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Time Varying Risk Premia in General Equilibrium with Production

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Preliminary
Comments welcome

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ABSTRACT

Endowment economies have generally been considered when trying to reproduce the empirical rejection of the expectation hypothesis of the term structure as an implication of equilibrium asset pricing models. Previous attempts have not been successful: large risk aversion parameters are needed to produce sizeable risk premia and even then, the expectation hypothesis is not rejected. We present an economy with a time-to-build technology, in which consumption is subject to cash-in-advance constraints, in which the expectations hypothesis of the term structure does not hold. Monetary shocks are much more important than real demand or supply shocks in producing the result.

RESUMEN

Cuando se ha tratado de explicar teóricamente el rechazo de la hipótesis de expectativas que de modo bastante robusto se obtiene para distintos países y mercados, se han utilizado modelos de dotación, con resultados negativos. En ellos, es preciso introducir coeficientes de aversión al riesgo muy elevado para obtener primas de riesgo apreciables, pero ni siquiera entonces se rechaza la hipótesis de expectativas. Presentamos una economía con tecnología time-to-build y restricciones de efectivo por adelantado, en la que la hipótesis de expectativas acerca de la formación de la estructura intertemporal se rechaza. Las perturbaciones monetarias son mucho más importantes que las perturbaciones reales, de demanda u oferta, en la generación de este resultado.

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1. INTRODUCTION

A long tradition of empirical work performed on a broad variety of markets has led to a widespread rejection of the expectations hypothesis in the formation of the term structure of interest rates: '*Empirical work has produced consensus on little more than that the... [expectations hypothesis]...can be rejected*', Shiller et al. (1983), p.61. [see also Fama (1976, 1984a, 1984b, 1986, 1990), Fama and Bliss (1987), and Shiller et al. (1983)].

With the exception of Den Haan(1995), endowment economies have been used when trying to explain the In those models, the work/leisure decisions of consumers play no role in determining equilibrium quantities and prices. Furthermore, since the size of the exogenous endowment is usually the main source of randomness in the economy, the assumed stochastic structure plays a crucial role in determining the equilibrium behaviour of decision and state variables and, in particular, of asset prices. Alternatively, a simple representation of the endowment process, estimated with actual output data is added to the model, at the cost of using restrictions not fully justified by the equilibrium model. In some other cases [Danthine and Donaldson(1986)] a given stochastic behaviour for inflation is imposed on the model, but in none of these instances are the assumed processes fully justified by their model.

That limitation tries to avoid the difficulties inherent to solving dynamic, stochastic, general equilibrium models. In some cases [Giovannini and Labadie(1991), Labadie(1989,1994)], solution methods, specific to the economies considered, have been developed. As an alternative, tractability of the model's equilibrium is often achieved by assuming a low-dimension state space Markov process for endowment.

In an endowment economy with two states of nature, Backus, Gregory and Zin (1989) conclude that the sign of the term premia is contrary to that of the autocorrelation in the marginal rate of substitution of the representative consumer. A similar result, for a somewhat different economy, was obtained by Salyer(1990) and Labadie (1994). But, in actual economies, the autocorrelation of detrended per capita consumption growth is essentially zero, so that equilibrium models that fit the behaviour of actual consumption data cannot explain the observed, positive term premia [den Haan(1995), Labadie(1994), Backus et al.(1989)].

In addition, to generate term premia of the magnitude estimated with US Treasury bills, it is necessary to use risk aversion coefficients higher than 8 or 10 [Backus et al.(1989)], which seem unlikely. This result is similar in spirit to the *equity premium* puzzle [Mehra and Prescott(1985)]. But, even when the autocorrelation of consumption growth and or the risk aversion coefficient are fixed

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at values that allow for reproduction of the observed term premia, equilibrium models of endowment economies are unable to produce enough volatility to explain the rejection of the expectations hypothesis in actual data [Backus et al.(1989)].

As a summary, these papers point out to the apparent inability of theoretical equilibrium models to explain the sign, magnitude and variability of term premia, specially in the shorter maturity end. Most authors are aware that the models may be too restrictive to reproduce some of the empirical observations concerning the term structure in actual economies. It is particularly interesting that sample standard deviations for the returns on longer maturities are substantially below those of the shorter maturities [Gibbons and Ramaswany(1986), Backus et al.(1989), Donaldson, Johnsen and Mehra(1990)]. This is against the evidence in post-war US data, where the volatilities in nominal and real bond returns smoothly decrease as we move to longer maturities. Den Haan (1995) points out the bias of traditional monetary models to alarmingly undervalue the variance of nominal returns at all maturities. In a paper with endogenous production and a shopping time transaction structure, he obtains return volatilities that smoothly decrease in the term to maturity, taking values close to those in actual US data.

The alternative of considering production economies, as in den Haan(1995), which we also adopt in this paper, leads to a full endogenous characterization of equilibrium prices and quantities. It precludes reaching closed form expressions for the decision and state variables on which to perform short and the long run sensitivity analysis, but existing numerical solution methodologies can be used to compute the empirical distribution of any characteristic of the stochastic equilibrium vector process that represents the economy.

As in the previous references, our analysis is designed to test whether the empirical evidence is consistent with a specific framework to explain term premia, the general equilibrium theory of the formation of asset prices, whose origins are Merton(1973), Lucas(1978), Breeden(1979), Brock(1982), and specially, Cox, Ingersoll and Ross (1985). Such a model implies restrictions that characterize the joint time evolution of quantities and prices, which allow for analyzing and testing some theoretical aspects of asset pricing beyond those considered in purely empirical work [like Brown and Dybvig(1986), Stambaugh(1986), and Gibbons and Ramaswamy(1986)].

Incorporating agents who choose time sequences for their decision variables so as to maximize their respective intertemporal objective functions, we can derive time series equilibrium to be confronted with those from actual economies. We bring money into our model through cash-in-advance restrictions as in Lucas(1982) and Svensson(1985). Private as well as public consumption must be paid for with cash. A time-to-build technology, as in Kydland and Prescott (1982) uses three production inputs: physical capital, inventories, and labor. While we believe inventories not to be crucial in interest rate determination, we think that the work/leisure decision plays a relevant role in

determining the behaviour of savings and its returns. Furthermore, a time-to-build technology provides us with a diversification of investment at t into different varieties, each with a distinct return.

The chronology of financial flows produces an inability to spend salaries and rents from capital to pay for consumption in the current period. That, in turn, pegs the equilibrium value of the shorter-term nominal interest rate to current real variables. As a consequence, monetary shocks have much more effect on longer term rates, which become related to inflation fluctuations, losing information on future returns. Longer term returns are, as in actual data, less volatile than short-term rates, but their volatility increases much more than that of short term returns in the presence of monetary shocks. The final result is that current longer term returns have less than perfect information on future short-term rates, to the point of frequently leading to rejection of the expectations hypothesis of the term structure.

Monetary shocks are much more likely than shocks in technology or preferences, to produce the variability in term premia that leads to rejection of the expectations hypothesis, as it is by and large the case in empirical work. The rejection is also more likely for longer forecast horizons. We present the structure of the model in Section 2, its deterministic steady-state being described in Section 3. We explain in Section 4 how to obtain time series equilibrium realizations. Section 5 contains a discussion on the equilibrium implications for the term structure of interest rates, and Section 6 analyzes the relative importance of each type of shock for the obtained results. The paper closes with some conclusions.

2. A MONETARY, GENERAL EQUILIBRIUM MODEL OF THE TERM STRUCTURE.

We consider an economy with a representative household, who owns the only firm in the economy. There is a Government, which spends some resources each period, financed through lump sum taxes, money creation and public debt issuing. Government expenditures do not play any role in production, nor do they affect household's preferences. The household is made up of a financial intermediary, a worker, a shopper and a firm manager. Private, as well as public consumption, must be paid for with cash. There are hence cash-in-advance constraints as in Lucas(1982) for the shopper, as well as for the Government, but just for the consumption good. Investment can be purchased on credit. At the beginning of each period t , the household holds money, M_t , which is divided between the intermediary and the shopper. Then, the financial intermediary goes to the financial market, the shopper goes to the commodity market, the manager to the firm, and the worker to the labor market.

Financial markets open first, and the intermediary, as well as the Government establish their

money demands. In addition, the financial intermediary decides the quantity of Government bonds she wishes. The Government decides at that point on its expenditures and financing mechanism, i.e., on how much to consume as well as on the distribution of its purchasing expenditures between tax collections, and net money and bond creation. Tax revenues are collected at this point.

After closing the financial markets, the labor market opens and production takes place. The firm manager hires some labor and produces output using labor, the stock of physical capital, and inventories as inputs. Afterwards, the market for the consumption/investment good opens and both, shopper and Government purchase consumption good using the money they acquired in the financial market. The firm retains some production to finance its investment and distributes the rest, as dividends, to the household. The commodity market closes for the day.

At the end of the session, the firm pays the worker for the labor he provided, and delivers the household the dividends obtained during the period. All markets are closed, so these funds are retained by the household until markets open next day.

Production: We consider a time to build technology of physical capital accumulation as in Kydland and Prescott(1982). Physical capital is subject to depreciation, and needs J periods to become productive. The firm pays a proportion φ_j of each project during the $j=1,2,\dots,J$ periods until it becomes productive, with $\varphi_1+\dots+\varphi_J=1$. We denote by $S_{j,t}$ the number of investment units which are, at time t , j periods away from completion, $j=1,2,\dots,J$. The law of motion for physical capital is therefore:

$$\begin{aligned} k_{t+1} &= (1-\delta)k_t + S_{1,t} \\ S_{j,t+1} &= S_{j+1,t} \end{aligned} \quad (1)$$

where δ is the rate of depreciation of productive capital. Projects already started are not left unfinanced. Choosing $S_{j,t}$ at time t , the firm is deciding the stock of capital k_{t+J} which will become productive at time $t+J$. The decision on k_t , the stock of capital which is productive at time t , was made at $t-J$ and before. Total investment, I_t , is each period the aggregate financing, in different proportions, of all investment projects which are not yet completed, together with the variation in inventories:

$$I_t = \sum_{j=1}^J \varphi_j S_{j,t} + y_{t+1} - y_t \quad (2)$$

where y_{t+1} is the stock of inventories at the end of t . It is a production factor at time $t+1$.

The production technology for time t output, q_t , is again as in Kydland and Prescott(1982):

$$q_t = F(\xi_{2,t}, k_t, n_t, y_t) = \xi_{2,t} n_t^\theta \left[(1-\sigma) k_t^{1-\sigma} + \sigma y_t^{1-\sigma} \right]^{\frac{1-\theta}{\sigma}} \quad (3)$$

where n_t denotes hours of employment, ξ_2 is a multiplicative shock in productivity that follows a stationary distribution with expectation one. The shape of the production function guarantees a positive demand for the three production inputs each period.

At each production point, the firm utilizes the stocks of inventories and physical capital accumulated from previous periods. It observes the realization of the random productivity shock, and decides how much labor to hire. When the realization of the shock is known to the firm, the stocks of physical capital and inventories are already given. Once output has been produced, the firm pays the labor factor, makes investment decisions, and distributes dividends D_t . Hence, the firm knows $\Omega_t = \{k_{t+J+1,t}, y_{t+1,t}, n_{t,t}, c_{t,t}, \xi_{2,t+1,t}\}$, $s \geq 1$, when it makes its decisions on labor, n_t , and investment, I_t .

This information scheme is in line with the stochastic structure assumed for the productivity shock in Kydland and Prescott(1982)¹. Our specification implies that the marginal rate of transformation between both types of capital at time t is already known at time $t-1$, since the shock $\xi_{2,t}$, which appears in the productivity of both types of capital, disappears in their ratio:

$$MRT_t^{k,y} = \frac{MP_t^k}{MP_t^y} = \frac{(1-\sigma)}{\sigma} \left[\frac{k_t}{y_t} \right]^{-(1+\theta)} \quad (4)$$

The firm distributes output between salary payments, investment and dividends D_t :

$$\omega_t n_t + D_t + I_t = q_t \quad (5)$$

where all variables, including wages, ω_t , are in real terms, using output as numeraire. They do so to maximize the expected present value of current and future dividends that will be delivered to the household:

¹ There is more complex. The productivity shock is split into several components which are sequentially observed by the firm. In that fashion, different decisions are made on the basis of distinct informational specifications, which allows for identifying investment on inventories apart from that on physical capital. We will see in section 5 that our assumption helps in the identification of our model as well.

$$\begin{aligned} & \underset{(D_t, n_t, k_{t+1}, s_{1,t}, s_{2,t}, \dots, s_{J-1,t})}{\text{Max}} E_0 \sum_{t=0}^{\infty} (\beta^t U_t^c) D_t \\ & \text{given } y_0, k_{-1}, s_{1,0}, s_{2,0}, \dots, s_{J-1,0} \end{aligned} \quad (6)$$

subject to (1), (2), (3) and (5). The firm discounts future profits using current information² and, in particular, the marginal utility of current consumption, in spite of the fact that dividends will not be used by the consumer until next period.

The optimality conditions are:

$$F_t^n = \omega_t \quad (7)$$

$$\varphi_j (U_t^c - (1-\delta)\beta E_t U_{t+1}^c) + \dots + \varphi_1 \beta^{j-1} E_t (U_{t+j-1}^c - \beta(1-\delta)U_{t+j}^c) = \beta^j E_t (F_{t+j}^k U_{t+j}^c) \quad (8)$$

$$U_t^c = \beta E_t ((1+F_{t+1}^y) U_{t+1}^c) \quad (9)$$

together with the transversality conditions:

$$\lim_{t \rightarrow \infty} \beta^{t+j} E_t (k_{t+j} U_{t+j}^c) = 0 \quad (10)$$

$$\lim_{t \rightarrow \infty} \beta^{t+1} E_t (y_{t+1} U_{t+1}^c) = 0 \quad (11)$$

where superindices indicate partial derivatives and E_t is the expectation conditional on the information set Ω_t .

Along the optimal path, labor is hired each period to the point where its marginal productivity is equal to the real wage. New investment projects are started so that the utility loss of devoting resources to finance all the projects under construction is equal to the expected future utility gain derived from the implied increase in output. Inventories are accumulated to the point where their future marginal product is expected to exactly compensate its owner for the current loss of utility. The transversality conditions select paths along which the expected current value of the terminal stocks of physical capital and inventories are each equal to zero.

The household: The household derives utility from the only consumption good, as well as from leisure. Total available time is normalized to one each period. The utility function is:

² Notice that the ratio of two successive discount factors is the marginal rate of substitution of consumption over time. If we started from a generic discount factor μ_t , we would obtain that condition as part of the characterization of equilibrium. In some cases, it is assumed that the discount factor used for the firm incorporates the fact that time t dividends will be used in consumption at time $t+1$. For instance, Christiano (1991) use U_{t+1}^c/P_{t+1} to discount nominal time t dividends.

$$\begin{aligned} U(c_t, l_t) &= \frac{1}{1-\gamma} \left[c_t^{\xi_{it}} l_t^{1-\xi_{it}} - 1 \right]^{1-\gamma} = \frac{1}{1-\gamma} \left[c_t^{\xi_{it}} (1-n_t)^{1-\xi_{it}} - 1 \right]^{1-\gamma} \\ E[\xi_{it}] &= \alpha \quad \forall t \end{aligned} \quad (12)$$

where c_t , l_t , and n_t denote consumption, leisure and working time, respectively. It is a constant relative risk aversion utility function, as in Kydland and Prescott, although with time separability of leisure. It includes a shock ξ_{it} that makes the marginal rate of substitution between consumption and leisure to randomly evolve over time:

$$\text{MRS}_t^{c,l-n} = \frac{\xi_{it}}{1-\xi_{it}} \frac{1-n_t}{c_t}$$

α indicating the relative importance of consumption and leisure in the utility function.

The household can transfer resources over time buying nominal bonds, B_{t+j}^j , issued by the Government each period t with maturity horizons $j=1, 2, \dots, J$. They offer to pay a nominal return i_t^j when they mature at time $t+j$, $j=1, 2, \dots, J$. Yields i_t^j on time t bonds are known by investors when they are issued and bought. At time t there is a portfolio of bonds maturing, those issued at time $t-j$ with maturity j , $j=1, 2, \dots, J$. The Government also imposes lump-sum taxes T_t on the household to finance its purchasing expenditures.

With our proposed chronological sequence of markets, the household owns at the beginning of each period: 1) a wide portfolio of nominal bonds with maturities at time t , $t+1, \dots, t+J-1$, purchased in previous periods, and 2) $M_t = P_{t-1} \omega_{t-1} n_{t-1} + P_{t-1} D_{t-1}$ monetary units which brings along as the result, at time $t-1$ prices, P_{t-1} , of the activities of the financial intermediary and the worker at time $t-1$: labor rents plus dividends. Financial markets open and the intermediary materializes his demand for money M_{t-1}^c and bonds, receiving the returns on maturing bonds and paying taxes:

$$\begin{aligned} & \frac{B_{t+1}^1}{P_t} + \dots + \frac{B_{t+1}^J}{P_t} + \frac{M_{t+1}^c}{P_t} - \left[\frac{M_t^c}{P_t} - c_{t-1} \frac{P_{t-1}}{P_t} \right] = \\ & \frac{P_{t-1} \omega_{t-1} n_{t-1} + P_{t-1} D_{t-1}}{P_t} + (1+i_{t-1}^1) \frac{B_t^1}{P_t} + \dots + (1+i_{t-1}^J) \frac{B_t^J}{P_t} - T_t \end{aligned} \quad (13)$$

After the financial markets close, the worker offers some of his time, output is produced, and the commodity market opens. There, the shopper faces a liquidity constraint that forces her to pay for consumption good with money:

$$c_t \leq \frac{M_{t+1}^c}{P_t} \quad (14)$$

where M_{t+1}^c is the quantity of money she brings from the money market, and P_t is the price of the consumption commodity. So long as nominal interest rates are positive, which will be the case in equilibrium, this cash-in-advance constraint is satisfied with equality. At the end of the period, the worker receives salary payments and dividends are given to the household, the only owner of the firm.

The household chooses consumption and leisure each period to maximize the expected present value of current and future utility, discounted at rate β , $0 < \beta < 1$, on the basis of the information set $\{n_{t+s}, c_{t+s}, D_{t+s}, \xi_{1,t+1-s}, s \geq 1\}$ and subject to the budget constraint (13) and the cash-in-advance constraint (14):

$$\begin{aligned} & \text{Max}_{\{c_t, M_{t+1}^c, B_{t+1}^1, \dots, B_{t+1}^J\}} \beta^t U(c_t, l_t) \\ & \text{given initial conditions: } M_0^c, B_0^1, \dots, B_{0-(J-1)}^J \\ & \text{taking as given: } i_t^1, \dots, i_t^J, r_t^1, \dots, r_t^J, T_t, P_t, \omega, \end{aligned}$$

leading to the optimality conditions:

$$\frac{U_t^{1-n}}{P_t} = \beta \omega_t E_t \left[\frac{U_{t+1}^c}{P_{t+1}} \right] \quad (15)$$

$$\frac{U_t^c}{P_t} = \beta^j (1+i_t^j) E_t \left[\frac{U_{t+j}^c}{P_{t+j}} \right], \quad j=1, 2, \dots, J \quad (16)$$

and transversality conditions:

$$\begin{aligned} \lim_{t \rightarrow \infty} E_t \left[\beta^t U_t^c \frac{M_{t+1}^c}{P_t} \right] &= 0 \\ \lim_{t \rightarrow \infty} E_t \left[\beta^t U_t^c \frac{B_{t+1}^j}{P_t} \right] &= 0 \quad j=1, 2, \dots, J \end{aligned} \quad (17)$$

Equation (15) has a clear interpretation: working one more hour today raises revenues by the nominal wage: $P_t \omega_t$, at the same time it decreases current utility by U_t^{1-n} . The proceedings can be used tomorrow to purchase the consumption commodity. The expected increase in tomorrow's utility of an additional unit of currency is given by the conditional expectation in (15). In terms of current utility, we have to discount by β .

Combining (15) and (16) for $j=1$, the optimal consumption/leisure decisions by the household is characterized by:

$$U_t^{1-n} = \frac{\omega_t U_t^c}{1+i_t^1} \quad (18)$$

which can be seen as the supply of labor function. The worker supplies hours of work to the point where the marginal rate of substitution between current consumption and leisure becomes equal to real wages. Labor payments are discounted by the return on one period bonds because they cannot be spent until $t+1$. By definition, the demand for leisure is the complement to one of n_t : $l_t = 1-n_t$.

We could have equivalently considered real bonds issued at time t , b_{t+j}^j , $j=1, 2, \dots, J$, paying real returns r_t^j at time $t+j$. We would then have optimality conditions involving real rates of interest:

$$U_t^c = \beta^j (1+r_t^j) E_t U_{t+j}^c, \quad j=1, 2, \dots, J \quad (19)$$

A condition similar to (18) does not arise with real returns, precisely because the friction not to be allowed to use current salary payments in today's consumption precludes the household to shelter from inflation.

The quantities demanded of the bonds at different maturities, $b_t^1, b_t^2, \dots, b_t^J$ are not fully identified, since they are all substitutable assets. Supply conditions would be needed to characterize them. We can however obtain the total amount of resources devoted to purchasing Government bonds each period, but not its allocation among the different maturities. Without loss of generality we treat the whole portfolio as a single on-period bond. That will not preclude us from determining the nominal and real returns of each individual bond, whose characteristics at the distinct horizons, $j=1, 2, \dots, J$, will be different in an endogenous way.

The Government: The Government realizes a consumption G_t , which finances raising taxes, T_t , and issuing money and bonds:

$$G_t - T_t = \frac{M_{t+1}^c - M_t^c}{P_t} + \frac{B_{t+1}^1 - (1+i_{t-1}^1) B_t^1}{P_t} + \dots + \frac{B_{t+1}^J - (1+i_{t-J}^J) B_t^J}{P_t} \quad (20)$$

where $G_t, T_t, M_t, P_t, B_{t-(J-1)}^J, i_{t-1}^1, \dots, i_{t-J}^J$ represent public consumption at t , lump-sum taxes, money at the end of period $t-1$, time t prices, the volume of bonds issued in period $t-j$, $j=1, 2, \dots, J$ all maturing at t , and their respective nominal rates of return³.

We assume the Government is successful at maintaining its planned expenditure policy, and

³ With bonds paying interest at all periods before maturity, this restriction gets more complicated, but our problem does not gain any generality and results do not change.

also that this adopts a very simple form, being constant over time:

$$G_t = G, \quad G > 0 \quad \text{given,} \quad \text{for all } t \quad (21)$$

We keep a simple public expenditure policy because we want to concentrate on the relevance of monetary policy, as well as random production and preferences, in explaining the main characteristics of the term structure.

Public consumption must be paid for with money, so that the government is subject to a cash-in-advance constraint similar to that of the consumer:

$$G = \frac{M_{t+1}^G}{P_t} \quad (22)$$

so that it needs to purchase money in the financial markets at time t .

On the other hand, we assume that the Government has a less than perfect control of the growth rate of money supply:

$$M_{t+1} = (g + \xi_{3t})M_t; \quad E[\xi_{3t}] = 0 \quad (23)$$

which is subject each period to a random deviation from its target. In real terms:

$$(1 + \pi_t) m_t = (g + \xi_{3,t-1}) m_{t-1} \quad (24)$$

Fiscal policy is defined by a constant level of public consumption and a lump-sum tax each period, which varies over time as a function of the stock of debt, and an equation is needed that determines either the evolution of the stock of bonds, or the tax rule. In the simpler case when there are just one period bonds and the money supply is controlled with no error, we get:

$$b_{t+1} = \frac{1+i_t}{1+\pi_t} b_t + \frac{1}{1+\pi_t} (G - T_t) - (g-1) \frac{m_t}{1+\pi_t} \quad (25)$$

where $b_t = B_t/P_t$ and we have used the fact that public consumption is paid for with Government money holdings. A well-known long-run equilibrium condition in the deterministic case which is expected to also hold here is that the gross real rate of interest $(1+r)$ be equal to the inverse of the discount parameter β , and being this less than one, the previous is an explosive first order autoregression in b_t , although the presence of taxes and money growth will tend to stabilize it⁴. To avoid this lack of stationarity, and ignoring the stabilizing effect of money growth, we can fix a tax schedule which responds to the stock of bonds:

⁴ Monetary contractions would tend to destabilize the time evolution of b_t .

$$T_t = T + a b_t \quad (26)$$

where lump-sum taxes have each period a constant component, T , plus a component that depends on the stock of bonds through a . A relationship of this kind guarantees stability of the system⁵ [see Leeper(1991) and Sims(1994)].

To summarize, the Government starts the period by deciding on public consumption, taxes and the amount (positive or negative) of money and bonds that wishes to put in circulation. When financial markets open, the government buys money, pays the consumer/investor the return on the maturing bonds, puts in circulation new money and bonds, and collects taxes. After that, the market for the consumption good opens, and the Government uses its money holdings to purchase the desired commodity units.

Equilibrium: Given parameter values, including public expenditures G and money growth g , and paths for taxes T_t and bonds $B_{t+1}^1, \dots, B_{t+1}^J$, a *competitive equilibrium* is a set of initial conditions: $P_0, M_0^c, M_0^g, y_0, k_0, s_{1,0}, s_{2,0}, \dots, s_{J,0}$ together with real functions defined on $(0, \infty)$: $\{c_t, l_t, n_t^d, k_{t+j}, y_{t+j}, D_t, M_{t+j}^c, M_{t+j}^g, M_{t+j}, i_t^1, \dots, i_t^J, r_t^1, \dots, r_t^J, P_t, \omega_t\}$ such that:

- i) given $i_t^1, \dots, i_t^J, r_t^1, \dots, r_t^J, P_t, \omega_t, T_t$ and the initial conditions $M_0, B_0^1, \dots, B_0^{J-1}$, the vector of functions $\{c_t, l_t, M_{t+j}^c, B_{t+1}^1, \dots, B_{t+1}^J\}$ solves the utility maximization problem of the consumer,
- ii) given $y_0, k_0, s_{1,0}, s_{2,0}, \dots, s_{J,0}, P_t, \omega_t$, the functions: $\{n_t^d, k_{t+j}, y_{t+j}, D_t\}$ solve the maximization problem of the firm,
- iii) $1 - l_t = n_t^d$ for all t , which determines equilibrium in the labor market,
- iv) $M_t^c + M_t^g = M_t$ for all t , the aggregate money demand by consumer and Government is equal to the money supply,
- v) the household purchases all bonds of different maturities $j=1, 2, \dots, J$, issued by the Government,
- vi) the budget and cash-in-advance constraints (20) and (22) for the Government are satisfied at all periods.

⁵ Substituting into (25):

$$b_{t+1} = \frac{1+i_t - a^T}{1+\pi_t} b_t$$

it is easy to see that so long as a^T falls inside the open intervals $(i_t - \pi_t, 1 + i_t)$ and $(1 + i_t, 2 + i_t + \pi_t)$, the resulting autoregressive process will be stationary.

Equilibrium in the labor market implies that the marginal rate of substitution between current consumption and leisure is equal to the marginal product of labor, normalized by short-term nominal rates:

$$F_l^n = RMS_l^{1-n,c} (1+i_t^1) \quad (27)$$

On the other hand, it is easy to see that Walras' law guarantees that equilibrium holds in the consumption commodity market: First, the budget constraints of household and Government, can be combined into:

$$\frac{M_{t+1}^c}{P_t} + G - \frac{M_{t+1}^g - M_t^g}{P_t} = \frac{P_{t-1} \omega_{t-1} n_{t-1} + P_{t-1} D_{t-1}}{P_t} \quad (28)$$

But, since the Government is subject to a cash-in-advance constraint, and the money market is in equilibrium, i.e., $M_t^c + M_t^g = M_t$, then (28) implies:

$$M_t = P_{t-1} \omega_{t-1} n_{t-1} + P_{t-1} D_{t-1} \quad (29)$$

so that the financial flow that the consumer receives at the end of $t-1$ is equal to the total money supply. If we write (29) at time $t+1$, divide through by P_t , and substitute the money market equilibrium condition *iv*) in the aggregate cash-in-advance constraints of both agents:

$$c_t + G = (g + \xi_{2t}) m_t \quad (30)$$

we get:

$$c_t + G_t = \omega_t n_t + D_t \quad (31)$$

The right hand term in (31) is equal, from the firm's constraint (5), to production, net of investment. That way, we get:

$$c_t + G_t + I_t = F(\xi_{2t}, n_t, K_t, y_t) \quad (32)$$

which implies equilibrium in the market for the consumption commodity, where output is split among private and public consumption, plus investment.

3. DETERMINISTIC STEADY-STATE

In the deterministic steady-state of this model with diminishing returns in each productive

input, all decision and state per capita variables are constant over time⁶, and random shocks are equal to their means. The optimality conditions of the consumer and the firm, together with their budget constraints and that of the Government plus fiscal and monetary policy rules, imply, in a deterministic steady state:

$$U_c^*(c^*, n^*) F_k^n(k^*, n^*, y^*) + (1+i^1) U_n^{1-n}(c^*, n^*) = 0 \quad (33)$$

$$\frac{1-\beta}{\beta} = F_n^y(k^*, n^*, y^*) \quad (34)$$

$$\frac{1-\beta(1-\delta)}{\beta^j} (\varphi_j + \beta\varphi_{j-1} + \dots + \beta^{j-1}\varphi_1) = F_k^*(k^*, n^*, y^*) \quad (35)$$

$$\frac{1}{\beta^j} = \frac{1+i^j}{(1+\pi^*)^j} = 1+r^j \quad (36)$$

$$G - T^* = \pi^* m^* + (\pi^* - i^*) b^* \quad (37)$$

$$1 + \pi^* = g \quad (38)$$

$$T^* = T + a^T b^* \quad (39)$$

$$c^* + \delta k^* + G = F(k^*, n^*, y^*) \quad (40)$$

$$c^* + G = (1 + \pi^*) m^* = g m^* \quad (41)$$

$$\omega^* = F_n^* \quad (42)$$

$$D^* = q^* - \omega^* n^* - \delta k^* = F(k^*, n^*, y^*) - \omega^* n^* - \delta k^* \quad (43)$$

$$I^* = \sum_1^j \varphi_j (\delta k^*) \quad (44)$$

where stars denote steady state values, meaning that, in steady state, the rate of inflation is equal to the rate of money creation. We also have in steady state: $s_{1,t}^* = \delta k^* = s_{2,t+1} = s_{3,t+2} = \dots = s_{j,t+(j-1)}$. The gross real rate of interest on j -periods is the inverse of β^j , while the gross nominal return is equal to the real rate times inflation rate accumulated over the maturity of the asset. The marginal product of inventories goes to 0 when β approaches 1, and increases without bound as β falls to zero. In that case, steady state investment would be equal to zero. Knowing interest rates, steady state stock of

⁶ In general, steady state would be the situation of constant growth rates for the relevant per capita variables can be maintained forever. In this neoclassical model with diminishing returns in all production inputs, the only sustainable value for those growth rates is zero.

physical capital, inventories, and the levels of employment and consumption are simultaneously derived from (33), (34), (35) and (40), for a given level of public consumption. Note that, through inflation, the rate of money creation affects nominal interest rates, which influence the steady state levels of consumption and leisure as well as those of physical capital, inventories and output. We get output from the production function, real balances from (41) and finally, (37) and (39) provide us with the long-run equilibrium values of lump-sum taxes and bonds as functions of the level of public consumption. G. Real wages are equal to the marginal product of labor, dividends are given by the difference between output and the aggregate of salary payments and investment, the latter being just the depreciation loss.

4. EQUILIBRIUM ANALYSIS

The stochastic properties of this general equilibrium model cannot be characterized analytically, since the optimality conditions contain conditional expectations of nonlinear functions of state and decision variables. They can however be estimated through analysis of equilibrium time series realizations, which is what we do in the following sections. By an equilibrium realization we mean a set of time series for the relevant variables that fluctuates around the deterministic steady state we just described.

4.1 Stability conditions

Numerical characterization of the equilibrium requires the use of stability conditions, since the optimality conditions and policy rules by themselves will not produce stable trajectories. For that, it is necessary to specify the stochastic processes followed by the structural perturbations in preferences, productivity, and money control, which will have to be stable. To obtain the stability conditions, we represent the system made up by all the mentioned equations, together with the processes for the structural shocks in technology, preferences, and money growth:

$$H(z_t, z_{t-1}, \dots, z_{t-J}) = 0 \quad (45)$$

$$z_t' = (c_t, n_t, k_{t+J}, y_{t+1}, m_t, b_t, \pi_t, T_t, i_{t-1}^1, i_{t-2}^2, \dots, i_{t-J}^J, r_{t-1}^1, r_{t-2}^2, \dots, r_{t-J}^J, \xi_{1t}, \xi_{2t}, \xi_{3t})$$

for which a first order Taylor approximation can be constructed and written as a multivariate autoregression in deviations with respect to steady state values:

$$H_0 \tilde{z}_t = -H_1 \tilde{z}_{t-1} - \dots - H_J \tilde{z}_{t-J} \quad (46)$$

where:

$$\tilde{z}_t' = \{c_t - c^*, n_t - n^*, k_{t+J} - k^*, y_t - y^*, m_t - m^*, b_t - b^*, \pi_t - \pi^*, T_t - T^*, i_{t-1}^1 - i^{1*}, \dots, i_{t-J}^J - i^{J*}, r_{t-1}^1 - r^{1*}, \dots, r_{t-J}^J - r^{J*}, \xi_{1t} - \xi_1^*, \xi_{2t} - \xi_2^*, \xi_{3t} - \xi_3^*\} \quad (47)$$

$$H_i = \partial H / \partial z_{t-i} \Big|_{z^*}, \quad i = 0, 1, \dots, J$$

which can be converted to a first order autoregression:

$$A \tilde{x}_t = B \tilde{x}_{t-1} \quad (48)$$

where:

$$\tilde{x}_t' = \{z_t', z_{t-1}', \dots, z_{t-(J-1)}'\}$$

$$A = \begin{bmatrix} H_0 & 0 & 0 & \dots & 0 \\ 0 & I & 0 & \dots & 0 \\ 0 & 0 & I & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & I \end{bmatrix} \quad B = \begin{bmatrix} -H_1 & -H_2 & -H_3 & \dots & -H_J \\ I & 0 & 0 & \dots & 0 \\ 0 & I & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & I & 0 \end{bmatrix} \quad (49)$$

where I is the identity matrix of order J and to the equations that form system (46), we have added, in a standard fashion, auxiliary equations to accommodate in the state vector the lags in that system. Stability conditions are obtained as orthogonality conditions between the state vector z_t and the eigenvectors corresponding to eigenvalues in the product matrix AB' with modulus greater than β^1 .

Solving the model, we lag all the optimality conditions that involve variables at different points in time: (8), (9), (16) and (19), so as not to find in z_t consumption or labor beyond time t . We also lag the Government budget constraint so as not to have future prices in the system. As a consequence, physical capital and inventories appear just in the global constraint of resources for the whole economy, so the columns in H_0 associated to these two variables have nonzero values just in the row corresponding to that constraint. Matrix H_0 , and hence, A , are then singular. We avoid that by also lagging one period the global resources constraint. The infinite eigenvalue which indicated singularity then disappears. The eigenvalues of the transformed system, all finite, numerically

coincide with the remaining eigenvalues of the original system⁷.

The number of stability conditions increases with the number of maturity periods considered, being equal to $J+1$:

$$g^i \left(c_t, n_t, k_{t+J-1}, y_t, m_t, r_{t-1}^1, r_{t-2}^2, \dots, r_{t-J-1}^{J-1}, \xi_{1t}, \xi_{2t}, \xi_{3t} / \text{lagged values} \right) = 0 \quad (50)$$

$$i = 1, \dots, J+1$$

The real rate of return on the longer bond never shows up in the stability conditions. Variables from the monetary sector: bonds, nominal rates and inflation and fiscal variables, public expenditures and taxes, do not appear in the stability conditions either. If we did not impose condition (26) that taxes were pegged to the stock of real bonds, then there would be an additional stability condition linking the stock of real bonds to state and decision variables. Any small deviation from its steady state would produce an explosive equilibrium trajectory.

Introducing in the model an active public expenditure policy, linking it to deviations of output from its steady state, for example, does not alter the number of stability conditions.

4.2 Equilibrium time series

To solve the model, we follow Sims suggestions (1984, 1989, 1990) to write the expectations of nonlinear functions of future state and decision variables that appear in the optimality conditions as being equal to their realized value plus the corresponding expectation error. There is no approximation error involved in that representation. We have $2J+2$ equations involving expectations: two sets of J optimality conditions in nominal and real bonds (16) and (19), plus optimality conditions (8) and (9) on K_{t+j} and y_{t+1} . There also are $2J+2$ expectation errors:

$$e_t^j = U_t^c - E_{t-j} U_t^c; \quad j = 1, 2, \dots, J;$$

$$u_t^j = \frac{U_t^c}{P_t} - E_{t-j} \left(\frac{U_t^c}{P_t} \right); \quad j = 1, 2, \dots, J;$$

$$a_t^1 = F_t^j U_t^c - E_{t-1} (F_t^j U_t^c); \quad \epsilon_t^j = F_t^k U_t^c - E_{t-j} (F_t^k U_t^c);$$

Since there are three structural shocks, then there can be at most three independent expectation errors. Errors associated to the expectations of the same function, like e_t^j and u_t^j ,

⁷ This difficulty does not mean that the two assets are redundant. For that, their optimality conditions should coincide, which happens just when the depreciation rate is zero. A possible redundancy could not possibly be solved by just lagging an equation, and some additional identifying assumption would be needed.

$j=1, 2, \dots, J$ with same origin and different forecasting horizon are moving average processes, having as innovation the one step ahead expectation error. In addition, the fact that the marginal rate of transformation between both types of capital belongs to the previous period information set implies the exact relationship:

$$\epsilon_t^1 = MRT_t^{k,y} a_t^1 \quad (51)$$

between the innovation ϵ_t^1 in the ϵ_t^1 -expectation error and a_t^1 . Hence, only three of the innovations are independent: u_t^1 , e_t^1 and either a_t^1 or ϵ_t^1 .

If we rewrite the conditional expectations as we said, then we have the system formed by: a) the $J+1$ stability conditions (50), b) the economy-wide cash in advance constraint (30), c) the constraint on global resources:

$$c_{t-1} + \varphi_J (k_{t+(J-1)} - (1-\delta)k_{t+(J-2)}) + \dots + \varphi_1 (k_t - (1-\delta)k_{t-1}) + y_t - y_{t-1} + G = F(\xi_{2(t-1)}, k_{t-1}, n_{t-1}, y_{t-1}) \quad (52)$$

which comes from combining (1), (2) and (32), d) the optimality conditions on capital stock and inventories:

$$\varphi_J + \frac{\varphi_{J-1} - (1-\delta)\varphi_J}{1+r_{t-J}^1} + \frac{\varphi_{J-2} - (1-\delta)\varphi_{J-1}}{1+r_{t-J}^2} + \dots - \frac{(1-\delta)\varphi_1}{1+r_{t-J}^J} = \beta^J \frac{F_t^k U_t^c - \epsilon_t^j}{U_t^c} \quad (8')$$

$$U_{t-1}^c \left[1 - \frac{1}{1+r_{t-1}^1} \right] = \beta (F_t^j U_t^c - a_t^1) \quad (9')$$

from dividing (8) and (9) by U_t^c , using (19) and making explicit the expectation error, e) equation (51) that links the innovations ϵ_t^1 and a_t^1 , and f) the moving average relation between ϵ_t^1 and ϵ_t^j produce a set of $J+7$ equations involving time- t decisions: k_{t+J-1} , y_t , c_t , n_t , m_t , the J real interest rates: $r_{t-1}^1, r_{t-2}^2, r_{t-3}^3, \dots, r_{t-J}^J$, 5 innovations: $a_t^1, \epsilon_t^1, \xi_{1t}, \xi_{2t}, \xi_{3t}$ and ϵ_t^j , a total of $J+11$ variables, so that 4 variables will be needed to obtain equilibrium sample realizations.

In this paper, we take $J = 4$ periods to be needed for capital to become productive. Applying the procedure in section 4.1, the resulting 5 stability conditions of type (50) can be written:

$$c_t = g^1 (k_{t+3}, k_{t+2}, k_{t+1}, k_t, y_t, m_t, \xi_{1t}, \xi_{2t}, \xi_{3t}) \quad (50.1)$$

$$n_t = g^2 (k_{t+3}, k_{t+2}, k_{t+1}, k_t, y_t, m_t, \xi_{1t}, \xi_{2t}, \xi_{3t}) \quad (50.2)$$

$$r_{t-1}^1 = g^3 (c_{t-1}, n_{t-1}, k_{t+3}, k_{t+2}, k_{t+1}, k_t, \xi_{1t}, \xi_{1t-1}, \xi_{2t}) \quad (50.3)$$

$$r_{t-2}^2 = g^4 (c_{t-2}, n_{t-2}, k_{t+3}, k_{t+2}, k_{t+1}, k_t, r_{t-2}^1, \xi_{1t}, \xi_{1t-2}, \xi_{2t}) \quad (50.4)$$

$$r_{t-3}^3 = g^5 \left(c_{t-3}, n_{t-3}, k_{t-3}, k_{t-2}, k_{t-1}, k_t, r_{t-3}^1, r_{t-3}^2, \xi_{1t}, \xi_{1t-3}, \xi_{2t} \right) \quad (50.5)$$

Equations (50.1), (50.2), (50.3) together with (30), (52), and (9') can be used to solve for $\{c_t, n_t, k_{t+2}, y_t, m_t, r_{t+1}^1\}$ $t=1, 2, \dots$, as functions of $\{c_{t-1}, n_{t-1}, k_{t+2}, k_{t+1}, k_t, k_{t-1}, y_{t-1}, \xi_{1t}, \xi_{1t-1}, \xi_{2t}, \xi_{2t-1}\}$. This block is solved taking as inputs sample realizations for four variables: $\{\xi_{1t}, \xi_{2t}, \xi_{3t}, a_t^j\}$ $t=0, 1, 2, \dots$, and initial conditions $\{k_0, s_{1,0}, s_{2,0}, s_{3,0}, y_0\}$. For the initial period ($t=0$), c_0, n_0 and m_0 are simultaneously obtained from (50.1), (50.2) and (30). Starting from them, the rest of initial conditions and the values of the input variables at $t=1$, we obtain $\{c_1, n_1, k_2, y_1, m_1, r_2^1\}$. Using the vector $\{c_t, n_t, k_2, y_t, m_t, r_t^1\}$ together with the realizations of the input variables at $t=2$ we obtain $\{c_2, n_2, k_3, y_2, m_2, r_3^1\}$ and so on.

Once we have time series for consumption, labor, capital, inventories, real balances and one period real interest rates, then two and three period real interest rates are calculated from (50.4) and (50.5). With all them and a sample realization for $\{e_t^j\}$, equilibrium real interest rates on four period investments are obtained from (8'). Inflation is obtained from (24) and the series for the price level derives from an initial price level and the inflation series. Expectation errors e_t^j , $j=1, 2, 3, 4$ are obtained from the lagged, expanded version of (19):

$$U_{t-j}^c = \beta^j (1 + r_{t-j}^1) (U_t^c - e_t^j) \quad j = 1, \dots, J \quad (19')$$

the one period nominal interest rate i_{t-1}^1 comes from (27) written at $t-1$, and taken to the similar version of (16) for $j=1$:

$$\frac{U_{t-j}^c}{P_{t-j}} = \beta^j (1 + i_{t-j}^1) \left[\frac{U_t^c}{P_t} - u_t^j \right] \quad (16')$$

produces u_t^j . Conditional on a MA process for expectation errors u_t^j , equation (16')

is used to obtain nominal rates for $j > 1$. Finally, bonds and taxes are derived from the Government budget constraint (20) and the fiscal policy rule (26). Hence, these two variables are not needed to generate equilibrium values for the rest of variables in the model. This only shows that the Ricardian proposition holds in this model.

This solution procedure was implemented for a baseline set of parameters, similar to those used in related research by different authors, shown in Table 1. Six experiments were run with either

⁸ This expectations error is not independent of a_t^j , which has been used to obtain series for consumption, labor, physical capital, inventories and one-period real interest rates. We generated time series realizations for it as a MA(3) process with innovation ϵ_t^1 , whose sample realization was obtained from that for a_t^j $t=0, 1, 2, \dots$ through (51).

just the shock in technology or in preferences, the two of them simultaneously, as well as the three same exercises, adding the money growth shock. The results of these experiments are summarized in Tables 2 to 5, which contain estimated regressions that we describe now.

5. THE EXPECTATIONS HYPOTHESIS OF THE TERM STRUCTURE.

Having derived equilibrium realizations for the nominal returns, implicit one-period forward returns at t for time $t+j$ are obtained by:

$$1 + f_t^j = \frac{1 + i_t^j}{1 + i_t^{j-1}} \quad j = 2, 3, \dots, J$$

while the return on holding at time t for one period an asset which is j periods away from maturity is⁹:

$$1 + h_t^j = \frac{1 + i_t^j}{1 + i_{t+1}^{j-1}} \quad j = 2, 3, \dots, J$$

The forward rate f_t^j is a proxy for i_{t+j}^j , while the holding return is a proxy for i_t^j . The former is known at time t , while the holding return h_t^j is realized at $t+1$. Their differences to the current short-term spot rate: $f_t^j - i_{t+j}^j$ and $HP_{t+1}^j = h_t^j - i_t^j$ are known as *ex-post term and holding premium*, while $f_t^j - i_t^j$ is labelled the *forward premium*. There is a tradition of interpreting these two premia as measures of risk, although the association is far from clear, specially in the case of the term premium. There are also *ex-ante* versions of the two premia, defined as: $f_t^j - E_t i_{t+j}^j$ and $HP_{t+1}^j = E_t h_t^j - i_t^j$. The *ex-post* premia include their *ex-ante* versions, plus an expectation error.

Fama(1984) shown that:

$$f_t^j = E_t (HP_t^j) + \left[E_t (HP_{t+1}^{j-1}) - E_t (HP_t^{j-1}) \right] + \dots + \left[E_t (HP_{t+j-2}^2) - E_t (HP_{t+j-3}^2) \right] + E_t i_{t+j-1}^1 \quad (53)$$

so that, under the assumption that the sum of the expectation changes is negligible, then the forward return is just the sum of expectations of future short-term spot returns and ex-ante holding premiums. The decomposition is exact for $j=1$.

The *expectations hypothesis* of the term structure states that holding premia are zero. Then,

⁹ The definition of implicit, k -period forward returns and the return on holding for k periods an asset that is j periods, $j > k$, away from maturity is defined similarly. We consider in our empirical exercises just one-period returns in both cases. Notice que $h_t^1 = f_t^1 = i_t^1$.

f_t^j would just be the expectation of the one-period return at time $t+j-1$, $E_t^j r_{t+j-1}$. The difference between both would then just be an expectation error which, under rational expectations, would be white noise for $j=1$, having a MA($j-1$) structure when $j > 1$. If we subtract i_t^j from both rates, we have a regression model:

$$i_{t+j-1}^j - i_t^j = \alpha + \beta (f_t^j - i_t^j) + u_{t+j-1} \quad j = 2, 3, \dots \quad (54)$$

on which the expectations hypothesis imposes the restrictions: $H_0: \alpha = 0, \beta = 1$. The alternative is that forward rates incorporate a premium to the expectation of future returns, that will be considered constant if $\alpha \neq 0$ but $\beta = 1$. Otherwise, if the risk premium fluctuates with some correlation with current forward rates, we will have $\beta < 1$ even under the null hypothesis since slope estimates will be biased towards the origin. Besides, there should not be any autocorrelation left in the residuals of that regression if we use the forward for $t+1$, otherwise acquiring the mentioned moving average structure due to the overlap of several forecast errors.

Empirical evidence is uniform in the rejection of the expectations hypothesis. Fama (1976, 1984a, 1984b, 1986, 1990), Fama and Bliss (1987) and Shiller et al. (1983), all find $\beta \neq 1$, even though such unanimity disappears when evaluating the ability of the term structure to forecast future short-term rates, since some authors find negligible estimates for β . The several papers by the former authors find some power in the current term structure to forecast future interest rates and holding returns. In contrast, no predictive power is found by Shiller et al. (1983) and Backus et al. (1989).

A second piece of evidence against the expectations hypothesis is that observed, ex-post, term premia are usually sizeable, being more volatile at the short end of the maturity structure. Backus et al. (1989) report significant average monthly term and holding premiums for the US T-bills market.

6.1 Forward rates as predictors of future spot returns.

The empirical literature has analyzed this issue using a whole array of variants of (54). Fama (1984a), Fama and Bliss (1987), Shiller et al. (1983) estimate (54) for different horizons, while Backus et al. (1989) use the *ex-post* premium, $i_{t+j-1} - f_t^j$ as dependent variable¹⁰. A significant constant in the regression is interpreted as a constant term premium, by itself a weak rejection of the expectation hypothesis. A slope different than one is interpreted as a stronger evidence against that hypothesis, leading to the belief that there is a term premium in current rates which evolves over time

¹⁰ Fama (1984a) works with monthly data, while Shiller et al. (1983) and Backus et al. (1989) use quarterly data. Fama and Bliss (1987) use annual data.

and is correlated with the forward premium.

Observed small but significant average ex-post term premia in actual markets initially suggested Roll (1970, ch. 5) and Fama (1976, table 1), among others, that at the least, constant term premiums existed. Additionally, estimated slopes different from one have been obtained when using the forward premium in (54) [Fama (1984a, table 4) and Shiller et al. (1983, table 3)]. In what might be the most popular study in this class, slope estimates in Fama (1984a) oscillate between 0.11 and 0.46. This leads to rejection of the expectation theory, although leaving some forecasting power in forward rates. With quarterly US Treasury bills data, and the same sample period, Backus et al. (1989) obtain similar results using prices, rather than returns. They also provide lower bounds of 0.00157 and 0.00164 for the standard deviations of the term and holding premiums, respectively¹¹. They summarize: '*Viewed as predictors, forward rates consistently overestimate future spot rates, and the 'forecast errors' are systematically related to variables that are known when the forecast is made. The consensus in the profession seems to be that forward rates contain, besides forecasts of future spot rates, risk premiums that change over time.*' Startz (1982) presents similar values for monthly data. This literature is reviewed in Shiller and McCulloch (1990).

We interpret our model as producing quarterly observations, which is consistent with the chosen parameterization, specially, the 0.99 value for the discount factor β . The results of estimating (54) with our simulated series appear in Table 2, for a baseline parameterization, and different specifications of economy-wide shocks, showing that:

- 1) Except in experiment 1, the slope estimates fall in the interval (0,1), being significantly different from both endpoints. Being different from 1, they lead to rejection of the *expectations hypothesis*, while, being different from zero, they concede some explanatory power to the forward rate as a predictor of future short-term interest rates. This is consistent with evidence in Fama (1984a, Table 4) and Fama and Bliss (1987, Table 3). In the first regard, they agree with the results in Shiller et al. (1983, Table 3), and Backus et al. (1989, Table 2), even though, as we have already mentioned, the latter find little explanatory power on the forward rate to predict future interest rates.
- 2) *The forecasting power of the forward rate for future short term rates decreases with the forecast horizon* in experiments 4 to 6. This result is consistent with the empirical evidence in Fama (1984a, Table 4), where monthly data are used, but not with Fama and Bliss (1987, Table 3) using annual data.

¹¹ obtained as the standard deviation of the fitted values in the regressions.

A different set of regressions between the first differences of future spot rates and forward rates is considered in Fama(1984a). He interprets the regressions in our Table 2 as measuring the accumulated change in the spot rate between t and $t+j$. The forecasting ability of this cumulative change may be due to the ability to forecast specific marginal changes which are part of the accumulated change. For instance, for $j = 2$, $i_{t+2}^1 - i_t^1$ can be decomposed into: $(i_{t+2}^1 - i_{t+1}^1) + (i_{t+1}^1 - i_t^1)$. Since $(i_{t+2}^1 - i_{t+1}^1)$ can be explained by $(f_t^2 - i_t^1)$ and $(i_{t+1}^1 - i_t^1)$ by $(f_t^2 - i_t^1)$, it is reasonable to presume that the difference between the dependent variables can be explained by the difference of the independent variables, i.e., that $(i_{t+2}^1 - i_{t+1}^1)$ may depend on $(f_t^2 - f_t^1)$. That way, we can determine the extent to which the forecasting ability of the forward rate on the accumulated change on spot rates depends on the ability of the forward premium to forecast some of the marginal changes in the global change in the spot rate. To do so, Fama(1984a) suggests regressions of differences of forward rates at t for different maturities.

The results in Table 3 show that:

- 1) *The forecasting ability is high and significant at all horizons. The coefficients associated to the different forecasting horizons are roughly equal. A similar statement applies to the coefficient of determination and the residual autocorrelation. In sum, the forecasting ability of the forward rate for $t+1$ on the future rate i_{t+1}^1 is the same than that of the forward rate for $t+2$ on the future rate i_{t+2}^1 . Fama found that, in actual data before 1974, the forecasting ability decreased with the forecasting horizon, being non-zero in all cases. After 1974, the significant forecasting horizon becomes much shorter, which he justified by the fact that the volatility of the first difference in spot rates was almost twice that of the differenced forward rates while, before 1974, both volatilities were similar.*
- 2) *Fama(1984a) finds a reduction in the estimated standard deviations, together with an increase in the coefficient of determination when using the first differences of future spot rates and the implicit forward rates in assets with successive maturity dates as dependent and independent variables, respectively. We also have, for each forecasting horizon, higher estimated standard deviations, and estimated standard error of the regression in Table 2 than in Table 3, while the coefficient of determination is lower.*

Figure 1 shows, for one sample realization with the three structural shocks, the time series of interest rate time differences versus the forward premium. The upper graph corresponds to the near future differential: $i_{t+1}^1 - i_t^1$, while the lower graph corresponds to the longer term difference: $i_{t+2}^1 - i_t^1$. The HE line corresponds to the expectations hypothesis, while the other line is the MCO estimate.

The distance between them is greater for the longer term change in interest rates, at the same time the dispersion in the sample seems to be wider. That is why the expectations hypothesis is rejected more clearly for the latter case.

6.2 The forward rate as an explanatory variable for the holding premium.

The similar conclusions are reached from regressions of the holding premium $h_t^{t,j} - i_t^1$, on the forward premium. Under rationality, according to (54), any detected relationship between the holding premium and any variable in the information set is necessarily capturing a relationship between that variable and the term premium. That is why one can be taken as evidence of the other. Table 4 contains estimates of the regressions between the holding premium and the forward premium, for different horizons:

$$h_t^j - i_t^1 = \alpha + \beta (f_t^j - i_t^1) + u_{t+1} \quad (55)$$

Like (54), this regression is specified as in Fama(1984a, Table 4), Fama and Bliss (1987, Table 1), and Backus et al.(1989), being different from those in Shiller et al.(1983). These authors find substantial variability in the holding premium. A positive slope in this equation would tell us that the current forward-spot differential has prediction power for future holding premiums, the same way a positive slope in the previous set of regressions was telling us that the forward-spot differential had explanatory power for future changes in the one-period spot rate. So, the slope in (55) tells us whether the expected premium component of the forward rate varies through time in a way that shows up reliably in the future premium. Likewise, the slope in (54) tells us whether the expected future spot rate component of the forward rate has variation that shows up reliably in future spot rates. We must bear in mind that regressions of the expected premium for different horizons are concerned with one-period holding premiums (all observed at $t+1$) on T-bills with different maturities. In contrast, the $i_{t+j-1}^1 - i_t^1$ regressions for different j were concerned with forecasts of changes in the one-period spot rate across different number of periods.

According to the pure expectations hypothesis [Meiselman(1962), Kessel(1965)], there are no expected premiums in forward rates, and all the variation in the forward premium: $f_t^j - i_t^1$ is due to expected changes in the spot rate: $E_t(i_{t+j-1}^1 - i_t^1)$. That would imply $\beta=0$ in (55), and $\beta=1$ in (54).

Our results in Table 4, for different horizons: $j=2,3,4$ and different combinations of shocks show that:

- 1) since we obtain in (54) and (55) values in the interval $(0,1)$, we have a richer situation where, in fact, expectations of future holding premiums, as well as those of future spot rates, both

contained in the forward premium vary in a way which shows up reliably in future holding premia and short term rates. Hence, the forward rate has prediction power for both, the holding premium and the change in the spot rate.

- 2) The estimated slope is positive, and increasing in the term to maturity. We then conclude, as these authors do, the existence of a term premium which evolves over time, and depends on the forward premium. Changes in estimated slopes over different maturities are wider in our model than in actual economies.
- 3) The slopes in tables 2 and 4 for all $j=2,3,4$ approximately add up to one. That expected relationship is analytically shown in Fama and Bliss(1987 [12], [13], pag.683).

Fama(1984a) suggested to also examine the relation between differences of holding excess returns, using assets maturing at adjacent periods, and differences in forward rates. The results of these regressions are presented in Table 5, where it can be seen that:

- 1) There is some explanatory power in the differences of the forward premia with respect to the differences of the implicit returns, but there is no specific relation between coefficients and horizon. This result is consistent, although weaker, than the empirical evidence in Fama(1984a)¹².
- 2) Fama(1984a) shows that the sum of the slopes in Tables 3 and 5 should be expected to be equal to 1, although such a condition does not hold in the empirical results beyond one period. In this negative aspect, the results in Tables 3 and 5 are also consistent with the empirical evidence.

Figure 2 presents a sample realization, this time plotting the holding premium versus the forward premium, for asset maturing 2 (upper graph) and 4 periods from now (lower graph). In this case, the horizontal axis corresponds to the expectations hypothesis. Even more clearly than in Figure 1, dispersion increases and the estimated line separates more from the expectations hypothesis reference the longer term we consider.

7. THE DIFFERENTIAL IMPORTANCE OF STRUCTURAL SHOCKS.

We have performed simulations of our model economy using different combinations of the shocks in preferences, technology, and the rate of monetary growth, in order to test the relative

¹² He observes a behaviour which is irregular in the forecast horizon, the lower slopes being those for two periods, increasing up to 4 periods, and then decreasing again as we forecast 6 periods ahead.

importance of each source of fluctuations on the characteristics we have generated for the term structure. A comparison of the six pannels in Tables 2 to 5 shows that the rejection of the expectations hypothesis in our general equilibrium model is far more clear when there is a monetary shock in the economy. Estimated slopes for (54) fall from values between 0.97 and 0.99 when just the shock in preferences is present, to values between 0.25 and 0.44 when, in addition, there is a monetary shock. The same coefficient is estimated at 0.95 when there is just a productivity shock, falling to between 0.59 and 0.76 when, in addition, a monetary shock is present. Under money growth shocks, estimated slopes decrease with the forecast horizon, showing an increased difficulty of forecasting longer term rates. We rejected the hypothesis $H_0: \beta=1$ in (54) in all simulations incorporating monetary shocks. With just the preference shock, we maintained H_0 with frequencies 33%, 82%, 92% at horizons of 1, 2 and 3 quarters, suggesting more predictability of longer term returns. The economy with both, preference and technology shocks, behaves like the one with just the latter, whose effects seem to dominate those of random shocks to preferences, at least for the values we have used for their variances.

Regressions in differences [Table 3] yielded results consistent with those in the levels of the excess returns: slopes close to 1 without the monetary shock, just to become much lower in presence of a monetary shock. The unit slope hypothesis is rejected in all simulations except when the shock in preferences is the only one present.

Estimation of (55) in Table 4 produced slopes close to zero with just the shock in preferences, increasing with the forecasting horizon, to fall in the range 0.56 to 0.72 when we added the money shock. The hypothesis $H_0: \beta=0$, consistent with the expectations theory of the term structure, was maintained with frequencies 33%, 42%, 24%, at horizons of 1, 2 and 3 quarters, but was always rejected with the monetary shock. Results with just the productivity shock were numerically similar, even though in that case we always rejected H_0 , with or without the money shock added to the model. The economy with the technology and preferences shocks behave again as that with just the productivity shock. Estimation of regressions in differences in Table 5 produced results consistent with the previous ones: forward rates predict a good deal of future holding returns fluctuations when there is a money growth shock in the economy. Without it, the hypothesis of no predictability is also maintained, although the estimated slope is close to zero.

These results show that it is hard not to reject the expectations hypothesis of the term structure in our model, except when the economy is just subject to a shock in preferences. However, the slope coefficients for both, the future spot rate and the excess holding return regressions, are much farther away from their hypothesized value when the monetary shock is present. Monetary shocks play an important role in producing enough time variation in the risk term that the expectations hypothesis is very clearly rejected.

In the absence of monetary shocks, volatility sharply decreases over the term structure. When the money shock is present, all returns become more volatile, but the increase in volatility is relatively much bigger for longer-term returns. Hence, long term interest rates become much harder to predict. Forward rates then lose predictive power for future spot rates, incorporating information on future holding returns, so that the slope in (55) moves away from zero. As a consequence, forward rates contain relatively less information on future spot rates, the slope coefficient in (54) falls significantly below one, and the R-squared clearly diminishes. Notice that it is not just the increased unpredictability of future interest rates that matters. The slope in (55) becomes different from zero just when fluctuations in holding premiums accurately reflect past fluctuations in forward rates.

To obtain evidence of time varying risk premia, not only the estimated constant term in the previous regressions must be zero but also, there has to be enough variation in the unobserved risk premium correlated with the forward premium that the estimated slope, being significant, is less than one. This is important, because rejection of the expectations hypothesis in our experiments is based on different causes: In absence of monetary shocks, estimated slopes are close to one and the null hypothesis fails because low volatility produces a tight sample and very precise MCO estimates with such small standard errors, that even slopes of .97 turn out to be statistically different from one. On the other hand, with monetary shocks, the expectations hypothesis fails to hold because there is enough correlation between the forward and the term premiums that the estimated slope clearly deviates from one. In the spirit of the expectations hypothesis that the forward rates be good predictors of future spot rates, the rejection is obvious just in the case of economies subject to monetary shocks.

Finally, there is nothing in our exercise that forces both slope coefficients to add up to one. According to (53), that would happen if the sum of the expectation changes in holding premiums is, in fact, negligible, which seems to be the case in our model, in consistency with Fama's suggestions.

These results have been obtained solving the model *forwards*, since we have taken as inputs sample realizations for the three structural shocks in technology, preferences and money growth control. However, we needed to add a sample realization for one of the expectations errors, a_t^1 , to be able to produce sample paths for the rest of endogenous variables. Hence, we cannot guarantee that the path for a_t^1 from which we started will fulfill the conditions of a true expectations error, of being orthogonal to all variables in the information set of the household.

We ran accuracy tests like in den Haan and Marcet (1989), based on sample correlations between the expectations errors and variables in Ω_t . We crossed for the tests current expectations errors with current 4 lagged values of: real balances, real bonds, the stocks of physical capital and inventories, and the three structural shocks, a total of 28 instruments. The left column in table 6

contains the values of the chi-squared statistic for errors u_t^1 , e_t^1 and ϵ_t^1 , for simulations with the shocks in technology and preferences, while the second column presents the values with the three shocks. The former case produces lack of orthogonality for two of the three independent expectations errors, while just one of them presents this problem when we add monetary shocks. Last column contains the statistics obtained excluding current inventories and physical capital from the instrument list, showing that these two variables were to blame for the detected correlation. Overall, the orthogonality criterium is quite closely satisfied.

8. CONCLUSIONS

We have analyzed whether general equilibrium asset pricing models can explain the empirical rejection of the expectations hypothesis of the term structure. In an economy with a time-to-build technology and a cash-in-advance constraint, where salary income cannot be spent when it is earned, there is each period a portfolio of investment projects not yet productive. Consumers make investment contributions to each project every single period until they become productive. Hence, we have a well defined term structure of returns on investment. In equilibrium, they coincide with the returns on real bonds, and are related, through inflation expectations, with nominal rates of interest at different maturities. In such an economy, shocks to the rate of money growth can generate enough time variability in risk terms which is correlated with current forward rates, that the expectations hypothesis is rejected, since forward rates become biased predictors of future spot rates. Shocks affecting the productivity of the inputs, or preferences do not seem able, by themselves, to produce the result in such a clear fashion.

The cash-in-advance liquidity constraint in a productive economy seems to be behind the model's ability to reproduce the empirical rejection of the expectations hypothesis. It implies that the shorter term nominal interest rate is linked to marginal utility of consumption and the marginal product of labor thereby not being so affected by monetary shocks as the longer term rates.

Labadie(1994) and Backus et al.(1989) have already provided evidence to the fact that the rejection does not arise in an endowment economy. Whether a time-to-build technology is also necessary to maintain the result is at this point still an open question.

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Table 1	
Parameter values	
Rate of time preference:	$\beta = 0.99$
Depreciation rate:	$\delta = 0.025$
Labor elasticity in production:	0.64
Elasticity of the composed input: physical capital + inventories:	0.36
Weight of physical capital, relative to inventories:	2.57
Number of periods to complete an investment project:	4
Proportions of investment projects financed each period:	$\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = 0.25$
Average consumption 'elasticity' in utility function:	1/3
Average 'elasticity' of leisure in utility function:	2/3
Coefficient of relative risk aversion:	1.50
MA parameters in expectation errors in nominal rates:	$\Theta_i = 0.9^i, i=1,2,3,4$
AR(1) parameter in technology shock:	0.90
AR(1) parameter in preferences:	0.90
Standard deviation of innovation in technology shock:	0.001
Standard deviation of innovation in preferences:	0.001
Standard deviation of innovation in money growth rate:	0.01

Table 2									
$i_{t+j}^1 - i_t^1 = \alpha + \beta (f_t^j - i_t^1) + u_{t+j,1}$									
	Experiment 1			Experiment 2			Experiment 3		
j	2	3	4	2	3	4	2	3	4
β	0.97	0.98	0.99	0.95	0.95	0.95	0.95	0.95	0.95
$s(\beta)$	0.01	0.02	0.02	0.00	0.00	0.01	0.00	0.01	0.01
R^2	0.98	0.97	0.96	0.99	0.99	0.99	0.99	0.99	0.98
r^1	0.05 (0.10)	0.50 (0.08)	0.67 (0.06)	0.62 (0.11)	0.74 (0.09)	0.86 (0.05)	0.34 (0.14)	0.66 (0.10)	0.81 (0.07)
H_0^1	0.33	0.82	0.92	0.00	0.00	0.00	0.00	0.00	0.00
	Experiment 4			Experiment 5			Experiment 6		
j	2	3	4	2	3	4	2	3	4
β	0.44	0.29	0.25	0.76	0.63	0.59	0.76	0.65	0.60
$s(\beta)$	0.05	0.05	0.06	0.04	0.06	0.06	0.05	0.07	0.07
R^2	0.45	0.30	0.25	0.78	0.65	0.61	0.79	0.67	0.62
r^1	-0.27 (0.09)	0.16 (0.07)	0.16 (0.11)	-0.10 (0.08)	0.33 (0.07)	0.35 (0.09)	-0.10 (0.09)	0.33 (0.07)	0.37 (0.08)
H_0^1	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00

Note: Experiment 1 contains just a shock in preferences and experiment 2 contains just a shock in productivity, while experiment 3 combines both shocks. Experiments 4 to 6 are the corresponding versions adding a money control shock.

The table summarizes the results from 100 regressions with simulated equilibrium series, using the parameter values in Table 1. For each experiment, we present: averages over the 100 simulations of: the slope β , its standard error of estimate, the R-squared, the first order residual autocorrelation r^1 , and the percentage of times the null hypothesis of a unit slope was not rejected. The number in brackets under r^1 is its empirical standard deviation.

Table 3									
$i_{t+j-1}^i - i_{t+j}^i = \alpha + \beta(f_t^i - f_t^{j-1}) + u_{t+j-1}$									
	Experiment 1			Experiment 2			Experiment 3		
j	2	3	4	2	3	4	2	3	4
β	0,97	0,97	0,97	0,95	0,95	0,95	0,95	0,95	0,95
$s(\beta)$	0,01	0,01	0,01	0,00	0,00	0,00	0,00	0,00	0,01
R^2	0,98	0,98	0,98	0,99	0,99	0,99	1,00	1,00	1,00
r^1	0,05 (0,10)	-0,03 (0,10)	-0,04 (0,09)	0,62 (0,11)	0,54 (0,12)	0,49 (0,11)	0,34 (0,14)	0,26 (0,14)	0,23 (0,13)
H_0^1	0,33	0,35	0,33	0,00	0,00	0,00	0,00	0,00	0,00

	Experiment 4			Experiment 5			Experiment 6		
j	2	3	4	2	3	4	2	3	4
β	0,44	0,44	0,44	0,76	0,76	0,75	0,76	0,76	0,76
$s(\beta)$	0,05	0,05	0,05	0,04	0,04	0,04	0,05	0,05	0,05
R^2	0,45	0,45	0,45	0,78	0,78	0,78	0,79	0,79	0,79
r^1	-0,27 (0,09)	-0,29 (0,09)	-0,29 (0,09)	-0,10 (0,08)	-0,16 (0,08)	-0,15 (0,08)	-0,10 (0,09)	-0,16 (0,09)	-0,15 (0,09)
H_0^1	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,01	0,01

Note: See note on Table 2.

Table 4									
$h_t^i - i_t^i = \alpha + \beta(f_t^i - i_t^i) + u_{t+i}$									
	Experiment 1			Experiment 2			Experiment 3		
j	2	3	4	2	3	4	2	3	4
β	0,03	0,06	0,10	0,05	0,09	0,13	0,05	0,09	0,12
$s(\beta)$	0,01	0,02	0,03	0,00	0,00	0,01	0,00	0,01	0,01
R^2	0,07	0,07	0,08	0,50	0,46	0,49	0,35	0,32	0,35
r^1	0,05 (0,10)	0,00 (0,12)	0,13 (0,08)	0,62 (0,11)	0,09 (0,26)	0,36 (0,15)	0,35 (0,14)	0,05 (0,23)	0,28 (0,13)
H_0^1	0,33	0,42	0,24	0,00	0,00	0,00	0,00	0,00	0,00

	Experiment 4			Experiment 5			Experiment 6		
j	2	3	4	2	3	4	2	3	4
β	0,56	0,71	0,72	0,24	0,38	0,43	0,24	0,37	0,42
$s(\beta)$	0,05	0,07	0,09	0,04	0,09	0,10	0,05	0,08	0,11
R^2	0,57	0,36	0,23	0,28	0,21	0,16	0,27	0,21	0,15
r^1	-0,27 (0,09)	-0,37 (0,08)	-0,16 (0,07)	-0,10 (0,09)	-0,21 (0,08)	-0,13 (0,08)	-0,10 (0,09)	-0,20 (0,09)	-0,13 (0,08)
H_0^1	0,00	0,00	0,00	0,00	0,02	0,00	0,01	0,02	0,04

Note: See note on Table 2.

In this table, H_0^1 shows the percentage of times that the null hypothesis of a zero slope was not rejected in the simulations.

Table 5									
$h_t^j - h_t^{j-1} = \alpha + \beta(f_t^j - f_t^{j-1}) + u_{t+1}$									
	Experiment 1			Experiment 2			Experiment 3		
j	2	3	4	2	3	4	2	3	4
β	0,03	0,00	0,00	0,05	0,00	0,00	0,05	0,00	0,00
$s(\beta)$	0,01	0,01	0,01	0,00	0,01	0,01	0,00	0,01	0,01
R^2	0,07	0,01	0,02	0,50	0,01	0,01	0,35	0,02	0,02
r^1	0,05 (0,10)	0,01 (0,10)	0,02 (0,09)	0,62 (0,11)	0,17 (0,14)	0,17 (0,15)	0,35 (0,14)	0,12 (0,13)	0,13 (0,15)
H_0^1	0,33	0,96	0,88	0,00	0,93	0,94	0,00	0,93	0,91
	Experiment 4			Experiment 5			Experiment 6		
j	2	3	4	2	3	4	2	3	4
β	0,56	-0,02	-0,04	0,24	-0,01	-0,01	0,24	-0,01	-0,01
$s(\beta)$	0,05	0,09	0,07	0,04	0,05	0,04	0,05	0,05	0,04
R^2	0,57	0,02	0,02	0,28	0,02	0,01	0,27	0,01	0,01
r^1	-0,27 (0,09)	0,02 (0,07)	0,00 (0,11)	-0,10 (0,09)	0,01 (0,08)	-0,01 (0,09)	-0,10 (0,09)	0,00 (0,08)	-0,02 (0,10)
H_0^1	0,00	0,96	0,91	0,00	0,92	0,98	0,01	0,95	0,95

Note: See note on Table 4.

Table 6			
ACCURACY TESTS IDENTIFICATION TESTS			
	A	B	C
u_t^1 (h_t^1)	45.19 (4.7)	29.00 (5.0)	28.46 (4.9)
	0% 0% 32%	97% 99% 100%	97% 99% 100%
e_t^1 (r_t^1)	45.99 (3.9)	48.26 (4.6)	29.23 (5.4)
	0% 0% 28%	1% 5% 52%	93% 99% 100%
ϵ_t^1 ($F_t^k U_t^c$)	24.02 (5.2)	27.72 (4.8)	27.73 (4.8)
	96% 100% 100%	99% 100% 100%	100% 100% 100%

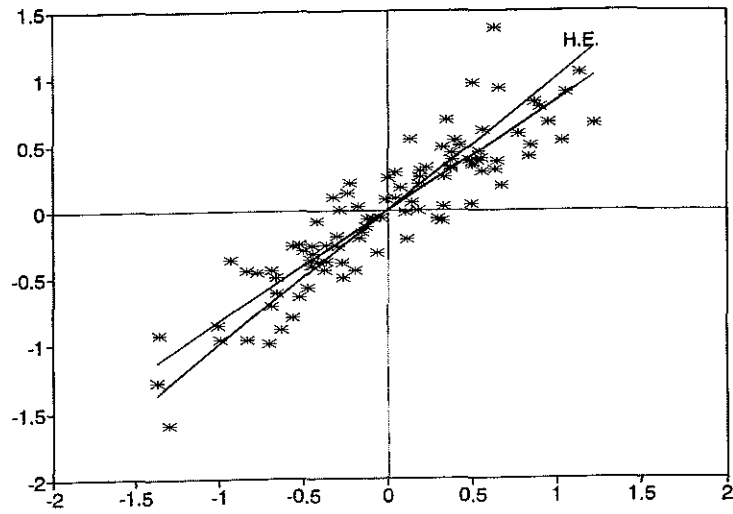
Note: Column A contains the results with shocks in technology and preferences. Column B contains the results with the three shocks. Column C excludes current values of physical capital and inventories from the list of instruments.

Empirical means for 100 simulations of the quadratic form in cross moments of each expectations error and current and 4 lagged values of: M_{t-1}/P_{t-1} , B_{t-1}/P_{t-1} , k_{t+1-1} , y_t , ξ_{t-1}^1 , ξ_{t-1}^2 , ξ_{t-1}^3 , a total of 28 instruments.

Each panel contains: the mean, across 100 simulations, of the chi-squared statistic, its empirical standard deviation, and the percentage of times the null hypothesis of orthogonality was rejected at 90%, 95% and 99% confidence levels.

FIGURE 1

Dif. spot vs forward premium (short)



Dif. spot vs forward premium (long)

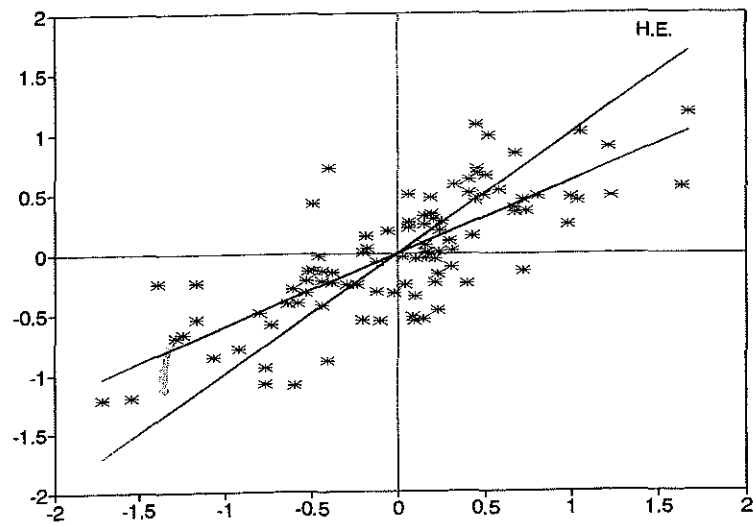
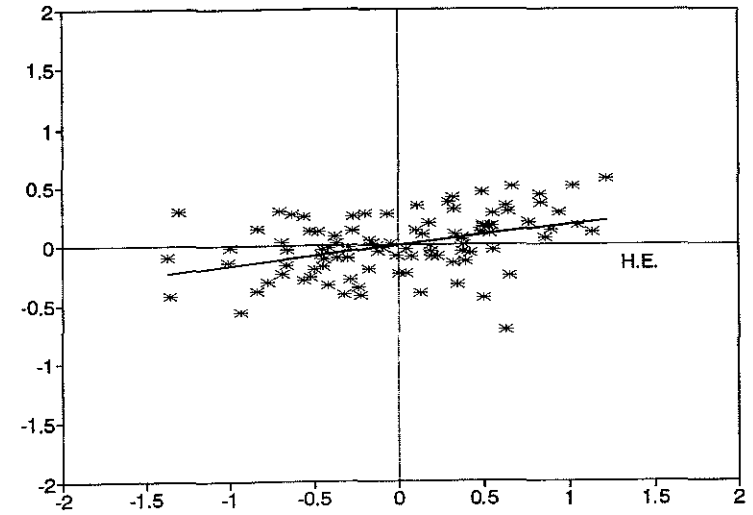


FIGURE 2

Holding vs forward premium (short)



Holding vs forward premium (long)

