Instituto Complutense de Análisis Económico
UNIVERSIDAD COMPLUTENSE
FACULTAD DE ECONOMICAS
Campus de Somosaguas
28223 MADRID
Teléfono 3942611 - FAX 3942613

Documento de Trabajo

Permanent components in seasonal variables

Rafael Flores
Alfonso Novales

No. 9406

Junio 1994

Instituto Complutense de Análisis Económico
UNIVERSIDAD COMPLUTENSE

PERMANENT COMPONENTS IN SEASONAL VARIABLES

Rafael Flores
Alfonso Novales
Instituto Complutense de Análisis Económico
Universidad Complutense
Campus de Somosaguas
28223 Madrid

ABSTRACT

We propose considering a seasonal time series as the realization of a s-variate stochastic process, s being the seasonal period. In this paper we propose a test statistic for the hypothesis of a univariate versus a multivariate representation of seasonality. We find evidence against the more standard univariate representation for some key variables of the U.S. economy. When a VAR representation is chosen for each of these variables and its residuals are properly orthogonalized, forecasting performance is improved, relative to univariate ARIMA models. Also, a Permanent-Transitory decomposition of each variable reveals that permanent components exhibit important seasonal fluctuations. This supports the view that seasonality should be considered as an integral part of agents' decision-making.

RESUMEN

Proponemos considerar a una serie temporal estacional como la realización de un proceso estocástico s-variante, siendo s el período estacional. En este artículo se propone un contraste para la hipótesis nula de representación univariante de la estacionalidad, versus la alternativa de representación multivariante. Para algunas variables importantes de la economía estadounidense se rechaza la representación univariante estandar. Los procesos VAR con residuos ortogonalizados, asociados a algunas de estas variables, proporcionan mejores previsiones que las obtenidas a partir de los sencillos modelos univariantes. Al mismo tiempo, la descomposición en componentes, Permanente y Transitoria, de cada variable revela que las componentes permanentes exhiben importantes fluctuaciones estacionales. Este hecho constituye una evidencia en favor de considerar el fenómeno de la estacionalidad como parte integral de la toma de decisiones de los agentes.

1. INTRODUCTION

Finding a proper way to handle the seasonal characteristics inherent to frequently observed variables is a crucial aspect of economic modeling. It is also an important source of controversy, and even the definition of seasonality is still subject to much discussion. [In addition to more classical references like Nerlove(1964), Box and Jenkins(1970), Sims(1974) and Wallis(1974), recent surveys of great interest are Hylleberg(1992) as well as the special issue of the Journal of Econometrics(1993)].

In this paper we are mainly concerned with the possibly multivariate nature of seasonality. Most of the work in modeling seasonality has been done in a univariate stochastic framework, i.e. a seasonal time series is considered as a realization of a univariate stochastic process (possibly including deterministic components). Since Box and Jenkins(1970), the univariate seasonal ARIMA model has been widely used, mainly due to its simplicity and its good forecasting performance, relative to other alternatives. Following a different approach, Osborn and Smith(1989) have used periodic autoregressive processes [see also Troutman(1979), Tiao and Gruppe(1980) and Osborn(1991)] and find that they may yield more accurate forecasts than univariate ARIMA specifications. Also, since periodic processes are multivariate in nature and arise when seasonality is considered as an integral part of the agents' decision-making problems[see Osborn(1988)], the multivariate approach to seasonality is appealing.

We propose a test for a univariate versus a multivariate representation of a seasonal variable, but rather than restricting our attention to the class of periodic autorregresive models, we work with a more general class of models, finite order VAR's with orthogonalized residuals.

We show that, for a univariate model to be satisfactory, some restrictions must be placed among the variances of the s seasons, as well as among the coefficientes of the

more general s-variate structure. Only when these two sets of restrictions are jointly satisfied, an univariate structure is acceptable for the seasonal variable.

When applied to some key macroeconomic U.S. variables, our test indicates that a multivariate representation should be preferred. We also show that multivariate models for these variables perform better than their univariate alternatives, in forecasting as well as in fitting the data.

In dealing with seasonal variables, a very important issue must be to determine which part of the seasonal fluctuations has a permanent nature and which one is transitory. Indeed, recently, Quah(1990) has shown that an appropriate Permanent - Transitory (P-T) decomposition may help to clarify the nature of relationships among economic variables as well as to explore whether their statistical properties agree with the implications from economic theory. The idea that seasonal fluctuations are mainly the result of optimal behaviour on the part of economic agents is becoming increasingly popular and hence, they should not be removed from the time series previously to be used either for forecasting purposes or to identify and estimate relationships. Only if seasonal fluctuations have a transitory character, their separate consideration might be justified. Hence, we are also interested in estimating the permanent and transitory components of a seasonal time series.

We propose the Gonzalo and Granger(1992) methodology to estimate season-specific (annual) **permanent** and **transitory** components for some macroeconomic quarterly U.S. variables. After estimating these annual components for a time series, the quarterly permanent and transitory components for the whole time series are computed. In all cases, these quarterly permanent components exhibit seasonal fluctuations, evidencing that the standard practice of seasonal adjusting of those variables would lead to an important loss of information.

The remainder of this paper is organized as follows. In section 2, we propose a test for a univariate versus a multivariate representation of a seasonal variable. The

Gonzalo-Granger (1992) methodology for estimating the permanent and transitory components in a vector of variables is surveyed in section 3. The results for four different seasonal variables: manufacturing employment and real wages, consumer prices and the money supply are presented in section 4. We carry out a forecasting exercise to investigate the ability of our multivariate models versus the univariate alternatives. The results confirm previous findings of Osborn and Smith(1989) that seasonally varying parameter processes may yield more accurate forecasts than conventional univariate ARIMA specifications. The paper closes with some conclusions and suggestions for further research.

2. A TEST OF A MULTIVARIATE VS. A UNIVARIATE UNDERLYING STOCHASTIC STRUCTURE

Let x_{ti} be an economic variable being observed at seasonal frequencies. Tipically, s = 4 or 12. For simplicity, we will denote by a time index t the year and by a second index i=1, ..., s the period of the year (a month or a quarter) to which the observation corresponds. Let $\mathbf{X}_t$ denote the vector: $\mathbf{X}_t' = (x_{t1} \ldots x_{ts})$ and let its VAR representation[1] be:

$$\Phi_x(B)\, \mathbf{X}_t = \mathbf{a}_t \qquad (1)$$

where $\Phi_x(B)$ is a sxs matrix of lag polynomials with $\Phi_x(0) = \mathbf{I}_s$, and all the roots of the characteristic equation for its determinant being on or outside the unit circle; $\mathbf{a}_t$ is an s-vector of random variables: $\mathbf{a}_t = (a_{t1}, a_{t2}, \ldots, a_{ts})'$.

Having introduced a seasonal variable as the realization of a multivariate vector, it is then of interest to test whether in some cases, the s-variate structure reduces to a

[1] Results for more general VARMA processes are currently under research.

univariate one. In this section we propose one such a test based on the orthogonalized residuals for a VAR model fitted to the s intra year series.

The Hillmer and Tiao(1979) representation of a VAR(p) model for the entire sample is:

$$\mathbf{D}_{\phi,n}\mathbf{X} = \mathbf{a} - \mathbf{G}_{\phi,n}\mathbf{X}^{*} \qquad (2)$$

where $\mathbf{X}$ denotes the ns-vector of sample observations, with n being the number of years, $\mathbf{a}$ is the $ns \times 1$ noise vector with a non scalar variance-covariance matrix $(\mathbf{I} \otimes \Sigma)$, where Σ is the $s \times s$ intra-year variance-covariance matrix, and $\mathbf{X}^{*}$ is the $ps \times 1$ vector of initial conditions. Finally, $\mathbf{D}_{\phi,n}$ is the $ns \times ns$ matrix of parameters:

$$\mathbf{D}_{\phi,n} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & .. & \mathbf{0} & \mathbf{0} & .. & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\phi_1 & \mathbf{I} & \mathbf{0} & .. & .. & .. & .. & .. & .. & \mathbf{0} \\ -\phi_2 & -\phi_1 & \mathbf{I} & .. & .. & .. & .. & .. & .. & \mathbf{0} \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ -\phi_p & .. & .. & .. & \mathbf{I} & .. & .. & .. & .. & \mathbf{0} \\ \mathbf{0} & -\phi_p & .. & .. & .. & \mathbf{I} & .. & .. & .. & \mathbf{0} \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ \mathbf{0} & .. & .. & .. & .. & .. & .. & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & .. & .. & .. & -\phi_p & .. & .. & -\phi_1 & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & .. & \mathbf{0} & -\phi_p & .. & -\phi_2 & -\phi_1 & \mathbf{I} \end{bmatrix} \qquad (3)$$

where $\mathbf{I}$ is the $s \times s$ identity matrix and Φ_i (i=1, ..., p) is the $s \times s$ matrix of parameters associated with lag i in the VAR(p) model.

$G_{\phi,n}$ is the ns×ps matrix of parameters:

$$G_{\phi,n}=\begin{bmatrix} -\phi_p & -\phi_{p-1} & \cdots & \cdots & -\phi_2 & -\phi_1 \\ 0 & \cdots & \cdots & \cdots & \cdots & -\phi_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots & -\phi_{p-1} \\ 0 & 0 & \cdots & \cdots & 0 & -\phi_p \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix} \qquad (4)$$

Let **T** be a s×s lower triangular matrix, constrained to have ones in the main diagonal, such that $T\Sigma T'=\Lambda$ is a diagonal matrix. Premultiplying (1) times $(I\otimes T)$ we get:

$$(I_n\otimes T)D_{\phi,n}X = (I_n\otimes T)a - (I_n\otimes T)G_{\phi,n}X^* \qquad (5)$$

or:

$$D^u_{\phi,n}X = \epsilon - G^u_{\phi,n}X^* \qquad (6)$$

where:

$$\begin{aligned} D^u_{\phi,n} &= (I_n\otimes T)D_{\phi,n} \\ G^u_{\phi,n} &= (I_n\otimes T)G_{\phi,n} \\ \epsilon &= (I_n\otimes T)a \end{aligned} \qquad (7)$$

with $Var(\epsilon) = (I_n\otimes\Lambda)$.

Expression (5) reduces to the Hillmer-Tiao representation for a univariate AR(r) process, $r\leq sp$, when: (a) the elements in any main lower diagonal of $D^u_{\phi,n}$ are all equal

to each other and (b) Λ is a scalar matrix. If these two conditions hold, then the univariate approach is appropriate. Our strategy for searching for the most appropriate framework can be stated as follows. We will first test for condition (b): if it is rejected, we then reject the univariate in favor of the multivariate framework; if (b) is not rejected, we then proceed to test (a). Rejecting (a) would again prove the univariate framework to be inadequate.

The Bartlett's statistic for the null hypothesis of equal variances:

$$H_0:\ \sigma_1^2 = \ldots = \sigma_s^2$$

has the expression:

$$Q = (n-s)\ \ln s^2 - \sum_{i=1}^{m}(n_i-1)\ \ln s_i^2$$

where s^2 is the sample variance computed with all observations, s_i^2 is the i-quarter sample variance, n is the sample size and n_i is the number of observations for quarter i-th. If the shocks in the s periodic series are i.,i.d., Normal random variables, Q follows a χ^2_{s-1} distribution.

Our suggestion consists on applying this test to the orthogonalized residuals of a VAR, fitted to the s intrayear time series. The rejection of the null hypothesis would suggest that Λ is not a scalar matrix and hence, the analysis of the seasonal variable should be carried out in a multivariate framework.

If condition (b) is not rejected, we then must test for condition (a). This condition implies a set of non linear constrains on the coefficients of the VAR model. For instance, in the case of a VAR(2) with s=4 we have that $D^u_{\phi,n}$ takes the general form:

$$D^{u}_{\phi,n}=\begin{bmatrix} T & 0 & 0 & \cdots & 0 & 0 & 0 \\ -T\phi_1 & T & 0 & \cdots & 0 & 0 & 0 \\ -T\phi_2 & -T\phi_1 & T & \cdots & 0 & 0 & 0 \\ 0 & -T\phi_2 & -T\phi_1 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & T & 0 & 0 \\ 0 & 0 & 0 & \cdots & -T\phi_1 & T & 0 \\ 0 & 0 & 0 & \cdots & -T\phi_2 & -T\phi_1 & T \end{bmatrix} \qquad (10)$$

with T and the ϕ_{ij} being 4x4 matrices of coefficients.

If the data generating process was indeed univariate, it will have to be an AR(8), and the matrices in $D^{u}_{\phi,n}$ will have a structure:

$$T\phi_1=\begin{bmatrix} \phi_{11,1} & \phi_{12,1} & \phi_{13,1} & \phi_{14,1} \\ \phi_{14,2} & \phi_{11,1} & \phi_{12,1} & \phi_{13,1} \\ \phi_{13,2} & \phi_{14,2} & \phi_{11,1} & \phi_{12,1} \\ \phi_{12,2} & \phi_{13,2} & \phi_{14,2} & \phi_{11,1} \end{bmatrix} \qquad (11)$$

$$T\phi_2=\begin{bmatrix} \phi_{11,2} & \phi_{12,2} & \phi_{13,2} & \phi_{14,2} \\ 0 & \phi_{11,2} & \phi_{12,2} & \phi_{13,2} \\ 0 & 0 & \phi_{11,2} & \phi_{12,2} \\ 0 & 0 & 0 & \phi_{11,2} \end{bmatrix} \qquad (12)$$

where $\phi_{ij,k}$ represents the element (i,j) of matrix ϕ_k.

The corresponding univariate AR(8) model would then be:

$$(1-\phi_{14,1}B-\phi_{13,1}B^2-\phi_{12,1}B^3-\phi_{11,1}B^4-\phi_{14,2}B^5-\phi_{13,2}B^6-\phi_{12,2}B^7-\phi_{11,2}B^8)X_t=a_t \qquad (13)$$

For the VAR(1) with s=4 case the constraints are:

$$T\phi_1=\begin{bmatrix} \phi_{11,1} & \phi_{12,1} & \phi_{13,1} & \phi_{14,1} \\ 0 & \phi_{11,1} & \phi_{12,1} & \phi_{13,1} \\ 0 & 0 & \phi_{11,1} & \phi_{12,1} \\ 0 & 0 & 0 & \phi_{11,1} \end{bmatrix} \qquad (14)$$

and the corresponding AR(4) is:

$$(1-\phi_{14,1}B-\phi_{13,1}B^2-\phi_{12,1}B^3-\phi_{11,1}B^4)X_t=a_t \qquad (15)$$

If the true process is indeed univariate, the implied constraints can be tested using the likelihood ratio test:

$$LR = n[\ln S^{*} - \ln S]$$

where S^{*} is the constrained residual sum of squares and S is the unconstrained residual sum of squares. The LR test has a χ^2 asymptotic distribution with J degrees of freedom, J being the number of constraints:

$$J = [s^2p + (\frac{s(s-1)}{2}) - ps]$$

Alternatively, since the residuals in (6) should be white noise under the null hypothesis, and should have the same variance as those from the corresponding AR(r) univariate model, a comparison of both simple and partial residuals correlograms will be very informative about the hypothesis.

3. A METHODOLOGY FOR COMMON PERMANENT COMPONENTS

Let X_t be a k-vector of I(1) time series containing r cointegrating relationships. This vector can be viewed as being generated by the following dynamic factor model [see, Peña and Box(1984,1987), Peña(1990) and Gonzalo and Granger(1991)]:

$$X_t = A_1 f_t + A_2 z_t$$

where:

- A_1 is a kx(k-r) matrix of parameters
- f_t is a (k-r)x1 vector of I(1) common factors
- A_2 is a kxr matrix of parameters
- z_t is a rx1 vector of I(0) common factors

Let α be a kxr matrix whose columns form a base of the cointegrating space, and let D be a (k-r)xk matrix whose rows are linearly independent from those of α'. Note that $\alpha' A_1$ must be equal to 0 since the rows of α' are cointegrating vectors. Among all the possible choices of D, we pick one such that the matrix:

$$M = \begin{bmatrix} D \\ \alpha' \end{bmatrix}$$

is invertible.

The linear transformation defined by M allows us to extract from X_t a set of k-r I(1) time series, as well as another set of r stationary ones:

$$DX_t = DA_2 z_t + DA_1 f_t$$
$$\alpha' X_t = \alpha' A_2 z_t$$

Let us assume that the vector X_t admits a finite VAR representation of order p:

$$X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \dots + \Pi_p X_{t-p} + a_t$$

and let Π denote the long-run multiplier kxk matrix:

$$\Pi = -(I - \Pi_1 - \Pi_2 - \dots - \Pi_p)$$

In the presence of the r cointegrating relationships, the matrix Π can be decomposed:

$$\Pi = \gamma \alpha'$$

In this general set-up, Gonzalo and Granger(1991) propose a choice:

$$D = \gamma'_\perp$$

where:

$$\gamma'_\perp \gamma = 0_{(k-r)xr}$$

which leads to decomposing X_t as the sum of two special components:

$$X_t = P_t + T_t$$

with:

$$P_t = C_1 \gamma'_\perp X_t$$
$$T_t = C_2 \alpha' X_t$$

where C_1 and C_2 are the kx(k-r) and kxr submatrices selected from:

$$(C_1 \ \ C_2) = \begin{bmatrix} \gamma'_\perp \\ \alpha' \end{bmatrix}^{-1} \qquad (14)$$

These two components P_t and T_t in X_t satisfy (see Quah (1989) and Gonzalo and Granger (1992):

(i) P_t is difference stationary, while T_t is covariance stationary

(ii) $var(\nabla P_t)$, $var(\nabla T_t) > 0$

(iii) $X_t = P_t + T_t$

(iv)

$$(a) \lim_{h \to \infty} \frac{\partial E_t(X_{t+h})}{\partial \epsilon_{Pt}} \neq 0$$

$$(b) \lim_{h \to \infty} \frac{\partial E_t(X_{t+h})}{\partial \epsilon_{Tt}} = 0$$

where ϵ_{Pt} and ϵ_{Tt} are the innovations to P_t and T_t, respectively.

Property (iv) means that a shock in P_t will generally modify the long-run forecast of the variable while a shock in T_t will not change it. That leads to Gonzalo and Granger(1991) to label P_t as the **permanent** component and T_t as the **transitory** component in X_t.

These authors show how to obtain the maximum likelihood estimator of $\gamma'_\perp$ in a sequence of steps:

(1) Regress ∇X_t and X_{t-1} on $(\nabla X_{t-1}, \ldots, \nabla X_{t-p+1})$. That provides us with k series of residuals R_{0t} and R_{1t} and with their cross product matrices:

$$S_{ij} = T^{-1} \sum_{t=1}^{T} R_{it} R'_{jt} \quad (i,j=0,1)$$

(2) Solve the equation:

$$|\lambda S_{00} - S_{01} S^{-1}_{11} S_{10}| = 0 \qquad (15)$$

giving the eigenvalues $\lambda_1 > \ldots > \lambda_k$ and eigenvectors $E = (e_1, \ldots, e_k)$ normalized such that $E'S_{00} E = I$.

(3) The ML estimator of $\gamma_\perp$ is then given by:

$$\hat{\gamma}_\perp = (\hat{e}_{r+1}, \ldots, \hat{e}_k)$$

with r being the rank of the cointegrating space.

(4) Finally, the vector α comes from a standard application of the Johansen(1988) procedure.

4. EMPIRICAL RESULTS

In this section, we apply the methods described above to some key macroeconomic U.S. variables: the narrow money measure M1, manufacturing employment (L), the consumer price index (P) and total wages in manufacturing deflated by the consumer price index (W/P). In all cases, we work with quarterly, non-seasonally adjusted observations, from 1949:1 to 1992:4, as obtained from the CITIBASE tape. Figures 1 and 2 show the logs of these variables as well as their quarters time series.

To get some insight into the nature of the underlying stochastic process, univariate ARIMA models were estimated, with the results shown in Table 1. All variables need regular as well as seasonal differencing to achieve stationarity, except the consumer price index series, which needs two regular differences. M1 follows an ARIMA(2 1 0)x(0 1 1)$_4$, employment an ARIMA(1 1 0)x(0 1 1)$_4$, consumer prices an ARIMA(2 2 0)x(2 0 0)$_4$ and real wages follow an ARIMA(1 1 0)x(2 1 1)$_4$.

As an initial step, we estimated univariate models for the four quarterly series x_{ts}, t=1959,.., 1992; s=1,2,3,4, that can be constructed for each variable, each of them having just 34 annual observations [see Tables 2.a to 2.d]. Under a univariate representation for the variable, the ARIMA models for its quarters should be very similar. While that is the case for money and prices, there are important differences across quarters in real wages and employment.

Note that equal variances and equal univariate models do not guarantee by themselves a univariate representation, since it might still be the case that the first and second quarters are related to each other differently than the third and the fourth quarters. The test proposed in section 2 accounts for these differences in behaviour among quarters.

4.1 Testing for a univariate versus a multivariate representation

The order of the VAR representation for X_t plays a crucial role in our proposed analysis. Table 3 contains the values of the Likelihood Ratio statistic, showing that a VAR(3) is appropriate for the logs of industrial employment, consumer prices and the narrow money supply, while a shorter VAR(2) would be enough for real wages. In all cases these multivariate autorregressive representations leave no serial correlation in the residuals, and lagged cross-correlations are zero.

We start by testing for a multivariate vs. a univariate structure in each variable. As discussed in section 2, after estimating a VAR representation for the four quarters, we will test whether the orthogonalized residuals share the same variance. If that hypothesis is rejected, there is no need to test any further, and we should work in a multivariate framework. The results in Table 4 lead to rejection of the null hypothesis of equality among the quarter variances in all cases. A multivariate representation is hence to be preferred to a univariate one for the four variables we considered.

4.2 Selecting the number of non-stationary factors

Once a quarterly time series has been accepted to have a multivariate representation then, in order to obtain an adequate multivariate model for the vector of quarters, it is necessary to test for common factors among them. The presence of such factors will lead to multivariate error correction models.

If there are some non-stationary factors, they correspond to the smaller eigenvalues of the matrix equation (15). The number of such factors can oscillate between zero and four in each variable. Having chosen the order of the vector of quarterly observations, we applied the Granger and Gonzalo (1991) approach, with the results shown in Table 5. By comparison with the 95% critical values in Johansen(1988), and conditional on the chosen lag length for the VAR representation, all variables seem to have two non-stationary (and hence two stationary) factors. The Gonzalo-Granger procedure provides the coefficients determining the non-stationary factors in each variable up to a nonsingular linear transformation, and we have scaled them to have the same mean values than the variable under consideration. Figure 3 shows the estimated non-stationary factors for each of the four variables under consideration. In them, f_{1t} denotes the factor associated to the smallest eigenvalue in (15), while f_{2t} denotes the factor associated to the next bigger eigenvalue. The demeaned stationary factors z_{1t} and z_{2t} are presented in Figure 4.

4.3 Estimation of the season-specific permanent and transitory components.

These estimated common factors allow us to proceed into analizing the **permanent and transitory components** in each quarter, as defined in Section 3. To obtain the **permanent components** in the four-dimensional vector that represents each variable, we estimated the linear projections of each quarter on the two non-stationary factors, the implied residual being the **transitory component**[2].

By construction, the transitory component of a given quarter does not have any permanent effect on the level of any quarter time series. In other words, if we denote by x_{it}, P_{it} and T_{it} the level, permanent and transitory components of the ith quarter, the lagged transitory component $T_{t-1\,i}$ does not have any effect on ∇P_{tj}, for all i,j = 1, ..., s.

Figure 5 shows the estimated permanent components for the four quarters of each variable. They look extremely similar for the four quarters; this feature is consistent with our idea of producing a robust, annual signal out of the original quarterly time series. In essence, we are applying a filter that transforms the somewhat heterogeneous behaviour of the four quarters in Figure 2, into the more homogeneous pattern shown in Figure 5. The corresponding transitory components are shown in Figure 6, and look reasonably stationary, as expected.

Up to this point, we have almost reached one of the goals of our analysis, having decomposed every quarter of each of the seasonal variables considered into a permanent and a transitory component, this being characterized by not having any permanent effect on the level of the quarter time series. Now, given a specific data point, we can determine which part of it is of a permanent nature, and which one is transitory. For some important economic questions, the latter one can be left out of the discussion, since its effects will die away quickly.

One could think of using the estimated components for the four quarters to compose a **'permanent'** component and a **'transitory'** component for the original set of time series, with the results shown in Figures 7 and 8. The estimated permanent component

[2] This is equivalent to inverting the M matrix as in (14). This regression approach is useful to reveal some specification errors. For instance, if we underestimate the number of non-stationary factors, the implied residuals will not be stationary.

is shown in units of the logged original variables[3], while the transitory one is given as a percentage of the original variables. An important feature in all cases is that seasonal fluctuations are present in both components, i.e. an important part of the seasonal fluctuations is of a permanent nature and very likely, a consequence of agents' optimal decisions. The immediate implication of this result is that seasonally adjusting a time series leads to an important loss of information about agents' behaviour.

Let us examine the interpretation of our estimated components. In the case of industrial employment in Figure 8, that component gives us the part of the employment change each quarter, in number of workers, that would have been of a transitory nature, and hence, less relevant for forecasting as well as for economic policy evaluation. For instance, let us suppose a situation in which the existing firms do not adjust their employment between quarters. Employment fluctuations between quarters would then respond to foreclosing and opening of firms. These are essentially of a permanent nature, so that the whole size of the employment fluctuations would be permanent. In such a world, our estimated permanent component should coincide with the original series, the transitory component being zero. If, as in real world economies, firms adjust their employment in the short-run, a part of the observed fluctuations would be of transitory nature. Therefore, it is not surprising that our estimated permanent component somewhat follows the pattern of the original variable, and suggests that it is important to examine the size of the difference between both of them, i.e., the transitory component.

4.4 Permanent-Transitory decompositions and forecasting performance

As a possible application of our analysis we now explore whether the permanent-transitory decomposition introduced in the previous section helps in improving forecasting performance. We focus here on univariate forecasting, although the ability to establish relationships between variables through the estimated components might also improve our multivariate forecasting performance. We use as a baseline the forecasts obtained from the univariate seasonal ARIMA (US) models in Table 1.

We compare the univariate forecasts with those obtained from the multivariate error correction model (MECM) associated to each vector of quarters:

[3] However, the composition of the estimated components for the four quarters does not guarantee that the similar permanent and transitory components for the whole series will satisfy the property of the transitory component not having any permanent impact on the level of the variable.

$$\nabla X_t = \delta + \gamma\alpha' X_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \nabla X_{t-i} + \epsilon_t \qquad (16)$$

$$\epsilon_t \sim N(0, \Sigma_\epsilon)$$

A reduction in the number of parameters can be obtained premultiplying by matrix M in Section 3 , with $D = \gamma_\perp'$:

$$\begin{pmatrix} \nabla\gamma_\perp' X_t \\ \nabla\alpha' X_t \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \alpha'\gamma \end{pmatrix} \alpha' X_{t-1} + \sum_{i=1}^{p-1} \begin{pmatrix} \Gamma_{11,i} & \Gamma_{12,i} \\ \Gamma_{21,i} & \Gamma_{22,i} \end{pmatrix} \begin{pmatrix} \nabla\gamma_\perp' X_{t-i} \\ \nabla\alpha' X_{t-i} \end{pmatrix} + \begin{pmatrix} \gamma_\perp'\epsilon_t \\ \alpha'\epsilon_t \end{pmatrix} \qquad (17)$$

Once (17) has been estimated, estimates of the MECM (16) can be obtained premultiplying by M^{-1}. The parsimonious MECM model was then pre-multiplied by the matrix that diagonalize Σ_ϵ.

Before carrying out the forecasting exercice we apply the test of section 2 to the orthogonalized residuals from the MECM models.

The first row of Table 6 shows the results for the Bartlett test. As expected, they are similar to those in Table 4, even though this time we have run it on the MECM. The second row shows the results of the likelihood ratio test (LR). The Bartlett test results support the multivariate representation for all variables while the LR test indicates that for both, P and W/P, some of the US constraints could be accepted. Results in Table 6 also suggest that the multivariate nature of P and W/P mainly comes from the difference in variances among the quarters while the multivariate nature of M1 and L comes from both, differences among the quarters variances as well as failure of the constraints among the coefficients for the different quarters.

Both the univariate and MECM models were estimated for the period 1959-1990, leaving 8 observations (1991-1992) for comparison with forecasts. The models were reestimated each quarter to obtain one-step-ahead forecasts.

Table 7 shows the root mean squared errors (RMSE) computed from the one quarter ahead forecasting errors, obtained with the two competing models (US RMSE in parenthesis). We obtain different results for M1 and manufacturing employment than for consumer price index and real wages. The MECM models provide better results for the former two variables while the US models perform better for the latter pair. This result is not surprising given the likely origin of the multivariate nature in both P and W/P .

Table 8 shows for each variable the RMSE adjusted for degrees of freedom. This adjusted RMSE has been computed from the residuals when the models were estimated with the whole sample (1959:1-1992:4). These RMSE (US in parenthesis) can be viewed as those that would be obtained extending the sample forecasting exercise to a large enough number of years. They show that the MECM model produces better in-sample RMSE's for all variables. Note that the gain from using a multivariate approach for P and W/P is not very large. Two quarters of P show a RMSE for the US model smaller than the RMSE obtained with the MECM. The same happens with the fourth quarter of W/P. Again given the special multivariate nature of P and W/P, this is not too surprising.

5. CONCLUSIONS

A seasonal time series can be viewed either as a realization of a univariate stochastic process or as a realization of a s-variate stochastic process, where s is the seasonal period.

The univariate approach leads to simple models which have been shown to provide good forecasting performance. Nevertheless, this approach implies constant seasonal parameters and this may lead to substantial dynamic misspecifications [see Osborn et al.(1988) and Birchenhall et al.(1989)]. On the other hand, although the multivariate approach allows for seasonally varying parameters, it leads to more complicated models that need longer time series to be elaborated.

We have been mainly concerned with the possible multivariate nature of seasonality and have proposed a procedure for testing the null hypothesis of a univariate versus an alternative multivariate representation of a seasonal time series. This is a two stage test based on the orthogonalized residuals from a VAR model. In the first stage, the hypothesis of equality of variances among orthogonalized residuals is tested. If variance homogeneity is not rejected, a likelihood ratio test for non linear constraints among the VAR parameters is carried out.

When this test is applied to some US quarterly macroeconomic series: manufacturing employment, consumer price index, real wages in manufacturing and M1, the null hypothesis is rejected and therefore a multivariate representation is preferred.

The acceptance of a multivariate approach leads to the analysis of common factors among seasons. This analysis not only helps in elaborating parsimonious multivariate models but also provides a way to obtain a Permanent-Transitory decomposition for the seasonal variable. To carry out this decomposition is important because seasonal fluctuations of a permanent nature can be considered as part of agents' decisions while those of a transitory nature could either be removed or used just for short run forecasting.

The analysis of the mentioned U.S. time series shows that the permanent component in all cases exhibits seasonal fluctuations. This supports the view that seasonality (or at least, a part of it) is the result of optimal agents' decisions. If this is the case, completely removing seasonal fluctuations may lead to losing an important piece of information on agents' behaviour.

Our forecasting exercise confirms previous results in Osborn and Smith(1989) that seasonally varying parameter processes may yield more accurate forecasts than conventional univariate specifications. Also, our testing procedure seems to be usefull in evaluating on a priori grounds the forecasting capabilities of a multivariate versus a univariate representation. These seem to be bigger when the likelihood ratio test strongly rejects the hypothesis of a univariate representation.

Searching for non-stationary common factors among different seasonal variables, discussing their economic interpretations and checking the likely forecasting gain in using this information are some of the topics on which further research needs to be done.

REFERENCES

BOX, G.E.P. and G.M. JENKINS (1970), 'Time Series Analysis Forecasting and Control', San Francisco:Holden Day.

BIRCHENHALL, C.R., R.C. BLADEN-HOVEL, A.P.L. CHUI, D.R. OSBORN and J.P. SMITH (1989), 'A Seasonal Model of Consumption', Economic Journal, 99, 837-843.

FRANSES, P.H. (1993), 'Common Periodic Features in Seasonal Time Series', manuscript.

GONZALO, J. and C.W.J. GRANGER (1991), 'Estimation of Common Long-Memory Components in Cointegrated Systems', Discussion Paper 91-33, University of California, San Diego.

HYLLEBERG, S. (1992), 'Modelling Seasonality', Oxford University Press.

HILLMER, S.C. and G.C. TIAO (1979), 'Likelihood Function of Stationary Multiple Autoregressive Moving Average Models', Journal of the American Statistical Association 74,652-660.

JOHANSEN,S. (1988), 'Statistical Analysis of Cointegrating Vectors', Journal of Economic Dynamics and Control, 12:231-254.

JOURNAL OF ECONOMETRICS (1993), number 55, North-Holland.

NERLOVE, M. (1964), 'Spectral Analysis of Seasonal Adjustment Procedures', Journal of Econometrics 32:241-286.

OSBORN D.R. (1988), 'Seasonality and Habit Persistence in a Life-Cycle Model of Consumption', Journal of Applied Econometrics 3, 255-266.

OSBORN D.R. (1991), 'The Implications of Periodically Varying Coefficients for Seasonal Time Series Processes', Journal of Econometrics 48, 373-384.

OSBORN D.R. and J.P. SMITH (1989), 'The Performance of Periodic Autoregressive Models in Forecasting Seasonal U.K. Consumption', Journal of Business and Economics Statistics 7, 117-127.

PEÑA, D. and G.P.E. BOX (1984), 'Hidden Relationships in Multivariate Time Series', Proc. of Bus. Ec. St. Section American Statistical Association, 494-499.

PEÑA, D. and G.P.E. BOX (1987), 'Identifying a Simplifying Structure in Time Series', Journal of the American Statistical Association 823, 836-843.

PEÑA, D. (1990), 'Cointegración y Reducción de Dimensionalidad en Series Temporales Multivariantes', Cuadernos Economicos del ICE, 44, 109-126.

QUAH, D. (1989), 'The Relative Importance of Permanent and Transitory Components: Identification and some Theoretical Bounds', Working paper, MIT and NBER, March 1989.

STOCK, J.H. and M.WATSON (1988), 'Testing for Common Trends', Journal of the American Statistical Association 83, 1097-1107.

TIAO, G.C. and M.R. GRUPE (1980), 'Hidden periodic Autoregressive-Moving Average Models in Time Series Data, Biometrika 67, 365-373.

TROUTMAN, B.M. (1979), 'Some Results in Periodic Autoregression', Biometrika 66, 219-228

WALLIS, K.F.(1974), 'Seasonal Adjustment and Relations Between Variables', Journal of American Statistical Association, 69, 18-31.

Table 1

Univariate Models (Quarterly Series)

	Differencing dD	Regular Structure ϕ_1	ϕ_2	Period	Seasonal Structure Φ_1	Φ_2	Θ_1	σ_a%	Ljung-Box statistic L.B.(15)
M1	$\nabla\nabla_4$	.23 (.09)	.18 (.09)	.	.	.	.77 (.06)	1.08	8.1
L	$\nabla\nabla_4$	.51 (.07)	.	.	.	.	.96 (.02)	1.18	11.6
P	∇^2	-.29 (.08)	-.47 (.08)	4.6	-.03 (.09)	-.16 (.08)	.	.44	17.0
W/P	$\nabla\nabla_4$	.15 (.09)	.	.	-.01 (.11)	-.23 (.09)	.77 (.08)	.63	20.4

Table 2a

Univariate Models (Quarters of Money)

	μ	ϕ_1	$\bar{a}$	σ_a %	L.B.(6)
∇m11	.0566 (.0083)	.44 (.15)	.00169 (.00467)	2.75	2.5
∇m12	.0571 (.0082)	.40 (.15)	.00159 (.00497)	2.91	2.2
∇m13	.0571 (.0093)	.54 (.14)	.00245 (.00436)	2.61	3.1
∇m14	.0598 (.0084)	.40 (.16)	.00138 (.00512)	2.99	2.7

Note: *m_{1j} denotes the jth-quarter of the log of M1, ϕ_1 is the parameter in an AR(1) term, μ is a constant and $\bar{a}$ denotes the sample mean of the residuals.L.B.(6) is the Ljung-Box statistic with 6 degrees of freedom and σ_a is the estimated standard error, in percentage terms.*

Table 2b

Univariate Models (Quarters of Labour)

	ϕ_1	ϕ_2	per	$\bar{a}$	σ_a %	L.B.(6)
∇l1	.	.	.	.00295 (.00707)	4.06	2.7
∇l2	.	.	.	.00237 (.00692)	3.98	4.4
∇l3	.	.	.	.00241 (.00628)	3.61	3.5
∇l4	.31 (.16)	-.36 (.16)	4.8	.00234 (.00582)	3.37	.9

Note: *lj denotes the log of the jth-quarter of industrial employment. ϕ_1 and ϕ_2 denote the two parameters in an AR(2) polynomial. "per" means period.*

Table 2c

Univariate Models (Quarters of Prices)

	ϕ_1	ϕ_2	per	$\bar{a}$	σ_a %	L.B.(6)
∇p1	.34 (.16)	-.46 (.16)	4.8	.00057 (.00309)	1.75	2.3
∇p2	.21 (.17)	-.34 (.17)	4.5	.00052 (.00323)	1.83	5.1
∇p3	.33 (.16)	-.47 (.16)	4.7	.00091 (.00298)	1.69	3.9
∇p4	.23 (.16)	-.45 (.16)	4.5	.00106 (.00339)	1.92	2.4

Note: *pj denotes the log of the jth-quarter of consumer prices.*

Table 2d

Univariate Models (Quarters of Real Wages)

	μ	ϕ_1	ϕ_2	per	$\bar{a}$	σ_a %	L.B.(6)
$\nabla w1$	.0023 (.0043)	.35 (.16)	.	.	-.00025 (.00288)	1.66	8.4
$\nabla w2$	.0019 (.0041)	.38 (.16)	.	.	-.00013 (.00263)	1.51	4.4
$\nabla w3$	.0025 (.0039)	.81 (.13)	-.30 (.16)	8.5	-.00033 (.00196)	1.14	5.3
$\nabla w4$	.0022 (.0050)	.52 (.15)	.	.	-.00046 (.00249)	1.44	8.0

Note: *wj denotes the jth-quarter of the log of real wages in manufacturing.*

Table 3

LIKELIHOOD RATIO TESTS FOR LAG ORDER
Distributed as chi-square(16)

LAGS	MONEY	EMPLOYMENT	PRICES	REAL WAGES
1 vs. 2	222.59	112.76	69.11	110.78
2 vs. 3	31.34	28.24	34.43	8.05
3 vs. 4	21.83	13.63	10.55	8.85
4 vs. 5	7.93	13.53	12.20	13.39

Note: *The critical values (for 16 degrees of freedom) are 26.3 and 32.0 at the 95% and 99% confidence levels, respectively.*

Table 4

BARTLETT TEST FOR EQUALITY OF VARIANCES ACROSS QUARTERS

MONEY VAR(3)	EMPLOYMENT VAR(3)	PRICES VAR(3)	REAL WAGES VAR(2)
10.4 (0.02)	30.6 (0.0)	15.2 (0.0)	12.6 (0.01)

Note: *H_0 is variance homogeneity. The critical value at the 95% is 7.81*

Table 5

JOHANSEN TEST FOR THE NUMBER OF NON-STATIONARY FACTORS

Number of Non-Stationary Factors (At least)	CRITICAL VALUES 95%	MONEY	EMPLOYMENT	PRICES	REAL WAGES
		VAR(3)	VAR(3)	VAR(3)	VAR(2)
1	8.083	2.538	3.820	0.249	3.347
2	17.844	11.359	16.605	15.444	13.672
3	31.256	34.080	36.309	41.610	38.326
4	48.419	66.074	67.269	80.021	70.862

Table 6

Bartlett and Likelihood Ratio Tests
(Error Correction Models)

TEST	M1	L	P	W/P
Bartlett	8.6(.03)	25.7(.00)	16.8(.00)	8.0(.04)
LR	93.8(.00)	79.6(.00)	58.3(.05)	43.2(.05)

Note: *Degrees of freedom of LR for M1, L, P and W/P are: 42, 42, 42 and 30. Probability of a* χ^2 *being larger than test value in parenthesis.*

Table 7

Root Mean Squared Forecasting Errors
(1991:1-1992:4)

M1	L	P	W/P
.01532 (.01643)	.00519 (.00567)	.00470 (.00275)	.00416 (.00365)

Note: *RMSE for US forecasts in parenthesis.*

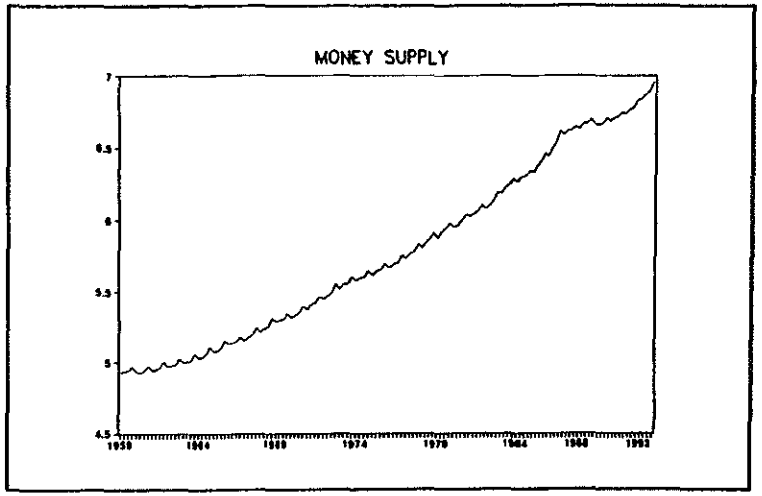
Table 8

In Sample Adjusted RMSE

QUARTER	M1	L	P	W/P
Q1	.00798 (.00958)	.00673 (.00994)	.00550 (.00507)	.00716 (.00781)
Q2	.01124 (.01179)	.01123 (.01239)	.00374 (.00435)	.00473 (.00548)
Q3	.00964 (.01095)	.00576 (.00785)	.00463 (.00399)	.00529 (.00561)
Q4	.00676 (.01068)	.01330 (.01330)	.00262 (.00408)	.00460 (.00441)
WHOLE SAMPLE	.00906 (.01078)	.00980 (.01176)	.00426 (.00437)	.00555 (.00627)

Note: *RMSE of US residuals in parenthesis.*

Figure 1a



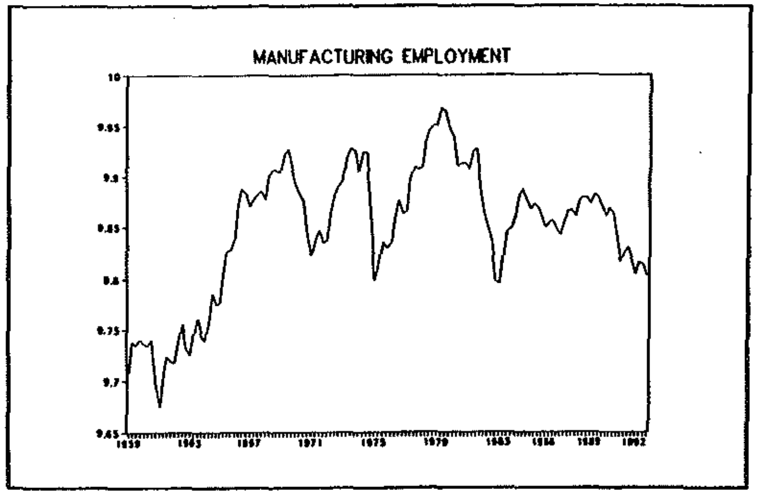


Figure 1b

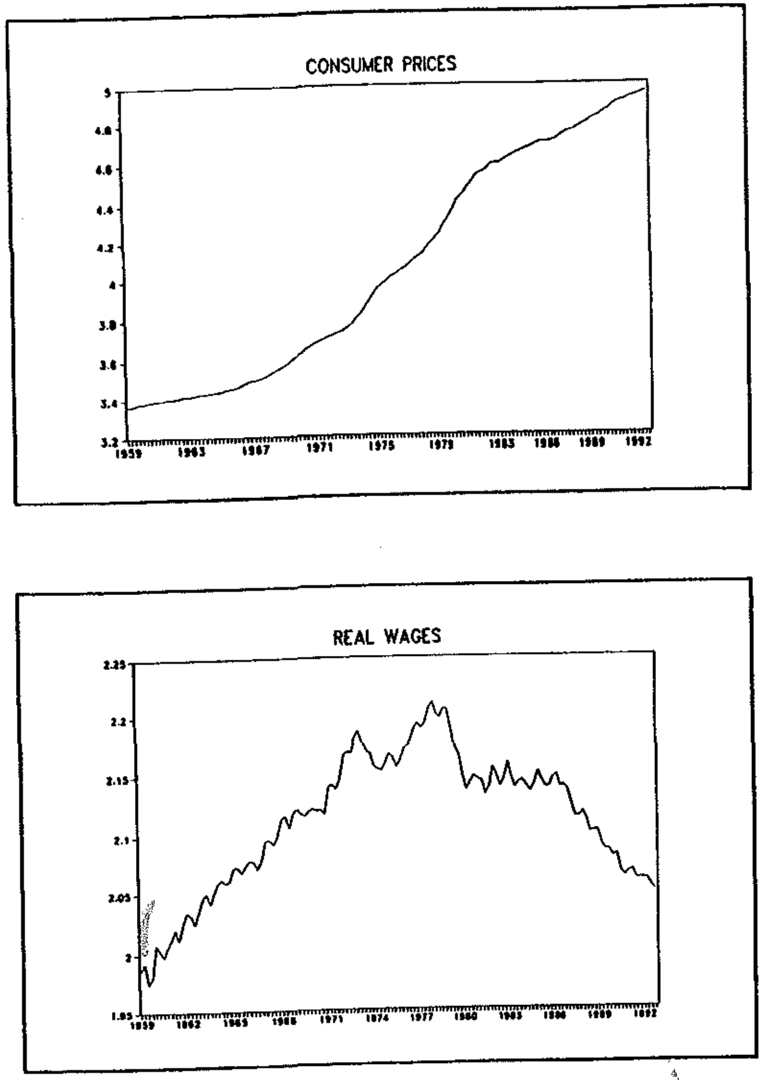


Figure 2a

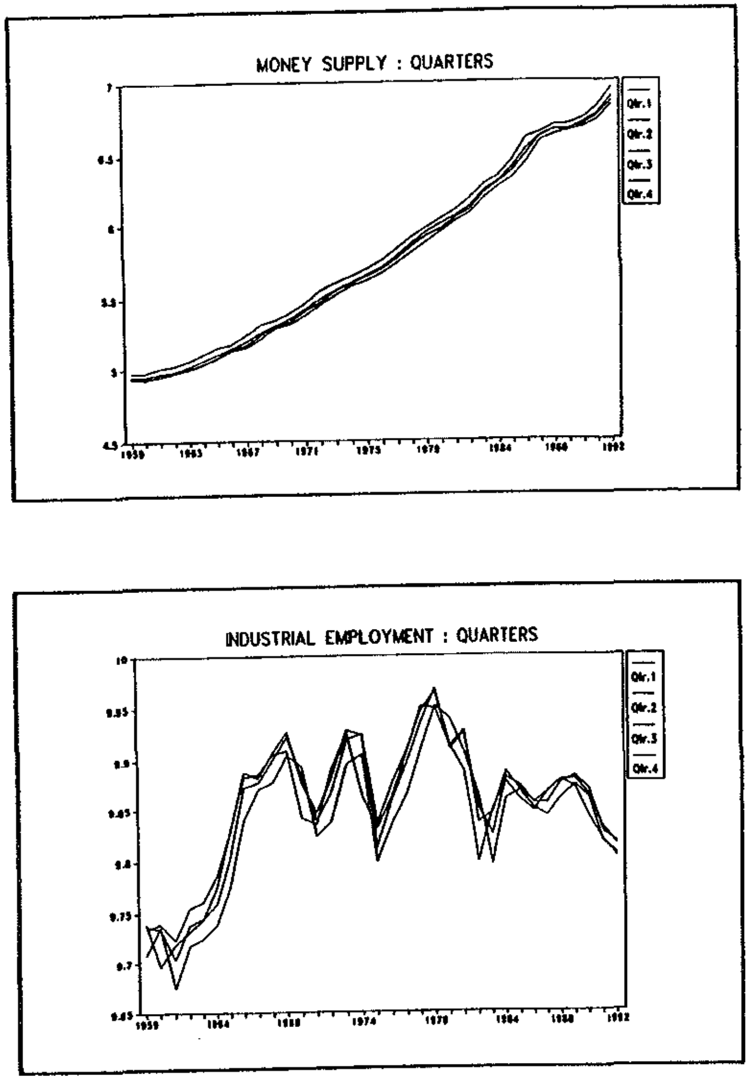
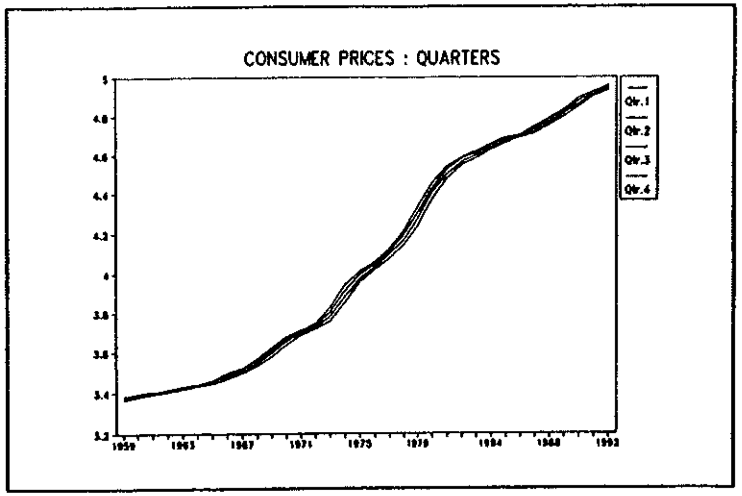


Figure 2b



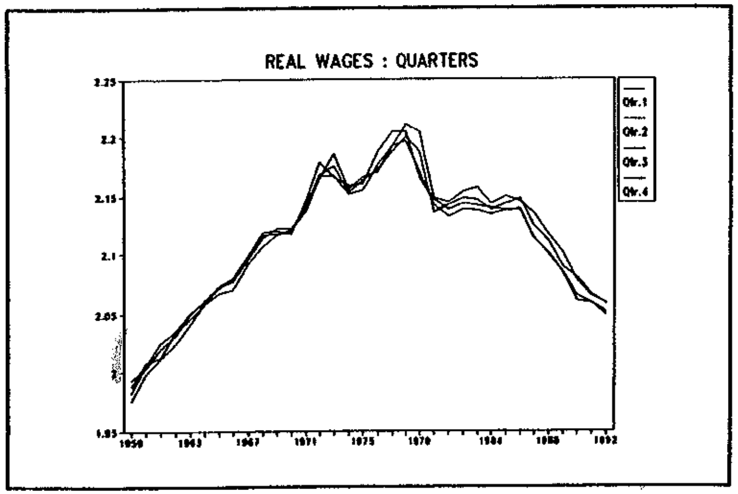
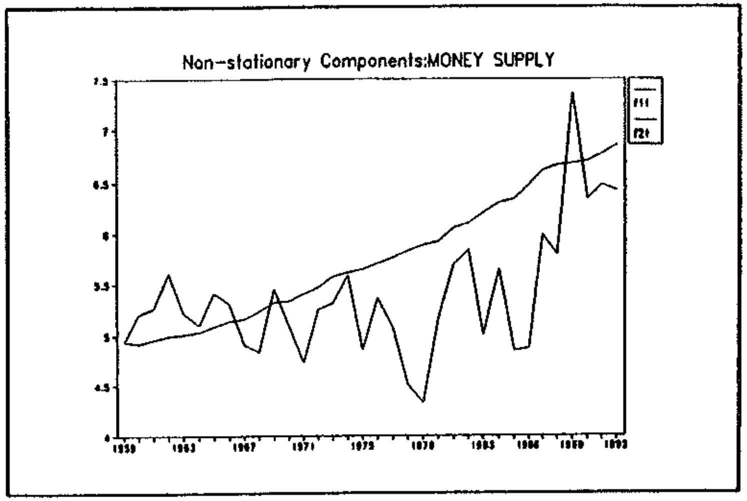


Figure 3a



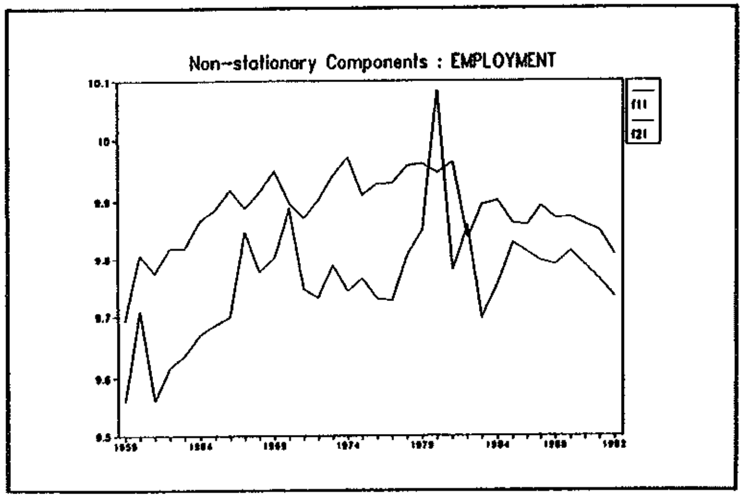
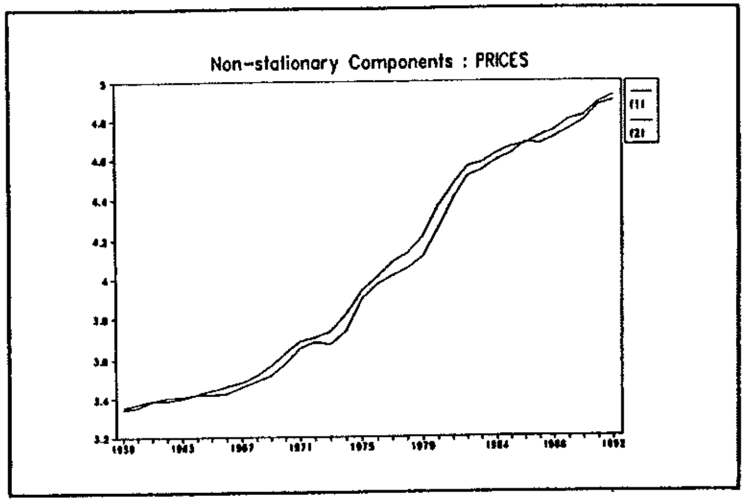


Figure 3b



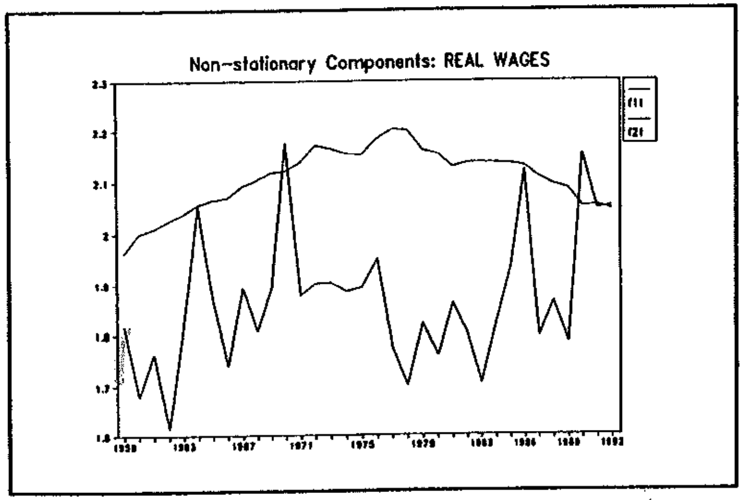
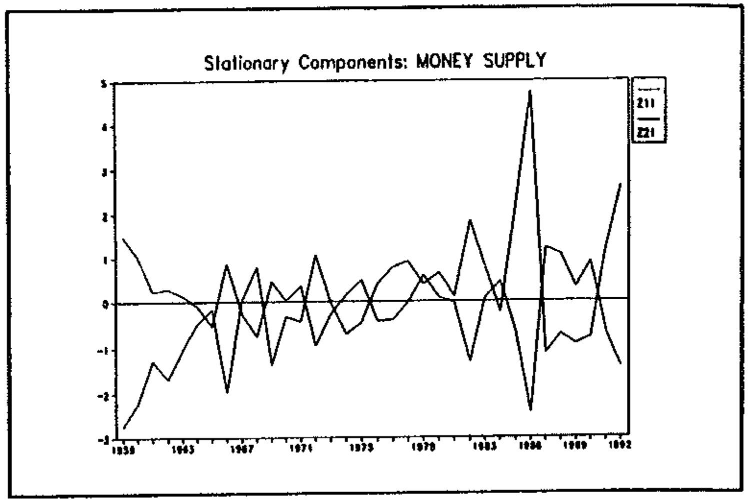


Figure 4a



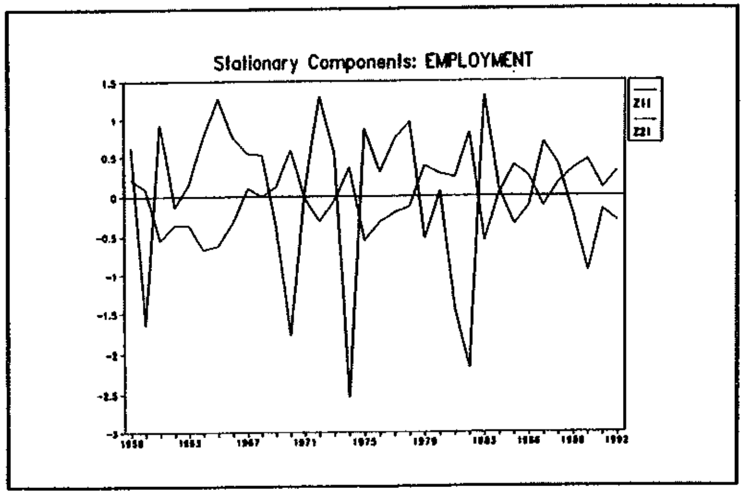
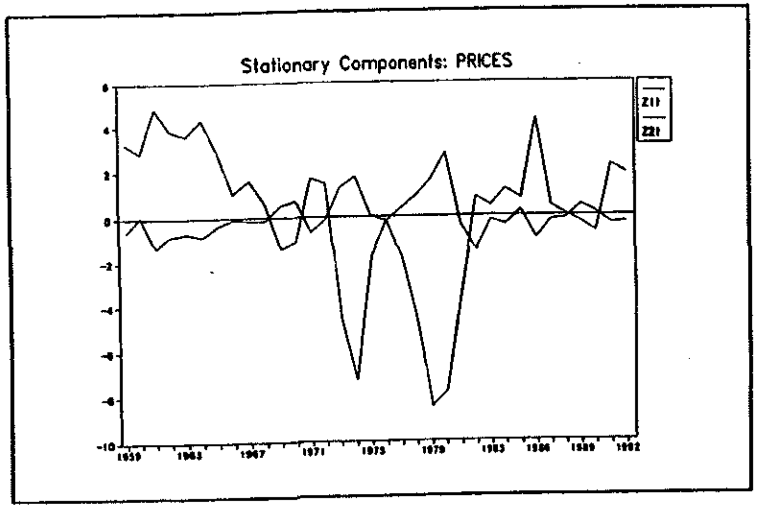


Figure 4b



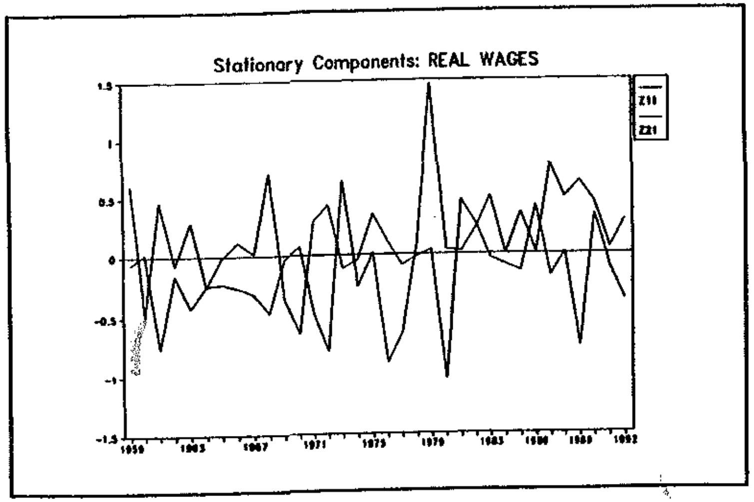
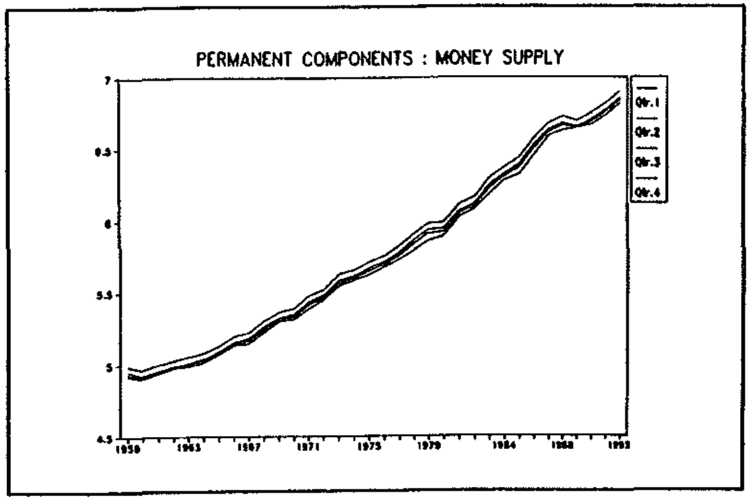


Figure 5a



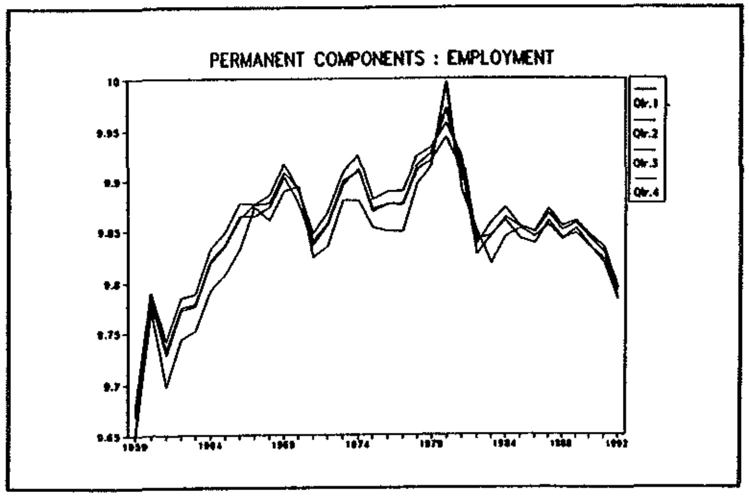
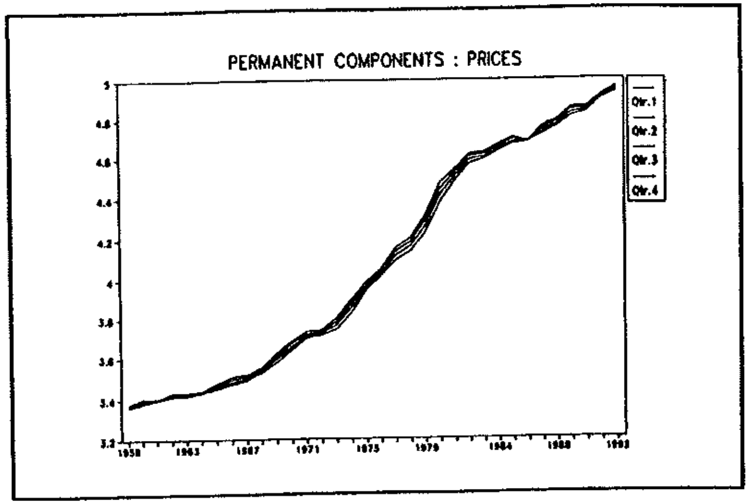


Figure 5b



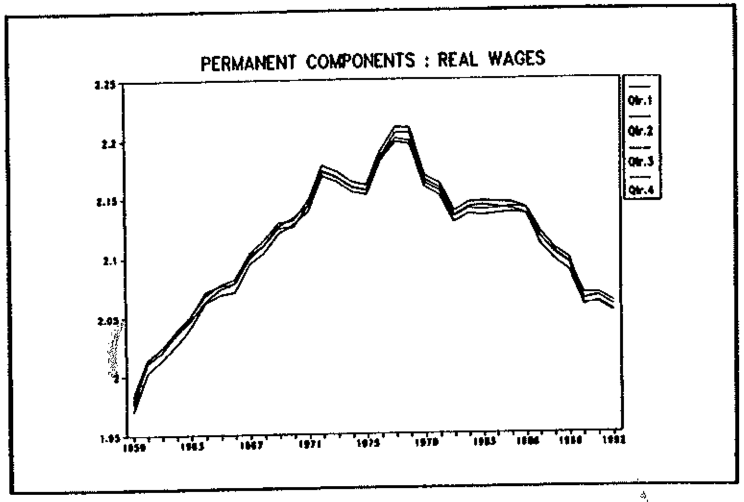
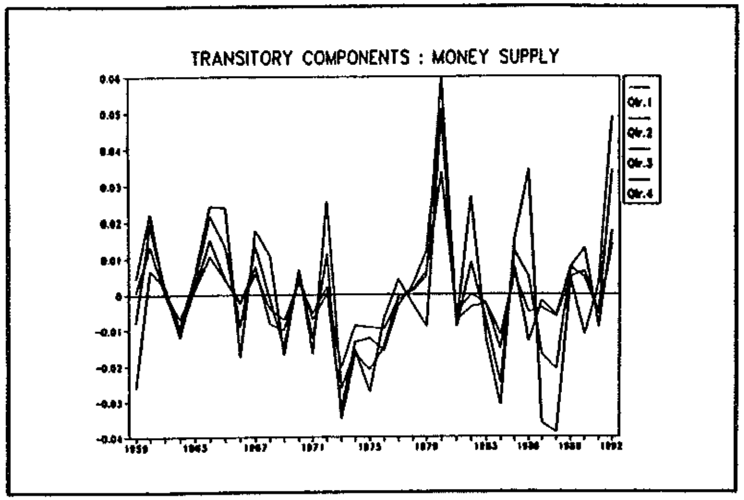


Figure 6a



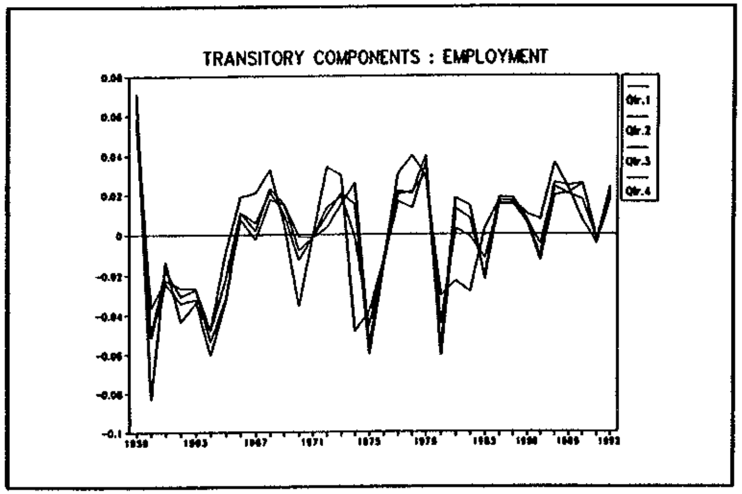
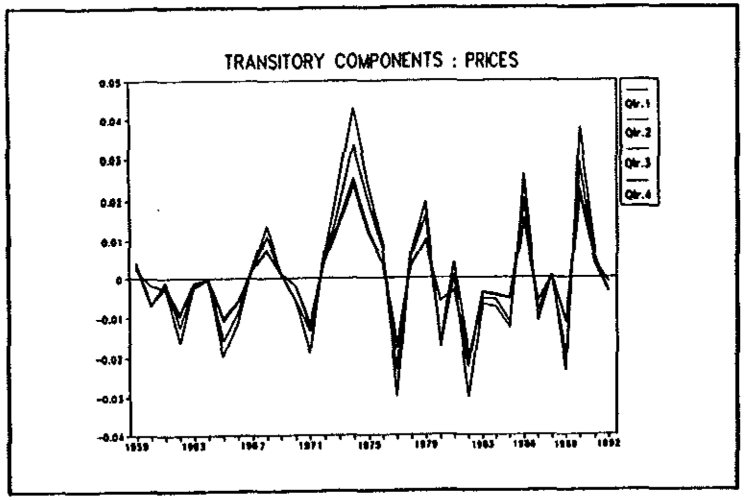


Figure 6b



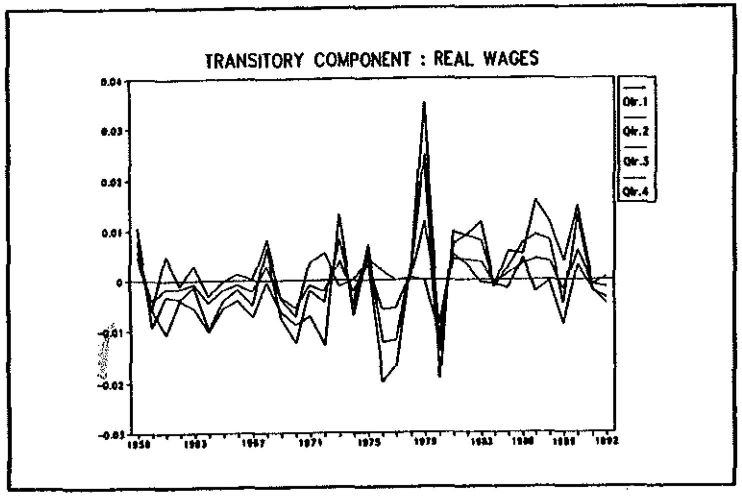
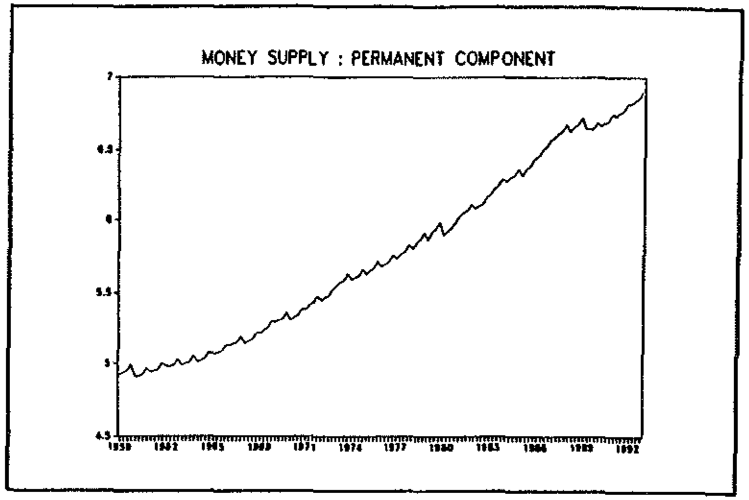


Figure 7a



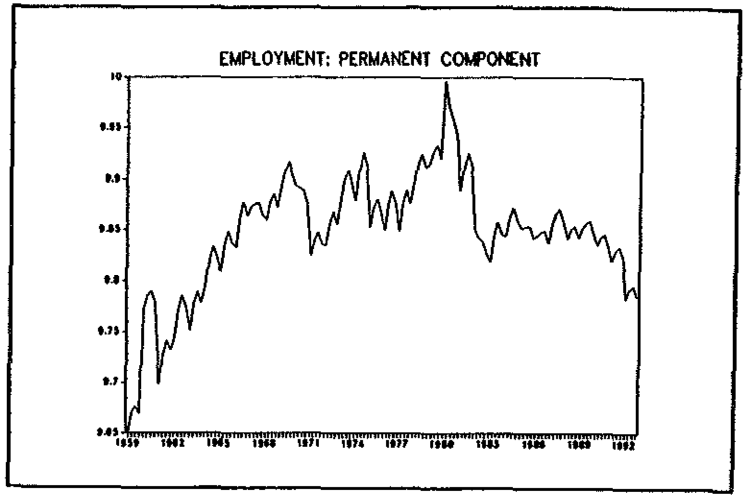
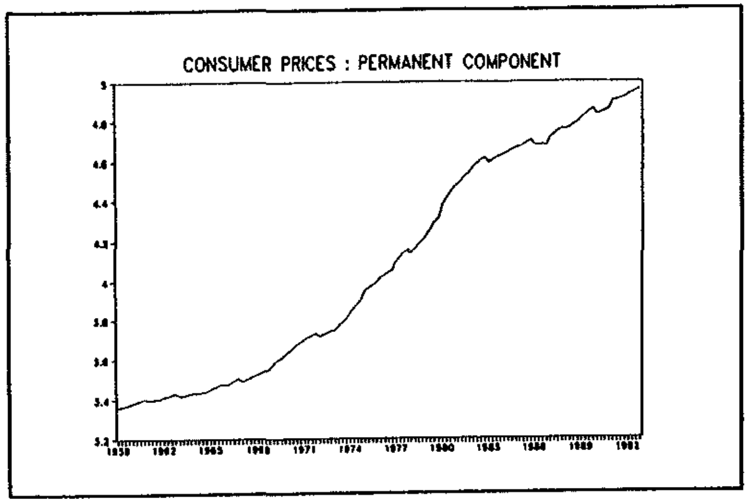


Figure 7b



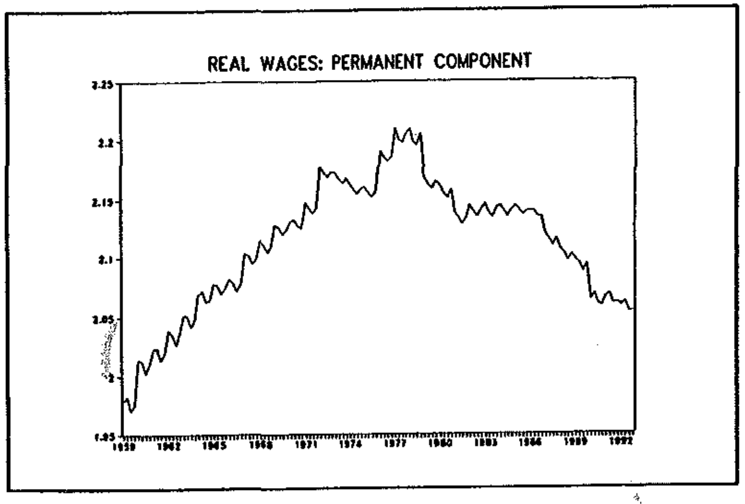
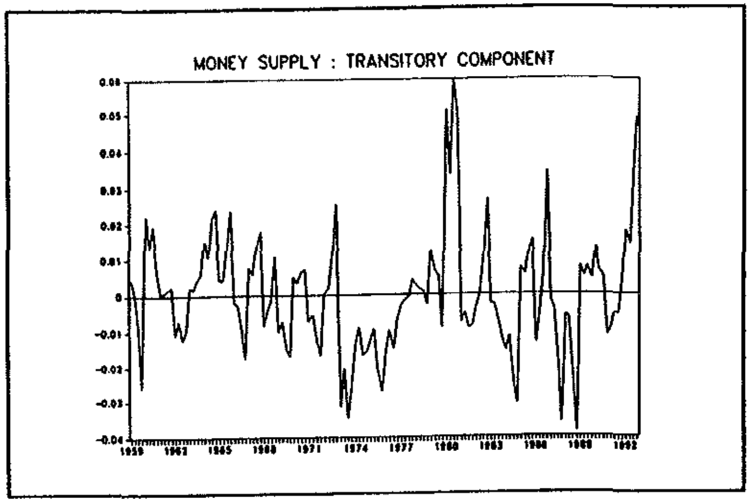


Figure 8a



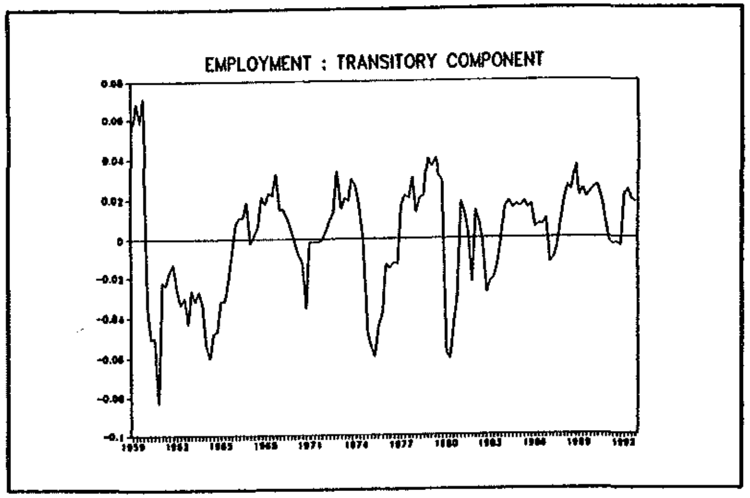
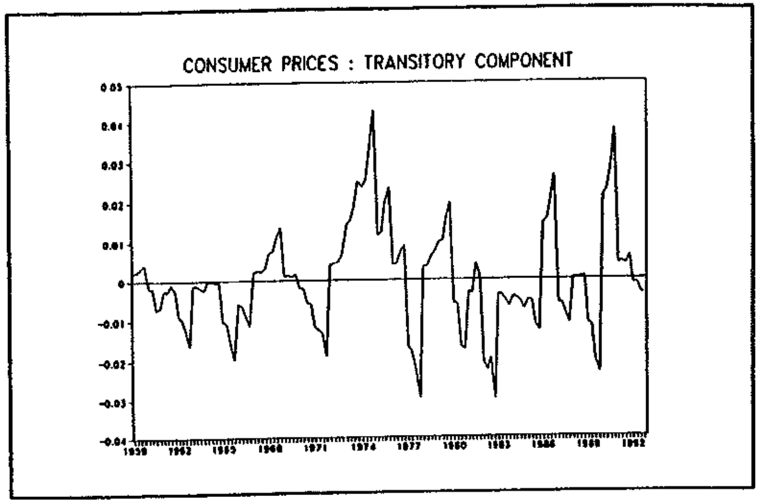
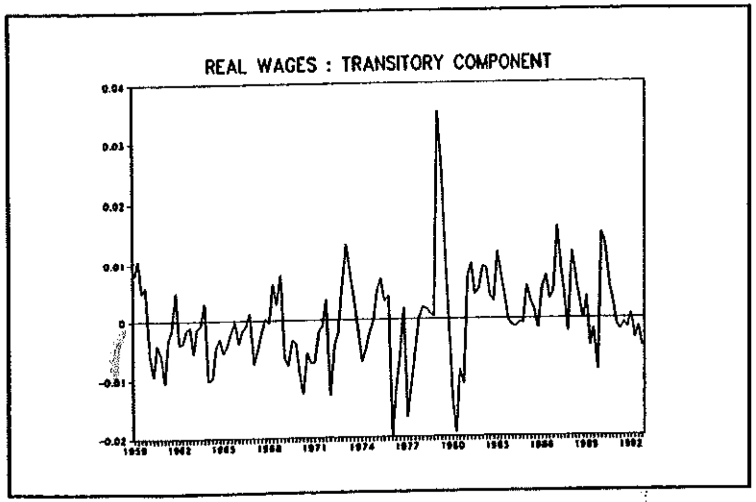


Figure 8b


CONSUMER PRICES : TRANSITORY COMPONENT


REAL WAGES : TRANSITORY COMPONENT

SERIE DE DOCUMENTOS DE TRABAJO DEL *ICAE*

9301 *"Análisis del Comportamiento de las Cotizaciones Reales en la Bolsa de Madrid bajo la Hipótesis de Eficiencia"*. Rafael Flores de Frutos. Diciembre 1992. (Versión final aceptada para publicación en Estadística Española)

9302 *"Sobre la Estimación de Primas por Plazo dentro de la Estructura Temporal de Tipos de Interés"*. Rafael Flores de Frutos. Diciembre 1992.

9303 *"Cambios de Estructuras de Gasto y de Consumo en el Cálculo del IPC"*. Antonio Abadía. Febrero 1993. (Versión final publicada en Revista de Economía Aplicada, Vol.1, Nº1)

9304 *"Tax Analysis in a Limit Pricing Model"*. Félix Marcos. Febrero 1993.

9305 *"El Tipo de Cambio Propio: Reformulación del Concepto y Estimación para el Caso Español"*. José de Hevia Payá. Junio 1993. (Versión final aceptada para publicación en Revista Española de Economía)

9306 *"Price Volatility Under Alternative Monetary Instruments"*. Alfonso Novales. Abril 1992.

9307 *"Teorías del Tipo de Cambio: Una Panorámica"*. Oscar Bajo Rubio. Simón Sosvilla Rivero. Junio 1993. (Versión final publicada en Revista de Economía Aplicada, Vol.1, Nº2).

9308 *"Testing Theories of Economic Fluctuations and Growth in Early Development (the case of the Chesapeake tobacco economy)"*. Rafael Flores de Frutos. Alfredo M. Pereira. Diciembre 1992.

9309 *"Maastricht Convergence Conditions: A Lower Bound for Inflation?"*. Jorge Blázquez. Miguel Sebastián. Marzo 1992.

9310 *"Recursive Identification, Estimation and Forecasting of Nonstationary Economic Time Series with Applications to GNP International Data"*. A. García-Ferrer. J. del Hoyo. A. Novales. P.C. Young. Marzo 1993.

9311 *"General Dynamics in Overlapping Generations Models"*. Carmen Carrera. Manuel Morán. Enero 1993. (Versión final aceptada para publicación en Journal of Economic Dynamics and Control)

9312 *"Further Evidence on Forecasting International GNP Growth Rates Using Unobserved Components Transfer Function Models"*. A. García-Ferrer, J. del Hoyo, A. Novales, P.C. Young. Septiembre 1993. (De próxima aparición en un volumen de homenaje a A. Zellner)

9313 *"Public Capital and Aggregate Growth in the United States: Is Public Capital Productive?"*. Rafael Flores de Frutos. Alfredo M. Pereira. Julio 1993.

9314 *"Central Bank Structure and Monetary Policy Uncertainty"*. José I. García de Paso. Abril 1993.

9315 *"Monetary Policy with Private Information: A Role for Monetary Targets"*. José I. García de Paso. Julio 1993.

9316 *"Exact Maximum Likelihood Estimation of Stationary Vector ARMA Models"*. José Alberto Mauricio. Julio 1993. (Versión final aceptada para publicación en Journal of the American Statistical Association)

9317 *"The Exact Likelihood Function of a Vector ARMA Model"*. José Alberto Mauricio. Julio 1993.

9318 *"Business Telephone Traffic Demand in Spain: 1980-1991, An Econometric Approach"*. Teodosio Pérez Amaral. Francisco Alvarez González. Bernardo Moreno Jiménez. Septiembre 1993. (Versión final aceptada para publicación en Information Economics and Policy)

9401 *"Contrastes de momentos y de la matriz de información"*. Teodosio Pérez Amaral. Junio 1994. (Versión final aceptada para publicación en Cuadernos Económicos del ICE)

9402 *"A partisan explanation of political monetary cycles"*. José I. García de Paso. Junio 1994.

9403 *"Estadísticos para la detección de observaciones anómalas en modelos de elección binaria: Una aplicación con datos reales"*. Gregorio R. Serrano. Junio 1994.

9404 *"Effects of public investment in infraestructure on the spanish economy"*. Rafael Flores de Frutos. Mercedes Gracia Díez. Teodosio Pérez Amaral. Junio 1994.

9405 *" Observaciones anómalas en modelos de elección binaria"*. Mercedes Gracia. Gregorio R. Serrano. Junio 1994. (Versión final aceptada para publicación en Estadística Española)

9406 *" Permanent components in seasonal variables"*. Rafael Flores. Alfonso Novales. Junio 1994.

9407 *" Money demand instability and the performance of the monetary model of exchange rates"*. Rodrigo Peruga. Junio 1994.

9408 *" Una nota sobre la estimación eficiente de modelos con parámetros cambiantes"*. Sonia Sotoca. Junio 1994.

9409 *" Distribución de la renta y redistribución a través del IRPF en España"*. Rafael Salas. Junio 1994.

9410 *" Trade balances: Do exchange rates matter?"*. Rodrigo Peruga. Junio 1994.