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# Recent results in quantum chaos and its applications to atomic nuclei

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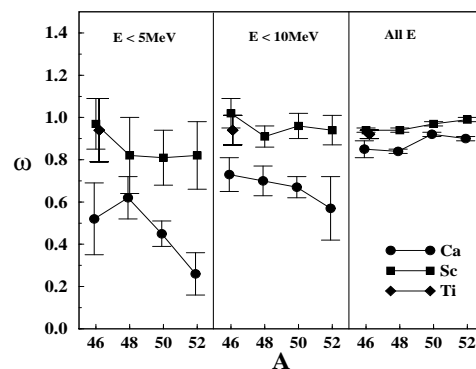
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**Abstract.** A survey of chaotic dynamics in atomic nuclei is presented, using on the one hand standard statistics of quantum chaos studies, and on the other a new approach based on time series analysis methods. The study of shell-model spectra in the  $pf$  shell shows that nuclear chaos is strongly isospin dependent and increases with excitation energy. On the other hand, it is found that chaotic quantum systems exhibit  $1/f$  noise and regular systems exhibit  $1/f^2$  behaviour. It is shown that the time series approach can be used to calculate quite accurately the fraction of missing levels and the existence of mixed symmetries in experimental level spectra.

## 1. Introduction

The understanding of quantum chaos has greatly advanced during the last two decades. It is well known that there is a clear relationship between the energy level fluctuation properties of a quantum system and the large time scale behaviour of its classical analogue. The spectral fluctuations of a quantum system whose classical analogue is fully integrable are well described by Poisson statistics, i. e. the successive energy levels are not correlated [1]. On the contrary the fluctuation properties of generic quantum systems, which are fully chaotic, coincide with those of random matrix theory (RMT) [2]. Comprehensive reviews can be found in [3, 4], and more recent developments are reviewed in [5].

The information on regular and chaotic nuclear motion available from the analysis of experimental data is rather limited, because it requires the knowledge of sufficiently large pure sequences, i.e. consecutive level samples all with the same quantum numbers ( $J, \pi, T$ ) in a given nucleus. The situation is clear above the one-nucleon emission threshold, where a large number of neutron and proton  $J^\pi = 1/2^+$  resonances are identified. The agreement between this Nuclear Data Ensemble (NDE) [6] and the GOE predictions is excellent. In the low energy domain, however, it is rather difficult (if not impossible) to get large enough pure sequences, and for this reason the conclusions are less clear [7, 8]. In order to get deeper understanding of what happens in the low energy region one can use the shell model with configuration mixing. Most shell-model studies on chaotic motion have been done in  $sd$ -shell nuclei. We present here calculations in  $pf$ -shell nuclei, where the configuration space is much larger, and therefore we can study the dependence of chaos on excitation energy and isospin. We present as well a different approach to quantum chaos based on traditional methods of time series analysis. The energy



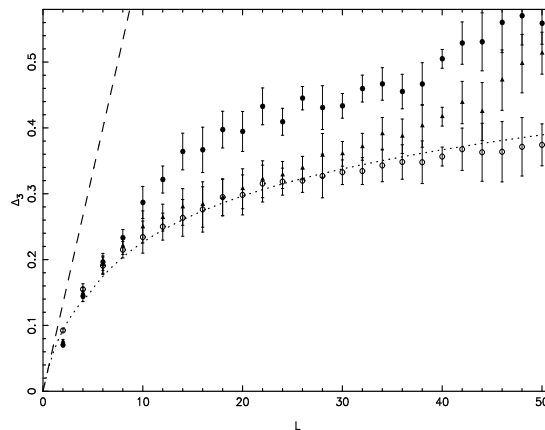
**Figure 1.** Brody parameter  $\omega$  for the  $A = 46, 48, 50$  and  $52$  Ca and Sc isotopes and  $^{46}\text{Ti}$ , using all the energy levels up to  $5$  MeV and  $10$  MeV over the yrast line, and the full spectrum.

spectrum can be considered as a discrete signal, and the sequence of energy levels as a time series. As we shall see, examination of the power spectrum of energy level fluctuations reveals very accurate power laws for completely regular or completely chaotic Hamiltonian quantum systems. It turns out that chaotic systems have  $1/f$  noise, in contrast to the brown noise of regular systems [9, 10, 11]. We also present applications of spectral fluctuations to the analysis of imperfect experimental spectra in nuclei [12].

## 2. Quantum chaos in $pf$ -shell nuclei

To study the isospin dependence of nuclear chaos, we have performed a detailed comparative analysis of spectral properties in the  $A = 46, 48, 50$  and  $52$  Ca and Sc isotopes and in  $^{46}\text{Ti}$  [13]. The  $T = T_z$  states are considered in all cases. The spectra are obtained by shell-model calculations in the  $pf$  shell using the KB3 interaction [14].

The nearest neighbour spacing distribution  $P(s)$  has been studied including all the levels up to  $5$  MeV and  $10$  MeV above the yrast line, and without any cutoff. The unfolding is performed for each  $J^\pi, T = T_z$  pure sequence separately and then the unfolded spacings are gathered into a single set for each nucleus to get better statistics. The values of the Brody parameter  $\omega$  are displayed in Fig. 1 separated in three sub-panels according to the energy cutoff. Up to  $5$  MeV above the yrast line, Ca isotopes show spectral fluctuations intermediate between those of regular and chaotic systems, except  $^{52}\text{Ca}$  which essentially is a regular system. On the contrary, all the Sc isotopes and  $^{46}\text{Ti}$  are very close to GOE fluctuations. For a given  $A$ , the big differences between Sc and Ca isotopes must be due to the residual two-body interaction, because the single particle energies are the same in both cases. We argue that the neutron-neutron interaction is much weaker than the neutron-proton interaction and thus the central field motion is less affected in Ca isotopes, where all the valence particles are neutrons. Another interesting feature observed in Fig. 1 is that in Ca isotopes  $\omega$  shows a strong fall from  $A = 48$  to  $A = 52$ , where  $\omega = 0.25$ . This astonishing result means that the two-body interaction is almost unable to perturb the single-particle motion in the low energy levels of  $^{52}\text{Ca}$ . The  $\omega$  value for  $^{46}\text{Ti}$  has been included in Fig. 1 to show that, at all energies,  $\omega$  reaches its maximum in  $^{46}\text{Sc}$ . Therefore, replacing a single neutron by a proton in Ca isotopes causes a transition from a quasi-regular to a chaotic regime. It is remarkable that this transition takes place abruptly at all excitation energies in all the isotopes. A second replacement of a neutron by a proton does not seem to produce appreciable effects on the  $P(s)$  statistic.



**Figure 2.** Average  $\Delta_3$  for all the  $J^\pi = 0^+, T = T_z$  levels of  $^{46}\text{Ca}$  (dots),  $^{46}\text{Sc}$  (squares) and  $^{46}\text{Ti}$  (diamonds). The dotted and dashed curves represent the GOE and Poisson  $\Delta_3$  values.

We have studied the behaviour of the  $\Delta_3(L)$  statistic as well. The results given below concern the full spectrum and not only the low energy region. Fig. 2 shows  $\Delta_3(L)$  values for  $L \leq 50$ , using the  $J^\pi = 0^+, T = T_z$  levels of  $^{46}\text{Ca}$ ,  $^{46}\text{Sc}$  and  $^{46}\text{Ti}$ . Of the three nuclei, only  $^{46}\text{Ti}$  follows the GOE line, at least until  $L = 50$ . For  $^{46}\text{Sc}$  the  $\Delta_3$  is close to GOE predictions up to a certain separation value,  $L_{sep} \simeq 30$  where it upbends from the GOE curve. In  $^{46}\text{Ca}$  the upbending starts at a smaller value  $L_{sep} \simeq 10$ . The upbending from the GOE curve and a linear growth of this statistic reveals a departure from the chaotic regime. The  $\Delta_3$  behaviour clearly shows a strong isospin dependence in the  $A = 46$  nuclei, with chaoticity increasing as  $T$  decreases. This happens not only from Ca to Sc, but also from Sc to Ti. The same phenomenon is observed for other  $J$  values.

Summarising, there exists a clear excitation energy and isospin dependence in the chaoticity degree of nuclear motion. It is observed not only in the ground state region, but along the whole spectrum. When the full spectrum is taken into account, the  $P(s)$  distribution is not very sensitive to the isospin dependence, but the effect is clearly seen in the  $\Delta_3$  statistic.

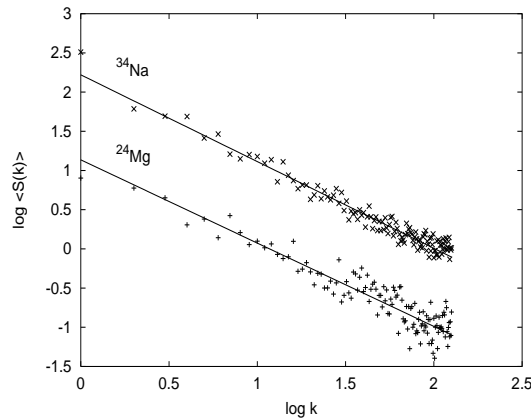
### 3. Time series approach to quantum chaos

A new approach to study spectral fluctuations in quantum systems has been recently presented. [9] The basic idea is to consider the energy spectrum as a time series where energy plays the role of time. Using this formal analogy spectral fluctuations can be studied by means of techniques borrowed from time series analysis. We can characterise the spectral fluctuations by the statistic  $\delta_n$  defined by  $\delta_n = \sum_{i=1}^n (s_i - \langle s \rangle)$ ,  $n = 1, 2, \dots, N - 1$ , where  $s_i = \epsilon_{i+1} - \epsilon_i$  is the spacing between two consecutive unfolded energy levels, and  $N$  is the total number of levels. Since  $\langle s \rangle = 1$ , the  $\delta_n$  function represents the deviation of the unfolded excitation energy from its mean value  $n$ .

One of the simplest methods to analyse correlations in a time series is the study of its power spectrum, which provides information on the correlations at all time scales. The power spectrum  $S(k)$  of a discrete and finite series  $\delta_n$  is given by  $S(k) = |\hat{\delta}_k|^2$ , where  $\hat{\delta}_k$  is the Fourier transform of  $\delta_n$ ,

$$\hat{\delta}_k = \frac{1}{N} \sum_n \delta_n \exp\left(\frac{2\pi i k n}{N}\right). \quad (1)$$

The first systems studied by this method were the spectra of atomic nuclei calculated using



**Figure 3.** Average power spectrum of the  $\delta_n$  function for  $^{24}\text{Mg}$  and  $^{34}\text{Na}$ , using 25 sets of 256 levels from the high level density region. The plots are displaced to avoid overlapping.

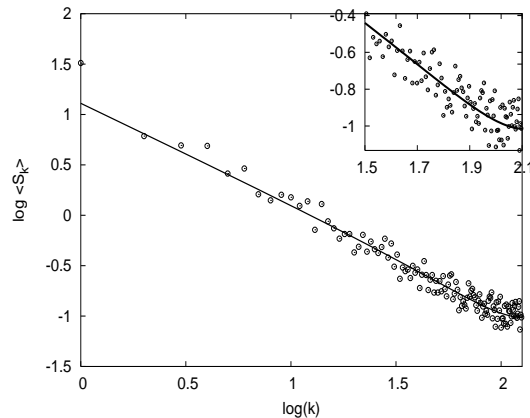
the shell model with configuration mixing [9]. Fig. 3 shows the results for a typical stable *sd*-shell nucleus,  $^{24}\text{Mg}$ , and for a very exotic nucleus,  $^{34}\text{Na}$ , in the *sd* proton and *pf* neutron shells. Clearly, the power spectrum of  $\delta_n$  follows closely a power law. We may assume the simple functional form  $\langle S(k) \rangle \sim 1/k^\alpha$ . A least-squares fit to the data of Fig. 3 gives  $\alpha = 1.11 \pm 0.03$  for  $^{34}\text{Na}$ , and  $\alpha = 1.06 \pm 0.05$  for  $^{24}\text{Mg}$ . These results raised the question of whether there is a general relationship between quantum chaos and the power spectrum of the  $\delta_n$  fluctuations of the system. A similar numerical calculation performed for the three classical random matrix ensembles GOE, GUE and GSE showed that the power spectrum  $\langle S(k) \rangle$  follows quite accurately a power law of type  $1/k$  for all of them. Therefore, the spectral fluctuations of chaotic quantum systems described by the  $\delta_n$  function exhibit  $1/f$  noise [9], independently of the symmetries of the system. As is well known, the existence of  $1/f$  noise is a remarkable and very ubiquitous property of many complex systems in nature and in social sciences. The origin of the  $1/f$  noise in the time fluctuations of these systems is an important open problem. In the case of quantum systems, an exact and complete proof of the  $1/f$  noise behaviour seems to be extremely difficult. However, it can be theoretically studied in semiclassical systems or random matrix ensembles, where the mathematical tractability of these systems may help to understand the origin of the  $1/f$  noise in chaotic quantum systems.

We have recently shown that the power spectrum of  $\delta_n$  for fully chaotic or integrable systems can be written in terms of the ensemble form factor  $K(\tau)$  as follows [10],

$$\langle S(k) \rangle_\beta = \frac{N^2}{4\pi^2} \left\{ \frac{K\left(\frac{k}{N}\right) - 1}{k^2} + \frac{K\left(1 - \frac{k}{N}\right) - 1}{(N - k)^2} \right\} + \frac{1}{4 \sin^2\left(\frac{\pi k}{N}\right)} + \Delta, \quad k = 1, 2, \dots, N-1, \quad N \gg 1, \quad (2)$$

where  $\beta$  is the repulsion parameter of RMT ensembles and takes the values  $\beta = 1$  for GOE,  $\beta = 2$  for GUE, and  $\beta = 4$  for GSE. Here  $\Delta = 0$  for integrable systems and  $\Delta = -1/12$  for chaotic systems. This equation, together with the appropriate values of  $K(\tau)$ , gives explicit expressions of  $\langle S(k) \rangle$  for specific ensembles or systems.

When  $k \ll N$  the first term of Eq. (2) becomes dominant and we can write  $\langle S(k) \rangle_\beta = \frac{N}{2\beta\pi^2 k}$



**Figure 4.** Numerical average power spectrum of the  $\delta_q$  function for  $^{34}\text{Na}$ , calculated using 25 sets of 256 consecutive levels from the high level density region, compared to the parameter free theoretical values (solid line) for GOE.

for chaotic systems and  $\langle S(k) \rangle_\beta = \frac{N^2}{4\pi^2 k^2}$  for integrable ones. These expressions show that, for small frequencies, the excitation energy fluctuations exhibit  $1/f$  noise in chaotic systems and  $1/f^2$  noise in integrable systems. As we shall see below, these power laws are also approximately valid through almost the whole frequency domain, due to partial cancellation of higher order terms. Only near  $k = N/2$  the effect of these terms becomes appreciable.

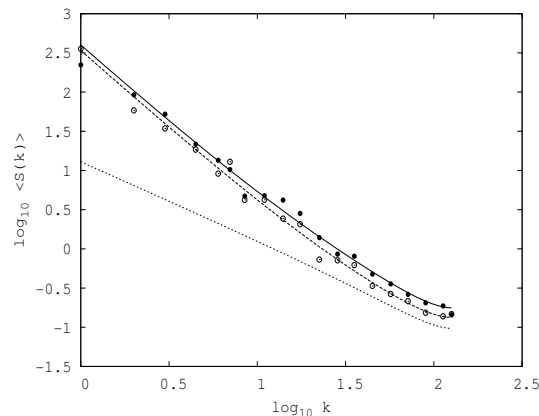
We have compared the analytical expression (2) to numerical results obtained for different RMT ensembles, shell model spectra of atomic nuclei and chaotic and regular quantum billiards. In all cases the agreement is excellent. As an example Fig. 4 displays the theoretical values of  $\langle S(k) \rangle$  compared with the shell-model results for  $^{34}\text{Na}$ . It can be seen that Eq. (2) gives  $1/k$  behaviour up to quite high values of the frequency counter  $k$ , and the agreement with the numerical power spectrum of  $^{34}\text{Na}$  is excellent.

#### 4. Applications

The time series approach to quantum chaos turns out to be quite fruitful to analyse imperfect spectra, i.e., quantum spectra with missing levels or mixed symmetries, which would induce misleading results about the chaotic or regular features of the spectra. It is possible to generalise Eq. (2) to a more complex situation in which every observed level sequence contains  $l$  pure sequences, each with a partial density  $\eta_i$  (the probability that a given level belong to the  $i$ -th sequence) and a fraction of observed levels  $\varphi_i$  (levels pertaining to the  $i$ -th sequence are observed with probability  $\varphi_i$ ). Then the power spectrum of  $\delta_n$  becomes [12]

$$\langle S(k) \rangle = \frac{N^2}{4\pi^2} \sum_{i=1}^l \eta_i \varphi_i \left[ \frac{K_i \left( \frac{\varphi_i k}{N \eta_i} \right) - 1}{k^2} + \frac{K_i \left( \frac{\varphi_i (N-k)}{N \eta_i} \right) - 1}{(N-k)^2} \right] + \frac{1}{4 \sin^2(\pi k/N)} + \langle \varphi^2 \rangle \Delta, \quad (3)$$

where  $K_i(\tau)$  is the original spectral form factor of the  $i$ -th pure level sequence and  $\langle \varphi^2 \rangle = \sum_{i=1}^l \eta_i \varphi_i^2$ . Clearly in the limit of a very large number of mixed symmetries or a very small fraction of observed levels one obtains again Poisson statistics. To apply (3) in practical cases one needs to assume a form of the spectral form factor  $K_i(\tau)$ . To see how well the method works in realistic situations [12], let us consider the shell-model spectrum of  $^{24}\text{Mg}$ , which exhibits  $1/f$  noise (see Fig. 4).



**Figure 5.** Numerical power spectrum of  $\delta_n$  with 20% missing levels ( $\varphi = 0.8$ ) using  $J = 3$  and  $J = 4$  pure sequences of length  $N = 256$  (open circles) and incomplete mixed sequences of the same length (filled circles). The solid and dashed lines correspond to Eq (3) with the best fit  $l$  and  $\varphi$  values. The theoretical curve for GOE (dotted line) is also shown.

Now Fig 5 shows the power spectrum of  $\delta_n$  when 20% of the levels are missing ( $\varphi = 0.8$ ), for the case of pure  $J^\pi = 3^+$  and  $J^\pi = 4^+$  sequences, and for the case of a mixed sequence of  $J^\pi = 3^+$  and  $J^\pi = 4^+$  states. Using (3), the parameters  $l$  and  $\varphi$  are calculated by means of a least-squares fit to the numerical power spectrum. In the case of pure  $J$  sequences, we obtain  $\varphi = 0.80 \pm 0.03, l = 1.1 \pm 0.3$ . Thus we see that the fit detects that the sequences belong to a single  $J$  value, and the fraction of observed levels is accurately reproduced. In the case of mixed  $J^\pi = 3^+$  and  $J^\pi = 4^+$  states, the fit gives  $\varphi = 0.77 \pm 0.03, l = 2.1 \pm 0.4$ . Therefore it detects that there are two mixed sequences ( $l = 2$ ) and the fraction of observed levels is also very close to the real 80% fraction. In general, the agreement between the results of the fit and the actual parameters used to generate the level sequences ranges from good to remarkable. Thus we conclude that this method can be very useful to extract relevant statistical information from experimental spectra.

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