

Primordial magnetic fields from metric perturbations

Antonio L. Maroto

CERN Theory Division, CH-1211 Geneva 23, Switzerland

and Departamento de Física Teórica, Universidad Complutense de Madrid, 28040 Madrid, Spain

(Received 14 May 2001; published 24 September 2001)

We study the amplification of electromagnetic vacuum fluctuations induced by the evolution of scalar metric perturbations at the end of inflation. Such perturbations break the conformal invariance of Maxwell equations in Friedmann-Robertson-Walker backgrounds and allow the growth of magnetic fields on super-Hubble scales. We relate the strength of the fields generated by this mechanism with the power spectrum of scalar perturbations and estimate the amplification on galactic scales for different values of the spectral index. Finally we discuss the possible effects of finite conductivity during reheating.

DOI: 10.1103/PhysRevD.64.083006

PACS number(s): 98.62.En, 98.80.Cq

I. INTRODUCTION

The existence of cosmic magnetic fields with large coherence lengths (>10 kpc) and a typical strength of 10^{-6} G still remains an open problem in astrophysics [1]. A partial explanation, widely considered in the literature, is based on the amplification of seed fields by means of the so called galactic dynamo mechanism. In this mechanism, the differential rotation of the galaxy is able to transfer energy into the magnetic field, but nevertheless it still requires a preexisting field to be amplified. The present bounds on the necessary seed fields to comply with observations are in the range $B_{seed} \gtrsim 10^{-17} - 10^{-22}$ G ($h = 0.65 - 0.5$) at decoupling time, coherent on a comoving scale of $\lambda_G \sim 10$ kpc, for a flat universe without a cosmological constant. For a flat universe with a nonvanishing cosmological constant, the limits can be relaxed up to $B_{seed} \gtrsim 10^{-25} - 10^{-30}$ G ($h = 0.65 - 0.5$) at decoupling for $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$ [2]. The observations of micro-Gauss magnetic fields in two high-redshift objects (see [1,2] and references therein) could, if correct, impose more stringent conditions on the seeds fields or even on the dynamo mechanism itself.

The cosmological origin of the seed fields is one of the most interesting possibilities, although some other mechanisms at the astrophysical level, such as the Biermann battery process, have also been considered [3,4]. In the cosmological case, in which we will be mainly interested in this work, it is natural to expect [5] that the same mechanism that gave rise to the large-scale galactic structure, i.e. amplification of quantum fluctuations during inflation, was also responsible for the generation of the primordial magnetic fields. However, it was soon noticed [5] that the gravitational amplification does not operate in the case of electromagnetic (EM) fields. This is because of the conformal triviality of Maxwell equations in Friedmann-Robertson-Walker (FRW) backgrounds, i.e. conformally invariant equations in a conformally flat space-time. In order to avoid this difficulty, several production mechanisms have been proposed in which Maxwell equations are modified in different ways. Thus for example, the addition of mass terms to the photon or higher-curvature terms in the Lagrangian was studied in [5]. The contribution of the conformal anomaly was included in [6]. In the context of string cosmology, the effects of a dynamical dilaton field were taken into account in [7]. Other examples

include non-minimal gravitational-electromagnetic coupling [8], inflaton coupling to EM Lagrangian [9], spontaneous breaking of Lorentz invariance [10] or back reaction of minimally coupled charged scalars [11–13]. Some of them are able to generate fields of the required strength to seed the galactic dynamo or even to account for the observations without further amplification.

In this paper we explore the alternative possibility, i.e. we avoid conformal triviality by considering deviations from the FRW metric (see [14] for a suggestion along these lines). This approach is rather natural since we know that galaxies formed from small metric inhomogeneities present at large scales and, in addition, it does not require any modification of Maxwell electromagnetism. In the inflationary cosmology, metric perturbations are generated when quantum fluctuations become super-Hubble sized and thereafter evolve as classical fluctuations, reentering the horizon during radiation or matter dominated eras [15]. The same mechanism would operate on large-scale EM fluctuations. However, if conformal invariance is not broken, each positive or negative frequency EM mode will evolve independently, without mixing. This implies that photons cannot be created and therefore magnetic fields are not amplified. However, in the presence of an inhomogeneous background, we will show that the mode-mode coupling between EM and metric perturbations generates the mixing. This in turn will allow us to relate the strength of the magnetic field created by this mechanism and the particular form of the metric perturbations described by the corresponding power spectrum. Those photons produced in the inflation-radiation transition with very long wavelengths can be seen as static electric or magnetic fields. Because of the high conductivity of the Universe in the radiation era, the electric components are rapidly damped whereas, thanks to magnetic flux conservation, the magnetic fields will remain frozen in the plasma and their subsequent evolution will be trivial, $Ba^2 = \text{const}$ [5,9]. The paper is organized as follows. In Sec. II we obtain the Maxwell equations in the presence of an inhomogeneous background and calculate the occupation number of the photons produced. In Sec. III we apply these results to calculate the corresponding magnetic field generated at galactic scales. Section IV is devoted to the analysis of the effects of finite conductivity in those results and finally, Sec. V includes the main conclusions of the paper.

II. MAXWELL EQUATIONS AND PHOTON PRODUCTION

Although there are previous works on the production of scalar and fermionic particles in inhomogeneous backgrounds [16,17], in this paper we will need to extend the analysis to the case of gauge fields. Let us then consider Maxwell equations

$$\nabla_\mu F^{\mu\nu} = 0, \quad (1)$$

in a background metric that can be split as $g_{\mu\nu} = g_{\mu\nu}^0 + h_{\mu\nu}$, where

$$g_{\mu\nu}^0 dx^\mu dx^\nu = a^2(\eta)(d\eta^2 - \delta_{ij} dx^i dx^j) \quad (2)$$

is the flat FRW metric in conformal time and

$$h_{\mu\nu} dx^\mu dx^\nu = 2a^2(\eta)\Phi(d\eta^2 + \delta_{ij} dx^i dx^j) \quad (3)$$

is the most general form of the linearized scalar metric perturbation in the longitudinal gauge and where it has been assumed that the spatial part of the energy-momentum tensor is diagonal, as indeed happens in the inflationary or perfect fluid cosmologies [15]. In this expression $\Phi(\eta, \vec{x})$ is the gauge invariant gravitational potential. Equation (1) can be written as:

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\mu} [\sqrt{g} g^{\mu\alpha} g^{\nu\beta} (\partial_\alpha A_\beta - \partial_\beta A_\alpha)] = 0, \quad (4)$$

which leads in this background to the following linearized equations

$$\frac{\partial}{\partial x^i} [(1 - 2\Phi)(\partial_i A_0 - \partial_0 A_i)] = 0, \quad (5)$$

for $\nu=0$ and

$$\begin{aligned} & \frac{\partial}{\partial \eta} [(1 - 2\Phi)(\partial_i A_0 - \partial_0 A_i)] \\ & + \frac{\partial}{\partial x^j} [(1 + 2\Phi)(\partial_j A_i - \partial_i A_j)] = 0, \end{aligned} \quad (6)$$

for $\nu=i$. In addition, we will use the Coulomb gauge condition $\vec{\nabla} \cdot \vec{A} = 0$.

In order to study the amplification of vacuum fluctuations, let us consider a particular solution of the above equations that we will denote by $A_\mu^{\vec{k},\lambda}(x)$ such that asymptotically in the past it behaves as a positive frequency plane wave with momentum \vec{k} and polarization λ , i.e.

$$A_\mu^{\vec{k},\lambda}(x) \xrightarrow{\eta \rightarrow -\infty} A_\mu^{(0)\vec{k},\lambda}(x) = \frac{1}{\sqrt{2kV}} \epsilon_\mu(\vec{k},\lambda) e^{i(\vec{k}\vec{x} - k\eta)}, \quad (7)$$

where $k^2 = \vec{k}^2$. For the two physical polarization states we have, $\vec{\epsilon}(\vec{k},\lambda) \cdot \vec{k} = 0$ and $\epsilon_0(\vec{k},\lambda) = 0$. We will work in a finite box with comoving volume V and we will take the continuum limit at the end of the calculation. We are assuming

that metric perturbations vanish before inflation starts, so that we can define an appropriate initial conformal vacuum state. Because of the presence of the inhomogeneous background, in the asymptotic future, this solution will behave as a linear superposition of positive and negative frequency modes with different momenta and different polarizations, i.e.,

$$\begin{aligned} A_\mu^{\vec{k},\lambda}(x) \xrightarrow{\eta \rightarrow \infty} & \sum_{\lambda'} \sum_q \left(\alpha_{kq\lambda\lambda'} \frac{\epsilon_\mu(\vec{q},\lambda')}{\sqrt{2qV}} e^{i(\vec{q}\vec{x} - q\eta)} \right. \\ & \left. + \beta_{kq\lambda\lambda'} \frac{\epsilon_\mu^*(\vec{q},\lambda')}{\sqrt{2qV}} e^{-i(\vec{q}\vec{x} - q\eta)} \right). \end{aligned} \quad (8)$$

It is possible to obtain an expression for the Bogolyubov coefficients $\alpha_{kq\lambda\lambda'}$ and $\beta_{kq\lambda\lambda'}$ to first order in the metric perturbations. With that purpose, we look for solutions of the equations of motion in the form:

$$A_\mu^{\vec{k},\lambda}(x) = A_\mu^{(0)\vec{k},\lambda}(x) + A_\mu^{(1)\vec{k},\lambda}(x) + \dots \quad (9)$$

where $A_\mu^{(0)\vec{k},\lambda}(x)$ is the solution in the absence of perturbations given by Eq. (7). Introducing this expansion in Eq. (5) and Fourier transforming, we obtain for the temporal component of the EM field to first order in the perturbations:

$$A_0^{(1)\vec{k},\lambda}(\vec{q}, \eta) = -\sqrt{\frac{2k}{V}} \frac{\vec{q} \cdot \vec{\epsilon}(\vec{k},\lambda)}{q^2} \Phi(\vec{k} + \vec{q}, \eta) e^{-ik\eta} \quad (10)$$

where, as usual, $\Phi(\vec{q}, \eta) = (2\pi)^{-3/2} \int d^3x e^{i\vec{q}\vec{x}} \Phi(\vec{x}, \eta)$. The zeroth order equation implies $A_0^{(0)\vec{k},\lambda}(\vec{q}, \eta) = 0$. The spatial equations (6) can be written to first order as:

$$\begin{aligned} & 2\Phi' A_i^{(0)'} + \partial_i A_0^{(1)'} - A_i^{(1)''} + 2\vec{\nabla} \Phi \cdot \vec{\nabla} A_i^{(0)} - 2\vec{\nabla} \Phi \cdot \partial_i \vec{A}^{(0)} \\ & + \vec{\nabla}^2 A_i^{(1)} + 4\Phi \vec{\nabla}^2 A_i^{(0)} = 0. \end{aligned} \quad (11)$$

Inserting again expansion (9), these equations can be rewritten in Fourier space as:

$$\frac{d^2}{d\eta^2} A_i^{(1)\vec{k},\lambda}(\vec{q}, \eta) + q^2 A_i^{(1)\vec{k},\lambda}(\vec{q}, \eta) - J_i^{\vec{k},\lambda}(\vec{q}, \eta) = 0 \quad (12)$$

where:

$$\begin{aligned} J_i^{\vec{k},\lambda}(\vec{q}, \eta) = & -\sqrt{\frac{2k}{V}} \left[i\Phi'(\vec{k} + \vec{q}, \eta) \right. \\ & \left. + \frac{k^2 - \vec{k} \cdot \vec{q}}{k} \Phi(\vec{k} + \vec{q}, \eta) \right] \epsilon_i(\vec{k},\lambda) e^{-ik\eta} \\ & + [\vec{\epsilon}(\vec{k},\lambda) \cdot \vec{q}] \Phi(\vec{k} + \vec{q}, \eta) \frac{k_i}{k} e^{-ik\eta} \\ & - i \frac{\vec{\epsilon}(\vec{k},\lambda) \cdot \vec{q}}{q^2} \frac{d}{d\eta} [\Phi(\vec{k} + \vec{q}, \eta) e^{-ik\eta}] q_i. \end{aligned} \quad (13)$$

Solving these equations we find, up to first order in the perturbations:

$$A_i^{\vec{k},\lambda}(\vec{q},\eta) = \frac{\epsilon_i(\vec{k},\lambda)}{\sqrt{2kV}} \delta(\vec{q}-\vec{k}) e^{-ik\eta} + \frac{1}{q} \int_{\eta_0}^{\eta} J_i^{\vec{k},\lambda}(\vec{q},\eta') \sin(q(\eta-\eta')) d\eta', \quad (14)$$

where η_0 denotes the starting time of inflation. Comparing this expression with Eq. (8), it is straightforward to obtain the Bogolyubov coefficients $\beta_{kq\lambda\lambda'}$, which are given by:

$$\beta_{kq\lambda\lambda'} = \frac{-i}{\sqrt{2qV}} \int_{\eta_0}^{\eta_1} \vec{\epsilon}(\vec{q},\lambda') \cdot \vec{J}^{\vec{k},\lambda}(\vec{q},\eta) e^{-iq\eta} d\eta \quad (15)$$

where η_1 denotes the present time. The total number of photons created with comoving wave number $k_G = 2\pi/\lambda_G$, corresponding to the relevant coherence length, is therefore given by [18]:

$$N_{k_G} = \sum_{\lambda,\lambda'} \sum_k |\beta_{kk_G\lambda\lambda'}|^2. \quad (16)$$

We will concentrate only in the effect of super-Hubble scalar perturbations whose evolution is relatively simple [15]:

$$\Phi(\vec{k},\eta) = C_k \frac{1}{a} \frac{d}{d\eta} \left(\frac{1}{a} \int a^2 d\eta \right) + D_k \frac{a'}{a^3}, \quad (17)$$

where the second term decreases during inflation and can soon be neglected. Thus, it will be useful to rewrite the perturbation as: $\Phi(\vec{k},\eta) = C_k \mathcal{F}(\eta)$. During inflation or preheating, these perturbations evolve in time, whereas they are practically constant during radiation or matter eras. We will neglect the effects of the perturbations once they reenter the horizon. This is a good approximation for modes reentering right after the end of inflation since they are rapidly damped. In addition, we will show that those modes are the more relevant ones in the calculation.

The power spectrum corresponding to Eq. (17) is given by:

$$\mathcal{P}_\Phi(k) = \frac{k^3 |C_k|^2}{2\pi^2 V} = A_S^2 \left(\frac{k}{k_C} \right)^{n-1}, \quad (18)$$

where for simplicity we have taken a power-law behavior with spectral index n and we have set the normalization at the Cosmic Background Explorer (COBE) scale $\lambda_C \approx 3000$ Mpc with $A_S \approx 5 \times 10^{-5}$. In the case of a blue spectrum, with positive tilt ($n > 1$), perturbations will grow at small scales and it is necessary to introduce a cutoff k_{max} in order to avoid excessive primordial black hole production [19]. Accordingly, only below the cutoff the perturbative method will be reliable. For negative tilt or scale-invariant spectrum there will be also a small scale cutoff related to the

minimum size of the horizon $k_{max} \lesssim a_I H_I$, where the I subscript denotes the end of inflation.

We can obtain an explicit expression for the total number of photons (16) in terms of the power spectrum. Taking the continuum limit $\sum_k \rightarrow (2\pi)^{-3/2} V \int d^3k$, we get:

$$N_{k_G} = \sum_{\lambda,\lambda'} V \int \frac{d^3k}{(2\pi)^{3/2}} |\beta_{kk_G\lambda\lambda'}|^2 = \sum_{\lambda,\lambda'} V \int \frac{d^3k}{(2\pi)^{3/2}} \frac{|C_{|k+k_G|}|^2}{2k_G V^2} \times \int d\eta \left\{ \sqrt{2k} \left[i\mathcal{F}' + \frac{k^2 - \vec{k} \cdot \vec{k}_G}{k} \mathcal{F} \right] \times [\vec{\epsilon}(\vec{k},\lambda) \cdot \vec{\epsilon}(\vec{k}_G,\lambda')] + [\vec{\epsilon}(\vec{k},\lambda) \cdot \vec{k}_G] \times [\vec{\epsilon}(\vec{k}_G,\lambda') \cdot \vec{k}] \frac{\mathcal{F}}{k} \right\} e^{-i(k_G+k)\eta} \Bigg|^2. \quad (19)$$

Notice that the last term in Eq. (13) does not contribute to $\beta_{kq\lambda\lambda'}$ because of the transversality condition of the polarization vectors. The integration in d^3k is dominated by the upper limit, i.e. $k \gg k_G$ and accordingly we can ignore the effect of the terms proportional to \vec{k}_G . In addition, for those modes k which are outside the Hubble radius at the end of inflation, we have $k\eta \ll 1$. With these simplifications we obtain:

$$N_{k_G} \approx \sum_{\lambda,\lambda'} \int \frac{dk d\Omega}{(2\pi)^{3/2}} \frac{|C_k|^2 k^2}{2k_G V} \times \left| \int d\eta (\sqrt{2k} \{ (i\mathcal{F}' + k\mathcal{F}) [\vec{\epsilon}(\vec{k},\lambda) \cdot \vec{\epsilon}(\vec{k}_G,\lambda')] \}) \right|^2. \quad (20)$$

Performing the integration in the angular variables and using the definition of the power spectrum in Eq. (18), we obtain:

$$N_{k_G} \approx \frac{4(2\pi)^{3/2}}{3k_G} \int dk A_S^2 \left(\frac{k}{k_C} \right)^{n-1} \left| \int d\eta (i\mathcal{F}' + k\mathcal{F}) \right|^2. \quad (21)$$

Finally, we will estimate the time integral. The behavior of scales that reenter the horizon during the radiation dominated era is oscillatory with a decaying amplitude [15], therefore, there is no long-time contribution to the integral that could spoil the perturbative method. Thus, for simplicity we will assume that the function \mathcal{F} vanishes for $\eta \gg 1/k$, and accordingly we estimate, $|\int d\eta (i\mathcal{F}' + k\mathcal{F})|^2 \sim \mathcal{O}(1)$. Our final expression for the occupation number is:

$$N_{k_G} \approx \frac{4(2\pi)^{3/2} A_S^2}{3k_G (k_C)^{n-1}} \int_{k_C}^{k_{max}} dk k^{n-1} \approx \frac{4(2\pi)^{3/2} A_S^2}{3n} \frac{k_{max}^n}{k_G k_C^{n-1}}. \quad (22)$$

III. MAGNETIC FIELD GENERATION

The energy density stored in a magnetic field mode B_k with wave number k is given by:

$$\rho_B(\omega) = \omega \frac{d\rho_B}{d\omega} = \frac{|B_k|^2}{2}, \quad (23)$$

with $\omega = k/a$ the physical wave number. In terms of the occupation number it reads

$$\rho_B(\omega) = \omega^4 N_k. \quad (24)$$

From Eq. (22), we can obtain the strength of the field at decoupling on a coherence scale corresponding to $k_G \sim 10^{-36}$ GeV as:

$$|B_{k_G}^{dec}| \approx \sqrt{2} (\omega_G^{dec})^2 N_{k_G}^{1/2} \approx \frac{2^{3/2} (2\pi)^{3/4} A_S k_{max}^{n/2} k_G^{3/2}}{\sqrt{3n} a_{dec}^2 k_C^{(n-1)/2}}. \quad (25)$$

In Fig. 1 we have plotted the strength of the magnetic field generated as a function of the comoving cutoff frequency k_{max} for different values of the spectral index n . Notice that the results are in general too weak to explain the observed fields without any amplification. However, for certain values of the cosmological parameters, the produced fields could act as seeds for a galactic dynamo.

We see that the spectrum of magnetic fields produced by this mechanism is thermal $B_k \sim k^{3/2}$, in the low-momentum region. We can then compare this spectrum with that corresponding to the thermal background radiation with a temperature $T_{dec} \approx 0.26$ eV present at decoupling time. The energy density in photons with comoving wave number k_G at decoupling is given by $\rho_R(\omega_G) \approx k_G^3 T_{dec} / a_{dec}^3$. Thus we find:

$$\frac{\rho_R(\omega_G)}{\rho_B(\omega_G)} = \frac{a_{dec} T_{dec}}{N_{k_G} k_G} \approx 1.4 \times 10^{36} \left(\frac{k_C}{k_{max}} \right)^n. \quad (26)$$

From this expression we see that the magnetic field energy density will dominate over the background thermal radiation whenever $\log(k_{max}/k_C) \geq 36/n$, i.e. for example, for $n=1$ this implies $k_{max} \geq 10^{-6}$ GeV.

The cutoff frequency k_{max} cannot be easily determined in general, since it depends on the specific mechanism that generates the perturbations and also on the evolution of the universe during reheating and thermalization. However we can estimate typical values in some particular regime. In the case in which metric perturbations are generated by inflation, it is natural to expect, $k_{max} \lesssim a_I H_I$, as commented before. Thus, let us take the simplest chaotic inflation model with potential $V(\phi) = \lambda \phi^4/4$ [20], with $\lambda \approx 10^{-12}$ fixed by COBE. In this model the Hubble parameter during inflation is $H_I \approx 10^{13}$ GeV. Owing to the uncertainties commented before, we will let the reheating temperature T_{RH} be a free parameter. After reheating the universe evolution is adiabatic $a_I/a_{dec} \sim T_{dec}/T_{RH}$, and we can calculate the cutoff frequency as $k_{max}/k_C \sim a_I H_I/k_C \sim a_{dec} T_{dec} H_I/(T_{RH} k_C)$, which yields $k_{max}/k_C \sim 10^{42} \text{ GeV}/T_{RH}$. Here we have assumed

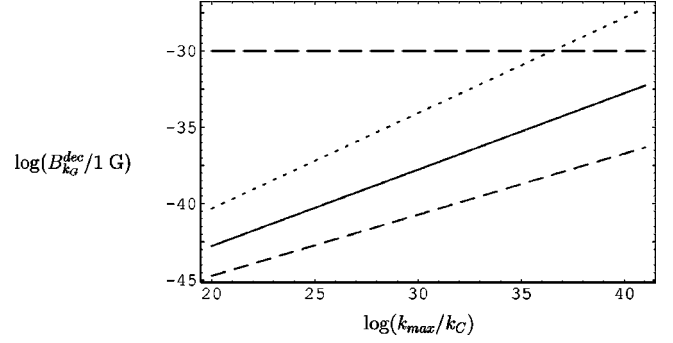


FIG. 1. $\log(B_{k_G}^{dec}/1 \text{ G})$ as a function of $\log(k_{max}/k_C)$. The continuous line corresponds to the scale-invariant Harrison-Zeldovich spectrum with $n=1$, the dashed line to $n=0.8$ and the dotted line to $n=1.25$. The dashed horizontal line represents the weakest galactic dynamo seed field limit corresponding to a flat universe with cosmological constant and $h=0.5$.

that the inflation-radiation transition takes place in a few inflaton oscillations [20] (see also [21]). Comparing with Fig. 1 we see that with these simple estimations for the $\lambda \phi^4/4$ model, the amplification could be above the requirements of the galactic dynamo if $T_{RH} \lesssim 10^6$ GeV for $n=1.25$.

IV. CONDUCTIVITY EFFECTS

In the previous discussion we have assumed that electric conductivity of the universe played no role in the generation of the magnetic fields. Although neglecting conductivity is a good approximation during inflation, it is not during the reheating or radiation periods. As commented before, the copious production of particles during reheating produces the growing of the conductivity which becomes very high during radiation [5]. This implies that the magnetic fields produced in the inflation-radiation transition will evolve conserving magnetic flux $\rho_B \sim a^{-4}$. However, it has been recently shown [13] that the growth of conductivity during reheating could affect the evolution of the EM modes. The effects of conductivity can be taken into account in a phenomenological way by introducing a current source $J_i = \sigma_c a A_i'$ in Eq. (11). This approach is only valid at sufficiently large scales [13] and in general the rigorous treatment would require to solve the set of coupled EM-matter fields equations (Vlasov equations) which is beyond the scope of this paper. In general, $\sigma_c(\vec{x}, \eta)$ is a time and space dependent function, and it has been shown that it grows exponentially during parametric resonance in a non-equilibrium plasma in QED [22]. The calculation of the actual function $\sigma_c(\vec{x}, \eta)$ in our case would be rather involved and model dependent, since it would require information about the reheating and thermalization processes. For that reason we will not take any particular model, but we will do a general discussion of the possible effects in different cases. We will also assume for simplicity that all the Fourier modes of the conductivity have the same time evolution, i.e. $\sigma_c(\vec{k}, \eta) = \Sigma(\eta) \sigma_k$ during reheating.

In previous works [12,13], the conductivity was considered as a homogeneous field, $\sigma_c(\eta)$ and because of the form of the current source term, its effect was the damping of each EM mode. In our production mechanism, the inhomogeneities are able to mix different EM modes and for this reason we need to have information about the complete spectrum σ_k and not only about the large-scale components. In addition, our analysis is perturbative and therefore we can only describe the initial stages in which the conductivity is still small. Let us then consider the modified spatial equation to first order in the metric perturbations and the conductivity:

$$(2\Phi' - a\sigma_c)A_i^{(0)'} + \partial_i A_0^{(1)'} - A_i^{(1)''} + 2\vec{\nabla}\Phi \cdot \vec{\nabla}A_i^{(0)} - 2\vec{\nabla}\Phi \cdot \partial_i \vec{A}^{(0)} + \vec{\nabla}^2 A_i^{(1)} + 4\Phi \vec{\nabla}^2 A_i^{(0)} = 0. \quad (27)$$

Following a similar analysis we find the same Eq. (12), but with the new current:

$$J_i^{\vec{k},\lambda}(\vec{q},\eta) = -\sqrt{\frac{2k}{V}} \left[\left(i\Phi'(\vec{k} + \vec{q}, \eta) - \frac{i}{2} a \sigma_c(\vec{k} + \vec{q}, \eta) + \frac{k^2 - \vec{k} \cdot \vec{q}}{k} \Phi(\vec{k} + \vec{q}, \eta) \right) \epsilon_i(\vec{k}, \lambda) e^{-ik\eta} + [\vec{\epsilon}(\vec{k}, \lambda) \cdot \vec{q}] \Phi(\vec{k} + \vec{q}, \eta) \frac{k_i}{k} e^{-ik\eta} - i \frac{\vec{\epsilon}(\vec{k}, \lambda) \cdot \vec{q}}{q^2} \frac{d}{d\eta} [\Phi(\vec{k} + \vec{q}, \eta) e^{-ik\eta}] q_i \right]. \quad (28)$$

In [13] the following lower limit on the (homogeneous) conductivity is obtained $a_I \sigma_c \sim a_I H_I / \alpha$, with α the fine structure constant, i.e. $a_I \sigma_c \sim k_{max} / \alpha$ and this implies that the conductivity term will dominate in $J_i^{\vec{k},\lambda}(\vec{k}_G, \eta)$ for $k \ll k_{max}$. However, as commented before, the dominant contribution to the EM amplification comes from the high-frequency modes, i.e. $k \sim k_{max}$. In such a case, the importance of the conductivity term is determined by the ratio σ_k / C_k when $k \rightarrow k_{max}$. Thus, if the conductivity is almost homogeneous, its spectrum will decline at short scales and

we expect its contribution to the EM field evolution to be negligible. In the opposite case, the analysis would become much more involved, since the above magnetohydrodynamical approximation would break down and the full set of microscopic equations would be needed. In any case, this simple analysis shows that the possible damping effects mainly affect those modes with $k \sim k_G$ and that, depending on the actual conductivity spectrum, the rest of the modes could be less severely affected than in other models.

V. CONCLUSIONS

In this work we have studied the production of photons in the presence of an inhomogeneous gravitational background. We have shown how the breaking of conformal invariance induced by the evolution of metric perturbations in the inflation-radiation transition is able to produce particles, and we have related the occupation number with the scalar metric perturbations power spectrum.

We have considered the possibility that this mechanism could have had some relevance in the problem of galactic magnetic fields and we have concluded that the total amplification is several orders of magnitude below the observed strengths. However, for certain values of the cosmological parameters and with the assistance of the dynamo mechanism, the amplification could be compatible with the current (low-redshift) galactic observations. We have also considered the effect of conductivity in a phenomenological way and we showed that although it could affect the evolution of EM modes, in some cases and depending on the particular form of the spectrum, the effects could be small.

The mechanism studied in this work only relies on the existence of a primordial spectrum of metric perturbations, described by the scalar spectral index n and a possible cutoff frequency k_{max} , which are the only free parameters in the model. Therefore, as a by-product we get that magnetic fields could also provide useful information about the metric perturbations spectrum and in particular about its small-scales region.

ACKNOWLEDGMENTS

This work has been partially supported by the Ministerio de Educación y Ciencia (Spain) (CICYT AEN 97-1693).

-
- [1] P.P. Kronberg, Rep. Prog. Phys. **57**, 325 (1994); E.N. Parker, *Cosmical Magnetic Fields* (Clarendon, Oxford, 1979); Y.B. Zeldovich, A.A. Ruzmaikin, and D. Sokolov, *Magnetic Fields in Astrophysics* (Gordon and Breach, New York, 1983).
- [2] A.C. Davis, M. Lilley, and O. Törnkvist, Phys. Rev. D **60**, 021301(R) (1999).
- [3] R.M. Kulsrud, R. Cen, J.P. Ostriker, and D. Ryu, Astrophys. J. **480**, 481 (1997).
- [4] E.R. Harrison, Phys. Rev. Lett. **30**, 188 (1973).
- [5] M.S. Turner and L.M. Widrow, Phys. Rev. D **37**, 2743 (1988).
- [6] A.D. Dolgov, Phys. Rev. D **48**, 2499 (1993).
- [7] M. Gasperini, M. Giovannini, and G. Veneziano, Phys. Rev. Lett. **75**, 3796 (1995).
- [8] R. Opher and U.F. Wichoski, Phys. Rev. Lett. **78**, 787 (1997).
- [9] B. Rathra, Astrophys. J. Lett. **391**, L1 (1992); F. Finelli and A. Gruppuso, Phys. Lett. B **502**, 216 (2001).
- [10] O. Bertolami and D.F. Mota, Phys. Lett. B **455**, 96 (1999).
- [11] A.C. Davis, K. Dimopoulos, T. Prokopec, and O. Törnkvist, Phys. Lett. B **501**, 165 (2001).
- [12] E. Calzetta, A. Kandus, and F. Mazzitelli, Phys. Rev. D **57**, 7139 (1998).
- [13] M. Giovannini and M. Shaposhnikov, Phys. Rev. D **62**, 103512 (2000); hep-ph/0011105.
- [14] B.A. Bassett, C. Gordon, R. Maartens, and D.I. Kaiser, Phys. Rev. D **61**, 061302(R) (2000).
- [15] V.F. Mukhanov, H.A. Feldman, and R.H. Brandenberger, Phys.

- Rep. **215**, 203 (1992).
- [16] J.A. Frieman, Phys. Rev. D **39**, 389 (1989); J. Céspedes and E. Verdaguier, *ibid.* **41**, 1022 (1990); A. Campos and E. Verdaguier, *ibid.* **45**, 4428 (1992).
- [17] V. Zanchin, A. Maia, Jr., W. Craig, and R. Brandenberger, Phys. Rev. D **57**, 4651 (1998); **60**, 023505 (1999); B.A. Bassett and S. Liberati, *ibid.* **58**, 021302(R) (1998).
- [18] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
- [19] E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990).
- [20] L. Kofman, A.D. Linde, and A.A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994); Phys. Rev. D **56**, 3258 (1997); J.H. Traschen and R.H. Brandenberger, *ibid.* **42**, 2491 (1990); Y. Shtanov, J. Traschen, and R. Brandenberger, *ibid.* **51**, 5438 (1995).
- [21] G. Felder, L. Kofman, and A. Linde, Phys. Rev. D **59**, 123523 (1999).
- [22] D. Boyanovsky, H.J. de Vega, and M. Simionato, Phys. Rev. D **61**, 085007 (2000).