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Chaos in hadrons

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Abstract. In the last decade quantum chaos has become a well established discipline with outreach to different fields, from condensed-matter to nuclear physics. The most important signature of quantum chaos is the statistical analysis of the energy spectrum, which distinguishes between systems with integrable and chaotic classical analogues. In recent years, spectral statistical techniques inherited from quantum chaos have been applied successfully to the baryon spectrum revealing its likely chaotic behaviour even at the lowest energies. However, the theoretical spectra present a behaviour closer to the statistics of integrable systems which makes theory and experiment statistically incompatible. The usual statement of missing resonances in the experimental spectrum when compared to the theoretical ones cannot account for the discrepancies. In this communication we report an improved analysis of the baryon spectrum, taking into account the low statistics and the error bars associated with each resonance. Our findings give a major support to the previous conclusions. Besides, analogue analyses are performed in the experimental meson spectrum, with comparison to theoretical models.

1. Introduction

Hadron spectroscopy has played a central role in the study of the strong interaction helping on its understanding and the development of Quantum Chromodynamics (QCD). Hadrons constitute bound states for quarks and gluons, and their accurate description is one of the principal aims of QCD. However, so far a quantitative and predictive theory of confined states has not been achieved, hence, in order to study the properties of hadrons we have to rely on models which have to be consistent with the underlying QCD. Constituent quark models [1] are examples of this kind of modeling.

On the other hand, as hadrons can be considered as aggregates of quarks and gluons, the mass spectrum of low-lying baryons or mesons can be understood as the energy spectrum of a quantum system, like an atomic nucleus, and it consists on all the possible states which stem from an interacting quantum system. Since Wigner discovered that the statistical properties of complex nuclear spectra are well described by the Gaussian Orthogonal Ensemble (GOE) of Random Matrix Theory [2], statistical methods have become a powerful tool to study the energy spectra of quantum systems [3, 4]. The most striking result in this field is that the statistical properties of the energy-level fluctuations determine if a system is chaotic, integrable

or intermediate. Moreover, the energy-level fluctuations of integrable and chaotic systems are universal. The former display a non-correlated sequence of levels, which follows the Poisson distribution [5], whereas the latter are characterized by a correlation structure described by GOE [6]. This kind of analysis has been already applied to the hadron mass spectrum in [7] obtaining a chaotic-like behavior. In [8] the spectral-statistic techniques have been used to compare the experimental baryon spectrum with theoretical ones, focusing on the problem of missing resonances.

In this communication we report an improved analysis of the baryon spectrum, taking into account the low statistics and the error bars associated with each resonance. Our findings give a major support to the previous conclusions. Besides, analogue analyses are performed in the experimental meson spectrum, with comparison to theoretical models.

2. Statistical analysis

Prior to any statistical analysis of the spectral fluctuations one has to accomplish some preliminary tasks. First of all, it is necessary to take into account all the symmetries that properly characterize the system. It is well known that mixing different symmetries deflects the statistical properties towards the Poisson statistics [9]. Hence, it is necessary to separate the whole spectrum into sequences of energy levels involving the same symmetries, that is, values of the good quantum numbers. The usual symmetries associated to baryons are spin (J), isospin (I), parity (P), and strangeness; and to mesons the same ones plus C -parity (C). Strangeness can be dropped due to the assumption of flavor $SU(3)$ invariance. Therefore, the baryon spectrum is split into sequences with fixed values of J , I and P , and the meson spectrum with fixed values of J , I , P and C .

The second preliminary task is the *unfolding* procedure. In any energy-level spectrum, the level density $\rho(E)$ can be split into a smooth part $\bar{\rho}(E)$ and a fluctuating part $\tilde{\rho}(E)$. The unfolding procedure allows one to extract the fluctuating part from the level density, removing the smooth component of the spectrum. Since the experimental hadron spectra are divided in very short sequences of levels, we have to use the so-called local unfolding procedure [10]. In each level sequence, the distances between consecutive levels, $S_i = E_{i+1} - E_i$, are rescaled using their average value $\langle S \rangle$ to obtain the quantities $s_i = S_i / \langle S \rangle$, called generically nearest neighbor spacings (NNS). For the rescaled spectrum the mean level density $\bar{\rho}(E) = 1$, and $\langle s \rangle = 1$. By arranging the set of NNS in a histogram we obtain what is called the nearest neighbor spacing distribution (NNSD) [11], denoted $P(s)$. The NNSD follows the Poisson distribution $P_p(s) = \exp(-s)$ for generic integrable systems [5] while chaotic systems with time reversal and rotational invariance are well described by the GOE of random matrices, whose NNSD follows the Wigner surmise $P_W(s) = \frac{\pi s}{2} \exp\left(-\frac{\pi s^2}{4}\right)$ [6].

However it is important to point out that performing the local unfolding on very short sequences of levels may cause a distortion on the actual $P(s)$, preventing a direct comparison with the theoretical predictions (GOE and Poisson). Inasmuch as $\langle s \rangle = 1$ for every spacing sequence no one of the spacings can be greater than $l - 1$, where l is the sequence length, and therefore the $P(s)$ distribution must exhibit a sharp cutoff at $s = l - 1$. When l is large enough this cutoff is irrelevant due to the exponential and Gaussian decays of the Poisson and Wigner distributions. Obviously, this is not the case for smaller values of l . This problem was taken into account in [8] by building GOE-like and Poisson-like spectra distorted in the same way by the unfolding procedure as the data distribution which is being studied in each case. That is, by dividing the GOE and Poisson spectra in the same number of sequences with the same lengths as the data spectrum and performing a local unfolding in the same way. These distributions built *ad hoc* for the spectrum which is being studied are thus more adequate as reference distributions for comparison than the theoretical predictions.

In this work we take a step further. Instead of building just one GOE-like and one Poisson-like

reference spectra to compare, we generate an ensemble of 1000 realizations, and we take their average as *theoretical predictions* for comparison with the data in each case. We will denote $P_{DW}(s)$ the distorted Wigner distribution and $P_{DP}(s)$ the distorted Poisson. Then, we can compare the results obtained with this improved analysis with those obtained in [8] in order to confirm the conclusions stated there on the baryon spectrum.

3. Results

3.1. Baryon spectrum

Fig. 1 shows the $P(s)$ distribution for the experimental spectrum provided by the Particle Data Group (PDG) and Fig. 2 shows the $P(s)$ distribution for one of the theoretical spectra from quark models analyzed in [8], the one by Capstick and Isgur [12], in both cases compared to the corresponding distributions $P_{DW}(s)$ and $P_{DP}(s)$. It can be seen that the experimental spectrum is closer to the Wigner distribution while the theoretical one is closer to Poisson (the same applies to the other two theoretical spectra by Löring *et al.* [14] analyzed in [8]). We have taken all the resonance states up to 2.2 GeV and only sequences with more than two levels are considered. Then the experimental spectrum has 40 levels distributed in 12 sequences, the one by Capstick and Isgur has 145 levels in 19 sequences.

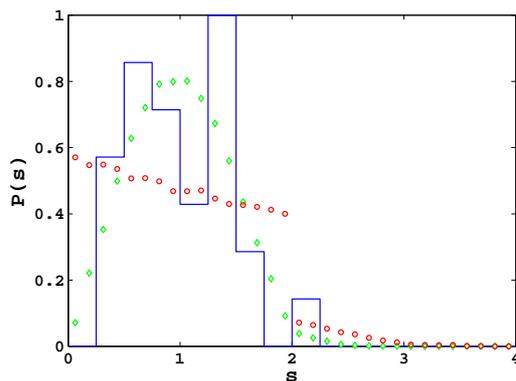


Figure 1. NNSD for the experimental spectrum from PDG (histogram), compared to the distorted distributions: Wigner, $P_{DW}(s)$ (diamonds) and Poisson, $P_{DP}(s)$ (circles).

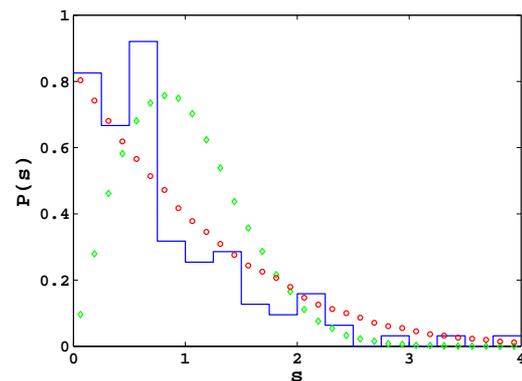


Figure 2. NNSD for the theoretical spectrum from the model by Capstick and Isgur (histogram), compared to the distorted distributions: Wigner, $P_{DW}(s)$ (diamonds) and Poisson, $P_{DP}(s)$ (circles).

In order to obtain a quantitative comparison we have performed, as in [8], the Kolmogorov-Smirnov test [13] with the null hypothesis that the studied NNSD coincides with the reference distribution $P_{DW}(s)$ or $P_{DP}(s)$, against the hypothesis that both distributions are different. The results (p -value) are shown in Table 1. EXP is the experimental spectrum, and the three theoretical spectra are from the model by Capstick and Isgur (CI) and from the model by Löring *et al.* (L1 and L2). It can be seen that all the theoretical spectra are incompatible with the Wigner distribution, whereas the experimental one seems to be closer to the Wigner than to the Poisson distribution.

These results confirm those obtained in [8], giving major support to the conclusions stated there. First, theory and experiment are statistically incompatible. Second, the usual statement of missing resonances cannot account for the discrepancies. As is well known, the existence of missing levels in a spectrum deflects the statistical properties towards Poisson [15, 9]. Thus, if the experimental spectrum is not complete due to missing states, it should be closer to the

Table 1. Probability to obtain, under the null hypothesis, a value of the Kolmogorov-Smirnov test statistic as extreme as that observed.

Spectrum	EXP	CI	L1	L2
Poisson	0.43	0.24	0.22	0.20
Wigner	0.82	$5 \cdot 10^{-4}$	10^{-4}	$2 \cdot 10^{-4}$

Poisson distribution than the theoretical ones. Hence, quark models, as they are presently built, may not be suitable to reproduce the low-lying baryon spectrum, and, therefore, to predict the existence of missing resonances.

A final test can be done in order to check if our analysis is robust against the inclusion of the error bars associated to the experimental data. We have generated an ensemble of 1000 “realizations” of the experimental spectrum whose levels are given by a Gaussian random variable with mean equal to the PDG estimation and variance equal to the error bar. Then we have built the NNSD and performed the K-S test for each of them. Figure 3 shows the histograms of the resulting p -values for the comparison to $P_{DW}(s)$ and $P_{DP}(s)$. The distribution of p_{DW} -values is narrowly concentrated in the region of high values with mean $\langle p_{DW} \rangle = 0.83$, that is, it remains practically unchanged with respect to the p_{DW} obtained for the original experimental spectrum, thus indicating that the result is robust. The distribution of p_{DP} -values is more spread and the mean value $\langle p_{DP} \rangle = 0.63$, which is higher than the one obtained for the original spectrum, but still reasonable because if the energy levels are allowed to fluctuate independently (in this case the fluctuation is induced by the error bars) the correlations are usually weakened and thus the statistics can be displaced towards Poisson.

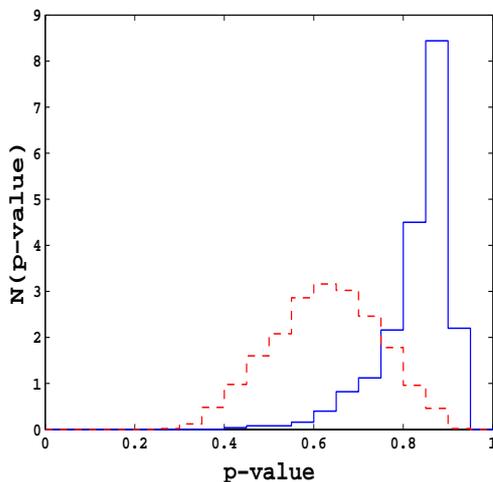


Figure 3. Distribution of the p -values of the K-S test for the 1000 “realizations” of the experimental spectrum within the error bars, for the null hypothesis that the distribution coincides with Poisson (dashed histogram) and with Wigner (solid histogram).

3.2. Mesons

A detailed analysis on the meson spectrum with comparison to theoretical models is in [16]. Here we show some of the preliminary results.

Fig. 4 shows the $P(s)$ distribution for the experimental spectrum provided by the PDG. As for baryons, the NNSD seems closer to the Wigner than to the Poisson distribution. This is confirmed by the result of the K-S test. When comparing to $P_{DW}(s)$ the p -value is $p = 0.47$

and when comparing to $P_{DP}(s)$ it is $p = 0.13$. Thus in this case we can almost safely reject the null hypothesis for the Poisson distribution. Fig. 5 shows the result of the same test as that performed for baryons to check the robustness of the analysis against the inclusion of the error bars associated to the experimental data. Again, as for baryons, we can say that the result is robust, $\langle p_{DW} \rangle = 0.46$ and $\langle p_{DP} \rangle = 0.28$.

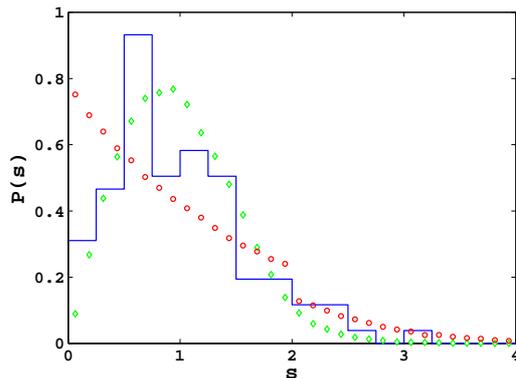


Figure 4. NNSD for the experimental spectrum from PDG (histogram), compared to the distorted distributions: Wigner, $P_{DW}(s)$ (diamonds) and Poisson, $P_{DP}(s)$ (circles).

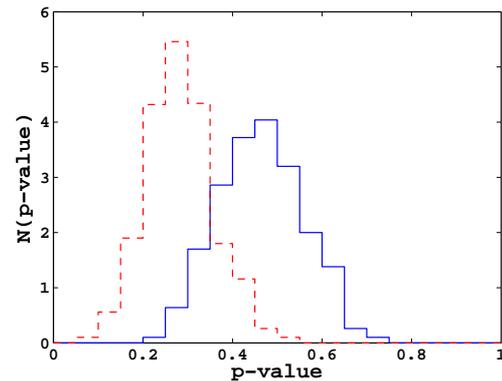


Figure 5. Distribution of the p -values of the K-S test for the 1000 “realizations” of the experimental spectrum within the error bars, for the null hypothesis that the distribution coincides with Poisson (dashed histogram) and with Wigner (solid histogram).

4. Conclusions

A statistical analysis of the low-lying baryon spectrum was carried out in [8], with comparison to theoretical spectra from quark models. There, it was shown that the statistical properties of the experimental spectrum are closer to those of a GOE spectrum, that is, to the theoretical prediction for chaotic systems, while the theoretical spectra analyzed are closer to a Poisson spectrum, that is, to the theoretical prediction for integrable systems. Then, it was concluded that theory and experiment are statistically incompatible and, moreover, the usual statement of missing resonances cannot account for the discrepancies. Thus, quark models, as they are presently built, may not be suitable to reproduce the low-lying baryon spectrum, and, therefore, to predict the existence of missing resonances.

In this work we perform an improved analysis of the baryon spectrum, taking into account the low statistics, that is, the possible distortion which can be produced on the NNSD when a local unfolding is performed in very short sequences of levels. In order to do that we have built GOE and Poisson-like reference spectra distorted in the same way by the unfolding procedure as the spectrum which is being analyzed. We take 1000 realizations of the distorted spectra in each case to obtain the average behavior of the NNSD. Comparison to these “theoretical predictions” built *ad hoc* for each spectrum confirms the previous results. Moreover, a test has been done to check the effect of the error bars associated to the experimental data on the analysis. We have generated 1000 “realizations” of the experimental spectrum with levels given by a Gaussian random variable with mean equal to the PDG estimation and variance equal to the corresponding error bar, and performed the analysis on each spectra. This check confirms that the result is robust. And thus, all this new analysis gives major support to the conclusions

stated in [8]. Finally, we report preliminary results for the meson spectrum, whose detailed analysis will be in [16].

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