RECURSIVE IDENTIFICATION, ESTIMATION AND FORECASTING OF NONSTATIONARY ECONOMIC TIME SERIES WITH APPLICATIONS TO GNP INTERNATIONAL DATA*

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ABSTRACT

In this paper, we propose a novel, unobserved components model for annual GNP variations in a number of countries. The model is formulated in state space terms and estimated using recursive methods of filtering and fixed interval smoothing. The annual real output for nine countries are analyzed under both univariate and transfer function versions of the unobserved components model, the latter using money supply as a leading indicator. The forecasting performance of these models is compared with the forecasting results obtained in previous research on the same data set.

RESUMEN

En este trabajo proponemos un modelo novedoso de componentes no observables para las variaciones en el PNB anual en varios países. El modelo se formula en espacio de los estados y se estima mediante procedimientos recursivos de filtrado y de suavizado con la muestra completa. Se analiza el producto real anual de nueve países a partir del modelo de componentes no observables en sus versiones univariante y de función de transferencia, utilizando en esta última versión la oferta monetaria como indicador adelantado. Se compara el comportamiento de las predicciones de estos modelos con las obtenidas en trabajos anteriores utilizando el mismo conjunto de datos.

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1. INTRODUCTION

In their seminal paper, Zellner and Palm (1974) established an important milestone for future research within the field of econometric identification, estimation and forecasting of macroeconomic models. The importance of such a contribution lies not only in clarifying some bones of contention between econometricians and time series analysts, but also in recognizing that different equation systems, with different uses, require different identification restrictions that should always be tested with actual data.

Their novel SEMTSA modelling approach, later emphasized by Zellner (1979), specified the components of a model using as much sound background information as possible. This strategy led to excellent performance in forecasting out-of-sample data. Finally, economic theory considerations were used to combine the components together into a sensible, multivariate, structural econometric model.

Unfortunately, as Zellner(1991) himself recognizes, "practical applications of this approach --see Zellner and Palm (1975) for examples of small scale structural models of the U.S. economy-- showed the difficulties of producing models which would, at the same time 1) forecast well; 2) have a sound economic interpretation; and 3) serve policy makers' needs adequately". The failure to obtain multivariate models in a so-called "one shot" approach, latter confirmed by others [McNees (1987), Litterman (1986), Highfield (1986)], motivated a different strategy for modelling macroeconomic relationships taking a variable-by-variable approach. In the first of a series of papers, Garcia-Ferrer et al. (1987) picked up an important key variable, real gross national or domestic product, and computed one-step-ahead forecasts for the U.S. and eight EEC countries for the 1974-1981 period, using an autoregressive model containing leading indicator variables and various forecasting procedures. It was found that Bayesian shrinkage forecasting techniques produced improved forecasts in terms of an out-of-sample root-mean-square criterion, relative to those provided by three naive models, as well as AR models with and without leading indicators. The precision of the forecasts obtained in this manner compared favourably with those of the OECD agency, derived from complicated "structural" econometric models.

Since there was a possibility that the nine countries chosen in the above sample, as well as the period employed, were "special" in some sense, an expanded sample of countries, and a longer time period, were then considered (Zellner and Hong (1989)) with similar results. Also, further developments within the Bayesian framework [Zellner at al. (1990) and (1991), Min and Zellner (1993)], as well as Zellner (1991)] showed important improvements in establishing a well tested modelling strategy whose forecasting results have also been put under test [see among others Mittnik (1990) and Otter (1990)].

The plan of this paper, which reports our subsequent progress to date, is as follows: In Section 2, we present our theoretical model based on some recursive methods for unobserved components models developed by Young (1984), in which any parameter variation is characterized by a stochastic state-space (SS) model. In Section 3, we analyze the annual data on real output for the nine countries considered in Garcia-Ferrer et al. (1987) using the methodology sketched above, and compare its forecasting performance with the univariate forecasting results of the previous papers. Section 4 deals with the forecasting comparison when the information set is increased by including a possible leading indicator variable such as the money supply. This is shown to be helpful in reducing forecast errors in the vicinity of turning points. Finally, in Section 5, we present a summary of results and some concluding remarks regarding future research possibilities.

2. THE THEORETICAL UNOBSERVED COMPONENTS MODEL.

The stochastic state-space model belongs to the class of unobserved components ARIMA (UC-ARIMA) models developed by Engle (1978) and Nerlove et al. (1979) that have been popular in the forecasting literature for some years. However, it has only been recently that papers which exemplify a time variable parameter estimation (TVP) approach [Harvey (1984), Kitagawa and Gersch (1984), Engle at al. (1988) and Ng and Young (1990)] have been utilized within the context of SS estimation. In particular, Young et al. (1990) use a novel spectral interpretation of the SS smoothing algorithms to decompose the series into various, quasi-orthogonal components, the models for which can be identified and estimated using recursive methods of estimation [Young(1984)] that can handle TVP models.

Following Young and Young(1990), we can write the "component" or "structural" model of a univariate time series $\mathbf{Y_t}$ as:

$$Y_t = T_t + P_t + \varepsilon_t \qquad (2.1)$$

where $\mathbf{T_t}$ is a low frequency or trend component, $\mathbf{P_t}$ is a perturbational component around the long run trend, which may be either a zero mean stochastic component with fairly general statistical properties, a sustained periodic or seasonal component, or a component dependent upon some exogenous (leading indicator) variable; and finally, ϵ_t is a zero mean, serially uncorrelated, white noise component with variance σ^2_ϵ.

2.1 The Trend Model

It is assumed here that the low-frequency or trend component can be represented by a local linear trend model of the form,

$$\begin{aligned} T_t &= T_{t-1} + S_{t-1} + \eta_t \\ S_t &= S_{t-1} + \xi_t \end{aligned} \qquad (2.2)$$

where $\mathbf{S_t}$ denotes the local slope or derivative of the trend, and η_t and ξ_t are zero mean, serially and mutually uncorrelated white noise inputs with variances σ_η^2 and σ_ξ^2, respectively. It is further assumed that these noise inputs are statistically independent of the white noise observational errors ϵ_t in equation (2.1), and therefore:

$$E(\varepsilon_t, \eta_s) = E(\varepsilon_t, \xi_s) = E(\eta_t, \xi_s) = 0 \qquad \forall t,s \qquad (2.3)$$

By introducing a trend model of this type, it is assumed that the time-series can be characterized by a varying mean value whose variability will depend upon the nature of the model (2.2). It has been argued that, for smooth trends, η_t is mainly necessary to handle sharp discontinuities of level or slope [Young and Ng(1989)] and, unless they exist, it can be constrained to be zero, which is the assumption made in what follows. In this case, the variance of ξ_t is the only unknown in (2.2) and it can be defined by the Noise Variance Ratio (NVR), which is the relation between σ^2_ξ and the variance of the observational noise σ^2_ϵ, that is:

$$NVR = \frac{\sigma_\xi^2}{\sigma_\varepsilon^2} \qquad (2.4)$$

This NVR uniquely defines this "Integrated Random Walk" (IRW) model for the trend, since all the other parameters in the SS model are constrained to unity or zero. The estimation of the NVR value is discussed later in Section 2.3.

2.2 The Stochastic Perturbation Model.

If $\mathbf{P}_t$ is assumed to be purely stochastic with rational spectral density, then it can be represented by a General Transfer Function (GTF) model, similar to the ARIMA model employed by Box and Jenkins (1970), although no stationarity restrictions are imposed here. It is assumed that the sum of the stochastic perturbation $\mathbf{P}_t$ and the white noise component ϵ_t follows an ARMA representation of the form:

$$P_t + \varepsilon_t = \frac{\gamma(L)}{\phi(L)}\, a_t \qquad (2.5)$$

where:

$$\phi(L) = \sum_{j=0}^{m} \phi_j L^j, \qquad \phi_0 \equiv 1$$

$$\gamma(L) = \sum_{j=0}^{m} \gamma_j L^j, \qquad \gamma_0 \equiv 1$$

For convenience, the order $\mathbf{m}$ is assumed in this presentation to be the same for both polynomials; however, different orders can be introduced in empirical work without any further problem. In the empirical applications of the GTF model described in the next section, we will concentrate on the use of the purely autoregressive (AR) form of (2.5). In this case, an AR or

subset AR model is identified for the perturbations, using the Akaike Information Criterion (AIC)[1].

2.3 Model Identification and Estimation.

Having defined the SS model structures for all the components, it is then straightforward to assemble them into an aggregate SS form, where the state vector is composed of all the states from the different submodels, and the observation vector is chosen to extract from the state vector the structural components $\mathbf{T}_t$ and $\mathbf{P}_t$ [Young et al. (1989)]. However, the problems of structural identification (similar to the ones that appear in standard econometric models) and subsequent parameter estimation for the complete SS model are clearly non-trivial. As regards identification, the imposition of certain restrictions (given a particular structure) has been, in general, the way to achieve identification in the statistical literature on signal extraction. For practical purposes, however, it is important to verify the actual degree of orthogonality among the estimated components in order to avoid spurious decompositions commonly found with these procedures [García-Ferrer and del Hoyo(1992)]. In such cases, it can be shown that, through an adequate choice of the NVR (in particular, selecting a NVR value for the trend so that its estimate does not contain higher frequency components associated with the perturbational behavior), the degree of orthogonality between components can be considerably enhanced [García-Ferrer et al.(1993)].

In order to estimate the proposed model, the most obvious approach is to formulate the problem in maximum likelihood (ML) terms. If the disturbances are normally distributed, the likelihood function for the observations can be obtained from the Kalman filter by "prediction error decomposition" [Harvey(1984); Harvey and Peters(1990)]. However, practical experience with this approach indicates that it can turn out to be rather complex, even for particularly simple structural models [García-Ferrer(1992)], with the likelihood function tending to be rather flat and indeterminate around the optimum.

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[1] Under certain conditions,the model proposed in (2.1) to (2.5) can be written as an ARIMA(m,2,m+2) model for Y_t. So, our model implies that Y_t is I(2), which can be tested. Following Dickey y Patula (1987), we performed ADF tests to individual countries for the whole sample period, 1950-1984. The results indicate that we could not reject the I(2) hypothesis, except by the UK and US.

Despite its problems, this ML approach has become the standard in recent years, following from the work of Harvey(1984), Kitagawa(1981) and others. More recently, Young and Tych(1993), have optimised the NVR values so that the logarithm of the pseudo-spectrum[2] (see below) matches the logarithm of either the AR spectrum or the periodogram of the data, in a least squares sense. In this paper, however, we utilise a rather different 'manual' approach based on the spectral filtering properties of the fixed interval smoothing (FIS) algorithms used in the state-space analysis [see Young(1988); Young(1993); Young and Tych(1993)].

In order to explain this approach, it is instructive to consider again the simplest example of equation (2.1); namely where $\mathbf{Y}_t$ is represented by a simple trend plus noise model, i.e.,

$$Y_t = T_t + e_t$$

in which $\mathbf{T}_t$ is assumed to evolve as a IRW process; i.e., in vector matrix terms,

$$\begin{bmatrix} T_t \\ S_t \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xi_t$$

This model can be written in the following alternative TF form,

$$Y_t = \frac{1}{(1-L)^2}\,\xi_t + e_t \qquad (2.6)$$

where L is the lag operator, i.e. $LY_t = Y_{t-1}$, so that the autocovariance generating function g(L) for the model is defined by the following expression,

$$g(L) = g_T(L) + \sigma_e^2$$

where $g_T(L)$ is the autocovariance generating function for the IRW component alone, i.e.

[2] pseudo because the IRW model (2.6) is nonstationary.

$$g_T(L) = \frac{\sigma_\xi^2}{(1-L)^2\,(1-L^{-1})^2}$$

Bell(1984) has shown that the classical Kolmogorov-Wiener-Whittle aproach to filtering and signal extraction can be aplied to nonstationary processes such as (2.6). Consequently, for a sample size of N, where N tends to infinity, the optimal smoothing (signal extraction) filter for estimating $\mathbf{T}_t$ is given by the ratio of $g_T(L)$ to g(L). In terms of the NVR this can be written simply as,

$$\hat{T}_{t/N} = \frac{NVR}{NVR + (1-L)^2(1-L^{-1})^2}\,Y_t \qquad (2.7)$$

where $\hat{\mathbf{T}}_{t/N}$ is the optimally smoothed estimate of $\mathbf{T}_t$ at sampling instant t based on all N samples. This is a symetric, two sided filter requiring only the specification of the NVR value. It is easy to verify that this is a lag-free, low-pass filter with a sharp cut off for smaller values of the NVR and excellent filtering properties which attenuate all higher frequency noise on the data. The associated FIS algorithm has been used for many years [see Jakeman and Young(1979)] in the various versions of the CAPTAIN and *micro*CAPTAIN programs [e.g. Young and Benner(1991) for a description of the latest version] where it is termed the "IRWSMOOTH" algorithm. Since, for large N, the asymptotically optimal smoothing filter (2.7) will yield the same results as the recursive IRWSMOOTH estimator (except for samples near the beginning and end of the series) the IRWSMOOTH estimates will naturally have similarly favourable properties.

The effect of the NVR value on the filter characteristics is clear from Fig.1, which shows the associated spectral density function for various NVR values: clearly, the NVR controls the *bandpass* of the filter, which is reduced progressively as the NVR is reduced in size. For this filter, the relationship between $\log_{10}(F_{50})$, where F_{50} is the 50% cut-off frequency, and $\log_{10}(NVR)$ is approximately linear over the useful range of NVR values, so that the NVR which provides a specified filter cut-off frequency can be obtained from the following approximate relationship [see Young(1987); Ng and Young(1990)],

$$NVR = 1600\ [F_{50}]^4 \tag{2.8}$$

[INSERT FIGURE 1]

Also shown on Fig.1 is the spectral density plot for the related smoothing filter used by Kydland and Prescott(1990), as discussed below, which is an IRWSMOOTH-type filter for the *quarterly* data, with the NVR constrained to 0.000625.

In the case of trend estimation, it is clearly important that the smoothed estimate $\hat{\mathbf{T}}_{t/N}$ follows the low frequency components of $\mathbf{Y}_t$, and rejects those components $\mathbf{P}_t$ and $\mathbf{e}_t$, that can be considered as higher frequency cycles or noise. To this end, the NVR value can be chosen by the analyst so that the resulting trend estimate represents the component of $\mathbf{Y}_t$ in an appropriate low frequency band, chosen by a reference to equation (2.8). For example, in the case of macroeconomic data such as those considered in this paper, it is likely that the detrended data will need to explain business cycle effects, and so the NVR should be chosen so that the IRWSMOOTH algorithm removes the low frequency effects without afecting the higher frequency behaviour that may be associated with such business cycles.

In the case of annual data, for example, an NVR = 1.0 yields an $F_{50} = 0.09$ cycles/year (associated cyclical period P_{50} approx. 11.2 years) and $F_{10} = 0.16$ cycles/year (P_{10} aprox. 6.3 years), so that the estimated trend will contain very little power at cycles with periods of 6 years or less that may be associated with business cycle behaviour. For example, in the case of the logarithm of the USA GNP series, the periodogram of the estimated trend using this NVR value matches the periodogram of the original data exactly at the lower frequency range; whilst the periodogram of the detrended data shows clearly how these low frequency components have been removed, leaving the components in the frequency range greater than 0.16 cycles virtually unaffected. This is the major justification for our choice of NVR = 0.1 for the analysis described in subsequent sections of this paper[3]. It is interesting to note, however, that this value tends to be confirmed by the optimisation approach of Young and Tych mentioned above. For example, optimisation of this type based on an AR(15) spectrum of the US and UK GNP series yields NVR values of 0.14 and 0.079, respectively.

[3] The choice of trend characteristics is also discussed in more detail by García-Ferrer and Queralt(1992).

It is also instructive to note from equation (2.7) that, at every sampling instant t, the IRWSMOOTH filter (2.7) provides an estimate of the trend which depends upon a (nominally infinite dimensional) centralised moving average (CMA) of the data either side of t [see Young and Tych(1993)]. In more general terms, the weighting pattern of the CMA for the FIS smoothing filter is dependent upon the kind of state-space model chosen to model to parameter variations and, in particular, on the value of the associated NVR parameters. Fig.2 shows a typical example of this CMA weighting pattern for the IRWSMOOTH algorithm with NVR=0.1. From Fig.2, we see that the trend at any sampling instant t depends on a symmetric, weighted average of the data either side of t.

[INSERT FIGURE 2]

Finally, one caveat: it should be noted that, with short series such as those considered in this paper, it is advisable to estimate the trend *at the same time* as the perturbational components. For example, either by estimating all the model parameters simultaneously, as in the optimisation approach; or, more simply, by using the Dynamic Harmonic Regression (DHR) mode, as discussed by Young(1988,1993) and Young and Tych(193). In general, this will yield a better estimate of the trend near both the beginning and end of the series, where distortions can occur. Because the perturbations about the trend are so small in the present context, however, this did not appear to yield any significant improvement in forecasting ability and simple IRWSMOOTH trend estimation with NVR=0.1 was used for all the GNP series.

2.3 The Kydland-Prescott (KP) filter.

The fourth author first used the IRWSMOOTH filter for estimating trends in macroeconomic data during a visit to the Federal Reserve Bank of Empirical Macroeconomics (IEM) in Mineapolis in 1988, where he established that the resulting small perturbational variations in quarterly $\log_e$(GNP) and $\log_e$(Unemployment) for the USA are both seen to exhibit a pronounced 'business cycle' as well as a strong inverse linear relationship [see Young(1989),(1993)]. Since 1988, Kydland and Prescott of the IEM have carried out a similar but more extensive study with a wider range of variables [Kydland and Prescott(1990)], in which they have demonstrated that clearly visible dynamic relationships appear to exist between many of the small perturbational 'business cycle' variables obtained in this manner.

Kydland and Prescott actually use an equivalent to the IRWSMOOTH algorithm with NVR = 0.000625 (see Fig.1) obtained via a form of Lagrange multiplier constrained optimisation (regularisation), which they attribute to Hodrick and Prescott(1980). Jakeman and Young(1979, 1984) have demonstrated the equivalence of this KP approach and recursive IRWSMOOTH filter [see also Young(1991)]. However, we believe that the state space formulation of the latter filter is inherently more flexible, since it allows for more transparent and easy extension to the more complex models discussed both in this paper and the related paper by Young(1993). It is interesting to note that, for quarterly data, the KP selected NVR of 0.000625 corresponds to a filter bandwidth which is very similar to that used for the annual data here, with P_{50} and P_{10} approximately 9.9 and 5.7 years, respectively.This is, of course, not surprising, since KP had similar basic objectives to our own.

3. ANALYSIS OF GNP/GDP ANNUAL DATA FOR NINE COUNTRIES.

In this section, we analyze the same annual data 1950-1984 for real gross national product in the nine countries considered in García-Ferrer et al. (1987). The data were taken from the International Monetary Fund's International Financial Statistics. The countries are: Belgium, Denmark, France, Germany, Ireland, Italy, the Netherlands, the United Kingdom and the United States.

The steps we followed in analyzing the data were:

1. Real GNP data up to 1973 were used to obtain fitted values for $\mathbf{T}_t$ and $\mathbf{P}_t$ according to (2.2) and (2.5), based on an approach discused in Section 2 and using the *micro*CAPTAIN software package.
2. The fitted models were used to generate eight and eleven one-step-ahead forecasts, to cover the same 1974-1981 and 1974-1984 periods studied in García-Ferrer et al.(1987) and Zellner and Hong(1979). In order to obtain the one-step-ahead forecasts, the models were re-estimated using all past data prior to each forecast period. Conversion from the original forecasts to growth rates followed immediately.
3. Forecast errors were computed for each forecast period and country. The root mean squared errors (RMSE's) by country, as well as overall measures of forecasting precision, were obtained in order to appraise the forecasting performance of the different approaches.

3.1 Estimation of the Individual Country Unobserved Components Models.

Plots of the estimated trends are shown in Figures 3.a to 3.i, while the trend derivatives appear in Figures 4.a to 4.i. The graphs show the smoothed trend and derivative estimates, based on the full sample through 1984, using the Kalman filter and fixed interval smoother, as discussed in Section 2.

The plots of the detrended (perturbational) data P_t are shown in Figures 5.a to 5.i. In order to model these series, we first consider an AR model using the Akaike Information Criterion (AIC). For the whole sample set this indicates that an AR(8) model is appropriate in most cases, yielding coefficients of determination between R^2_T = .372 and R^2_T = .665 . In some cases, however, further examination indicates that a subset AR(8) with some parameters constrained to zero provides superior AIC, with R_T^2-values only marginally lower than the full AR(8) model. A summary of the identification and the last period estimation results is presented in Table 1.

[INSERT FIGURES 3, 4 AND 5]

[INSERT TABLE 1]

3.2 Forecasting Performance Comparisons in the Univariate Models.

In García-Ferrer et al.(1987) and subsequent papers, several univariate forecasting models were investigated. In the first place, the forecasting performance of 'naive models'(NM's) was analyzed in order to serve as a benchmark in evaluating the forecasting performance of more complicated models and procedures. The three naive models used to forecast the output growth rate were:

$$NM\ I \ :\ \hat{y}_t = 0 \qquad (3.1)$$

$$NM\ II \ :\ \hat{y}_t = y_{t-1} \qquad (3.2)$$

$$NM\ III \ :\ \hat{y}_t = past\ average \qquad (3.3)$$

For each country, the three NM forecasts in (3.1)-(3.2) were calculated in García-Ferrer at al.(1987) and Zellner and Hong(1989) for the periods 1974-81 and 1974-84, and are reproduced in lines A to C of Table 2.

[INSERT TABLE 2]

In general, the NM's forecast errors are rather large, particularly in the vicinity of the turning points in the rate of growth of output, which occurred in the neighborhoods of 1974-75, 1979-80 and 1983-84 for some countries. As a first step in attempting to improve on the forecast performance of these naive models, an AR(3) model on y_t was fitted to each country. Such a model was chosen to allow for the possibility of the autoregressive polynomial having two complex roots associated with a cyclical solution, plus one real root associated with a trend on the growth rate. The AR(3) model's forecasts are reproduced in lines D of Table 2, from the same sources as before. In general, these latter models did not produce a substantial improvement in overall performance in relation to the simpler ones, possibly due to the inclusion of unnecessary parameters. It was also noted that the AR(3) models had large forecasting errors in the vicinity of turning points.

Univariate, one-step-ahead forecasts for our unobserved component models (UCM) represented in equations (2.1), (2.2) and (2.5) can be obtained via the Kalman filter, by repeated application of the prediction equations developed in Young(1988) as simple by-product of the recursive forecasting and smoothing algorithms. In lines E of Table 2, the RMSE's for these model forecasts are shown. They range from 2.05 percentage points for Ireland, to 4.92 for Italy for the 1974-81 forecast period, and from 2.11 for France to 4.44 for Italy for the 1974-1984 period. Although the UCM models produce some improvement in comparison with the other univariate alternatives A to D (it has the smallest RMSE's for Belgium, France and the Netherlands), their overall performance did not produce a significant gain over the other univariate

models. In spite of having the smallest median of the nine countries RMSE's, once again the UCM models show large forecasting errors in the vicinity of turning points. It seem obvious that the reasons for this failure lie in the two models used for the trend T_t and the perturbations P_t. In the first case, a different, univariate alternative to the IRW seems necessary, while in the case of the perturbations, we will see that adding a leading indicator variable (namely, each country's money supply) helps to improve the forecasting performance. These possibilities are explored in the next section.

[INSERT TABLE 3]

4. TRANSFER FUNCTION MODELING BETWEEN ANNUAL GNP AND THE MONEY SUPPLY FOR THE NINE COUNTRIES.

There are two potential sources of problems with our univariate forecasting results in the previous section. One is related to the type of model used to forecast future trend values based on the IRW model described in (2.2). The other problem is concerned with the possibility of enlarging our information set by allowing for the presence of a leading indicator (LI) variable, such as the money supply, which can help to improve the information on the turning points [as in García-Ferrer et al.(1987) as well as Zellner and Hong(1989)].

4.1 The Trend Model Reconsidered

In many situations, the IRW model is particularly useful for describing large, smooth changes in the trend; other alternatives, like the RW, provide for smaller scale, less smooth variations [Young(1984)]. However, since the NVR value uniquely defines the performance of the algorithm, the trend estimates are strongly dependent on the chosen NVR.

On purely theoretical grounds, the reduced form equation corresponding to the IRW trend model would be an ARIMA(0,2,0). However, when we computed the autocorrelation (acf) and partial autocorrelation (pacf) functions for the second differences (∇^2) of our estimated trends in the previous section, the evidence (in all cases) was against the white noise hypothesis. On the contrary, as Table 4 shows, there is a well defined AR(2) structure in all countries in the sample, allowing for the possibility of a pseudo-cycle within the trend. In a sense, the NVR = 0.1

produces estimates similar to those of a cyclical trend model, as we can see from the trend derivative estimates in Figures 4.a to 4.i, which reveal the long term oscillatory behavior in the trend.

Bearing the above results in mind, the IRW model for the trend in (2.2) can now be extended in the following manner (see Young et al. 1992):

$$\begin{aligned} T_t &= T_{t-1} + S_{t-1} \quad (i) \\ S_t &= S_{t-1} + D_{t-1} \quad (ii) \\ \phi(L)\, D_t &= a_t \quad (iii) \end{aligned} \qquad (4.1)$$

where $\phi(\mathbf{L})$ is the AR polynomial:

$$\phi(L) = 1 + \phi_1 L + \phi_2 L^2 + \ldots + \phi_m L^m$$

which is equivalent to an ARIMA model:

$$\phi(L)\, \nabla^2 T_t = a_t$$

of the kind estimated for each of the trend models in Table 4. Equation 4.1(iii) can also be considered as a Double Integrated Autoregressive (DIAR) model. This model can be transformed easily into state space form [see Young (1993)], and adjoined with (i) and (ii) to yield a complete state space model for the trend, which then replaces the simpler IRW model.

[INSERT TABLE 4]

The estimation results for such cyclical trend models are shown in Table 4, together with measures of statistical fit (R^2) and the Ljung-Box (LB) test for the estimated residuals. In all countries, the AR structure produced complex roots which caused the expected long term oscillatory behavior, but of such a long period that it has no economic interpretation given our sample size[4]. In this table, the cycles embedded in the trend show periodicity ranging between around 10 years for Germany, Italy, U.K. and U.S.A., to 23 years for Netherlands. The last column in Table 4 shows the CCF between the estimated $\{\xi_t\}$ and $\{\varepsilon_t\}$ from (2.1)-(2.2).

4.2 Transfer Function Modelling Between the GNP and Money Supply Perturbations for Individual Countries.

Our second option in trying to overcome the problems discussed in the univariate forecasting exercises is the use of a leading indicator (LI) variable. In García-Ferrer et al.(1987), as well as in Zellner and Hong(1989), an autoregressive leading indicators (ARLI) model (using three lags of output growth rates, two lags of stock rates of return, one lag of the world stock rate of return, and one lag of the money supply growth rate, all in real terms) was employed to generate one year ahead forecasts for the growth rate of real output y_{it}, for eight and eleven periods: (1974-81 and 1974-84), for the nine countries considered in this paper. When this ARLI model was estimated by least squares, it was found that there was a clear improvement in forecasting performance relative to the use of both AR(3) and naive models. Further additional computations were performed to check the effects of using two types of Stein-like shrinkage techniques in forecasting, which generally produced better overall results.

As an alternative to the ARLI model, Mittnik(1990) examined (just for the 1974-81 period) the forecasting performance of a linear, time invariant state-space model. After determining the state-space dimension n (the rank of the Hankel matrix), the system matrices were derived with singular value decomposition techniques. It turns out that the difference equation representation implied by the resulting state-space model is similar to the previous ARLI model except that the latter does not posses an autoregressive component; rather, the second lag of the money supply growth rate appears as an explanatory variable.

In line with these models, in this paper we use the money supply as a potential leading indicator of real ouput growth. Consequently, we estimated the perturbations for the money supply

[4] This reflects the fact that our NVR choice just allows for some cyclical effects to start showing up in the estimated trends. Hong(1989) provides a careful study of the periodicities of the cyclical fluctuations in this GNP data set.

(PMS) using the same detrending procedure as with the GNP data, and the same NVR = 0.1 [5]. We then identified and estimated individual country TF models between these two perturbations (PMS and PGNP), which took the folowing general form,

$$PGNP_{it} = \frac{B(L)}{A(L)} PMS_{it} + \frac{D(L)}{C(L)} e_{it} \qquad \begin{array}{l} i = 1,...,9 \\ t = 1,...,T \end{array} \qquad (4.2)$$

where A(L), B(L), D(L) and C(L) are polynomials in the backward operator L.

Since both, $PGNP_t$ and PMS_t are mean stationary variables, the identification and estimation of the TF models (4.2) can be obtained directly by application of the Simplified Recursive Instrumental Variable (SRIV) algorithm developed by Young(1985) and included in the input-output option of microCAPTAIN [6]. The cross-correlation functions between the $PGNP_t$ and PMS_{t-s} components showed important contemporaneous and first lagged values, suggesting a dynamic relationship between the two variables.

However, a preliminary empirical analysis showed that: a) there is evidence of simultaneity in several countries, b) the dynamics were not homogeneous across countries, and c) the money supply was not always a leading indicator. We have focused on the one-sided relationship for consistency with the above mentioned references, leaving the analysis of the feedback relation for future research.

When the identified TF suggested a contemporaneous relationship between the two variables, we estimated an univariate ARIMA model for the input (PMS_t) and used their one-step-ahead forecasts to obtain future output values ($PGNP_t$) through the TF model (4.2).

[INSERT TABLE 5]

Table 5 shows the identification results: R_T^2 is the coefficient of determination based

[5] Notice that in obtaining the PMS data, we only need to specify the NVR for the trend, and there is no need to further identify its univariate model.

[6] At present, the I-O mode available in *micro*CAPTAIN does not have a forecasting option ready. We have, therefore, used the package mainly as a specification tool, while the estimation and prediction with the full model (the TF plus the noise model) have been carried out using the SCA statistical package.

on the full TF model and YIC [see Young(1988)] which is an identification criterion based on balancing the degree to which the model explains the data with how well the model parameter estimates are statistically defined. As with other identification criteria, the YIC is aimed at identifying models which explain the data well within an efficient parameterization.

4.3. Forecasting Performance of the TF Models.

Our purpose in this section is to produce one year ahead forecasts for GNP, in order to compare the forecasting performance of our TF models with the results obtained by the previously cited works. Our procedure for generating TF forecasts is similar to that discussed in Young(1988) and follows from the previous univariate (UCM) procedure discussed in Section 3.2. The estimated ARIMA model for the trend is combined with the TF model (4.2) within the Kalman filter framework, and the forecast is then obtained directly from the Kalman filter prediction equations. In essence, this is equivalent to combining the individual forecasts from the two, assumed quasi-orthogonal component models, as follows:

1. Based on the estimated ARIMA models for the GNP trends in Table 4 up to and including 1973, eight and eleven one-step-ahead forecasts for T_t for the years in our forecast periods, 1974-81 and 1974-84 are generated. As before, in making one-step-ahead forecasts, the models are re-estimated using all past data, prior to each forecast period[7].
2. Based on our TF model (4.2) and the empirical results of Table 5, similar one-step-ahead forecasts for the perturbations (P_t) of GNP are generated, once again with the models re-estimated prior to each forecast period.
3. Individual forecasts for the components (T_t and P_t) are added to obtain the aggregate output (Y_t) forecasts. Conversion from the original data forecasts to growth rates follows imediately and forecast errors can be computed for each forecast period and country.

[7] Note that this adaptive re-estimation is unusual in the economic aplications of the Kalman filter but is justified in this case since the latest parameter estimates are required for forecasting at any forecasting origin. In this case, the re-estimation was carried out separately for each component sub-model, since full recursive estimation of all the parameters, simultaneously, is a heavily nonlinear problem, not well suited for recursive estimation.

Root mean squared errors (RMSE) by country and forecast periods for the different alternatives are shown in Table 6. In rows F, we present the forecasting results obtained by least squares estimation of the AR(3) model with the leading indicators. Rows G1 and G2 show the AR(3)LI forecast results under different shrinkage alternatives. Both, F and G models are reproduced from Zellner and Hong (1989, Table 1, p.193). For the 1974-81 forecast period, row H shows the forecast results of the SSM analyzed by Mittnik(1990), and row I shows his forecast results with shrinkage parameter 0.5 . Finally, we present in rows J.1 and J.2 the RMSE's from the ARIMA(2,2,0) for the trend (TM) in Table 4, and our TF model (TFUCM) developed in sections 4.1 and 4.2 .

[INSERT TABLE 6]

Row J.1 shows that the autoregressive trend model (TM) by itself has an acceptable forecasting performance, beating the univariate models in Table 2 for some countries. Although it clearly underperforms the leading indicators models in Table 6, we have to remember that it is only one of the components of the GNP series. With the exception of France, the addition of the GNP/Money stock perturbation transfer models significantly improves the forecasting results, which become quite comparable with the rest of the models in Table 6. In terms of the median RMSE, the combined model (TFUCM in row J.2) decreases the median RMSE relative to the trend model (TM) by 11% (2.27 versus 2.55) in the 1974-1981 forecast period, and by 20% (2.04 versus 2.55) in the 1974-1984 period. Relative to our own univariate model (UCM in Table 2) the reduction in the median RMSE is of 18% (2.27 versus 2.78) for the 1974-1981 forecast period, and of 27% (2.04 versus 2.55) for the 1974-1984 period.

Our TFUCM model worked reasonably well in comparison with the other TF alternatives in Table 6. The comparison is specialy favourable in the longer forecast period, where our TFUCM model has the lowest RMSE's in six out of the nine countries. In that period, it does better than the AR(3)LI, which uses more sample information, in all countries but Belgium. The different relative performance between both forecast periods may well be due to the fact that our model does not capture very well the dramatic change that took place in the growth rates of most countries in 1975 and 1976. When averaging over the longer period, that forecast error gets more easily diluted.

Summary forecasting performance measures for the TF models are presented in Table

7. In the 1974-1984 forecast period, our TFUCM model produced the lowest median and average RMSE's among the different alternatives in Table 6. Plots of our TFUCM forecasts, calculated from (4.1) and (4.2) and observed annual growth rates are presented in Figures 6.a to 6.i.

5. SUMMARY AND CONCLUDING REMARKS

We have taken the stimulating research work started by Zellner and Palm (1974) on the steps of identification, estimation and forecasting with macroeconomic models, as a motivation for our own state-space approach to modelling the different components of an unobserved components model. In a long sequence of papers, professor Zellner and his collaborators have analyzed a data set on the key macroeconomic variables for the U.S. and eight european countries, which we ourselves have taken as the standard for comparison of our proposal.

Our strategy is based on an unobserved components model formulated in state space terms, in which the first component (trend) is second order stationary and includes some cyclical properties; whereas the second component (perturbation about the trend) is stationary and has an AR representation. The recursive estimation of the components in our procedure crucially hinges on the choice of the NVR parameters for the trend model, but there are objective rules based on the spectral nature of the recursive smoothing algorithms that result in a choice common to various GNP data sets.

Our unobserved components model is first used to obtain univariate forecasts for real output growth rates. We find that although the model produces (in some cases) improvement in comparison with the other univariate alternatives, its overall performance does not show a significant gain. In particular, as happend with its competitors, our univariate model shows large forecasting errors in the vicinity of turning points.

In order to overcome these forecasting failures, an alternative was formulated in Section 4 with two major modifications. On the one hand, the trend model was reconsidered by allowing a more flexible DIAR representation than the original IRW trend model. On the other hand, we employed a transfer function model for the GNP perturbations, using the real money supply as a leading indicator variable. The forecasting results using this latter approach improved our previous univariate results, and produced results comparable to other alternatives, using similar or even larger information sets. Our good forecasting results must emerge from the fact that our strategy is data based, so that a GNP model is chosen for each country on the basis of its time series peculiarities. However, we have also provided evidence on a number of common characteristics, like the trend NVR choice, which make this analysis quite practical and immediate, even if the number of subsamples (i.e., countries), is quite large.

These encouraging results suggest further extension of this work: a) estimates of the real money supply trend could be used in an attempt to improve on our trend forecasts; b) the feedback dynamic relation between GNP and the money supply needs to be incorporated in the analysis, e.g. by introducing a VAR model between GNP and money supply perturbations; c) other variables like the stock price index should be considered as a source of further improvement in the forecasting performance; d) given the visible correlation between all the GNP perturbations in Figure 5, a full multivariable model is clearly possible; and e) having been obtained with a rather novel methodology, our forecasts could be combined with those from some of the other approaches mentioned in this paper, to try to gain further forecasting efficiency.

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Table 1

AR Identification Results for the Perturbations P_t

Country	AR Model order	AIC	R^2_T
Belgium	8	6.797	0.393
Denmark	8	2.454	0.489
France	8	5.557	0.372
Germany	8	5.580	0.484
Ireland	8	8.457	0.665
Italy	6	4.935	0.372
Netherlands	8	2.358	0.493
U.K.	8	1.540	0.538
U.S.A.	8	6.969	0.354

Note: The estimation period was 1950-1983 for all countries.

Table 2

RMSE for the one-year-ahead forecast errors

Model	Belgium	Denmark	France	Germany	Ireland	Italy	Netherlands	U.K.	U.S.A.	Median
Forecast Period : 1974 - 1981										
A. NMI (y_t = 0)	3.09	2.83	2.96	2.95	4.38	3.72	3.77	2.21	3.48	3.09
B. NMII (y_t = y_{t-1})	4.25	3.73	2.43	3.26	2.06	4.88	4.04	3.91	3.60	3.73
C. NMIII (y_t = past average)	3.23	3.48	3.05	3.87	1.88	3.90	3.74	2.95	2.81	3.23
D. AR(3)	3.66	3.46	2.89	3.39	1.69	4.75	3.52	3.50	2.49	3.46
E. UCM (NVR = 0.1)	2.49	3.20	2.32	2.78	2.05	4.92	2.82	2.95	2.52	2.78
Forecast Period : 1974 - 1984										
A. NMI (y_t = 0)	2.59	2.78	2.65	2.69	4.02	3.27	3.32	2.30	3.79	2.78
B. NMII (y_t = y_{t-1})	3.53	3.56	2.20	2.91	2.85	4.26	3.57	3.69	3.89	3.56
C. NMIII (y_t = past average)	3.01	3.05	3.08	3.85	2.35	3.98	3.87	2.62	3.09	3.08
D. AR(3)	2.58	2.96	2.47	3.10	2.29	4.34	3.35	3.21	3.01	3.01
E. UCM (NVR = 0.1)	2.71	2.86	2.11	2.78	2.66	4.44	3.22	2.57	3.11	2.78

Table 3

Summary Forecasting Performance Measures for the Univariate Models

Model	Largest Country RMSE	Smallest Country RMSE	Median of Nine Countries RMSE
Forecasting Period: 1974 - 1981			
A. NMI (y_t = 0)	4.38	2.21	3.09
B. NMII (y_t = y_{t-1})	4.88	2.06	3.73
C. NMIII (y_t = past average)	3.90	1.88	3.23
D. AR(3)	4.75	1.69	3.46
E. UCM (NVR = 0.1)	4.92	2.05	2.78
Forecasting Period: 1974 - 1984			
A. NMI (y_t = 0)	4.02	2.59	2.78
B. NMII (y_t = y_{t-1})	4.26	2.20	3.56
C. NMIII (y_t = past average)	3.98	2.35	3.08
D. AR(3)	4.34	2.29	3.01
E. UCM (NVR = 0.1)	4.44	2.11	2.78

Note: Based on information in Table 2.

Table 4						
ARIMA Models for the Estimated Trend of GNP for the Nine Countries						
Country	Model	R^2	LB_3 , LB_6	CCF		
				-1	0	1
Belgium	$(1-1.533L+.689L^2)\nabla^2T_t = a_t$ (.14) (.14)	0.900	1.0 , 6.7	-.07	-.08	.02
Denmark	$(-1.195L+.373L^2)\nabla^2T_t = a_t$ (.17) (.17)	0.786	4.4 , 5.6	-.09	-.09	-.15
France	$(1-1.560L+.703L^2)\nabla^2T_t = a_t$ (.14) (.14)	0.913	4.5 , 6.2	.11	-.18	.07
Germany	$(1-1.416L+.818L^2)\nabla^2T_t = a_t$ (.12) (.12)	0.830	3.1 , 5.9	.03	.10	.07
Ireland	$(1-1.5740+.834L^2)\nabla^2T_t = a_t$ (.12) (.13)	0.890	2.9 , 4.9	-.07	.19	.18
Italy	$(1-1.446L+.783L^2)\nabla^2T_t = a_t$ (.13) (.14)	0.830	2.2 , 6.0	-.08	.20	-.15
Netherlands	$(1-1.413L+.647L^2)\nabla^2T_t = a_t$ (.15) (.15)	0.899	3.6 , 5.4	-.24	-.23	.26
U.K.	$(1-1.187L+.565L^2)\nabla^2T_t = a_t$ (.15) (.15)	0.701	2.9 , 6.6	.20	-.34	-.18
U.S.A.	$(1-1.464L+.804L^2)\nabla^2T_t = a_t$ (.12) (.12)	0.850	6.9 , 8.6	-.06	-.10	.09

Notes: a) LB_n denotes the Lunj-Box statistics for n degrees of freedom.
b) The estimation period was 1950-1983 in all cases.
c) The standard error for the CCF values between $\{\xi_t\}$ and $\{\epsilon_{t-s}\}$ is: $\hat{\sigma} = 0.183$

Table 5					
Transfer Function Models for GNP / Money Perturbations					
Country	Best Time Delay	A's	B's	YIC	R^2_T
Belgium	0	1	3	-0.654	.254
Denmark	1	2	2	-1.067	.468
France	1	2	2	-0.230	.194
Germany	1	2	1	-2.572	.561
Ireland	0	2	1	-2.642	.651
Italy	2	2	3	-1.915	.526
Netherlands	0	2	2	-1.040	.570
U.K.	2	3	3	-1.249	.641
U.S.A.	1	1	2	-3.616	.618

Table 6

RMSE for the one-year-ahead TF Forecast Errors: 1974-81 and 1974-84
(Percentage Points)

Model	Belgium	Denmark	France	Germany	Ireland	Italy	Netherlands	U.K.	U.S.A.	Median
Forecast period: 1974 - 1981										
F. AR(3)LI [a]	1.56	2.92	2.43	1.47	1.83	2.57	2.63	2.23	1.82	2.23
G. AR(3)LI [a]										
1. Shrinkage(1)	1.69	2.37	1.35	2.03	1.77	2.22	2.87	2.26	2.75	2.22
2. Shrinkage(2)	1.68	2.21	1.61	1.25	1.52	2.01	2.52	2.46	1.78	1.78
H. SSM [b]	2.24	2.68	2.43	1.41	1.35	1.64	2.62	2.23	1.30	2.23
I. SSM [c]										
1. Shrinkage(2)	2.04	2.07	1.52	1.11	1.48	1.51	2.42	2.40	1.47	1.52
J. UCM										
1. TM	2.82	3.05	2.11	2.39	2.50	3.29	2.55	2.83	2.29	2.55
2. TFUCM	2.31	2.27	2.14	2.08	1.77	2.64	2.28	2.43	1.75	2.27
Forecast Period: 1974 - 1984										
F. AR(3)LI [a]	1.73	2.73	2.52	2.28	2.80	3.40	2.41	2.32	2.14	2.41
G. AR(3)LI [a]										
1. Shrinkage(1)	1.96	2.26	1.66	2.00	2.14	2.45	2.53	2.39	2.79	2.26
2. Shrinkage(2)	1.81	2.37	2.07	1.94	2.31	2.73	2.50	2.63	2.03	2.31
J. UCM										
1. TM	2.55	2.67	1.89	2.22	2.47	3.03	2.57	2.49	2.59	2.55
2. TFUCM	2.04	2.37	1.91	1.78	1.88	2.36	2.22	2.13	1.84	2.04

NOTES: a) Zellner and Hong (1989, Table 1, p.193)
b) Mittnik (1990, Table 1, p.206)
c) Mittnik (1990, Table 3, p.207)

Table 7

Summary Performance Forecasting Measures for the TF Models

Model	Largest RMSE	Smallest RMSE	Median RMSE	Average RMSE
Forecasting Period : 1974 - 1981				
F. AR(3)LI	2.92	1.47	2.23	2.16
G. AR(3)LI				
1. Shrinkage(1)	2.87	1.35	2.22	2.14
2. Shrinkage(2)	2.52	1.25	1.78	1.89
H. SSM	2.68	1.30	2.23	1.99
I. SSM				
1. Shrinkage(2)	2.42	1.11	1.52	1.78
J. TFUCM	2.64	1.75	2.27	2.19
Forecasting Period : 1974 - 1984				
F. AR(3)LI	3.40	1.73	2.41	2.48
G. AR(3)LI				
1. Shrinkage(1)	2.79	1.66	2.26	2.24
2. Shrinkage(2)	2.73	1.81	2.31	2.26
J. TFUCM	2.37	1.84	2.04	2.06

Note: See footnotes a, b and c to Table 6.

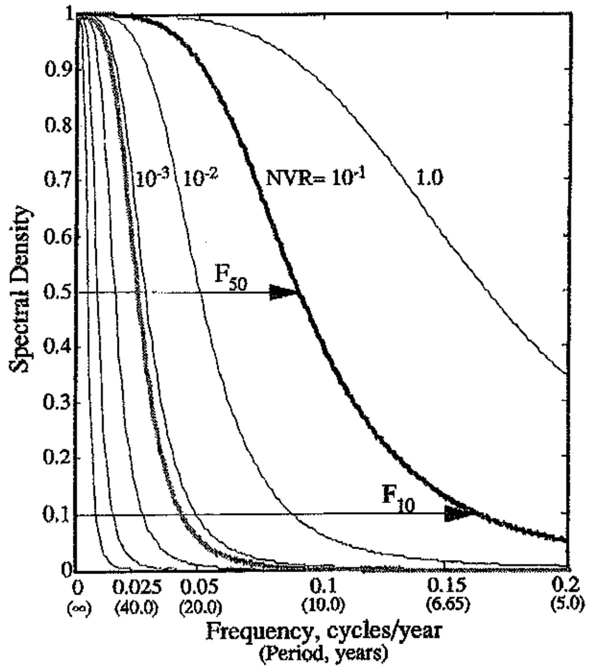

Fig. 1 Spectral characteristics of the IRWSMOOTH filter: the NVR=0.1 filter used in the present analysis is shown (thick dark) with the 50% and 10% bandwidths indicated by arrows; the filter characteristics for other NVR values (1.0, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}) are shown by fine lines. Also shown (thick light) for comparison is the KP filter for *quarterly* data (see text), where the frequency (period) axis should be read in cycles/quarter (quarters).

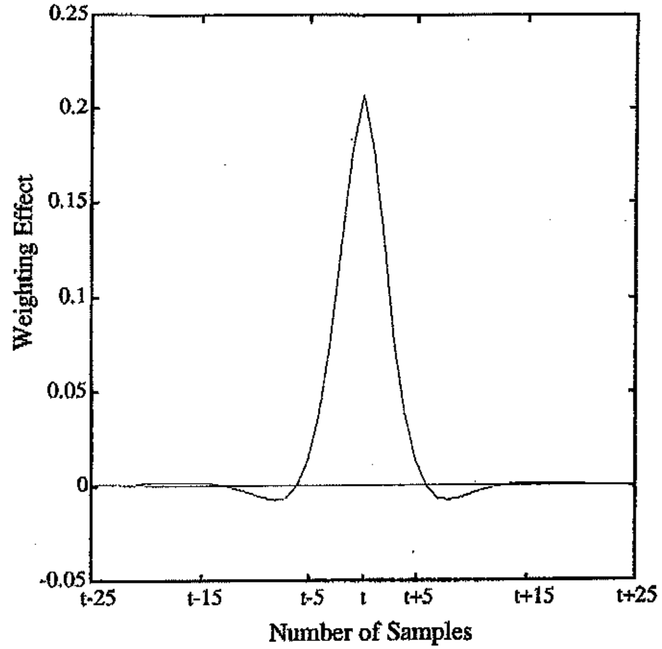

Fig. 2 Weighting effect of the IRWSMOOTH filter considered as a Centralised Moving Average (CMA) filter

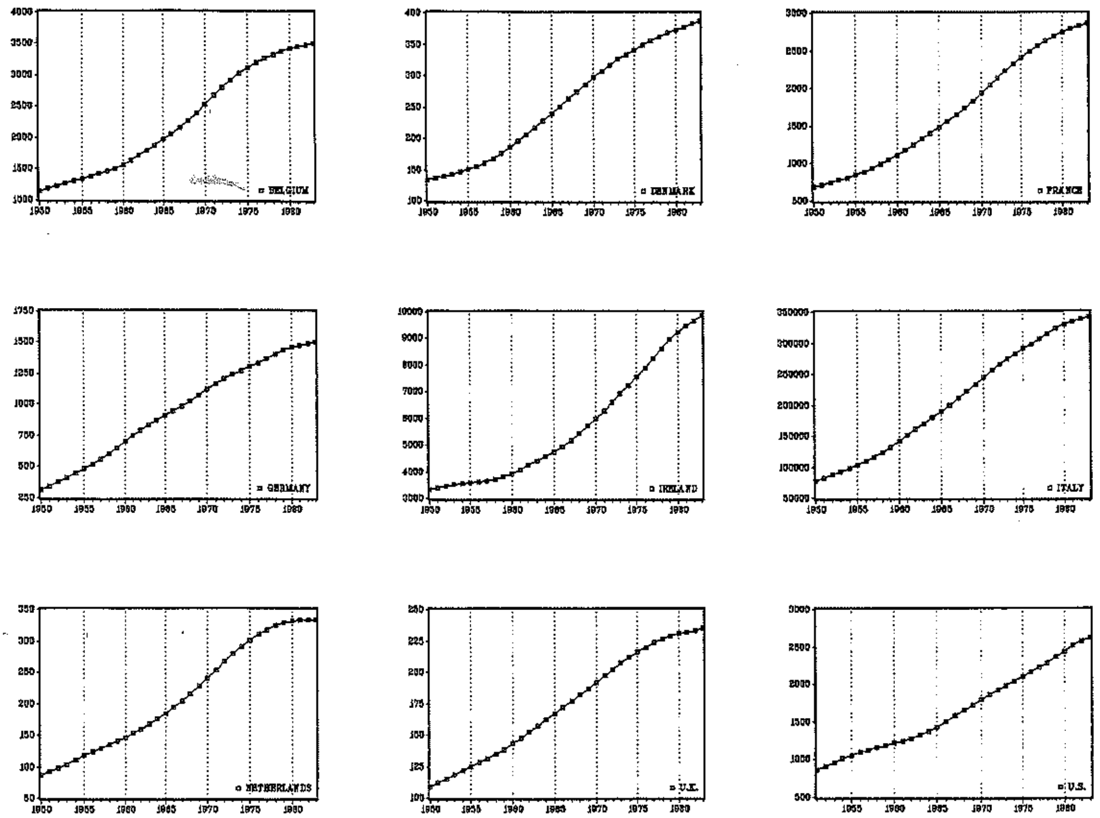

Figure 3. Plots of the Estimated GNP Trends for 9 Countries, 1950–1983

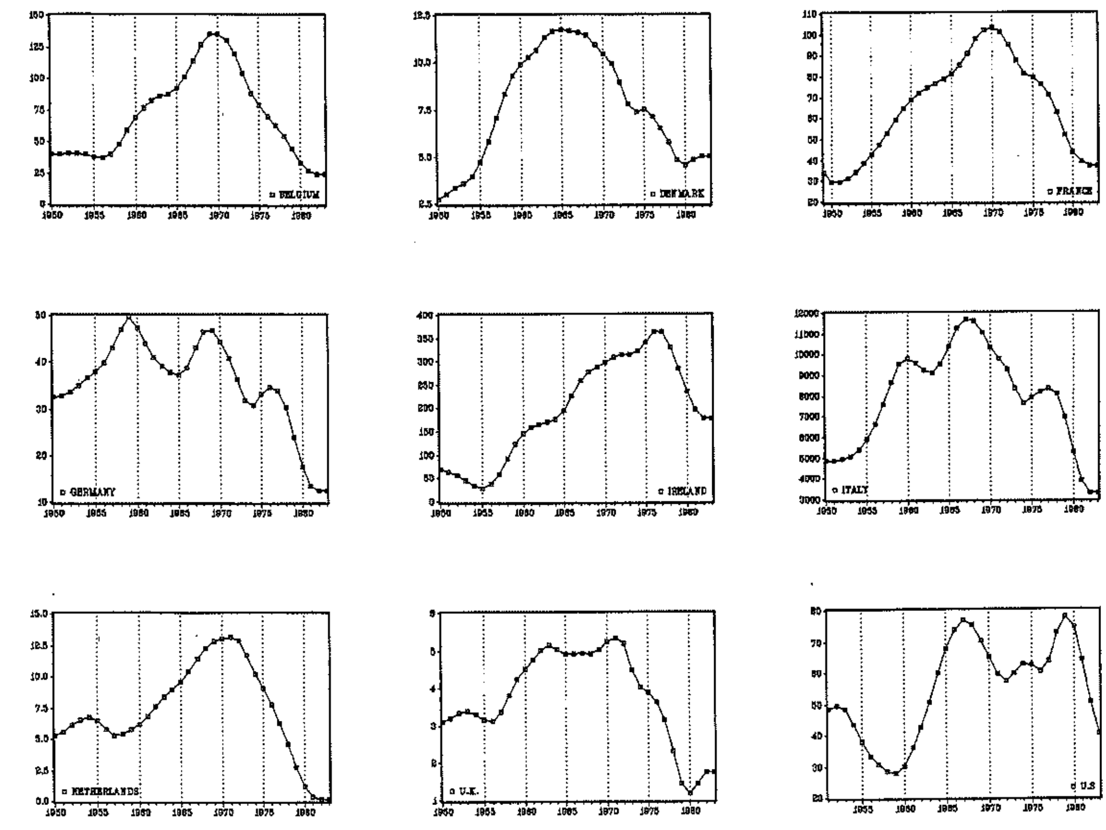

Figure 4. Plots of the Estimated GNP Trend Derivatives, 1950–1983

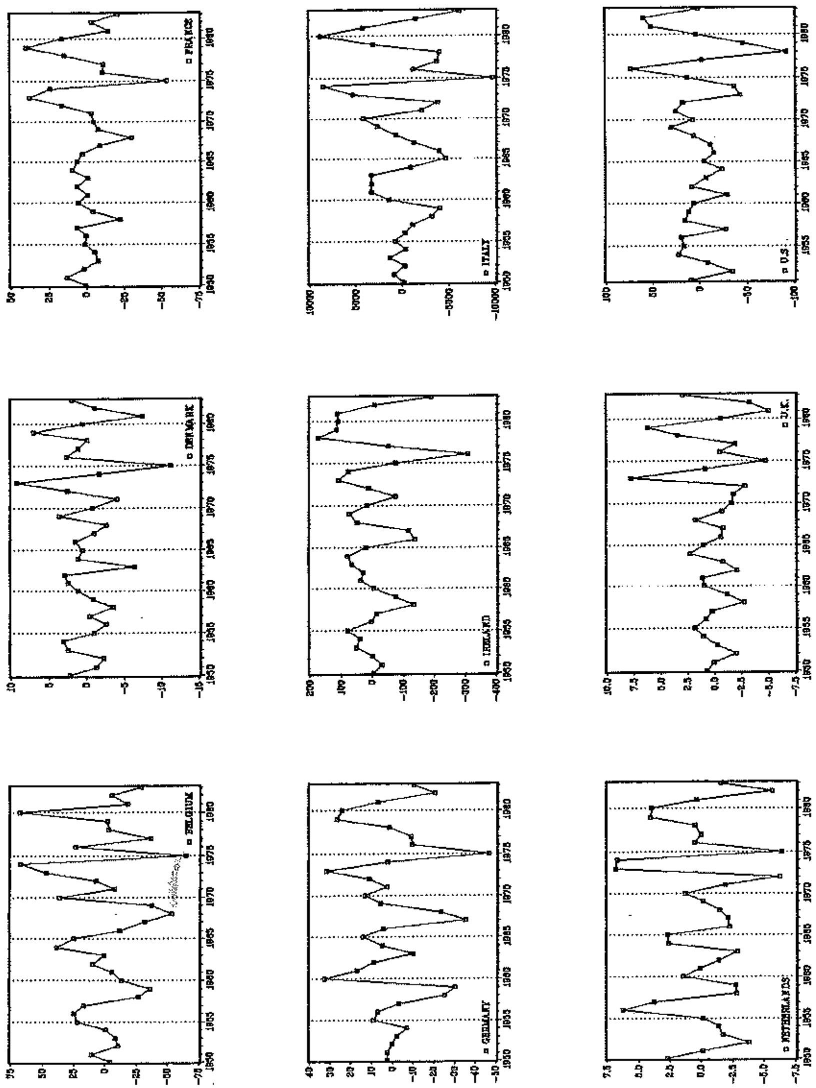

Figure 5. Plots of the GNP Perturbations for 9 Countries, 1950–1983

Figure 6.a: Observed and Predicted GNP Growth Rates for BELGIUM

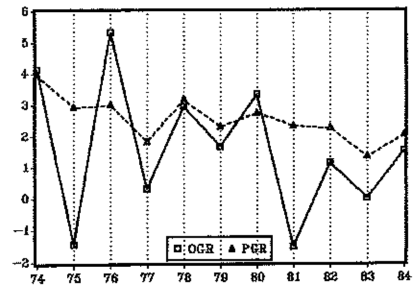

Figure 6.b: Observed and Predicted Growth Rates for DENMARK

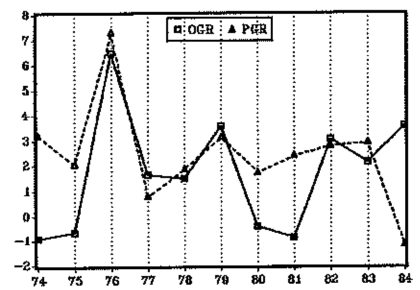

Figure 6.c: Observed and Predicted GNP Growth Rates for FRANCE

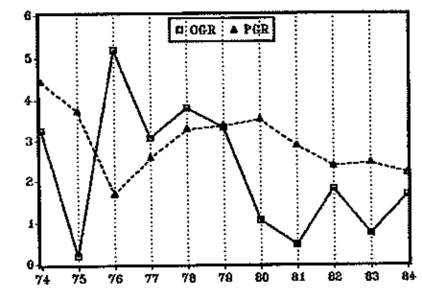

Figure 6.d: Observed and Predicted GNP Growth Rates for GERMANY

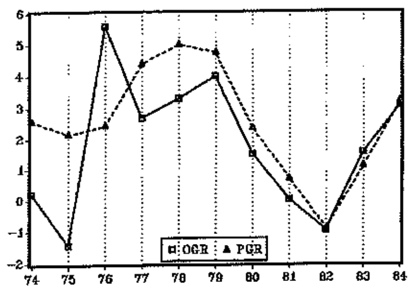

Figure 6.e: Oserved and Predicted GNP Growth Rates for IRELAND

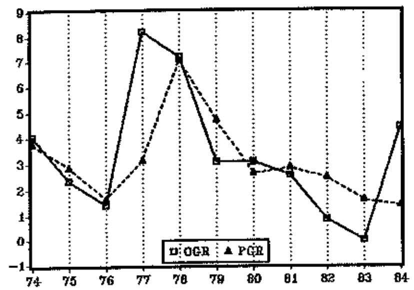

Figure 6.f: Observed and Predicted GNP Growth Rates for ITALY

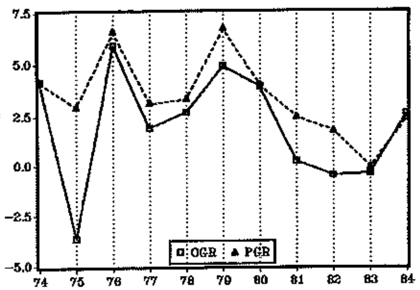

Figure 6.g: Observed and Predicted GNP Growth Rates for NETHERLANDS

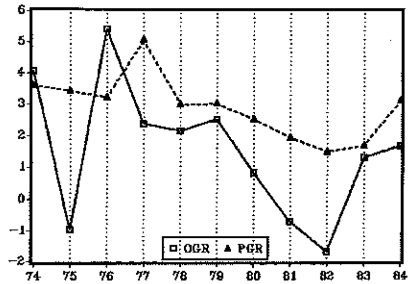

Figure 6.h: Observed and Predicted GNP Growth Rates for the U.K.

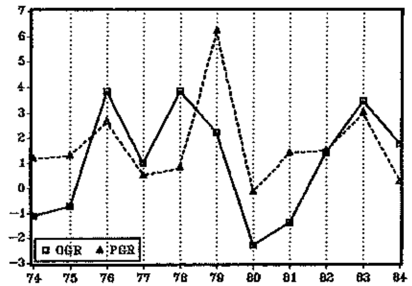

Figure 6.i: Observed and Predicted GNP Growth Rates for the U.S.