RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 31, NUMBER 10 15 MAY 1985

Spinorial solitary wave dynamics of a (1+3)-dimensional model

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(Received 18 December 1984)

The interaction dynamics for the minimum solitary waves of the classical Dirac field with scalar self-interaction in 1+3 dimensions is exhibited. It is shown that the possibility to use these waves as models of elementary particles, as has been suggested before, is ruled out by the radiative nature of the above interaction.

Stable particlelike solutions of Poincaré-invariant nonlinear spinorial equations have been shown to exist in 1+3 dimensions.[1] The recent discovery[2] of renormalization properties of a spinorial field with Fermi self-interactions in the (1+3)-dimensional case suppresses a possible handicap in order to propose its solitary waves as interesting models to describe the properties of elementary particles.[3] The present work is a contribution to the study of these objects as possible models for elementary constituents of matter.

The field equation of the model is

$$i\gamma^\mu \partial_\mu \psi - m\psi + 2\lambda(\bar{\psi}\psi)\psi = 0 \ , \tag{1}$$

where λ is a positive arbitrary parameter. Hereafter, without loss of generality, we take $m=1$ and $\lambda=\frac{1}{2}$. Solutions of Eq. (1) with minimum angular momentum can be found having the form in spherical coordinates

$$\psi_s(\mathbf{r},t) = \begin{bmatrix} G(r)\begin{pmatrix}1\\0\end{pmatrix} \\ iF(r)\begin{pmatrix}\cos\theta \\ \sin\theta\, e^{i\varphi}\end{pmatrix}\end{bmatrix} e^{-i\Lambda t} \ ,$$

with F and G satisfying

$$\begin{aligned} &dG/dr + (1+\Lambda)F - (G^2-F^2)F = 0 \ , \\ &dF/dr + 2F/r + (1-\Lambda)G - (G^2-F^2)G = 0 \ , \end{aligned} \tag{2}$$

where $0 < \Lambda \leq 1$. There is a family of solitary waves which depends continuously on the frequency Λ. The charge and energy spectra of this family present a minimum at $\Lambda = 0.936$. Soler[4] used a minimum-energy principle in order to break this degeneracy. Furlan and Rączka[5] invoked the same principle in their study of renormalization of four-Fermi interactions.

The interaction dynamcis of solitary waves of (1+3)-dimensional models is, so far, entirely a subject to be studied via computer experiments. In order to simulate numerically Eq. (1) in cylindrical coordinates a code was constructed. It consists of two parts: the initial and the evolution modules. The former looks for the solitary waves of Eq. (2), changes spherical to cylindrical geometry, and boosts the solitary waves along the z axis. The output of this module are the initial conditions for the evolution module. These conditions are

$$\psi_s(\mathbf{r},t=0) = \begin{pmatrix} u_1(\rho,z) \\ u_2(\rho,z)e^{i\varphi} \\ iu_3(\rho,z) \\ iu_4(\rho,z)e^{i\varphi}\end{pmatrix} \ , \tag{3}$$

with

$$u_1 = [G(\rho_+)\cosh(\phi/2) + iz_+F(\rho_+)\sinh(\phi/2)]\Lambda_+ + [G(\rho_-)\cosh(\phi/2) - iz_-F(\rho_-)\sinh(\phi/2)]\Lambda_- \ , \tag{4}$$

$$u_2 = i[-F(\rho_+)\rho_+{}^{-1}\Lambda_+ + F(\rho_-)\rho_-{}^{-1}\Lambda_-]\rho\sinh(\phi/2) \ , \tag{5}$$

$$u_3 = [-iG(\rho_+)\sinh(\phi/2) + z_+F(\rho_+)\cosh(\phi/2)]\Lambda_+ + [iG(\rho_-)\sinh(\phi/2) + z_-F(\rho_-)\cosh(\phi/2)]\Lambda_- \ , \tag{6}$$

$$u_4 = [F(\rho_+)\rho_+{}^{-1}\Lambda_+ + F(\rho_-)\rho_-{}^{-1}\Lambda_-]\rho\cosh(\phi/2) \ , \tag{7}$$

$$\rho_\pm = [\rho^2 + (z\pm z_0)^2\cosh\phi]^{1/2}, \quad z_\pm = (z\pm z_0)\cosh(\phi/2)\rho_\pm{}^{-1}, \quad \Lambda_\pm = \exp[\pm i\Lambda(z\pm z_0)\sinh\phi] \ . \tag{8}$$

These conditions simulate two "particles" initially centered at positions $\pm z_0$ on the z axis and moving along that axis with velocities $\pm\tanh\phi$. The evolution module uses an implicit splitting scheme for time stepping. The mathematical details of this code will be published elsewhere.

Define

$$\tilde{\Lambda} = \left\{(E_s{}^2 - P_s{}^2)^{1/2} - 0.5\int_0^{20}\int_0^{20} d\rho\, dz\, \rho[1-(P_s/E_s)^2]^{-1/2}[\bar{\psi}_s(\mathbf{r},t)\psi_s(\mathbf{r},t)]^2\right\} Q_s{}^{-1} \ , \tag{9}$$

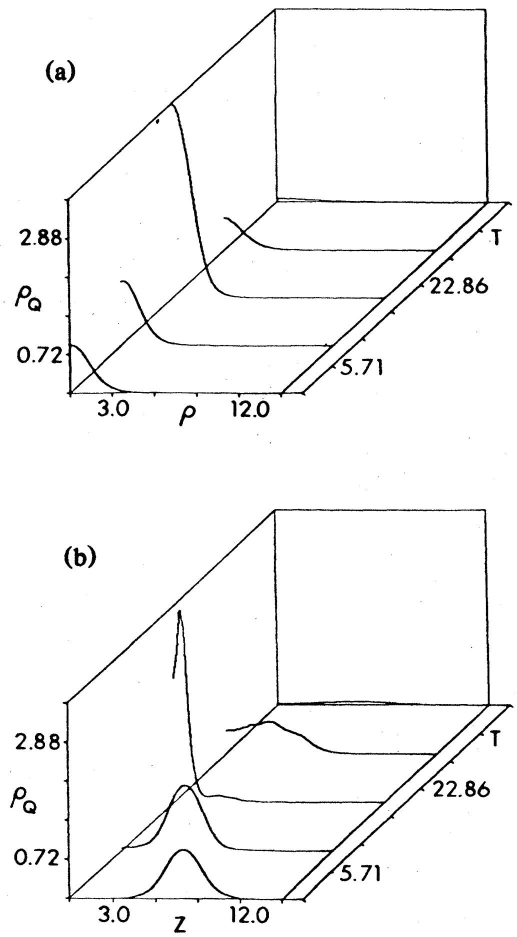

FIG. 1. Decay of two minimum solitary waves after interaction at $v=\tanh^{-1}0.4$: (a) Cross section in the ρ axis at point z where $\rho_Q(\rho=0,z)$ is maximum. (b) Cross section in the z axis with $\rho=0$.

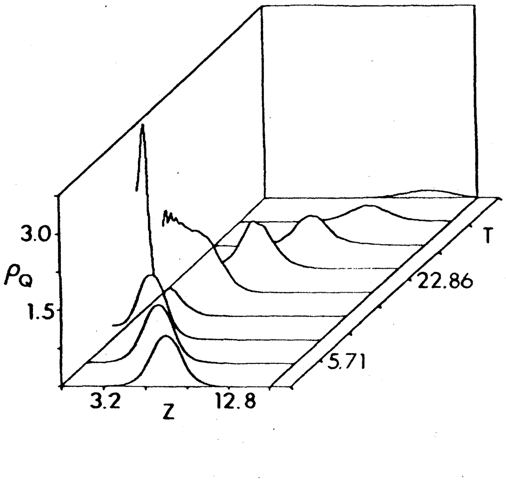

FIG. 2. Cross section in the z axis ($\rho=0$) of the decay, via lump formation, of two minimum solitary waves with initial velocities $v=\tanh^{-1}0.6$.

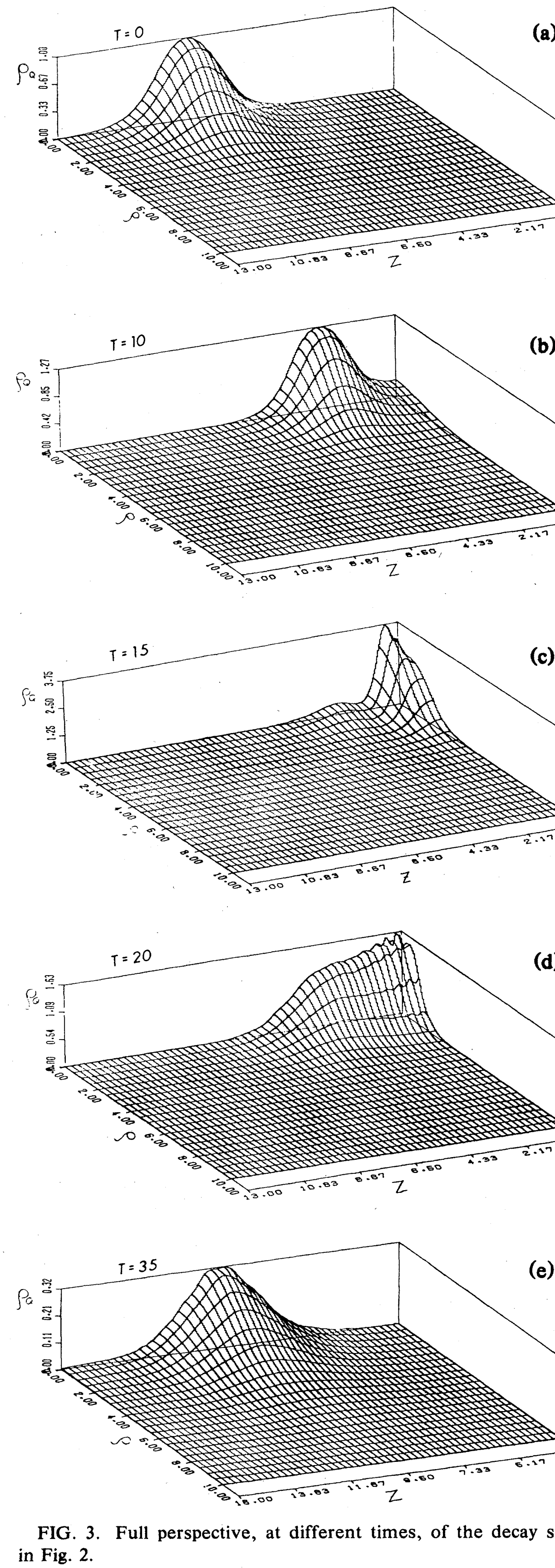

FIG. 3. Full perspective, at different times, of the decay shown in Fig. 2.

where Q_s, E_s, and P_s are the charge, energy, and z linear momentum of the boosted solitary wave ψ_s. For accuracy control of the initial module, two tests were employed: $|\Lambda-\tilde{\Lambda}|\Lambda^{-1}$ was less than 0.002, and the difference between Q_s and $\int_0^{20} drr^2(G^2+F^2)$ was less than 1%. The evolution module was monitored against charge and energy conservation. The relative error of these quantities was less than 1%.

The charge density $\rho_Q(\rho,z,t)$ is plotted in the figures, where we show the z half-line, since that is enough by symmetry. The computer results reveal the decay of two interacting minimum solitary waves with initial velocities $v > v_c \approx \tanh^{-1}0.2$. For

$$\tanh^{-1}0.3 \lesssim v \lesssim \tanh^{-1}0.5$$

the final state is reached quickly [Figs. 1(a) and 1(b)]; when $\tanh^{-1}0.6 \lesssim v$, two well-defined lumps emerge [Figs. 2, 3(a)–3(e)]. However, the lump tops go to zero slowly with increasing time. As in the (1+1)-dimensional case,[6] for $v \lesssim v_c$ a bound-state formation takes place. Similar results are obtained with $\Lambda \lesssim 0.936$.

From the above results we deduce the following conclusions.

(i) The minimum-solitary-wave interaction of the Dirac equation with the above specific form of the self-coupling is radiative and therefore these waves cannot be used as models of elementary fermions.

(ii) The strong nature of this interaction suggests that the exponent $q=2$ in the self-coupling Lagrangian term is too high for a (1+3)-dimensional suitable model.

(iii) The charge density of Eq. (1) is positive definite and hence the radiative character of the minimum-solitary-wave interaction implies the disappearance of these waves after the collision. Bearing in mind the dynamical stabilization of the solitary waves via a negative-energy radiation,[1] it seems reasonable to conjecture that the coupling, in an appropriate form, of the spinor field with its charge-conjugate field would prevent the minimum-solitary-wave destruction. Recently, a special form of this coupling was used in Ref. 7 in order to obtain a nonlinear model of confined Dirac quarks.

We are indebted to Professor M. Soler and Professor A. F. Rañada for discussions, and the Fusion Division (Junta de Energia Nuclear), Madrid, for use of computer facilities. This work is part of a program partially supported by Comision Asesora de Investigacion Cientifica y Tecnica (CAICYT).

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