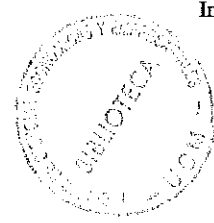# TAX ANALYSIS IN A LIMIT PRICING MODEL

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ABSTRACT

This paper deals with taxation, profits of firms and welfare. More specifically it analyses the effect of a lump sum tax on a noncompetitive market with free entry. The main result is that there are relevant situations in which the tax increases the profits of the incumbent firms. Unfortunately this goes with a reduction in consumer suplus (and in social welfare measured by consumer surplus, plus profits of firms plus revenues of the Government). But a way is suggested and some examples given in which that problem can be overcome.

RESUMEN

El artículo trata problemas relacionados con imposición, beneficios de las empresas y bienestar. Más específicamente, analiza el efecto de un impuesto de cuantía fija en un mercado no competitivo con libertad de entrada. El resulado más importante es que existen situaciones relevantes en las cuales el impuesto incrementa los beneficios de las empresas establecidas. Desgraciadamente este incremento va acompañado de una disminución del bienestar social; no obstante, se sugiere una vía, y se dan algunos ejemplos, a través de la cual dicho problema puede superarse.

TAX ANALYSIS IN A LIMIT PRICING MODEL

I. INTRODUCTION

The subject of this paper is to analyse the effect of taxes on profits of firms in noncompetitive markets with free entry. We principally approach the case of a lump sum tax on firms operating in a market where the technology is freely available and therefore all firms have the same cost structure and economies of scale (existence of a fixed cost that every firm which works in the market has to pay) are the only barriers to entry. However, we extend the analysis to more complex cases combining the lump sum tax with a subsidy.

To make the analysis simpler and the explanation clearer, we approach the problem using a linear model, although in the Appendix the results are generalised.

The most important conclusion is that relevant situations exist in which the incumbent firms maximize profits if they pay a lump sum tax. It implies that it is possible to design a taxation system where the position of direct payers is not damaged [*].

This paper has its precedents in Dixit (1979), Omori and Yarrow (1982) and Corchon and Marcos (1988). It retains the basic idea that the prevention of entry is a good strategy.

The main conclusion of the latter is that under similar conditions to those we will establish here, particularly the decrease of average cost the best option for the incumbent firms is to prevent entry, and only one firm remains in the market

supplying the limit output[*] or the monopoly output (the largest of the two amounts).

The plan of the paper is as follows. Section II stablishes the model and gets the main results. Section III carries out an extension introducing a subsidy and makes some suggestions for futher research. Finally an appendix generalises the main result of Section II.

II. THE MODEL.

As previously mentioned a linear model with a single firm in the market will be used in order to analyse the effect of the lump sum tax on its profits.

We depart from the conclusion drawn by Omori and Yarrow and Corchón and Marcos that preventing entry maximizes the profits of the incumbent firm, and therefore it should produce a quantity equal to or greater than the limit output.

Let's design the demand

$$z = a - p$$

where $z = y + x$, y being the output of the incumbent firm and x that of the potential entrant.

The cost functions are equal for both

$$c = K + vy \quad , \quad c = K + vx$$

(it is important to note that this kind of cost function implies the decrease of average cost which is a necessary condition for the conclusion of Omori and Yarrow and Corchón and Marcos).

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[*] The limit output is the central concept of this paper and it is the lesser ouput which prevents entry. We say that an output (of the incumbent firm) prevents entry if given this output another firm cannot work in the market with positive profits. It is obvious, if the profit function is quasi-concave (which is a usual assumption), that the limit output provides a greater profit than any other output which prevents entry (except if monopoly output is larger than limit output : in that case the former prevents entry and maximizes profits).

Now the problem is

$$\max \ \pi \ = \ (a-y)y - K - vy - T$$

restricted to $\quad y \geq \overline{y} = a - v - 2\sqrt{K+T}$ *

$$T \geq 0$$

The Lagragian of the problem is

$$L(\ y,\ T,\ \lambda_1,\ \lambda_2,\ r,\ s)\ =\ (a-v)y - y^2 - K - T + \lambda_1(y-\overline{y}-r^2) + \lambda_2(T-s^2)$$

where r, s are slack variables.

The first order conditions are

$$\partial L \ / \ \partial y = a - v - 2y + \lambda_1 = 0 \tag{1}$$

$$\partial L \ / \ \partial T = -1 - \lambda_1 \, \partial \overline{y} \ / \ \partial T + \lambda_2 = -1 + \lambda_1 \ / \ \sqrt{K+T} + \lambda_2 = 0 \tag{2}$$

$$\partial L \ / \ \partial \lambda_1 = y - \overline{y} - r^2 = 0 \tag{3}$$

$$\partial L \ / \ \partial \lambda_2 = T - s^2 = 0 \tag{4}$$

$$\partial L \ / \ \partial r = -2\lambda_1 r = 0 \tag{5}$$

$$\partial L \ / \ \partial s = -2\lambda_2 s = 0 \tag{6}$$

$$\lambda_1 \geq 0 \qquad \lambda_2 \geq 0$$

Condition (2) involves the impossibility of λ_1 and λ_2 simultaneously being zero. Therefore there are only three possible cases : 1) $\lambda_1 = 0$; $\lambda_2 > 0$; 2) $\lambda_1 > 0$; $\lambda_2 > 0$, and 3) $\lambda_1 > 0$, $\lambda_2 = 0$.

1) $\quad \lambda_1 = 0 \quad , \ \lambda_2 > 0$

In this case condition (2) implies $\lambda_2 = 1$ and then conditions (6) and (4) lead us to

$$s = T = 0$$

and for condition (1)

$$Y^* = (a-v) / 2$$

which is the monopoly output.

This is the case of blockaded entry in Bain's terminology. The established firm maximizes profits without having to worry about a possible entrant. It produces the monopoly output that being greater than limit ouput[*] prevents entry.

In this case a lump sum tax has the effect of reducing the profits of the firm by the same value as the tax. Because limit output is less than monopoly output, we have

$$\frac{(a-v)}{2} \geq a-v-2\sqrt{K}$$

that is

$$a \leq v + 4\sqrt{K}$$

On the other hand, the profits of the firm are

$$\pi^* = (a-v)^2 / 4 - K$$

and then the feasibility of the industry needs

$$a > v + 2\sqrt{K}$$

Therefore the scope for this case is

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[*] Condition (3) $y - \bar{y} - t_2 = 0 \rightarrow y^* \geq \bar{y}$

$$v + 2\sqrt{K} < a < v + 4\sqrt{K}$$

2) $\lambda_1 > 0, \ \lambda_2 > 0$

Conditions (5) and (6) imply

$$r = s = 0$$

and (3) , (4)

$$T = 0, \quad y = \bar{y} : a - v - 2\sqrt{K}$$

$$\bar{p} = v + 2\sqrt{K} \qquad \bar{\pi} = 2(a-v)\sqrt{K} - 5K$$

In this case limit output is greater than monopoly output and it is the output which maximizes profits. On the other hand, because profits of the firm are at a maximum if T = 0, lump sum tax diminishes profits.

Besides condition (1) says ($\lambda_1 > 0$)

$$-\lambda_1 = a - v - 2y < 0$$

That is

$$a - v - 2a + 2v + 4\sqrt{K} < 0$$

$$a > v + 4\sqrt{K}$$

And condition (2)

$$\lambda_2 = 1 + \frac{(a-v-2y)}{\sqrt{K}} > 0$$

$$\sqrt{K} + a - v - 2a + 2v + 4\sqrt{K} > 0$$

$$a < v + 5\sqrt{K}$$

Therefore the scope for this solution is

$$v + 4\sqrt{K} < a < v + 5\sqrt{K}$$

3) $\lambda_1 > 0$, $\lambda_2 = 0$

Conditions (5) and (3) lead us to

$$r = 0, \quad y = \overline{y}$$

and (6) and (4) to

$$s \geq 0 \qquad T \geq 0$$

on the other hand, removing λ_1 between (1) and (2)

$$\lambda_1 = 2a - 2v - 4\sqrt{K + T} - a + v = \sqrt{K + T}$$

$$5\sqrt{K + T} = a - v$$

we find $T = (a - v)^2 / 25 - K$

limit output is

$$\overline{y} = a - v - 2\sqrt{K + \frac{(a-v)^2}{25}} - K$$

$$\overline{y} = \frac{3}{5}(a - v)$$

price
$$\overline{p} = \frac{(2a + 3v)}{5}$$

and profits
$$\overline{\pi} = \frac{1}{5}(a - v)^2 - K$$

Finally condition $T \geq 0$ provides the scope where this case is relevant

$$T = [(a-v)^2 / 25] - K \geq 0$$

implies
$$(a - v)^2 > 25 K$$

that is
$$a > v + 5\sqrt{K}$$

To summarise, if the demand is great enough
$$(a > v + 5\sqrt{K})$$

a lump sum tax can increase the profits of the firm.

We must point out that any expanding industry will sooner or later fall into this last case and therefore its theoretical relevance to have been shown.

A brief comment on this result

Problems with taxes arise because everybody wants to pay as little as possible and fight against any increase but in this case the tax does not damage the people who pay it and the Government will encounter less opposition.

For instance, if the Government burdens profits of luxury restaurants, hotels or casinos with a tax it may have to face powerful pressure. But with this kind of tax it does not attack their profits and therefore will come up against less opposition.

The same applies for firms making sophisicated luxury cars, yachts or private planes. From another point of view this tax could be useful for those goods, the consumption of which the Government would like to reduce (Tobacco, alcohol, etc).

Of course, there is no reason for the Government to increase the profits of firms. If so it can choose the size of the tax to maximise its revenues or simply leave profits at the same level as they were without tax.

EFFECTS ON SOCIAL WELFARE

Reducing the limit output it increases the price and this increase is the cause for the rise in profits. But unfortunately this change encompasses a diminishing in consumer surplus* great enough to reduce the social welfare measured by consumer surplus, plus profits of the firms plus revenues of Government.

Only if it burdens goods consumed by people whose surplus can be considered not significant for social welfare (such as, may be, those examples quoted previously) the latter is not negatively affected.

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*It is easy to show that in this linear model a tax, T , provides a fall in the social surplus equal to 2T.

III. EXTENSION: A LUMP SUM TAX COMBINED WITH A SUBSIDY PER UNIT

Now, an attempt will be made to overcome some of the limitations of the previous analysis: If a subsidy per unit sold (or bought) is introduced it is found that social welfare can be improved.

Following on with the linear model, if we combine the tax with a subsidy to the producers per unit sold (or equivalently to the consumers per unit bought) the social surplus can be written

$$E = E_c + \pi + T - ty$$

$$E = \frac{y^2}{2} + ay - y^2 - vy - K - T + ty \, T - ty$$

that is

$$E = (a - v)y - y^2 / 2 - K$$

If we maximize this expression supposing that the monopoly output is less than limit output* and therefore that this one is the solution, the formal expresion of the problem is

$$\max E = (a - v)y - y^2 / 2 - K$$

restricted to

$$y = \overline{y} = a - v + t - 2\sqrt{K + T}$$

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*We must remember that the case in which the monopoly output is greater is not of interest to our propposals.

$$T \geq 0$$

$$t \geq 0$$

The correspondent Lagrangian introducing slack variables is

$$L = (a-v)y - \frac{y^2}{2} - K + \lambda_1[y - (a-v+t-2\sqrt{K}+T)] + \lambda_2(T-r^2) + \lambda_3(t-s^2)$$

and the first order conditions

$$\frac{\partial L}{\partial y} = a - v - y + \lambda_1 = 0 \tag{1}$$

$$\frac{\partial L}{\partial T} = \frac{\lambda_1}{(\sqrt{K+T})} + \lambda_2 = 0 \tag{2}$$

$$\frac{\partial L}{\partial t} = -\lambda_1 + \lambda_3 = 0 \tag{3}$$

$$\frac{\partial L}{\partial \lambda_1} = y - (a-v+t-2\sqrt{K+T}) = 0 \tag{4}$$

$$\frac{\partial L}{\partial \lambda_2} = T - r^2 = 0 \tag{5}$$

$$\frac{\partial L}{\partial \lambda_3} = t - s^2 = 0 \tag{6}$$

$$\frac{\partial L}{\partial r} = -2r\lambda_2 = 0 \tag{7}$$

$$\frac{\partial L}{\partial s} = -2s\lambda_3 = 0 \tag{8}$$

$$\lambda_2 \geq 0 \ , \ \lambda_3 \geq 0$$

Conditions (2) and (3) imply that any λ_i cannot be positive, therefore

$$\lambda_1 = \lambda_2 = \lambda_3 = 0$$

then for condition (1)

$$y = a - v$$

and for condition (4)

$$t = 2\sqrt{K+T}$$

so, we find the competitive solution which is what we could have expected. Profits

$$\pi = (a-v) \ (v-v) - K + 2\sqrt{K+T} \ (a-v) - T$$

$$\pi = [2(a-v) - \sqrt{K+T}] \sqrt{K+T}$$

are positive and greater than profits without taxes

$$\pi = 2(a-v)\sqrt{K} - 5K$$

except if T is unussually great.

As the value of social welfare

$$E = 1/2 \ (a-v)^2 - K \quad *$$

does not depend on the value of the tax (it only must verify the relation ship $t = 2\sqrt{K+T}$ we can select the latter to achieve another goal**

SOME SUGGESTIONS FOR FURTHER RESEARCH

Up to now we have considered only increasing returns, but similar results can be achieved, for some particular situations, when the average cost curve is U-shaped.

In that case, it depends on the size of demand wether the prevention of entry is the best option for the incumbent firm or not. If demand is narrow enough to make the entry deterrent profitable the tax system will provide the same outcome that in the preceding case, but even introducing only the lump sum tax social welfare can be improved, by allowing a better use of economies of scale***.

Finally, it can be suggested that if there are several firms in the industry, the lump sum tax can reduce their number allowing a more effective application of economies of scale, and therefore improving social welfare.

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* We can remember that the social surplus without taxes was

$$E = 1/2 \ (a-v)^2 - 3K$$

** For instance, it is easy to show that if we want to maximize the profits of the firm the value of the tax has to be

$$T = (a - v)^2 - K$$

*** An example of this can be obtained from the author on request.

APPENDIX : THE EFFECT OF A LUMP SUM TAX ON THE PROFITS OF FIRMS IN THE GENERAL CASE

Now, we leave the linear model and approach the problem assuming that demand and average cost functions are decreasing and twice differenciable and the profit function is quasi-concave.

To obtain the output and the tax which maximize the profits of the firm the problem to solve is, as we have seen before

$$\max \quad \pi = yf^{-1} \ (y) - K - T - v(y)$$

$$\text{restricted to} \quad y \geq \bar{y} \ (T)$$

$$T \geq 0$$

where $f^{-1}(y)$ is the inverse demand function and $v(y)$ the variable costs.

The Lagrangian of the problem introducing slack variables can be written

$$L = yf^{-1}(y) - K - T - v(y) + \lambda_1 \ (y - \bar{y} - r^2 \ ) + \lambda_2 \ (T - s^2 \ )$$

and the first order conditions

$$\partial L/ \ \partial y = f^{-1} \ (y) + y \ / \ f' - v' + \lambda_1 = 0 \tag{1}$$

$$\partial L/ \ \partial T = -1 - \lambda_1 \ \partial \bar{y}/ \ \partial T \ + \lambda_2 = 0 \tag{2}$$

$$\partial L/ \ \partial r = -2r\lambda_1 \ = 0 \tag{3}$$

$$\partial L/ \ \partial s = -2s\lambda_2 \ = 0 \tag{4}$$

$$\partial L / \ \partial \lambda_1 = y - y - r^2 = 0 \qquad (5)$$

$$\partial L / \ \partial \lambda_2 = T - s^2 = 0 \qquad (6)$$

$$\lambda_1 \ \geq 0, \quad \lambda_2 \ \geq 0$$

Condition (2) implies that one restriction , at least, is active. Therefore, three possible cases may arise : 1) $\lambda_1 = 0$; $\lambda_2 > 0$; 2) $\lambda_1 > 0, \lambda_2 > 0$; 3) $\lambda_1 > 0, \quad \lambda_2 = 0$.

1) $\lambda_1 = 0$; $\quad \lambda_2 > 0$.

Conditions (4) and (6) imply

$$T = S = 0$$

and condition (1)

$$f^{-1} (y) + y / f' = v'$$

marginal revenue equals to marginal cost, provide us with output y^* which maximizes profits (the monopoly output).

In this case, the parameters of demand and costs are such that monopoly output is greater than or equal to limit output : it is the case of blockaded entry in Bain's terminology. Here the incumbent firm maximize profits without worrying about potencial competitors. To burden it with a lump sum tax would neither modify output nor price, but would reduce the profits by the value of the tax.

2) $\quad \lambda_1 > 0, \quad \lambda_2 > 0$.

Condition (4) and (6) imply

$$T = s = 0$$

and conditions (3) and (5)

$$y = \bar{y}$$

besides , condition (1)

$$f^{-1} (y) + y / f' < v'$$

implies that monopoly output is less than limit output[*], which is the solution.

In this case the tax would increase the limit price and possibly the profits. But the increase of profits would always be less than tax and therefore it is not profitable for the firm.

3) $\quad \lambda_1 > 0, \quad \lambda_2 = 0$.

Conditions (3) and (5) imply

$$y = \bar{y}$$

and condition (1) that limit output is greater than monopoly output.

In this case, the lump sum tax can increase the profits of the incumbent firm. We can obtain the best value of the tax removing λ_1 from equation (1) and (2). Then we get

$$f^{-1} [y(T)] + [\ y \ (T)] / f' \ - v' - 1 / (dy/dT) = 0$$

and by solving it we obtain the value of T.

Of course, whether or not the tax would increase the profits of the firms, and whether this increase would be

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[*] This can be easily seen remembering the second order conditions.

meaningful is an empirical question. The model that we have presented is general enough to allow for any result. However, we have seen some arguments in favour of the relevance of the third case

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