Instituto Complutense de Análisis Económico
UNIVERSIDAD COMPLUTENSE
FACULTAD DE ECONOMICAS
Campus de Somosaguas
28223 MADRID
Teléfono 3942611 - FAX 3942613

Documento de Trabajo

A retrial model at nostationary regime

Mercedes Vázquez

No. 9412 Octubre 1994

Instituto Complutense de Análisis Económico
UNIVERSIDAD COMPLUTENSE

A RETRIAL MODEL AT NONSTATIONARY REGIME

Mercedes Vázquez
Departamento de Análisis Económico
Facultad de Económicas y Empresariales
Universidad Complutense de Madrid
Campus de Somosaguas
28223 Madrid, Spain

ABSTRACT

In this article we analyze a model of a retrial queueing system where customers in the orbit join a queue with FCFS discipline. We adopt a nostationary regime. We derive some probabilities using the theory of semiregenerative processes. We obtain an integral estimator for the blocking probability. We also find lower and upper bounds for the l^1 distance between the probability distributions in an $M/G/1/\infty$ retrial model and the M/G/1/ queue at stationary regime.

RESUMEN

En este artículo analizamos un modelo con reintentos en el cual los clientes en órbita pasan a formar parte de una cola con disciplina FIFO. Adoptaremos régimen no estacionario. Obtendremos algunas probabilidades usando la Teoría de procesos Semiregenerativos y un estimador integral para la probabilidad de bloqueo. Por último, mostramos una cota inferior y otra superior para la distancia l^1 entre la distribución del número de individuos en el sistema en el caso M/G/1 con reintentos y el sistema clásico $M/G/1/\infty$ en estado estacionario.

1. INTRODUCTION

Consider a single channel system with arrivals generated by a Poisson process at the rate λ. These arrivals are called primary customers. If the service area is free when a customer arrives, the service starts immediately. If the customer finds the channel busy he will reapply for service after a random period assumed to be exponentially distributed with parameter μ. Customers standing by to reapply are said to be in orbit. When a customer arrives into the orbit he must join a queue with FCFS discipline, i.e. only the first customer in the queue can repeat an attempt. We will assume that the flow of primary customers and the flow of repeated attempts are mutually independent. The service time distribution S(x), with Laplace transform $\beta(s)$, is assumed to be the same for both type of customers. Fayolle [4] studied this type of retrial system in the M/M/1 case. An extensive analysis for retrial systems can be found in Falin [2] and Young and Templeton [6].

Farahmand [3] analyzes this model in the steady state. He obtains a generating function for the number of customers in orbit when the channel is busy and when the channel is free. We study the behavior of the system in a nonstationary regime. To do this we first introduce some results that we will use in the analysis that follows.

Let N_n denote the number of customers in the orbit immediately after the nth departure and η_n the time of the nth departure. We assume that $Pr(\eta_0=0)=1$. It is easy to see that N_n satisfies

$$N_{n+1} = K \quad , \text{ if } \; N_n = 0$$

$$N_{n+1} = \begin{cases} N_n+K-1, & N_n>0 \text{ and a retried call occupies the channel} \\ N_n+K\ , & N_n>0 \text{ and a primary call occupies the channel} \end{cases}$$

where K represents the number of arrivals during the nth service time. Note that the variable K is independent of n.

It is easy to prove that in this model the pair (N_{n+1}, η_{n+1}) is a markovian renewal process with transition matrix

$$Q(x) = \begin{pmatrix} B_0(x) & B_1(x) & \ldots\ldots \\ A_0(x) & A_1(x) & \ldots\ldots \\ 0 & A_0(x) & A_1(x) & . \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

where

$$A_i(x) = \lambda\int_0^x e^{-(\lambda+\mu)t}k_{i-1}(x-t)dt+\mu\int_0^x e^{-(\lambda+\mu)t}k_i(x-t)dt,\ n>0 \qquad (1)$$

$$A_0(x) = \mu\int_0^x e^{-(\lambda+\mu)t}k_0(x-t)dt;\ B_i(x)=\lambda\int_0^x e^{-\lambda t}k_i(x-t)dt\ \forall n\geq 0; \qquad (2)$$

$$K_i(x) = Pr(k=i, S\leq x) = \int_0^x e^{-\lambda t}\ \frac{(\lambda t)^i}{i!}\ S(dt)$$

Given the structural form of the transition matrix $Q(x)$ our model belongs to the group called "of the M/G/1 type". An extensive analysis of these systems can be found in Neuts [5].

One of the most important characteristics of a markovian renewal process is the fundamental period, i.e, the time it takes the process to go the first time from state i to state i-1. Let T denote the length of a fundamental period and V the number of customers served during T. The transform $G(z,s)= E[e^{-sT}z^V]$, $|z|<1$, $Re(s)>0$ satisfies the equation

$$G(z,s) = zA(G(z,s),s)$$

where

$$A(z,s) = \sum_{i=0}^{\infty}\int_0^{\infty} e^{-sx}z^i dA_i(x),\ \ |z|<1,\ \ Re(s)>0.$$

See [5]. In our case we obtain the following lemmma for the variables T and V:

Lemma 1

The joint generating function for T and V is

$$G(z,s) = z\,\frac{\mu+\lambda G(z,s)}{\lambda+\mu+s}\beta(s+\lambda-\lambda G(z,s)) \qquad (3)$$

and the first order moments are

$$E(T) = \frac{\lambda+\mu\rho}{\lambda(1-\rho)\mu} \quad ; \quad E(V) = \frac{\lambda+\mu}{(1-\rho)\mu}$$

Proof

Using formulas (1) and (2) we have

$$A(x,z) = (\lambda z+\mu)\int_0^{\infty} e^{-(\lambda+\mu)t}K(x-t,z)dt \Rightarrow A(z,s) = \frac{\lambda z+\mu}{\lambda+\mu+s}\beta(\lambda-\lambda z+s) \quad (4)$$

where

$$K(x,z) = \sum_{i=0}^{\infty}\int_0^x e^{-\lambda t}\frac{(\lambda tz)^i}{n!}\,dS(t) = \int_0^x e^{-(\lambda-\lambda z)t}dS(t)$$

is the generating function for the number of arrivals during a service time not longer than x. Taking derivatives in (3) with respect to s and z, at the point s=1 and z=0, we obtain E(T) and E(V). ∎

Observe that when μ tends to infinity the joint Laplace transform and generating function given in the previous lemma coincides with Tackás' formula for the joint transform of the busy period and the number of customers served in it for a classical M/G/1/∞ queue.

The variable T_0 representing the length of time elapsed in between two consecutive visits to state zero is closely connected to the fundamental period. Let $T_0(s)$ be the Laplace transform of this variable. It can be proved [5] that $T_0(s)$ satisfies the relation

$$T_0(s) = \sum_{i=0}^{\infty}B_i(s)G^i(1,s)=B(G(1,s),s)$$

where

$$B(z,s) = \sum_{i=0}^{\infty}\int_0^{\infty} e^{-sx}z^i dB_i(x),\ \ |z|<1,\ \ Re(s)>0$$

We now find the corresponding expressions for $T_0(s)$ and $B(z,s)$ in our system.

Lemma 2

The Laplace transform $T_0(s)$ is

$$T_0(s) = \frac{\lambda}{\lambda+s}\beta(\lambda-\lambda G(1,s)+s) \tag{5}$$

Proof

The transform $B(z,s)$ can be written as

$$B(z,s) = s\int_0^{\infty} e^{-sx}\int_0^{x}\lambda e^{-\lambda t}\int_0^{x-t} e^{-\lambda u}e^{\lambda uz}dS(u)dtdx$$

Thus

$$B(z,s) = \frac{\lambda}{\lambda+s}\beta(\lambda-\lambda z+s) \tag{6}$$

from which (5) is easily derived.

■

Let $C(t)$ represent the number of busy channels and $N(t)$ the number of customers in the orbit at time t. In the next section we obtain a formula for the joint generating function of the process $(C(t),N(t))$ in a nonstationary regime.

2. THE NONSTATIONARY REGIME

We assume that initially i customers occupy the orbit. Let

$$P^*_{10}(z,s) = \int_0^{\infty} e^{-st}\, P_{10}(t,z)\, dt$$

$$P^*_{11}(z,s) = \int_0^{\infty} e^{-st}P_{11}(t,z)dt$$

be the Laplace transforms of the partial generating functions

$$P_{10}(z) = \sum_{n=0}^{\infty}\Pr(C(t)=0,N(t)=n/N_0=i)z^n$$

$$P_{11}(z) = \sum_{n=0}^{\infty}\Pr(C(t)=1,N(t)=n/N_0=i)z^n$$

We now obtain explicit formulas for $P^*_{10}(z,s)$ and $P^*_{11}(z,s)$ in terms of the Laplace transforms of the fundamental period and of the service time.

THEOREM 1

In our M/G/1/∞ retrial model the Laplace transforms $P^*_{10}(z,s)$, $P^*_{11}(z,s)$ are given by

$$P^*_{10}(z,s) = \frac{\mu G^i(1,s)(z-\beta(\lambda-\lambda z+s))}{(\lambda-\lambda\beta(\lambda-\lambda G(1,s)+s)+s)((\lambda+\mu+s)z-(\lambda z+\mu)\beta(\lambda-\lambda z+s))} + \frac{z^{i+1}}{(\lambda+\mu+s)z-(\lambda z+\mu)\beta(\lambda-\lambda z+s)}$$

$$P^*_{11}(z,s) = \left[\frac{\mu G^i(1,s)(\lambda z-\lambda-s)}{(\lambda-\lambda\beta(\lambda-\lambda G(1,s)+s)+s)((\lambda+\mu+s)z-(\lambda z+\mu)\beta(\lambda-\lambda z+s))} + \frac{z^{i+1}(\lambda+\mu z^{-1})}{z(\lambda+\mu+s)-(\lambda z+\mu)(\beta(\lambda-\lambda z+s))}\right]\times\left[\frac{1-\beta(\lambda-\lambda z+s)}{\lambda-\lambda z+s}\right]$$

Proof

The process $X(t)=(C(t),N(t))$ is semiregenerative with respect to the markovian process (N_n,η_n). Hence (see [1]),

$$\Pr(X(t)/N_0=i) = \sum_{j\in E}\int_0^t R(i,j,ds)\Pr(X(t-s),T_1>t-s/N_0=j) \tag{7}$$

where $T_1=\eta_1$ represents the time of the first instant of departure after $t=0$ and $R(i,j,x)$ is the markovian renewal function of the process (N_n,η_n).

The conditional probabilities are given by

a) When $C(t-s)=0$,

1. $\Pr(C(t-s)=0, N(t-s)=0, T_1>t-s/N(0)=0) = e^{-\lambda(t-s)}$

2. $\Pr(C(t-s)=0, N(t-s)=n, T_1>t-s/N(0)=n) = e^{-(\lambda+\mu)(t-s)}$, $n>0$.

3. $\Pr(C(t-s)=0, N(t-s)=n, T_1>t-s/N(0)=j) = 0$, $n\neq j$.

b) When $C(t-s)=1$

1. $\Pr(C(t-s)=1, N(t-s)=n, T_1>t-s/N(0)=0) =$

$$= \int_0^{t-s} \lambda\, e^{-\lambda(t-s)} \frac{(\lambda x)^n}{n!} (1-S(x))\, dx$$

2. $\Pr(C(t-s)=1, N(t-s)=n, T_1>t-s/N(0)=j)=$

$$= \int_0^{t-s} e^{-(\lambda+\mu)(t-s-x)} e^{\lambda x} \left[\lambda \frac{(\lambda x)^{n-j}}{(n-j)!} + \mu \frac{(\lambda x)^{n-j+1}}{(n-j+1)!} \right] (1-S(x))\, dx$$

when $n\geq j-1$.

3. Zero otherwise.

Let

$$R_i(z,s) = \int_0^\infty e^{-st} \sum_{n=0}^{\infty} z^n\, dR(i,n,t), \qquad R_{i,0}(s) = \int_0^\infty e^{-st} dR(i,0,t)$$

be the Laplace transforms of the partial generating functions associated to the renewal function. We now obtain the generating functions and Laplace transforms associated to (7) which, together with the conditional probabilities stated above, give:

$$P^*_{i0}(z,s) = \frac{\mu R_{i,0}(s)}{(\lambda+s)(\lambda+\mu+s)} + \frac{R_i(z,s)}{(\lambda+\mu+s)} \tag{8}$$

$$P^*_{i1}(z,s) = \left[\frac{(\lambda+\mu z^{-1})}{(\lambda+\mu+s)} R_i(z,s) + \frac{\lambda\mu-(\lambda+s)\mu z^{-1}}{(\lambda+s)(\lambda+\mu+s)} R_{i,0}(s) \right] \frac{(1-\beta(\lambda-\lambda z+s))}{(\lambda-\lambda z+s)} \tag{9}$$

We next find an expression for both $R_{i,0}(s)$ and $R_i(z,s)$. The first of these two functions represents the Laplace transform of the mean number of visits to the state zero given that at $t=0$ there are i customers in the orbit. Hence,

$$R_{i,0}(s) = \frac{G^i(1,s)}{1-T_0(s)}$$

where $G(1,s)$ and $T_0(s)$ are given in formulas (3) and (5) respectively. On the other hand $R(t)$, with elements $R(i,j,t)$, satisfies the renewal equation

$$R(t) = U(t) + R(.)*Q(t)$$

where $U(t)$ is the identity matrix. Using this equation the generating function and Laplace transform of $R(t)$ is

$$R_i(z,s) = \left[z-A(z,s)\right]^{-1} \left\{ z^{i+1} + R_{i,0}(s) \left[zB(z,s)-A(z,s) \right] \right\} \tag{10}$$

Taking (4) and (6) and the expression for $R_{i,0}(s)$ into account we obtain $R_i(z,s)$ from (10) and, given (8) and (9), we are done. ■

The Laplace transform of the number of customers in orbit $P^*_i(z,s)$ can be found by adding $P^*_{i0}(z,s)$ to $P^*_{i1}(z,s)$ for each i. We then obtain

$$P^*_i(z,s) = \frac{\mu G^i(1,s)(z-1)}{(\lambda-\lambda\beta(\lambda-\lambda G(1,s)+s)+s)((\lambda+\mu+s)z-(\lambda z+\mu)\beta(\lambda-\lambda z+s))}$$

$$+ \frac{z^{i+1}}{((\lambda+\mu+s)z-(\lambda z+\mu)\beta(\lambda-\lambda z+s))} \left[1 + \frac{(\lambda+\mu z^{-1})(1-\beta(\lambda-\lambda z+s))}{\lambda-\lambda z+s} \right] \tag{11}$$

3. INTEGRAL ESTIMATOR FOR THE BLOCKING PROBABILITY

Assume now that $i=0$ almost everywhere. After some algebra (11) can be written as:

$$P^*(z,s) = (1-z)\,\frac{P^*_{00}(z,s)}{\beta(\lambda-\lambda z+s)-z} + \frac{1-\beta(\lambda-\lambda z+s)}{(\lambda-\lambda z+s)(z-\beta(\lambda-\lambda z+s))}$$

Taking derivatives of $P^*(z,s)$ with respect to z at the point z=1 we obtain the Laplace transform for the mean number of customers in orbit in a nonstationary regime. This is

$$\int_0^\infty e^{-st}E(N(t))dt = \frac{P_0^*(s)}{1-\beta(s)} + \lambda s^{-2} - \frac{1}{s(1-\beta(s))} = \lambda s^{-2} - \frac{P_1^*(s)}{1-\beta(s)}$$

where $P_0^*(s)=P_{00}^*(1,s)=s^{-1}-P_{01}^*(s)$. $P_{01}^*(s)$ represents the Laplace transform for the blocking probability $P_1(t)$ given that the system is initially empty.

For a stationary regime (i.e. when $\rho=\lambda(\lambda+\mu)\beta_1\mu^{-1}<1$) it is known [3] that

$$\lim_{t\to\infty} E(N(t)) = \left\{2\lambda^2\mu^{-1}\beta_1+\lambda\rho(\beta_1+\sigma^2\beta_1^{-1})\right\}/2(1-\rho) \qquad (12)$$

where β_1 and σ^2 are the mean and variance of the service time respectively. The blocking probability in steady state is given by

$$Pr(C_t=1) = \lim_{t\to\infty} P_1(t) = \lim_{s\to 0} sP^*_{01}(s) = \lambda\beta_1$$

These results enable us to obtain an integral estimator for the blocking probability. To do this we use Abel's Theorem to get

$$\lim_{t\to\infty} E(N(t)) = \lim_{s\to 0}\int_0^\infty se^{-st}E(N(t))dt = \lim_{s\to 0}\left[\lambda s^{-1}-\frac{sP_1^*(s)}{1-\beta(s)}\right]$$

$$= \lim_{s\to 0}\frac{s}{1-\beta(s)}\lim_{s\to 0}\left[(\lambda(1-\beta(s)))s^{-2}-P_1^*(s)\right]$$

$$= \beta_1^{-1}\lim_{s\to 0}\left[(\lambda(1-\beta(s)))s^{-2}-\lambda\beta_1^{-1}s^{-1}\right] + \beta_1^{-1}\lim_{s\to 0}\left[\lambda\beta_1 s^{-1}-P_1^*(s)\right]$$

$$= -\lambda\beta_2(2\beta_1)^{-1} + \beta_1^{-1}\int_0^\infty(\lambda\beta_1-p_1(t))dt.$$

and, given (12), we obtain

$$\int_0^\infty(\lambda\beta_1-p_1(t))dt = \lambda\beta_2/2 + \left\{2\lambda^2\mu^{-1}+\lambda\rho(1+\sigma^2\beta_1^{-2})\right\}/2(1-\rho) =$$

$$= \left\{\lambda^2\mu+\lambda\beta_2(2\beta_1^2)^{-1}\right\}/(1-\rho) \qquad (13)$$

This formula provides an estimator for the l^1 distance between the blocking probabilities in stationary and nonstationary regimes. A similar result can be found in Falin [2] for a different retrial queueing model.

As an application of this result consider the integral estimator of the traffic intensity $\lambda\beta_1$

$$\xi_T = T^{-1}\int_0^T C(t)dt$$

where [0,T] is the observation interval. The expectation of this estimator is

$$E(\xi_T) = T^{-1}\int_0^T p_1(t)dt = \lambda\beta_1 + T^{-1}\int_0^T(\lambda\beta_1-p_1(t))dt$$

$$= \lambda\beta_1 - T^{-1}\int_0^\infty(\lambda\beta_1-p_1(t))dt - T^{-1}\int_T^\infty(\lambda\beta_1-p_1(t))dt$$

$$= \lambda\beta_1 - T^{-1}\left\{\lambda^2\mu+\lambda\beta_2(2\beta_1^2)^{-1}\right\}(1-\rho)^{-1}+o(T^{-1})$$

given (13). Therefore the integral estimator is shown to be both biased and asymptotically unbiased.

4. THE NUMBER OF CUSTOMERS IN THE SYSTEM IN STEADY STATE

The goal of this section is to study the behaviour of the system in steady state, when the parameter of retrial tends to infinity, by using a stochastic decomposition. More precisely, we wish to obtain a lower and an upper bound for the l^1 distance between the probability distribution in steady state of the number of customers in the system in our M/G/1 retrial model and the probability distribution of the number of customers in the system in the context of the classical $M/G/1/\infty$ queue. In the following theorem we find the generating function Q(z) for the number of customers in the system in the retrial model.

THEOREM 2

If our M/G/1 retrial model is in steady state the generating function Q(z) is given by

$$Q(z) = (1-\lambda(\lambda+\mu)\beta_1\mu^{-1})\,\frac{\beta(\lambda-\lambda z)\mu(1-z)}{(\mu+\lambda z)\beta(\lambda-\lambda z)-(\lambda+\mu)z}$$

Proof

Let $\lambda(\lambda+\mu)\beta_1\mu^{-1}<1$. It is easily proved that

$$P_0(z) = \sum_{n=0}^{\infty}\Pr(C_t=0,N_t=0)z^n = \lim_{s\to 0} sP^*_{10}(z,s)$$

and

$$P_1(z) = \sum_{n=0}^{\infty}\Pr(C_t=1,N_t=0)z^n = \lim_{s\to 0} sP^*_{11}(z,s)$$

and, given [3], $P_0(z)$ and $P_1(z)$ are

$$P_0(z) = \frac{\mu(1-\rho)(\beta(\lambda-\lambda z)-z)}{(\lambda z+\mu)\beta(\lambda-\lambda z)-(\lambda+\mu)z} \quad\text{and}\quad P_1(z) = \frac{\mu(1-\rho)(1-\beta(\lambda-\lambda z))}{(\lambda z+\mu)\beta(\lambda-\lambda z)-(\lambda+\mu)z}$$

Taking into account that Q(z) can be expressed in terms of $P_0(z)$ and $P_1(z)$ as $Q(z) = P_0(z) + zP_1(z)$, the theorem is proved. ■

We denote by $\Pr(Q=n)$ and $Q_n(\infty)$ the steady state probabilities that there are n customers in the system for the M/G/1 retrial model and the classical $M/G/1/\infty$ queue respectively. We denote by $\hat{Q}_n$ the steady state probability that n customers are in the orbit given that the channel is free.

The generating function Q(z) can be rewritten as follows

$$Q(z) = (1-\lambda\beta_1)^{-1}\frac{\mu(1-\rho)(\beta(\lambda-\lambda z)-z)}{(\lambda z+\mu)\beta(\lambda-\lambda z)-(\lambda+\mu)z} \times \frac{\beta(\lambda-\lambda z)(1-z)}{(\beta(\lambda-\lambda z)-z)}(1-\lambda\beta_1)$$

where the first term is the generating function of $\hat{Q}_n$ and the second term is the Pollaczek-Khinchin formula associated to the number of customers in the system in the classical $M/G/1/\infty$ queue. Hence the probability $\Pr(Q=n)$ can be written as a convolution between the probabilities $Q_n(\infty)$ and $\hat{Q}_n$. Taking this stochastic decomposition into account we get the following theorem:

THEOREM 3

The l^1 distance between $\Pr(Q=n)$ and $Q_n(\infty)$ tends to zero as $\mu\to\infty$. Furthermore, such distance satisfies

$$2\lambda^2\beta_1\mu^{-1} \le \sum_{n=0}^{\infty}\left|\Pr(Q=n)-Q_n(\infty)\right| \le \frac{2\lambda^2\beta_1}{(1-\lambda\beta_1)}\,\mu^{-1}$$

Proof

Observe that

$$\left|\Pr(Q=n)-Q_n(\infty)\right| = \left|\sum_{k=0}^{n}Q_k(\infty)\hat{Q}_{n-k}-Q_n(\infty)\right| = \left|Q_n(\infty)\hat{Q}_0-Q_n(\infty)+\sum_{k=0}^{n-1}Q_k(\infty)\hat{Q}_{n-k}\right|$$

So

$$\left|\Pr(Q=n)-Q_n(\infty)\right| \le Q_n(\infty)(1-\hat{Q}_0) + \Pr(Q=n)-Q_n(\infty)\hat{Q}_0 = Q_n(\infty)(1-2\hat{Q}_0) + \Pr(Q=n)$$

Hence

$$\sum_{n=0}^{\infty}\left|\Pr(Q=n)-Q_n(\infty)\right| \le 1-2\hat{Q}_0+1 = 2(1-\hat{Q}_0)$$

Moreover

$$\hat{Q}_0 = (1-\lambda\beta_1)^{-1}\left.\frac{\mu(1-\rho)(\beta(\lambda-\lambda z)-z)}{(\lambda z+\mu)\beta(\lambda-\lambda z)-(\lambda+\mu)z}\right|_{z=0} = \frac{(1-\rho)}{(1-\lambda\beta_1)}$$

and, given that $\rho = \lambda(\lambda+\mu)\beta_1\mu^{-1}$,

$$\sum_{n=0}^{\infty} |Pr(Q=n)-Q_n(\infty)| \le \frac{2\lambda^2\beta_1}{(1-\lambda\beta_1)} \mu^{-1}$$

On the other hand

$$\sum_{n=0}^{\infty} |Pr(Q=n)-Q_n(\infty)| = |Pr(Q=0)-Q_0(\infty)| + \sum_{n=1}^{\infty} |Pr(Q=n)-Q_n(\infty)|$$

Thus

$$\sum_{n=0}^{\infty} |Pr(Q=n)-Q_n(\infty)| \ge |Pr(Q=0)-Q_0(\infty)| - Pr(Q=0) + Q_0(\infty) = 2(Q_0(\infty)-Pr(Q=0))$$

where $Q_0(\infty)=1-\lambda\beta_1$ and $Pr(Q=0)=1-\rho$. ■

Observe that the rate of convergence when μ tends to infinity is independent of the specific service time distribution. It only depends on its first moment.

REFERENCES

[1] E. Çinlar, Introduction to stochastic processes, Prentice-Hall, (1975).

[2] G.I. Falin, A survey of retrial queues, Queueing Systems 7 (1990) 127-168.

[3] K. Farahmand, Single line queue with repeated demands, Queueing Systems 6 (1990) 223-228.

[4] G. Fayolle, A simple exchange with delayed feedbacks, in: *Teletraffic Analysis and Computer Performance Evaluation*, O.J. Boxma, J.W. Cohen, H.C. Tijms, eds. (North-holland, Amsterdam, 1986)

[5] M.F. Neuts, Structured stochastic matrices of M/G/1 type and their applications, Marcel Dekker (1989).

[6] T. Yang and J.G.C. Templeton, A survey on retrial queues, Queueing Systems 2 (1987) 203-233.

SERIE DE DOCUMENTOS DE TRABAJO DEL *ICAE*

9301 *"Análisis del Comportamiento de las Cotizaciones Reales en la Bolsa de Madrid bajo la Hipótesis de Eficiencia"*. Rafael Flores de Frutos. Diciembre 1992. (Versión final aceptada para publicación en Estadística Española)

9302 *"Sobre la Estimación de Primas por Plazo dentro de la Estructura Temporal de Tipos de Interés"*. Rafael Flores de Frutos. Diciembre 1992.

9303 *"Cambios de Estructuras de Gasto y de Consumo en el Cálculo del IPC"*. Antonio Abadía. Febrero 1993. (Versión final publicada en Revista de Economía Aplicada, Vol.1, N°1)

9304 *"Tax Analysis in a Limit Pricing Model"*. Félix Marcos. Febrero 1993.

9305 *"El Tipo de Cambio Propio: Reformulación del Concepto y Estimación para el Caso Español"*. José de Hevia Payá. Junio 1993. (Versión final publicada en Revista Esñañola de Economía, Vol.11, N°1, 1994)

9306 *"Price Volatility Under Alternative Monetary Instruments"*. Alfonso Novales. Abril 1992.

9307 *"Teorías del Tipo de Cambio: UnaPanorámica"*. Oscar Bajo Rubio. Simón Sosvilla Rivero. Junio1993. (Versión final publicada en Revista de Economía Aplicada, Vol.1, N°2).

9308 *"Testing Theories of Economic Fluctuations and Growth in Early Development (the case of the Chesapeake tobacco economy)"*. Rafael Flores de Frutos. Alfredo M. Pereira. Diciembre 1992.

9309 *"Maastricht Convergence Conditions: A Lower Bound for Inflation?"*. Jorge Blázquez. Miguel Sebastián. Marzo 1992.

9310 "Recursive Identification, Estimation and Forecasting of Nonstationary Economic Time Series with Applications to GNP International Data". A. García-Ferrer. J. del Hoyo. A. Novales. P.C. Young. Marzo 1993. (De próxima aparición en un volumen de homenaje a A. Zellner)

9311 *"General Dynamics in Overlapping Generations Models"*. Carmen Carrera. Manuel Morán. Enero 1993. (Versión final aceptada para publicación en Journal of Economic Dynamics and Control)

9312 *"Further Evidence on forecasting International GNP Growth Rates Using Unobserved Components Transfer Function Models"*. A. García-Ferrer, J. del Hoyo, A. Novales, P.C. Young. Septiembre 1993.

9313 *"Public Capital and Aggregate Growth in the United States: Is Public Capital Productive?"*. Rafael Flores de Frutos. Alfredo M. Pereira. Julio 1993.

9314 *"Central Bank Structure and Monetary Policy Uncertainty"*. José I. García de Paso. Abril 1993.

9315 *"Monetary Policy with Private Information: A Role for Monetary Targets"*. José I. García de Paso. Julio 1993.

9316 *"Exact Maximum Likelihood Estimation of Stationary Vector ARMA Models"*. José Alberto Mauricio. Julio 1993. (Versión final aceptada para publicación en Journal of the American Statistical Association)

9317 *"The Exact Likelihood Function of a Vector ARMA Model"*. José Alberto Mauricio. Julio 1993.

9318 *"Business Telephone Traffic Demand in Spain: 1980-1991, An Econometric Approach"*. Teodosio Pérez Amaral. Francisco Alvarez González. Bernardo Moreno Jiménez. Septiembre 1993. (Versión final aceptada para publicación en Information Economics and Policy)

9401 *"Contrastes de momentos y de la matriz deinformación"*. Teodosio Pérez Amaral. Junio 1994. (Versión final publicada en Cuadernos Económicos del ICE, Nº 55, 1993/3)

9402 *"A partisan explanation of political monetary cycles"*. José I. García de Paso. Junio 1994.

9403 *"Estadísticos para la detección de observaciones anómalas en modelos de elección binaria: Una aplicación con datos reales"*. Gregorio R. Serrano. junio 1994. (Versión final aceptada en Cuadernos de Estadística Española)

9404 *"Effects of public investment in infraestructure on the spanish economy"*. Rafael Flores de Frutos. Mercedes Gracia Díez. Teodosio Pérez Amaral. Junio 1994.

9405 *" Observaciones anómalas en modelos de elección binaria"*. Mercedes Gracia. Gregorio R. Serrano. Junio 1994. (Versión final aceptada para publicación en Estadística Española)

9406 *" Permanent components in seasonal variables"*. Rafael Flores. Alfonso Novales. Junio 1994.

9407 *" Money demand instability and the performance of the monetary model of exchange rates"*. Rodrigo Peruga. Junio 1994.

9408 *" Una nota sobre la estimación eficiente de modelos con parámetros cambiantes"*. Sonia Sotoca. Junio 1994.

9409 *" Distribución de la renta y redistribución a través del IRPF en España"*. Rafael Salas. Junio 1994.

9410 *" Trade balances: Do exchange rates matter?"*. Rodrigo Peruga. Junio 1994.

9411 *"A retrial system with constant attempts"*. M. Vázquez. Octubre 1994.

9412 *" A retrial model at nonstationary regime"*. M. Váquez. Octubre 1994.

9413 *"Equívocos y singularidades en el sistema financiero español"*. Jaime Terceiro Lomba. Noviembre 1994.

9414 *"On univariate forecasting comparisons:The case of the spanish automobile industry"*. A.García-Ferrer, J. del Hoyo, A.S. Martín-Arroyo, P.C. Young Junio 1994. (Versión final aceptada para publicación en Journal of Forecasting)

SERIE DE DOCUMENTOS DE TRABAJO DEL *ICAE*

9301 *"Análisis del Comportamiento de las Cotizaciones Reales en la Bolsa de Madrid bajo la Hipótesis de Eficiencia"*. Rafael Flores de Frutos. Diciembre 1992. (Versión final aceptada para publicación en Estadística Española)

9302 *"Sobre la Estimación de Primas por Plazo dentro de la Estructura Temporal de Tipos de Interés"*. Rafael Flores de Frutos. Diciembre 1992.

9303 *"Cambios de Estructuras de Gasto y de Consumo en el Cálculo del IPC"*. Antonio Abadía. Febrero 1993. (Versión final publicada en Revista de Economía Aplicada, Vol.1, N°1)

9304 *"Tax Analysis in a Limit Pricing Model"*. Félix Marcos. Febrero 1993.

9305 *"El Tipo de Cambio Propio: Reformulación del Concepto y Estimación para el Caso Español"*. José de Hevia Payá. Junio 1993. (Versión final publicada en Revista Esñañola de Economía, Vol.11, N°1, 1994)

9306 *"Price Volatility Under Alternative Monetary Instruments"*. Alfonso Novales. Abril 1992.

9307 *"Teorías del Tipo de Cambio: UnaPanorámica"*. Oscar Bajo Rubio. Simón Sosvilla Rivero. Junio1993. (Versión final publicada en Revista de Economía Aplicada, Vol.1, N°2).

9308 *"Testing Theories of Economic Fluctuations and Growth in Early Development (the case of the Chesapeake tobacco economy)"*. Rafael Flores de Frutos. Alfredo M. Pereira. Diciembre 1992.

9309 *"Maastricht Convergence Conditions: A Lower Bound for Inflation?"*. Jorge Blázquez. Miguel Sebastián. Marzo 1992.

9310 "Recursive Identification, Estimation and Forecasting of Nonstationary Economic Time Series with Applications to GNP International Data". A. García-Ferrer. J. del Hoyo. A. Novales. P.C. Young. Marzo 1993. (De próxima aparición en un volumen de homenaje a A. Zellner)

9311 *"General Dynamics in Overlapping Generations Models"*. Carmen Carrera. Manuel Morán. Enero 1993. (Versión final aceptada para publicación en Journal of Economic Dynamics and Control)

9312 *"Further Evidence on forecasting International GNP Growth Rates Using Unobserved Components Transfer Function Models"*. A. García-Ferrer, J. del Hoyo, A. Novales, P.C. Young. Septiembre 1993.

9313 *"Public Capital and Aggregate Growth in the United States: Is Public Capital Productive?"*. Rafael Flores de Frutos. Alfredo M. Pereira. Julio 1993.

9314 *"Central Bank Structure and Monetary Policy Uncertainty"*. José I. García de Paso. Abril 1993.

9315 *"Monetary Policy with Private Information: A Role for Monetary Targets"*. José I. García de Paso. Julio 1993.

9316 *"Exact Maximum Likelihood Estimation of Stationary Vector ARMA Models"*. José Alberto Mauricio. Julio 1993. (Versión final aceptada para publicación en Journal of the American Statistical Association)

9317 *"The Exact Likelihood Function of a Vector ARMA Model"*. José Alberto Mauricio. Julio 1993.

9318 *"Business Telephone Traffic Demand in Spain: 1980-1991, An Econometric Approach"*. Teodosio Pérez Amaral. Francisco Alvarez González. Bernardo Moreno Jiménez. Septiembre 1993. (Versión final aceptada para publicación en Information Economics and Policy)

9401 *"Contrastes de momentos y de la matriz deinformación"*. Teodosio Pérez Amaral. Junio 1994. (Versión final publicada en Cuadernos Económicos del ICE,N° 55, 1993/3)

9402 *"A partisan explanation of political monetary cycles"*. José I. García de Paso. Junio 1994.

9403 *"Estadísticos para la detección de observaciones anómalas en modelos de elección binaria: Una aplicación con datos reales"*. Gregorio R. Serrano. Junio 1994. (Versión final aceptada en Cuadernos de Estadística Española)

9404 *"Effects of public investment in infraestructure on the spanish economy"*. Rafael Flores de Frutos. Mercedes Gracia Díez. Teodosio Pérez Amaral. Junio 1994.

9405 *" Observaciones anómalas en modelos de elección binaria"*. Mercedes Gracia. Gregorio R. Serrano. Junio 1994. (Versión final aceptada para publicación en Estadística Española)

9406 *" Permanent components in seasonal variables"*. Rafael Flores. Alfonso Novales. Junio 1994.

9407 *" Money demand instability and the performance of the monetary model of exchange rates"*. Rodrigo Peruga. Junio 1994.

9408 *" Una nota sobre la estimación eficiente de modelos con parámetros cambiantes"*. Sonia Sotoca. Junio 1994.

9409 *" Distribución de la renta y redistribución a través del IRPF en España"*. Rafael Salas. Junio 1994.

9410 *" Trade balances: Do exchange rates matter?"*. Rodrigo Peruga. Junio 1994.

9411 *"A retrial system with constant attempts"*. M. Vázquez. Octubre 1994.

9412 *" A retrial model at nonstationary regime"*. M. Váquez. Octubre 1994.

9413 *"Equívocos y singularidades en el sistema financiero español"*. Jaime Terceiro Lomba. Noviembre 1994.

9414 *"On univariate forecasting comparisons:The case of the spanish automobile industry"*. A.García-Ferrer, J. del Hoyo, A.S. Martín-Arroyo, P.C. Young Junio 1994. (Versión final aceptada para publicación en Journal of Forecasting)