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APPLICATIONS OF GAVER-STEHFEST METHOD OF INVERTING LAPLACE TRANSFORMS TO RUIN THEORY

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APPLICATIONS OF GAVER-STEHFEST METHOD OF INVERTING LAPLACE TRANSFORMS TO RUIN THEORY

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Applications of Gaver-Stehfest method of inverting Laplace transforms to Ruin Theory

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ABSTRACT The so-called Renewal Theory is a frequently used methodology in applied mathematics. Renewal Theory is mainly focussed on solving a Volterra integral equation of the second kind known as Renewal Integral Equation:

$$\Phi(u) = h(u) + \int_0^u \Phi(u-x) d_x F(x)$$

Gaver-Stehfest algorithm of inverting Laplace transform willl be applied to the calculation of ultimate ruin probabilities in the classical case of Risk Theory. We will show that this method could be considered as extraordinary fast and accurate specially when dealing with heavy-tailed claim size distributions.

1. INTRODUCTION

The non-ruin probability could be expressed using the following integral equation:

$$\Phi(U) = \frac{\theta}{1+\theta} + \frac{1}{(1+\theta)\,p_1} \int_0^U \Phi(U-x)\,(1-F(x))\,dx \qquad (1.1)$$

where $F(x)$ is the d.f. of the claim size and p_1 is the expected value of the claim size.

Using the Laplace transform of the former expression:

$$\Phi^*(s) = \int_0^{+\infty} e^{-sx}\Phi(x)\,dx = \frac{\theta}{1+\theta}\left(\frac{1}{s}\right) + \frac{1}{(1+\theta)\,p_1}\Phi^*(s)\left(\frac{1}{s} - F^*(s)\right) \qquad (1.2)$$

finally:

$$\Phi^*(s) = \frac{\frac{\theta}{1+\theta}\left(\frac{1}{s}\right)}{1 - \frac{1}{(1+\theta)p_1}\left(\frac{1}{s} - F^*(s)\right)} \qquad (1.3)$$

where:

$$F^{*}(s)=\int_{0}^{+\infty} e^{-sx}F(x)\,dx \tag{1.4}$$

once that we got the expression of the Laplace transform, we will use the Gaver-Stehfest method (Communications of the ACM. Volume 13. Number 1. January, 1970) to approximate the value of the non-ruin probability function:

$$\Phi(U)\simeq z\sum_{n=1}^{N}k_{n}^{N}\Phi^{*}(nz) \tag{1.5}$$

where:

$$z=\frac{\ln(2)}{U}$$

$$k_{n}^{N}=(-1)^{n+\frac{N}{2}}\sum_{i=\left[\frac{(n+1)}{2}\right]}^{Min\left(n,\frac{N}{2}\right)}\frac{i^{\frac{N}{2}}(2i)!}{\left(\frac{N}{2}-i\right)!i!(i-1)!(n-i)!(2i-n)!}$$

and N is even.

The Gaver-Stehfest method turned out to be a very useful method of inverting Laplace transforms in the paper of Davies and Martin (Numerical Inversion of the Laplace transform. Journal of Computational Physics 33. pgs: 1-32 1979).

One of the advantages of this method is that we only need to evaluate the Laplace transform values in the real axis. The efficiency in terms of computational time is another interesting fact, we only need to obtain the value of the Laplace transform ($\Phi^{*}(s)$) N times to get the approximation and the coefficients k_{n}^{N} $(n=1,..,N)$ are constant for any value of U. This figure N should be equal to the maximum number of significant digits with which our programming language works (20 with extended type in Turbo Pascal 6.0) as stated in the original paper by Stehfest pg. 48. The accuracy observed using this method in 50 sample functions ranged from 8 to 17 significant digits .

Finally, using (1.3) in (1.5) :

$$\Phi(U)\simeq z\sum_{n=1}^{20}k_{n}^{20}\left(\frac{\frac{\theta}{1+\theta}\left(\frac{1}{nz}\right)}{1-\frac{1}{(1+\theta)p_{1}}\left(\frac{1}{nz}-F^{*}(nz)\right)}\right) \tag{1.6}$$

We can conclude that we **only need to use the Laplace transform of the claim size distribution function (1.4) and**

substitute into (1.6) in order to obtain the approximation.
. In the next section we will get expression (**1.4**) for several models of the claim size.

2. Laplace transform of the distribution function of the claim size.

2.1. Exponential Claim size(Table 1).

The Laplace transform of the claim size distribution function is:

$$F^*(s) = \int_0^{+\infty} e^{-sx}\left(1 - e^{-\frac{1}{p_1}x}\right)dx = \frac{1}{s} - \frac{p_1}{sp_1 + 1}$$

2.2. Pareto Claim size(Table 2).

We will use the following family of Pareto distributions with common expected value $p_1 = 1$:

$$F(x) = 1 - \left(\frac{\lambda}{x+\lambda}\right)^{\lambda+1} \qquad x \geq 0 \qquad \lambda \geq 1 \quad \text{integer}$$

and the Laplace transform:

$$F^*(s) = \left(\frac{1}{s}\right) - \int_0^{+\infty} e^{-sx}\left(\frac{\lambda}{x+\lambda}\right)^{\lambda+1} dx \qquad s \geq 0 \qquad (2.1)$$

the former integral could be expressed in terms of the exponential integral (Table of integrals, series and products. I.S. Gradshteyn. pg. 359. Expression 3.353.2.):

$$\int_0^{+\infty} e^{-sx}\left(\frac{\lambda}{x+\lambda}\right)^{\lambda+1} dx = \frac{\lambda^{\lambda+1}}{\lambda!}\left(\sum_{k=1}^{\lambda}(k-1)!\,(-s)^{\lambda-k}\lambda^{-k} - \frac{(-s)^{\lambda}}{\lambda!}e^{\lambda s}E_i(-\lambda s)\right)$$

$$s \geq 0 \qquad (2.2)$$

we will now use the series expansion of the exponential integral in order to guarantee highly accurate results (Handbook of mathematical functions. Abramovitz and Stegun. pgs. 228/9. Exp. 5.1.7. and 5.1.10.):

$$E_i(-x) = -E_1(x) = \gamma + Ln(x) + \sum_{n=1}^{\infty}\frac{(-1)^n x^n}{nn!} \qquad x > 0 \quad (2.3)$$

where γ is Euler's constant. The infinite sum of expression(2.3) could be truncated when the desired accuracy is obtained and the number of terms will not be very large when x is small.

Finally, substituting (2.3) in (2.2), expression (2.1) could be written:

$$F^*(s) = \left(\frac{1}{s}\right) - \frac{\lambda^{\lambda+1}}{\lambda!}\left(\sum_{k=1}^{\lambda}(k-1)!\,(-s)^{\lambda-k}\lambda^k - \left(\gamma + Ln(\lambda s) + \sum_{n=1}^{\infty}\frac{(-1)^n(\lambda s)^n}{nn!}\right)\right) \tag{2.4}$$

If we study the range of values of the argument of the function $F^*(s)$ used in the approximation of expression (1.6) , it is obvious that the maximum is $\frac{20Ln(2)}{U}$; so the maximum value in brackets of the infinite sum of (2.4) will be $\frac{\lambda 20Ln(2)}{U}$. When $U \geq \lambda 20Ln(2)$ (most interesting case in heavy-tailed distributions) the number of terms of the infinite sum that can guarantee a high degree of accuracy will not be large.

2.3. Log-Normal claim size.

In this case :

$$F(x) = \phi\left[\frac{\ln(x)-\mu}{\sigma}\right] \qquad x \geq 0$$

and the Laplace transform:

$$F^*(s) = \int_0^{+\infty} e^{-sx}\phi\left[\frac{\ln(x)-\mu}{\sigma}\right]dx$$

can be approximated using composite Gaussian integration.

2.4. Inverse-Gaussian claim size.

The distribution function is:

$$F(x) = \phi\left[(\beta x)^{-\frac{1}{2}}(x-\mu)\right] + e^{\frac{2\mu}{\beta}}\phi\left[-(\beta x)^{-\frac{1}{2}}(x+\mu)\right] \qquad x>0 \qquad \mu,\beta>0$$

and the Laplace transform using (1.4) and integrating by parts:

$$F^*(s) = \int_0^{+\infty} e^{-sx}F(x)\,dx = \left(\frac{1}{s}\right)[F(0)+f^*(s)] = \left(\frac{1}{s}\right)e^{\frac{\mu}{\beta}\left(1-(1+2\beta s)^{\frac{1}{2}}\right)} \qquad s \geq -\frac{1}{2\beta}$$

where $f^*(s)$ is the Laplace transform of the density function. (Insurance risk models. Panjer and Willmot. pg.114).

3. NUMERICAL EXAMPLES.

Our purpose is testing the accuracy of this method with the non-ruin probability function.

The number of significant digits(t) will be defined as the largest non negative integer for which:

$$\frac{|exact - approx.|}{exact} < 5 \times 10^{-t}$$

(Numerical Analysis. Burden and Faires. pg. 12) and the approximation is obtained using (1.6).

Table 1 is devoted to the case of exponential claim size ($p_1 = 1$) and in this case a close formula for the non-ruin probability exists:

$$\Phi(U) = 1 - \left(\frac{1}{1+\theta}\right) e^{\left\{-\frac{\theta}{1+\theta}\frac{U}{p_1}\right\}} \tag{3.1}$$

(Bowers et al. Chapter 12. pg.354. exp. 12.3.8.).

In Table 2 the claim size follows a Pareto distribution ($\lambda = 1$) and for the exact value we used the result of the product integration with at least 8 significant digits guaranteed.

Table 3 uses a log-normal distribution with parameters $\mu = -1.61$ and $\sigma = 1.8$ ($p_1 = 1$)and the exact value is again obtained via product integration..

Finally, in Table 4 we show approximations of the non-ruin probability for inverse-gaussian claim size (another heavy-tailed distribution frequently used) with parameters $p_1 = \mu = 1$ $\beta = 100$.

4. CONCLUSIONS

The Stehfest-Gaver method of inverting Laplace transforms is a very useful tool in approximating non-ruin probabilities.

An accuracy of 6 to 10 significant digits is obtained in every case studied (Tables 1,2 and 3) except for Log-normal claim size and large initial reserves where the accuracy of the "exact" values (using Product integration) is not guaranteed to be more than 5 digits . The efficiency in terms of computational time is also outstanding because we only need to evaluate 20 times the Laplace transform of the c.d.f. of the claim size as shown in (1.6).

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Table 1.Exponential claim size

Initial reserves	exact value	Approximation	significant digits
	security loading 0.10		
1	1.69908439743398E-0001	1.69908439770805E-0001	10
2	2.42042801749836E-0001	2.42042801766076E-0001	10
3	3.07908739366478E-0001	3.07908740508401E-0001	9
4	3.68050974182838E-0001	3.68050974265166E-0001	10
5	4.22966891872471E-0001	4.22966892433273E-0001	9
6	4.73110655650173E-0001	4.73110660956152E-0001	8
7	5.18896962272682E-0001	5.18896965783159E-0001	8
8	5.60704471645613E-0001	5.60704471100288E-0001	9
9	5.98878938400015E-0001	5.98878936542536E-0001	9
10	6.33736071337152E-0001	6.33736071038652E-0001	10
15	7.67518945533545E-0001	7.67518945448946E-0001	10
20	8.52435808016502E-0001	8.52435808424416E-0001	10
25	9.06335633216578E-0001	9.06335576359720E-0001	7
30	9.40547815245582E-0001	9.40547722365047E-0001	7
35	9.62263533150804E-0001	9.62263569444643E-0001	8
40	9.76047290168683E-0001	9.76047509864536E-0001	7
45	9.84796342737754E-0001	9.84796710497752E-0001	7
50	9.90349685034567E-0001	9.90350195945269E-0001	6
55	9.93874593637195E-0001	9.93875015539217E-0001	7
60	9.96111981500719E-0001	9.96112149101849E-0001	7
65	9.97532133060993E-0001	9.97531869629976E-0001	7
70	9.98433554976714E-0001	9.98432899133832E-0001	6
75	9.99005720295452E-0001	9.99004704397909E-0001	6
80	9.99368894460910E-0001	9.99367602649885E-0001	6

Initial reserves	exact value	Approximation	significant digits
	security loading 0.25		
1	3.45015397537615E-0001	3.45015399081559E-0001	9
2	4.63743963171489E-0001	4.63743961964280E-0001	9
3	5.60950691124779E-0001	5.60950694178492E-0001	8
4	6.40536828706223E-0001	6.40536834205998E-0001	8
5	7.05696447062846E-0001	7.05696441221273E-0001	8
6	7.59044630470238E-0001	7.59044630545609E-0001	10
7	8.02722428846715E-0001	8.02722431381115E-0001	9
8	8.38482785604276E-0001	8.38482786815076E-0001	9
9	8.67760889422731E-0001	8.67760888837957E-0001	9
10	8.91731773410710E-0001	8.91731752556876E-0001	8
15	9.60170345305709E-0001	9.60170337048993E-0001	8
20	9.85347488889013E-0001	9.85347820524478E-0001	7
25	9.94609642400732E-0001	9.94610012649656E-0001	7
30	9.98016998258667E-0001	9.98016727291440E-0001	7
35	9.99270494427556E-0001	9.99269506010181E-0001	6
40	9.99731629897678E-0001	9.99730370694638E-0001	6
45	9.99901272156731E-0001	9.99900349051558E-0001	6
50	9.99963680056190E-0001	9.99963563762474E-0001	7
55	9.99986638639368E-0001	9.99987308056237E-0001	6
60	9.99995084630117E-0001	9.99996326428694E-0001	6
65	9.99998191736474E-0001	9.99999664643154E-0001	6
70	9.99999334777025E-0001	1.00000078159110E+0000	6
75	9.99999755278144E-0001	1.00000099445572E+0000	6
80	9.99999909971860E-0001	1.00000075988239E+0000	6

	security loading 0.50		
Initial reserves	exact value	Approximation	significant digits
1	5.22312459617474E-0001	5.22312456093566E-0001	8
2	6.57721920644939E-0001	6.57721923568126E-0001	9
3	7.54747039219038E-0001	7.54747038494120E-0001	9
4	8.24268574589515E-0001	8.24268571040400E-0001	9
5	8.74082931441625E-0001	8.74082933793631E-0001	9
6	9.09776477842258E-0001	9.09776467135502E-0001	8
7	9.35352021423730E-0001	9.35351987636523E-0001	8
8	9.53677691184799E-0001	9.53677659891495E-0001	8
9	9.66808621088091E-0001	9.66808607275445E-0001	8
10	9.76217337768498E-0001	9.76217402891796E-0001	7
15	9.95508035333943E-0001	9.95508346518228E-0001	7
20	9.99151577465773E-0001	9.99150957848973E-0001	6
25	9.99839753682387E-0001	9.99838748949949E-0001	6
30	9.99969733380158E-0001	9.99969595059271E-0001	7
35	9.99994283373932E-0001	9.99995208225726E-0001	6
40	9.99998920268805E-0001	1.00000023726641E+0000	6
45	9.99999796065120E-0001	1.00000081673326E+0000	6
50	9.99999961481677E-0001	1.00000044215873E+0000	7
55	9.99999992724828E-0001	9.99999063344616E-0001	7
60	9.99999998625898E-0001	9.99999473636730E-0001	6
65	9.99999999740466E-0001	9.99999252776395E-0001	6
70	9.99999999950980E-0001	9.99999201706092E-0001	6
75	9.99999999990741E-0001	9.99999270045184E-0001	6
80	9.99999999998251E-0001	9.99999395283873E-0001	6

	security loading 0.75		
Initial reserves	exact value	Approximation	significant digits
1	6.27749109982254E-0001	6.27749112649679E-0001	9
2	7.57501231041743E-0001	7.57501231567304E-0001	9
3	8.42026830497392E-0001	8.42026828965644E-0001	9
4	8.97090107344027E-0001	8.97090114432603E-0001	8
5	9.32960476517571E-0001	9.32960456246904E-0001	8
6	9.56327836005275E-0001	9.56327806527464E-0001	8
7	9.71550246646935E-0001	9.71550251817150E-0001	8
8	9.81466719488688E-0001	9.81466801124384E-0001	7
9	9.87926697210752E-0001	9.87926897207125E-0001	7
10	9.92134979009685E-0001	9.92135275041564E-0001	7
15	9.99077282296103E-0001	9.99076852080426E-0001	7
20	9.99891747528438E-0001	9.99890957657916E-0001	6
25	9.99987299910309E-0001	9.99987623495843E-0001	7
30	9.99998510036068E-0001	9.99999572447939E-0001	6
35	9.99999825198674E-0001	1.00000073631082E+0000	6
40	9.99999979492454E-0001	1.00000026368528E+0000	7
45	9.99999997594072E-0001	9.99999681939309E-0001	7
50	9.99999999717739E-0001	9.99999406577659E-0001	6
55	9.99999999966885E-0001	9.99999348817937E-0001	6
60	9.99999999996115E-0001	9.99999403143367E-0001	6
65	9.99999999999544E-0001	9.99999562796234E-0001	7
70	9.99999999999947E-0001	9.99999700951081E-0001	7
75	9.99999999999994E-0001	9.99999849037557E-0001	7
80	9.99999999999999E-0001	9.99999995591021E-0001	9

security loading 1.00			
Initial reserves	exact value	Approximation	significant digits
1	6.96734670143683E-0001	6.96734663440184E-0001	8
2	8.16060279414279E-0001	8.16060285579423E-0001	8
3	8.88434919925785E-0001	8.88434929034146E-0001	8
4	9.32332358381694E-0001	9.32332362053336E-0001	9
5	9.58957500688051E-0001	9.58957474128034E-0001	8
6	9.75106465816068E-0001	9.75106464908977E-0001	9
7	9.84901308288841E-0001	9.84901389579720E-0001	7
8	9.90842180555633E-0001	9.90842393412590E-0001	7
9	9.94445501730879E-0001	9.94445777195433E-0001	7
10	9.96631026500457E-0001	9.96631258842912E-0001	7
15	9.99723457814926E-0001	9.99722700888716E-0001	6
20	9.99977300035119E-0001	9.99977242889812E-0001	7
25	9.99998136673414E-0001	9.99999040063815E-0001	6
30	9.99999847048840E-0001	1.00000059090059E+0000	6
35	9.99999987445004E-0001	1.00000012518390E+0000	7
40	9.99999998969423E-0001	9.99999644108660E-0001	7
45	9.99999999915405E-0001	9.99999427121726E-0001	6
50	9.99999999993056E-0001	9.99999474566312E-0001	6
55	9.99999999999430E-0001	9.99999606284206E-0001	7
60	9.99999999999953E-0001	9.99999741752512E-0001	7
65	9.99999999999996E-0001	9.99999899551511E-0001	7
70	1.00000000000000E+0000	9.99999988476208E-0001	8
75	1.00000000000000E+0000	1.00000008024546E+0000	7
80	1.00000000000000E+0000	1.00000012637816E+0000	7

Table 2.Pareto claim size			
	security loading 0.10		
Initial reserves	exact value	Approximation	significant digits
20	5.01857708974819E-0001	5.01857715299281E-0001	8
30	5.88563571621537E-0001	5.88563585567970E-0001	8
40	6.52106951743631E-0001	6.52106964239527E-0001	8
50	7.00845024776556E-0001	7.00845026577035E-0001	9
60	7.39355095054357E-0001	7.39355098591586E-0001	9
70	7.70449374831646E-0001	7.70449379117980E-0001	8
80	7.95982642134711E-0001	7.95982642981311E-0001	9
90	8.17239226137549E-0001	8.17239238013056E-0001	8
100	8.35140858666314E-0001	8.35140862479533E-0001	9
200	9.23675096917496E-0001	9.23675095209415E-0001	9
300	9.53383654592380E-0001	9.53383644621885E-0001	8
400	9.67168340283154E-0001	9.67168343285667E-0001	9
500	9.74872475650746E-0001	9.74872498487229E-0001	8
600	9.79722486398103E-0001	9.79722531564864E-0001	8
700	9.83033876782793E-0001	9.83033930952635E-0001	7
800	9.85430088035199E-0001	9.85430126183240E-0001	8
900	9.87240850119273E-0001	9.87240873783269E-0001	8
1000	9.88655662941776E-0001	9.88655678331483E-0001	8
	security loading 0.25		
Initial reserves	exact value	Approximation	significant digits
20	7.54739590847465E-0001	7.54739595158802E-0001	8
30	8.21662206274195E-0001	8.21662216332812E-0001	8
40	8.62440779212670E-0001	8.62440773983404E-0001	8
50	8.89480964771816E-0001	8.89480972754970E-0001	8
60	9.08476102607961E-0001	9.08476100842297E-0001	9
70	9.22405819130120E-0001	9.22405817320186E-0001	9
80	9.32971122082658E-0001	9.32971113957290E-0001	8
90	9.41206657674171E-0001	9.41206653537479E-0001	9
100	9.47773445210180E-0001	9.47773430796054E-0001	8
200	9.76199164076019E-0001	9.76199172885222E-0001	8
300	9.84844516993188E-0001	9.84844542099294E-0001	8
400	9.88928036634233E-0001	9.88928035760415E-0001	9
500	9.91291367907108E-0001	9.91291365580876E-0001	9
600	9.92828471169574E-0001	9.92828467470989E-0001	9
700	9.93906825503735E-0001	9.93906820887616E-0001	9
800	9.94704580123632E-0001	9.94704579476607E-0001	9
900	9.95318384341103E-0001	9.95318381191583E-0001	9
1000	9.95805146201094E-0001	9.95805146916452E-0001	9
	security loading 0.50		
Initial reserves	exact value	Approximation	significant digits
20	8.80725924318327E-0001	8.80725924060629E-0001	10
30	9.18574342093764E-0001	9.18574344954664E-0001	9
40	9.39144151675035E-0001	9.39144142375805E-0001	8
50	9.51836554154512E-0001	9.51836556544714E-0001	9
60	9.60350554495986E-0001	9.60350552715880E-0001	9
70	9.66412341414192E-0001	9.66412341641262E-0001	10
80	9.70925468531211E-0001	9.70925468946166E-0001	10
90	9.74404323019026E-0001	9.74404331653490E-0001	8
100	9.77161287485837E-0001	9.77161293073499E-0001	8

200	9.89139405508884E-0001	9.89139399477127E-0001	8
300	9.92916441531810E-0001	9.92916440115167E-0001	9
400	9.94752216449975E-0001	9.94752212794132E-0001	9
500	9.95834885661014E-0001	9.95834885148425E-0001	9
600	9.96548302747255E-0001	9.96548305688672E-0001	9
700	9.97053593787067E-0001	9.97053597337605E-0001	9
800	9.97430127356595E-0001	9.97430129649061E-0001	9
900	9.97721500343071E-0001	9.97721501473917E-0001	9
1000	9.97953638362732E-0001	9.97953640632050E-0001	9
	security loading 0.75		
Initial reserves	**exact value**	**Approximation**	**significant digits**
20	9.24091619464066E-0001	9.24091619225284E-0001	10
30	9.48944218072594E-0001	9.48944218767804E-0001	9
40	9.61961693637236E-0001	9.61961689683318E-0001	9
50	9.69858167084519E-0001	9.69858178496077E-0001	8
60	9.75116079482640E-0001	9.75116085160837E-0001	8
70	9.78850526769400E-0001	9.78850531378569E-0001	9
80	9.81631398539353E-0001	9.81631400233146E-0001	9
90	9.83778287209427E-0001	9.83778295154682E-0001	8
100	9.85483476947812E-0001	9.85483478704120E-0001	9
200	9.92971984026168E-0001	9.92971977563415E-0001	8
300	9.95379193508647E-0001	9.95379195737621E-0001	9
400	9.96561306169256E-0001	9.96561304347409E-0001	9
500	9.97262868519292E-0001	9.97262870226889E-0001	9
600	9.97727126454776E-0001	9.97727128760111E-0001	9
700	9.98056958502775E-0001	9.98056960727492E-0001	9
800	9.98303317988463E-0001	9.98303319367819E-0001	9
900	9.98494309217282E-0001	9.98494309150223E-0001	10
1000	9.98646699282168E-0001	9.98646699983818E-0001	9
	security loading 1.00		
Initial reserves	**exact value**	**Approximation**	**significant digits**
20	9.44950563848686E-0001	9.44950561982919E-0001	9
30	9.63112721578000E-0001	9.63112720279469E-0001	9
40	9.72490746810597E-0001	9.72490742850436E-0001	9
50	9.78152903834896E-0001	9.78152914237104E-0001	8
60	9.81920186441225E-0001	9.81920187994962E-0001	9
70	9.84598323889612E-0001	9.84598323306923E-0001	9
80	9.86595798143588E-0001	9.86595801348140E-0001	9
90	9.88140738493122E-0001	9.88140741491448E-0001	9
100	9.89370141664381E-0001	9.89370140899414E-0001	9
200	9.94805794725541E-0001	9.94805791113258E-0001	9
300	9.96571340842106E-0001	9.96571344337730E-0001	9
400	9.97442844912322E-0001	9.97442843900108E-0001	9
500	9.97961678214558E-0001	9.97961677878601E-0001	10
600	9.98305735152982E-0001	9.98305736795252E-0001	9
700	9.98550540740143E-0001	9.98550541056619E-0001	10
800	9.98733602309202E-0001	9.98733602762589E-0001	10
900	9.98875650154306E-0001	9.98875650483990E-0001	10
1000	9.98989071910635E-0001	9.98989072708050E-0001	9

Table 3.Log-normal claim size

security loading 0.10

Initial reserves	exact value	Approximation	significant digits
10	2.60231767415007E-0001	2.60231774718607E-0001	8
20	3.43307391403681E-0001	3.43307396737384E-0001	8
30	4.06445339994330E-0001	4.06445280227508E-0001	7
40	4.58266037683928E-0001	4.58265904414921E-0001	7
50	5.02360401166465E-0001	5.02360456111438E-0001	7
60	5.40689459567940E-0001	5.40689336681108E-0001	7
70	5.74485387675449E-0001	5.74485412086493E-0001	8
80	6.04592487880642E-0001	6.04592323192437E-0001	7
90	6.31624307340222E-0001	6.31623414465740E-0001	6
100	6.56045972712942E-0001	6.56044878234203E-0001	6
200	8.11878061922804E-0001	8.11877864658241E-0001	7
300	8.86829729396766E-0001	8.86828696554749E-0001	6
400	9.27525251502344E-0001	9.27522721049831E-0001	6
500	9.51290279533989E-0001	9.51293071150573E-0001	6
600	9.65930164132491E-0001	9.65933436981531E-0001	6
700	9.75345280148664E-0001	9.75348638193318E-0001	6
800	9.81624888367240E-0001	9.81614691046808E-0001	5
900	9.85948154404996E-0001	9.85950916081833E-0001	6
1000	9.89009255109262E-0001	9.89027529085923E-0001	5

security loading 0.25

Initial reserves	exact value	Approximation	significant digits
10	4.81168350140423E-0001	4.81168358182100E-0001	8
20	5.89218482893505E-0001	5.89218487735016E-0001	8
30	6.60460776587699E-0001	6.60460682859782E-0001	7
40	7.12601739564540E-0001	7.12601546135142E-0001	7
50	7.52806098468802E-0001	7.52806225530324E-0001	7
60	7.84830743233767E-0001	7.84830635258233E-0001	7
70	8.10926130287055E-0001	8.10926297900931E-0001	7
80	8.32555927536240E-0001	8.32555883761316E-0001	7
90	8.50727097793419E-0001	8.50726162331297E-0001	6
100	8.66161350012510E-0001	8.66160226835302E-0001	6
200	9.44436722635996E-0001	9.44437578230303E-0001	6
300	9.70844772646244E-0001	9.70845167699259E-0001	7
400	9.82469767708101E-0001	9.82468854161480E-0001	6
500	9.88460725064066E-0001	9.88464772957218E-0001	6
600	9.91899907610643E-0001	9.91903817218640E-0001	6
700	9.94036375359624E-0001	9.94040118853447E-0001	6
800	9.95445829589504E-0001	9.95438733999540E-0001	5
900	9.96420546339272E-0001	9.96423426378534E-0001	6
1000	9.97120590898172E-0001	9.97133833094402E-0001	5

security loading 0.50

Initial reserves	exact value	Approximation	significant digits
10	6.63125565762044E-0001	6.63125571174837E-0001	8
20	7.59812160018472E-0001	7.59812157785500E-0001	9
30	8.15459721879987E-0001	8.15459614726930E-0001	7
40	8.52285951468661E-0001	8.52285755224006E-0001	7
50	8.78486062539128E-0001	8.78486221664701E-0001	7
60	8.98008497669762E-0001	8.98008423922967E-0001	7
70	9.13041582583855E-0001	9.13041807497194E-0001	7

80	9.24910484377334E-0001	9.24910512780498E-0001	8
90	9.34468937549234E-0001	9.34468146083458E-0001	6
100	9.42292931060382E-0001	9.42291978507397E-0001	6
200	9.77868124825461E-0001	9.77869070380845E-0001	6
300	9.88430703805331E-0001	9.88431245215513E-0001	6
400	9.92930594008588E-0001	9.92930104173450E-0001	7
500	9.95251795656231E-0001	9.95254946451083E-0001	6
600	9.96602043974300E-0001	9.96604875939009E-0001	6
700	9.97455198136325E-0001	9.97457820172960E-0001	6
800	9.98027826705774E-0001	9.98023190101867E-0001	6
900	9.98430283125567E-0001	9.98432220960342E-0001	6
1000	9.98723588132130E-0001	9.98731855897922E-0001	5

security loading 0.75

Initial reserves	exact value	Approximation	significant digits
10	7.54251127423556E-0001	7.54251135330604E-0001	8
20	8.34330801793760E-0001	8.34330809478091E-0001	8
30	8.77058890153055E-0001	8.77058771267206E-0001	7
40	9.03921997590360E-0001	9.03921820145585E-0001	7
50	9.22322467887867E-0001	9.22322620289726E-0001	7
60	9.35637345521158E-0001	9.35637287409364E-0001	7
70	9.45655130080967E-0001	9.45655348220171E-0001	7
80	9.53417875915481E-0001	9.53417929191694E-0001	7
90	9.59575022607857E-0001	9.59574366724138E-0001	6
100	9.64552307954011E-0001	9.64551519678122E-0001	6
200	9.86516019142821E-0001	9.86516839033201E-0001	6
300	9.92886366644296E-0001	9.92886851795721E-0001	7
400	9.95609154837372E-0001	9.95608800051050E-0001	7
500	9.97025349521727E-0001	9.97027809785434E-0001	6
600	9.97856218170870E-0001	9.97858359654397E-0001	6
700	9.98385243042018E-0001	9.98387211767425E-0001	6
800	9.98742679603666E-0001	9.98739239463134E-0001	6
900	9.98995323014785E-0001	9.98996741336225E-0001	6
1000	9.99180343359648E-0001	9.99186324718729E-0001	5

security loading 1.00

Initial reserves	exact value	Approximation	significant digits
10	8.07845660083023E-0001	8.07845668430972E-0001	8
20	8.74770462429058E-0001	8.74770462364928E-0001	10
30	9.08838363325595E-0001	9.08838266209765E-0001	7
40	9.29628406905812E-0001	9.29628253184441E-0001	7
50	9.43575671195437E-0001	9.43575804898764E-0001	7
60	9.53515221084589E-0001	9.53515167749462E-0001	7
70	9.60907817487743E-0001	9.60908022587528E-0001	7
80	9.66585811727130E-0001	9.66585864806707E-0001	7
90	9.71058503135077E-0001	9.71057933759778E-0001	6
100	9.74654674233282E-0001	9.74653992686472E-0001	6
200	9.90347970105184E-0001	9.90348680654282E-0001	6
300	9.94875597495195E-0001	9.94876010616251E-0001	7
400	9.96819329054347E-0001	9.96819049020656E-0001	7
500	9.97835648482940E-0001	9.97837651234892E-0001	6
600	9.98434722865815E-0001	9.98436445409247E-0001	6
700	9.98817679873131E-0001	9.98819256831704E-0001	6
800	9.99077288065111E-0001	9.99074569535028E-0001	6
900	9.99261298868423E-0001	9.99262443003321E-0001	6

1000	9.99396377356102E-0001	9.99401061218795E-0001	6

Table 4. Inverse Gaussian claim size

Security Loading 0.10	
Initial reserves	**Approximation**
10	1.38362637371266E-0001
20	1.62463267415600E-0001
30	1.82574435496147E-0001
40	2.00590577319753E-0001
50	2.17239991850557E-0001
60	2.32893480976210E-0001
70	2.47769924720695E-0001
80	2.62011038407794E-0001
90	2.75714828682924E-0001
100	2.88952285097071E-0001
200	4.03456514614382E-0001
300	4.96057128995442E-0001
400	5.73243557053335E-0001
500	6.38216302546431E-0001
600	6.93132172200201E-0001
700	7.39637265754829E-0001
800	7.79059987188606E-0001
900	8.12496377305226E-0001
1000	8.40863204850918E-0001
Security Loading 0.25	
Initial reserves	**Approximation**
10	2.87760435267190E-0001
20	3.29471826702437E-0001
30	3.62958394601707E-0001
40	3.91995009413165E-0001
50	4.18056314271001E-0001
60	4.41906074865805E-0001
70	4.64003534194507E-0001
80	4.84651940266488E-0001
90	5.04065391938175E-0001
100	5.22402359710606E-0001
200	6.64923604714433E-0001
300	7.60483320169216E-0001
400	8.27496052227981E-0001
500	8.75284659466562E-0001
600	9.09638691575403E-0001
700	9.34442160773609E-0001
800	9.52397677716804E-0001
900	9.65416160009448E-0001
1000	9.74863959530718E-0001
Security Loading 0.50	
Initial reserves	**Approximation**
10	4.48801445872244E-0001
20	4.99535468280400E-0001
30	5.38504304245915E-0001
40	5.71079835268514E-0001
50	5.99385597298886E-0001
60	6.24534133082644E-0001
70	6.47201721793779E-0001
80	6.67839466640306E-0001

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90	6.86768740330230E-0001
100	7.04229343614023E-0001
200	8.26155062600754E-0001
300	8.93914934438543E-0001
400	9.34162974709046E-0001
500	9.58750681516647E-0001
600	9.73999904434889E-0001
700	9.83544019897867E-0001
800	9.89554985874867E-0001
900	9.93356295434686E-0001
1000	9.95766729598524E-0001
Security Loading 0.75	
Initial reserves	**Approximation**
10	5.51139978674942E-0001
20	6.02138110212740E-0001
30	6.40166462838918E-0001
40	6.71197935920359E-0001
50	6.97600008027543E-0001
60	7.20615221699657E-0001
70	7.40999396953950E-0001
80	7.59256597332664E-0001
90	7.75745442844107E-0001
100	7.90732916681927E-0001
200	8.88980582447983E-0001
300	9.37897872691112E-0001
400	9.64380381249556E-0001
500	9.79265147958059E-0001
600	9.87811102930965E-0001
700	9.92784250576174E-0001
800	9.95706724821683E-0001
900	9.97435561155229E-0001
1000	9.98462909516296E-0001
Security Loading 1.00	
Initial reserves	**Approximation**
10	6.21728061629658E-0001
20	6.70390691559684E-0001
30	7.05916381516969E-0001
40	7.34415427927980E-0001
50	7.58308078030225E-0001
60	7.78862683321492E-0001
70	7.96849129585499E-0001
80	8.12779335282960E-0001
90	8.27016231072987E-0001
100	8.39828985753956E-0001
200	9.20428781609052E-0001
300	9.57807931112911E-0001
400	9.76909863782672E-0001
500	9.87119830384968E-0001
600	9.92721792317982E-0001
700	9.95848134795495E-0001
800	9.97615147869054E-0001
900	9.98622644988679E-0001
1000	9.99200562291648E-0001