Rotating beams in isotropic optical system

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Abstract: Based on the ray transformation matrix formalism, we propose a simple method for generation of paraxial beams performing anisotropic rotation in the phase space during their propagation through isotropic optical systems. The widely discussed spiral beams are the particular case of these beams. The propagation of these beams through the symmetric fractional Fourier transformer is demonstrated by numerical simulations.

OCIS codes: (070.2590) ABCD transforms; (070.2575) Fractional Fourier transforms; (070.3185) Invariant optical fields; (070.2580) Paraxial wave optics.

References and links

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1. Introduction

The analysis and synthesis of spiral paraxial beams, whose transversal intensity distribution rotates around the axis without changing its form (except for some scaling) during its propagation in free space, have been treated in many publications [1–5]. Here we propose an alternative approach for the description of spiral beams based on the ray transformation matrix formalism which is suitable for the analysis of their propagation through any isotropic paraxial optical system (IOS). Moreover, this method is also valid for the design of beams which perform other types of phase-space rotations during their propagation through an IOS.

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Beam propagation through a lossless paraxial optical system is described by the canonical integral transformation [6], represented by operator $\mathcal{R}^{\mathbf{T}}$, whose kernel is parameterized by the real symplectic ray transformation matrix **T**, which relates the position $\mathbf{r} = (x, y)^t$ and direction $\mathbf{p} = (p_x, p_y)^t$ of an incoming ray to those of the outgoing ray

$$\begin{pmatrix} \mathbf{r}_o \\ \mathbf{p}_o \end{pmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{pmatrix} \mathbf{r}_i \\ \mathbf{p}_i \end{pmatrix} = \mathbf{T} \begin{pmatrix} \mathbf{r}_i \\ \mathbf{p}_i \end{pmatrix}.$$

Here we use dimensionless variables. Note that the variables $(\mathbf{r}, \mathbf{p})$ form, in paraxial approximation, optical phase space. Using the modified Iwasawa decomposition [7], the properly normalized ray transformation matrix **T** can be written as a product of three matrices,

$$\mathbf{T} = \mathbf{T}_L \mathbf{T}_S \mathbf{T}_O = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{G} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{X} & \mathbf{Y} \\ -\mathbf{Y} & \mathbf{X} \end{bmatrix}, \quad (1)$$

where the first matrix $\mathbf{T}_L$ with the symmetric matrix $\mathbf{G} = -(\mathbf{CA}^t + \mathbf{DB}^t)(\mathbf{AA}^t + \mathbf{BB}^t)^{-1}$ represents a lens, the second matrix $\mathbf{T}_S$, defined by the positive definite symmetric matrix $\mathbf{S} = (\mathbf{AA}^t + \mathbf{BB}^t)^{1/2}$, corresponds to a scaler, and the third matrix $\mathbf{T}_O$ is orthogonal and describes rotations in phase space. Instead of $\mathbf{T}_O$ one can use the unitary matrix $\mathbf{U} = \mathbf{X} + i\mathbf{Y} = (\mathbf{AA}^t + \mathbf{BB}^t)^{-1/2}(\mathbf{A} + i\mathbf{B})$, regarding that $(\mathbf{r}_o - i\mathbf{p}_o) = \mathbf{U}(\mathbf{r}_i - i\mathbf{p}_i)$. There are three basic transforms related to the rotations in phase space: the fractional Fourier transform (FrFT), signal rotator and gyrator, associated with unitary matrices

$$\mathbf{U}_f(\gamma_x, \gamma_y) = \begin{bmatrix} \exp(i\gamma_x) & 0 \\ 0 & \exp(i\gamma_y) \end{bmatrix}, \quad \mathbf{U}_r(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \quad \mathbf{U}_g(\alpha) = \begin{bmatrix} \cos\alpha & i\sin\alpha \\ i\sin\alpha & \cos\alpha \end{bmatrix},$$

respectively. In particular, the FrFT is responsible for the rotation in (x, p_x) and (y, p_y) planes at angles γ_x and γ_y, respectively. The signal rotator produces the rotation in (x, y) and (p_x, p_y) planes. The gyrator operator describes cross (y, p_x) and (x, p_y) rotations. Note that a cascade of two rotators defined by $\mathbf{U}_1$ and $\mathbf{U}_2$ corresponds to a phase-space rotator described by the matrix $\mathbf{U}_2\mathbf{U}_1$. The experimental setups capable to perform these phase-space rotators were discussed in Ref [8].

2. Isotropic and anisotropic phase-space rotators

All the transforms associated with orthogonal ray transformation matrix produce rotation in phase space. Nevertheless, one of them, corresponding to the symmetric FrFT, $\mathcal{R}^{\mathbf{T}_f(\varphi,\varphi)}$, is inherently different. Indeed, its unitary matrix $\mathbf{U}_f(\varphi,\varphi)$ is scalar, $\mathbf{U}_f(\varphi,\varphi) = \exp(i\varphi)\mathbf{I}$, and commutes with any unitary matrix. The $\det \mathbf{U}_f(\varphi,\varphi) = \exp(2i\varphi)$ while the determinant of the matrix describing other phase-space rotators, indicated as $\mathbf{U}_{ar}$ which can be expressed as a product of $\mathbf{U}_f(\gamma, -\gamma)$, $\mathbf{U}_r(\theta)$ and $\mathbf{U}_g(\alpha)$ equals one: $\det \mathbf{U}_{ar} = 1$. The last transforms $\mathcal{R}^{\mathbf{T}_{ar}}$, which we will further call as anisotropic rotators, describe the movements on the orbital Poincaré sphere [9,10].

Any IOS, which may consist of centered spherical lenses, mirrors and free space intervals, is described by the ray transformation matrix with scalar $\mathbf{A} = a\mathbf{I}$, $\mathbf{B} = b\mathbf{I}$, $\mathbf{C} = c\mathbf{I}$ and

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$\mathbf{D} = d\mathbf{I}$. Therefore the matrices $\mathbf{G} = g\mathbf{I}$, $\mathbf{S} = s\mathbf{I}$, and $\mathbf{U} = \mathbf{U}_f(\varphi,\varphi) = \exp(i\varphi)\mathbf{I}$ are also scalar ones, where $g = -(ac+bd)/(a^2+b^2)$, $s = (a^2+b^2)^{1/2}$, and $\varphi = \gamma_x = \gamma_y = \arg(a+ib)$. In particular, for free space propagation (Fresnel diffraction) $a = d = 1$, $c = 0$ and the angle φ is limited: $\varphi \in [0, \pi/2]$. We observe that in the case of IOS, the orthogonal matrix in the decomposition (1) corresponds to the symmetric FrFT. Since the lens and scaler transformations don't change the form of the beam intensity, then the intensity distribution at the output plane of the IOS is described by the symmetric FrFT power spectrum with a proper scaling $\left|\mathcal{R}^{\mathbf{T}_{IOS}}[f_i(r_i)](r_o)\right|^2 = \left|\mathcal{R}^{\mathbf{T}_f(\varphi,\varphi)}[f_i(r_i)](r_o/s)\right|^2$. Correspondingly, the construction of the spiral beams during the propagation through the IOS reduces to the generation of such beams for the symmetric fractional Fourier transformer. We recall that beam propagation through optical fiber with a quadratic refractive index profile corresponds to the symmetric FrFT of its complex field amplitude at angles φ defined by the propagation distance z and the refractive index gradient g: $\varphi = gz$ [11]. Note that in this case φ can cover the interval of several periods of 2π. Other fractional Fourier transformers can be constructed using one or two spherical lenses.

The realization of other phase-space rotators, $\mathcal{R}^{\mathbf{T}_{ar}}$, requires the application of asymmetric optical elements such as cylindrical lenses or mirrors. Nevertheless, it is possible, as for the spiral beams, to design beams Ψ(r) for which the evolution of their intensity distribution during the propagation through the IOS (symmetric FrFT, $\mathcal{R}^{\mathbf{T}_f(\varphi,\varphi)}$) and the anisotropic phase-space rotator $\mathcal{R}^{\mathbf{T}_{ar}}$ will be identical

$$\left|\mathcal{R}^{\mathbf{T}_{ar}}[\Psi(r_i)](r_o)\right|^2 = \left|\mathcal{R}^{\mathbf{T}_f(\varphi,\varphi)}[\Psi(r_i)](r_o)\right|^2. \quad (2)$$

Below the operator for the canonical transform $\mathcal{R}^{\mathbf{T}}$ associated with orthosymplectic matrix **T** will be denoted by $\mathcal{R}^{\mathbf{U}}$.

3. Design of rotating beams

Let us consider the way how to generate the beams which satisfy Eq. (2). Note that any $\mathbf{U}_{ar}$ may be factored as $\mathbf{U}_{ar}(\gamma,\mathbf{U}_0) = \mathbf{U}_0\mathbf{U}_f(\gamma,-\gamma)\mathbf{U}_0^{-1}$, where $\mathbf{U}_0$ is also unitary matrix [12]. It has been shown [13,14] that for any unitary matrix $\mathbf{U}_0$ with elements U_{jk} ($j,k = 1,2$) there exists a complete orthonormal set of Gaussian modes $\left\{\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r}),\, m,n = 0,1,\ldots\right\}$,

$$\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r}) = \frac{(-1)^{m+n}\exp(x^2+y^2)}{2^{m+n-1/2}(\pi m!\,n!)^{1/2}}\left(U_{11}\frac{\partial}{\partial x} + U_{12}\frac{\partial}{\partial y}\right)^m\left(U_{21}\frac{\partial}{\partial x} + U_{22}\frac{\partial}{\partial y}\right)^n \exp(-2x^2-2y^2), \quad (3)$$

which are eigenfunctions for the transform $\mathcal{R}^{\mathbf{U}_{ar}(\gamma,\mathbf{U}_0)}$. The corresponding eigenvalues are given by $\exp[-i(m-n)\gamma]$. Moreover, these modes are also eigenfunctions for the symmetric FrFT, defined by $\mathbf{U}_f(\varphi,\varphi)$, with eigenvalues $\exp[-i(m+n+1)\varphi]$. For $\mathbf{U}_0 = \mathbf{U}_g(\alpha)$ the formula (3) reduces to Hermite-Laguerre-Gaussian modes [15]: $\mathcal{H}_{m,n}^{\mathbf{U}_g(\alpha)}(\mathbf{r}) = (\pi 2^{m+n-1} m!\,n!)^{-1/2} i^n \mathcal{G}_{m,n}(\mathbf{r}|\alpha)$. In particular, $\mathcal{H}_{m,n}^{\mathbf{U}_g(0)}(\mathbf{r})$ and $\mathcal{H}_{m,n}^{\mathbf{U}_g(\pm\pi/4)}(\mathbf{r})$ are Hermite-Gaussian (HG) and Laguerre-Gaussian (LG) modes respectively:

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$$\mathcal{H}_{m,n}^{\mathbf{U}_g(0)}(\mathbf{r}) = \mathcal{H}_{m,n}^{\mathbf{I}}(\mathbf{r}) = \frac{i^n}{(\pi 2^{m+n-1} m!\,n!)^{1/2}} \exp(-x^2 - y^2) H_m(\sqrt{2}x) H_n(\sqrt{2}y),$$

$$\mathcal{H}_{m,n}^{\mathbf{U}_g(\pm\pi/4)}(\mathbf{r}) = \frac{(\pm i)^n (-1)^{\min}}{(\pi/2)^{1/2}} \sqrt{\frac{\min!}{\max!}} \exp\left(-r^2 \pm i(m-n)\psi\right)(\sqrt{2}r)^{|m-n|} L_{\min}^{|m-n|}(2r^2), \tag{4}$$

where $\mathbf{r} = (x, y)^t = (r\cos\psi, r\sin\psi)^t$, $\min = \min(m,n)$, $\max = \max(m,n)$.

HG modes are eigenfunctions for the FrFT at angles $(\gamma,-\gamma)$ and therefore at any pair of angles γ_x and γ_y. LG modes are eigenfunctions for the signal rotator $\mathbf{U}_r(\pm\theta) = \mathbf{U}_g(\pm\pi/4)\mathbf{U}_f(\theta,-\theta)\mathbf{U}_g(\mp\pi/4)$ with eigenvalue $\exp[\pm i(m-n)\theta]$. Note that mode $\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r})$ satisfies Eq. (2) for $\mathbf{U}_{ar}(\gamma,\mathbf{U}_0)$.

A linear combination of the orthosymplectic modes $\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r})$ of the same order $m+n$ and $\mathbf{U}_0$ is also an eigenfunction for the symmetric FrFT for all possible angles φ. It means that all the modes in this decomposition accumulate the same Gouy phase during the propagation through an IOS or the corresponding symmetric fractional Fourier transformer. Similarly, a linear superposition of the modes $\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r})$ with the same difference of the indices, $m-n$, is an eigenfunction for the transform associated with $\mathbf{U}_{ar}(\gamma,\mathbf{U}_0)$ for any γ.

Analogously, a beam $\Psi(\mathbf{r})$ which undergoes the same transformation during propagation through the symmetric fractional Fourier transformer, except for a constant phase factor, as during the propagation through the optical system described by the one parametric unitary matrix $\mathbf{U}_{ar}(\gamma,\mathbf{U}_0)$,

$$\mathcal{R}^{\mathbf{U}_f(\varphi,\varphi)}[\Psi(\mathbf{r})] = \exp(i\phi)\mathcal{R}^{\mathbf{U}_{ar}(\gamma,\mathbf{U}_0)}[\Psi(\mathbf{r})], \tag{5}$$

can also be represented as a linear combination of modes $\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r})$. Using the relation $\mathbf{U}_{ar}^{-1}(\gamma,\mathbf{U}_0) = \mathbf{U}_{ar}(-\gamma,\mathbf{U}_0)$ the last equation can be rewritten in the following form

$$\mathcal{R}^{\mathbf{U}_{ar}(-\gamma,\mathbf{U}_0)}\mathcal{R}^{\mathbf{U}_f(\varphi,\varphi)}[\Psi(\mathbf{r})] = \exp(i\phi)\Psi(\mathbf{r}).$$

We observe that $\Psi(\mathbf{r})$ has to be an eigenfunction of the transform described by the unitary matrix $\mathbf{U}_{ar}(-\gamma,\mathbf{U}_0)\mathbf{U}_f(\varphi,\varphi)$. As we have mentioned above the modes $\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r})$ are eigenfunctions for the symmetric FrFT, $\mathcal{R}^{\mathbf{U}_f(\varphi,\varphi)}$, and for the phase-space rotator $\mathcal{R}^{\mathbf{U}_{ar}(-\gamma,\mathbf{U}_0)}$ with eigenvalues $\exp[-i(m+n+1)\varphi]$ and $\exp[i(m-n)\gamma]$ respectively and therefore

$$\mathcal{R}^{\mathbf{U}_{ar}(-\gamma,\mathbf{U}_0)}\mathcal{R}^{\mathbf{U}_f(\varphi,\varphi)}\left[\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r})\right] = \exp(i\phi)\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r}), \tag{6}$$

where $\phi = (m-n)\gamma - (m+n+1)\varphi$. Representing γ as $\gamma = v\varphi$, where v indicates the velocity of the phase-space rotation associated with $\mathbf{U}_{ar}(-\gamma,\mathbf{U}_0)$ during symmetric FrFT at angle φ, we can rewrite ϕ as $\phi(v,\varphi) = -[m(1-v)+n(1+v)+1]\varphi$. Then a linear combination of these modes

$$\Psi^{\mathbf{U}_0}(\mathbf{r},v) = \sum_{m,n} c_{mn}\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r}) \tag{7}$$

with the arbitrary complex c_{mn} and the mode indices m and n, which satisfy the relation

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$$m(1-v)+n(1+v)=\text{const}, \tag{8}$$

is also an eigenfunction of the canonical integral transform $\mathcal{R}^{\mathbf{U}_{ar}(-v\varphi,\mathbf{U}_0)}\mathcal{R}^{\mathbf{U}_f(\varphi,\varphi)}$:

$$\mathcal{R}^{\mathbf{U}_{ar}(-v\varphi,\mathbf{U}_0)}\mathcal{R}^{\mathbf{U}_f(\varphi,\varphi)}\left[\Psi^{\mathbf{U}_0}(\mathbf{r},v)\right]=\exp[i\phi(v,\varphi)]\Psi^{\mathbf{U}_0}(\mathbf{r},v). \tag{9}$$

If v is irrational we have a trivial case where only one mode $\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r})$ satisfies relation (8). Moreover, it is easy to see that for $v=1$ the beam anisotropically rotating in phase space during symmetric FrFT is given by

$$\Psi_n^{\mathbf{U}_0}(\mathbf{r},1)=\sum_{m=0}^{\infty}c_{mn}\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r}), \tag{10}$$

where n is fixed. Similar expression for fixed m and arbitrary n can be obtained for $v=-1$. Since any field amplitude $f(\mathbf{r})$ can be represented as a linear superposition of the orthonormal modes $\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r})$, it also can be written as a sum of the beams rotating in the phase space with the same velocity $v=1$ as

$$f(\mathbf{r})=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}f_{mn}\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r})=\sum_{n=0}^{\infty}\Psi_n^{\mathbf{U}_0}(\mathbf{r},1), \tag{11}$$

where $\Psi_n^{\mathbf{U}_0}(\mathbf{r},1)$ is given by Eq. (10) and $f_{mn}=c_{mn}$.

Note that the sign of the parameter v indicates the direction of the rotation, while its absolute value corresponds to the number of complete loops which beam makes during the 2π-interval of φ. It can be bigger or less than one if a beam has certain symmetry in the phase space. The velocity of rotation is defined by the indices of any pair of modes $\mathcal{H}_{m_1,n_1}^{\mathbf{U}_0}(\mathbf{r})$ and $\mathcal{H}_{m_2,n_2}^{\mathbf{U}_0}(\mathbf{r})$ in the composition (7) as a ratio of the differences of the eigenvalues of these modes for symmetric FrFT and anisotropic phase-space rotator

$$m_1(1-v)+n_1(1+v)=m_2(1-v)+n_2(1+v)\ \Rightarrow\ v=\frac{(m_2+n_2)-(m_1+n_1)}{(m_2-n_2)-(m_1-n_1)}=\frac{k}{l}$$

with nonzero integers k and l. Note that a linear combination of any two modes $\mathcal{H}_{m,n}^{\mathbf{U}_0}(\mathbf{r})$ with fixed $\mathbf{U}_0$ and such indices that $m_1+n_1\neq m_2+n_2$ and $m_1-n_1\neq m_2-n_2$ always forms an anisotropically rotating beam. Since phase-space rotators are periodic with period 2π (the symmetric FrFT except for the phase): $\mathbf{U}_f(\varphi+2\pi,\varphi+2\pi)=\mathbf{U}_f(\varphi,\varphi)$ and $\mathbf{U}_{ar}(\gamma+2\pi,\mathbf{U}_0)=\mathbf{U}_{ar}(\gamma,\mathbf{U}_0)$ and $\mathbf{U}_f(2\pi,2\pi)=\mathbf{U}_{ar}(2\pi,\mathbf{U}_0)=\mathbf{I}$, we observe from Eq. (9) that $\Psi_n^{\mathbf{U}_0}(\mathbf{r},v)=\Psi_n^{\mathbf{U}_0}(\mathbf{r},k/l)$ is an eigenfuction of the symmetric FrFT at angles $2\pi/v=2\pi l/k$. Taking into account the periodicity of the rotators [16] we can conclude that $\Psi_n^{\mathbf{U}_0}(\mathbf{r},k/l)$ is an eigenfunction for the operator $\mathcal{R}^{\mathbf{U}_f(2\pi/k,2\pi/k)}$. Similarly, one can prove that $\Psi_n^{\mathbf{U}_0}(\mathbf{r},k/l)$ is an eigenfunction for the operator $\mathcal{R}^{\mathbf{U}_{ar}(2\pi/l,\mathbf{U}_0)}$.

As an example we consider a family of rotating beams with the rotation velocity $v=1$, which can be expressed in the integral form (used for numerical simulations) or according with Eq. (10) by series

$$\Psi_2^{\mathbf{U}_g(\alpha)}(\mathbf{r},1)=\sum_{n=0}^{\infty}\frac{\sqrt{(3n+2)!}}{(3n)!}3^{3n}\mathcal{H}_{3n+2,2}^{\mathbf{U}_g(\alpha)}(\mathbf{r}). \tag{12}$$

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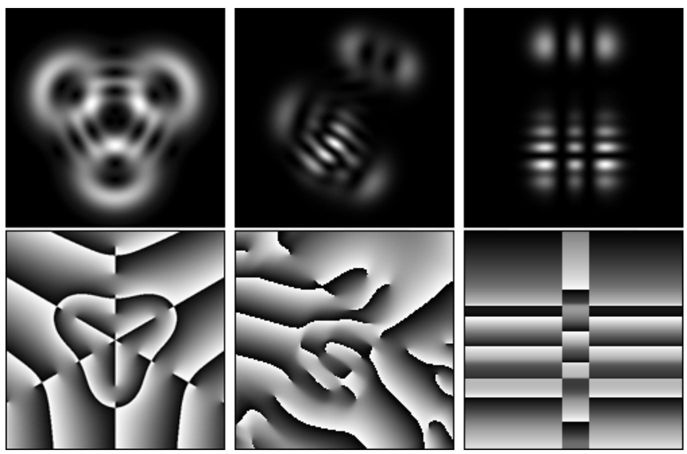
The gyrator matrix $\mathbf{U}_0 = \mathbf{U}_g(\alpha)$ defines anisotropic phase-space rotator $\mathbf{U}_{ar}\left(\gamma, \mathbf{U}_g(\alpha)\right)$. For $\alpha = \pi/4$ it reduces to the signal rotator, $\mathcal{H}_{3n+2,2}^{\mathbf{U}_g(\pi/4)}(\mathbf{r})$ are LG modes, and $\Psi_2^{\mathbf{U}_g(\pi/4)}(\mathbf{r},1)$ is a spiral beam shown in Fig. 1a. For $\alpha = \pi/8$ the beam rotating in other phase-space planes is obtained (see Fig. 1b). If $\alpha = 0$ then $\mathcal{H}_{3n+2,2}^{\mathbf{U}_g(0)}(\mathbf{r})$ are HG modes and the beam $\Psi_2^{\mathbf{U}_g(0)}(\mathbf{r},1)$, displayed in Fig. 1c, performs the anisotropic rotations in (x, p_x) and (y, p_y) planes during the propagation. In the video connected to the figure the evolution of the intensity distribution of these beams during their propagation through the symmetric fractional Fourier transformer, which can be an optical fiber, is shown. Note that $\Psi_2^{\mathbf{U}_g(\alpha)}(\mathbf{r},1)$ are eigenfunctions for the symmetric FrFT for angle $\varphi_0 = 2\pi/3$ and we display only a part of the loop corresponding to the intensity evolution in the interval $\varphi \in [0, 2\pi/3]$ (observe that the bottom circle of the spiral beam is at the left side in the end of the video). For the spiral beam we observe the simple rotation of the intensity distribution at the transversal plane (x, y), while for other ones the intensity distribution is changing since the anisotropic phase-space rotation is performed in another plane.



a b c

Fig. 1. Intensities (top row) and phases (bottom row) of the beam $\Psi_2^{\mathbf{U}_g(\alpha)}(\mathbf{r},1)$, given by Eq. (12), for various values of the angle α: (a) $\alpha = \pi/4$ (a spiral beam), (b) $\alpha = \pi/8$, (c) $\alpha = 0$ (combination of HG modes). (Media 1) Intensity and phase evolution of the beam $\left|\mathcal{R}^{\mathbf{U}_f(\varphi,\varphi)}[\Psi_2^{\mathbf{U}_g(\alpha)}(\mathbf{r},1)]\right|^2$ for $\varphi \in [0, 2\pi/3]$.

3. Conclusion

We have shown that beams performing anisotropic rotation in phase space during their propagation in isotropic systems may be represented as a linear combination of the Gaussian modes expressed by Eq. (3). While the parameter $\mathbf{U}_0$ defines the plane of the phase-space rotation, the indices of the modes participated in the beam synthesis determine the velocity of the rotation *v*. These beams can be generated using spatial light modulators. The application of these beams for light-matter interaction is under investigation. We only mention that spiral beams have been found useful for optical trapping.

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