. The central maximum of the autocorrelation signal is a triangle whose width in the base is twice the width of the slits of the ZRC. Optimum ZRS has the largest effective amplitude and the full-width at half-effective-signal-amplitude closest to the width of the slits. The characterization and design of suitable ZRCs to obtain good autocorrelation signals has been previously studied [2], [3]. The main limitation of these ZRCs designing techniques is that the necessary calculations are laborious and there is not a systematic method for obtaining codes larger than 30 elements. Recently, we presented [4], [5] a new approach to the design of ZRCs based on optimization techniques. We used a deterministic sampling optimization method [4] to obtain optimum ZRCs up to 100 elements and we used a genetic algorithm (GA) [5] to obtain optimum ZRCs with thousands of elements. Using these techniques, we have extended the design capabilities of optimum ZRCs. In all these works, the authors consider a parallel ray beam and neglect the distance between the two ZRCs. Accordingly, they consider the autocorrelation approximation to the problem. Usually, the period of the gratings of displacement measurement systems determines the width of the ZRS and, therefore, the width

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of the slits of the ZRC. Today, ultrahigh resolution systems impose the use of very narrow slits (a few micrometers). This reduction of the width of the slits, along with the significant distance between codes (in some cases, a few millimeters) makes diffraction effects much more noticeable and the autocorrelation approximation is not valid. The shadow of the first code onto the second one is distorted by diffraction, and the signal becomes the correlation between the distorted shadow and the transmittance of the ZRC. This situation produces two drawbacks in the signal: the effective amplitude decreases and the width of the central peak dramatically increases. In this work, we remove the autocorrelation approximation and propose a method to design ZRCs robust to diffractive effects based on a GA. Our method considers the propagation of an incident ray beam through ZRCs and it calculates the diffracted ZRS. The GA find the ZRC that generates the fewest degraded signal. The ZRS generated have the maximum effective amplitude and a suitable width of the central peak. Mathematically, a ZRC consists of a group of unequally spaced slits which can be described by the following sequence of binary data, $\mathbf{c} = [c_1, \ldots, c_n]$, where n is the length of the ZRC, $c_i = 1$ if a transparent slit is located at the i-position, and $c_i = 0$ elsewhere. The light and dark regions in the ZRC are integer multiples of the width of a single slit, named b. Accordingly, the transmittance of a ZRC can be expressed as the following sum:

$$t(x) = \sum_{j=1}^{n} c_j \cdot \mathrm{rect}\left(\frac{x - j \cdot b}{b}\right) \qquad (1)$$

where rect is the rectangle function. In order to calculate the ZRS, we assume that the illuminating light is a parallel ray beam and we calculate the propagated beam using the angular spectrum of plane waves [7]. The light passes through the first ZRC and then, it incides on the other ZRC. The field amplitude in the second ZRC is

$$U(x) = \int_{-\infty}^{+\infty} T(\nu) e^{jkz\sqrt{1-(\lambda\nu)^2}} e^{j2\pi\nu x} d\nu \qquad (2)$$

where $T(\nu)$ is the Fourier transform of the transmittance of the first ZRC (1), λ is the wavelength of the light beam, $k = 2\pi/\lambda$ is the wave number, and z is the gap between the ZRCs. Then, the light passes through the second ZRC and the intensity behind it is $I(x) = |U(x)|^2 |t(x)|^2$, where $t(x)$ is a binary transmittance. When one code laterally shifts with respect to the other, the signal registered in the photodiode is the correlation between the light that passes through the first ZRC and the other ZRC

$$S(u) = \int_{-\infty}^{+\infty} |U(x)|^2 \, t(u + x) dx \qquad (3)$$

where u is the relative displacement between ZRCs. If the distance z between ZRCs is neglected, the diffraction is not considered and (3) becomes the autocorrelation approximation. Fig. 2 shows the effect of the diffraction. A ZRC is optimized with autocorrelation approximation, by means of the method shown in [5]. The discontinuous graph is the signal obtained with this code and without diffraction. The continuous one

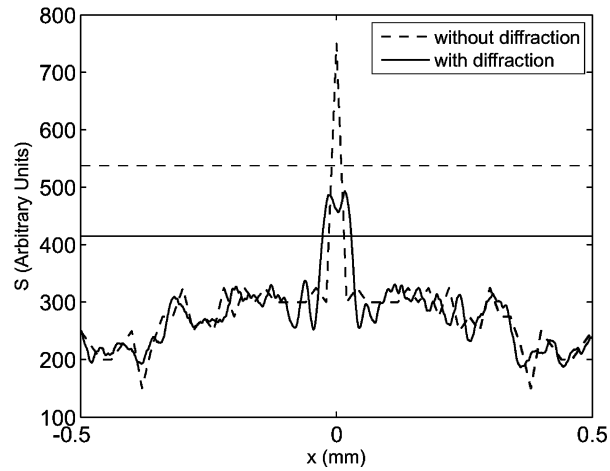

Fig. 2. Effect of the diffraction. Discontinuous graph is the reference signal calculated without diffraction (autocorrelation approximation) and continuous one is the calculated signal taking into account the diffraction in the ZRC. Also, the electronic threshold situated at half the height of the effective signal amplitude of each signal can be seen.

is the signal taking into account the diffractive effects in the ZRC (3). Also, the electronic threshold situated at half the height of the effective signal amplitude of each signal can be seen. Because of the diffraction, the effective signal amplitude decreases and the full-width at half-effective-signal-amplitude increases. The ZRC has 60 elements, the width of the slit is $b = 20$ μm, and it is described by the binary vector $\mathbf{c} = [1\,1\,0\,0\,1\,0\,0\,0\,1\,1\,1\,1\,0\,0\,0\,1\,1\,1\,0\,1\,0\,1\,0\,0\,0\,1\,1\,0\,0$ $1\,0\,1\,1\,0\,1\,0\,0\,0\,1\,0\,0\,1\,0\,1\,0\,1\,1\,0\,0\,1\,0\,0\,1\,0\,1\,0\,1\;1\,1\,1]$. The wavelength used is $\lambda = 635$ nm and the distance between ZRCs is $z = 2.5$ mm. In the present example, the effective signal amplitude of the autocorrelation approximation is 425 units, compared to 156 units in the diffracted signal (discontinuous and continuous signals, respectively). The full-width at half-effective-signal-amplitude in the autocorrelation approximation is 18.9 μm and the diffracted signal has 56.2 μm. A quantitative comparison between these signals shows that the autocorrelation approximation is not valid; the diffraction makes this code useless. On the other hand, the central peak is split in two lobes. These lobes can produce errors in the detection of the position, which is a serious problem. A ZRS with a great effective amplitude is a sufficient condition for a narrow full-width at half-effective-amplitude. Accordingly, the objective is to obtain diffracted signals with the maximum possible quantity of effective signal amplitude and the suitable width of the signal is assured. Because of the binary character of the variables of the ZRC and the efficiency of the genetic algorithms with binary variables [4], [5], we propose to use a GA for this task.

III. Design Methodology

GAs are robust problem-solving techniques based on the principles of natural evolution and selection [6]. GAs are population-based algorithms, in which a set of potential solutions to the problem are evolved by means of the application of several operators. In order to apply a GA to a given optimization problem, an objective function and a set of genetic operators are needed, which will be applied to the population of solutions to improve it. The objective function is the effective signal amplitude

$$f(\mathbf{c}) = S(0) - S(u_0) \tag{4}$$

where $S(u_0)$ is the second maximum of the ZRS. This value is obtained removing the central maximum of the signal and calculating the maximum of the new signal. The optimization problem is

$$\max_{\mathbf{c}} f(\mathbf{c}), \qquad c_j \in \{0, 1\} \tag{5}$$

where the only constraint is the binary character of the variables. The dependence with $\mathbf{c}$ is given by the transmittance of the ZRC, (2) and (3). The GA we implement for finding optimum binary ZRCs starts with a randomly generated initial population of binary codes (individuals) of length n. Each ZRC represents a possible solution to the problem, which must be evaluated to obtain an objective function value associated to it, also known as fitness. The population of ZRCs is evolved through the successive application of the genetic operators, basically selection, crossover, and mutation [6]. Selection is the process by which individuals are randomly sampled with probabilities proportional to their fitness values, which, in this case, is the value of f. An elitist strategy, consisting of passing the highest fitness codes to the next generation, is applied in order to preserve the best solution encountered thus far in the evolution. The selected set, of the same size of the initial population, is subjected to the crossover operation with a probability P_c. First, the ZRCs are coupled with a predetermined probability. Second, for each pair of codes, an integer position along the code is selected uniformly at random. Two new codes are then composed by swapping all elements between the selected position and the end of the code. The last operator in the standard GA is the mutation operator. By means of this operation, every element in every ZRC of the population may be changed from 1 to 0, or vice versa, with a very small probability named P_m. This operator prevents the GA from prematurely convergence to suboptimal solutions. The GA evolution stops when a given stop criterion is fulfilled, usually, the number of generations (number of times that the genetic operators are applied).

IV. Results and Discussion

The GA was run with $P_c = 0.6$, $P_m = 0.015$, 5000 generations, and a population of 100 individuals. The length of the ZRCs is 60 elements. In Fig. 3, we show two diffracted reference signals obtained with two different ZRCs. The continuous graph is the signal obtained with a ZRC obtained with the autocorrelation approximation, this is the continuous signal shown in Fig. 2. The discontinuous graph is the signal obtained with the new ZRC robust to diffractive effects. Also, the electronic threshold situated at half the height of the effective signal amplitude of each signal can be seen. The quantity of the effective signal with the autocorrelation approximation is 156 units, and by means of the new technique, the effective signal obtained is 194.5 units. The full-width at half-effective-signal-amplitude with the autocorrelation approximation is 56.2 μm and the new signal has 23.6 μm. The new signal has a greater quantity of effective signals and the width of the new signal is similar to the width of the ZRC slits (20 μm), since this ZRC is less sensitive to the diffraction effects. Furthermore, the new signal has one single central lobe. The optimum diffractive ZRC is $\mathbf{c} =$ [1 1 0 0 1 1 0 0 0 0 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0]. Note that in this code, the ones are more grouped than autocorrelation case.

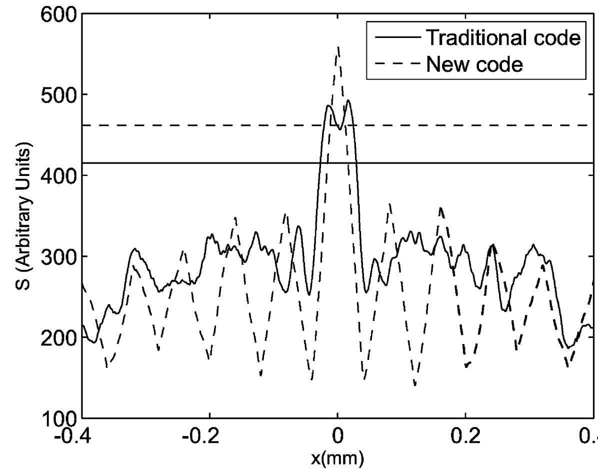

Fig. 3. Comparison between the diffractive effects in different ZRCs. Continuous graph is the signal calculated with a traditional ZRC. Discontinuous graph is the calculated signal obtained with the new ZRC, robust to diffractive effects. Also, the electronic threshold situated at half the height of the effective signal amplitude of each signal can be seen.

V. Conclusion

The application of GAs has been demonstrated to be a suitable technique to the design of reference signals robust to diffractive effects. The degradation of the reference signal produced by the diffraction of the light beam has been shown as it propagates between the codes. A method to model the diffractive effects and the design of reference signals robust to these effects has been proposed. Because of the binary nature of the problem and the parallel processing of the GAs, these algorithms are efficient tools for obtaining optimum codes. It has been shown that optimum codes have especially grouped transparent and opaque areas that minimize the diffractive effects.

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